EJERCICIO 1

**1. Una deuda se expresa en sentido negativo. Luego inicalmente el estado económico de Pedro es - 60 bs.
Al recibir 320 bs. Aumenta su capital tantos bs. como ha recibido; entonces la operaciòn queda expresada como se indica**

$-60+320=+260$ *bs.*

2. $1.170-1.515=-345$ *sucres*

3. $200+56-189=+\$67$

4. $-665-1.178+2.280=+437$ *soles*

5. $20-15+40-75=-\$30$

6. $-67+72-16+2=-\$9$

7. $200-78-81-93+41-59=-70$ *colones*

8. $-45-66-79+200-10=0$

EJERCICIO 2

1. $12-15=-3^\circ$

2. $-3+8-6=-1^\circ$

3. $15-(-3)=15+3=18^\circ$

4. $-(-8)+5=8+5=13^\circ$

5. $-4+7+2-11=-6^\circ$

6. $-8+4\cdot1=-8+4=-4^\circ \rightarrow 7am$
$-8+4\cdot2=-8+8=0^\circ \rightarrow 8am$
$-8+4\cdot5=-8+20=+12^\circ \rightarrow 11am$

7. $-1-2\cdot2=-1-4=-5^\circ \rightarrow 10am$
$-1-2\cdot3=-1-6=-7^\circ \rightarrow 11am$
$-1-2\cdot3+3\cdot1=-1-6+3=-4^\circ \rightarrow 12am$
$-1-2\cdot3+3\cdot3=-1-6+9=+2^\circ \rightarrow 2pm$

8. $-56+7=-49^\circ$

9. $-71+5=-66^\circ$ *long*
$-15+(-5)=-20^\circ$ *lat.*

10. $18+3=+21^\circ$ *long*
$65-4=+61^\circ$ *lat.*

11. $-75+135=+60$ *años*

EJERCICIO 3

1. $+32m\,;\,-16m$

2. $+10m\,;\,-4m$

3. $50-85=-35m$

4. $-6\cdot11=-66m$

5. $-8\times6=-48m$
$9\times6=+54m$

6. $400\cdot2=+800m \rightarrow corredor$
$-400\cdot3=-1.200m \rightarrow yo$

**7. Los 40 pies de longitud del poste se reparten asi: 15 pies que sobresalen - se asumen en sentido positivo- .
25 pies que se encuentran enterrados. - se asumen en sentido negativo - .
Al introducir 3 pies màs, se adicionan a los que estan bajo el suelo y se deben descontar de los que estan por encima.
Aritmèticamente significa:**

$40=15+25$
$-25-3=-28$ *pies*
$+15-3=+12$ *pies*

8. $55-52=+3m$

9. $-32+15=-17m$

10. $35-47=-12m$

11. $-39+56=+17m$

12. $90-58-36=-4m$

13. $72-30\cdot1=72-30=+42m$
$72-30\cdot2=72-60=+12m$
$72-30\cdot3=72-90=-18m$
$72-30\cdot4=72-120=-48m$

14. $-120-(-60)\cdot1=-120+60=-60\,Km$
$-120-(-60)\cdot2=-120+120=0$
$-120-(-60)\cdot3=-120+180=+60\,Km$
$-120-(-60)\cdot4=-120+240=+120\,Km$

EJERCICIO 7.

1. $x+2x=3x$
2. $8a+9a=17a$
3. $11b+9b=20b$
4. $-b-5b=-6b$
5. $-8m-m=-9m$
6. $-9m-7m=-16m$
7. $4a^x+5a^x=9a^x$
8. $6a^{x+1}+8a^{x+1}=14a^{x+1}$
9. $-m^{x+1}-5m^{x+1}=-6m^{x+1}$
10. $-3a^{x-2}-a^{x-2}=-4a^{x-2}$
11. $\frac{1}{2}a+\frac{1}{2}a=\frac{1+1}{2}a=\frac{2}{2}a=a$
12. $\frac{3}{5}ab+\frac{1}{10}ab=\frac{6ab+ab}{10}=\frac{7}{10}ab$
13. $\frac{1}{3}xy+\frac{1}{6}xy=\frac{2+1}{6}xy=\frac{3}{6}xy=\frac{1}{2}xy$
14. $-\frac{1}{5}xy-\frac{4}{5}xy=\frac{-1-4}{5}=-\frac{5}{5}xy=-xy$
15. $-\frac{5}{6}a^2b-\frac{1}{8}a^2b=\frac{-20-3}{24}a^2b=-\frac{23}{24}a^2b$
16. $-a-\frac{7}{8}a=\frac{-8-7}{8}a=-\frac{15}{8}a$
17. $8a+9a+6a=23a$
18. $15x+20x+x=36x$
19. $-7m-8m-9m=-24m$
20. $-a^2b-a^2b-3a^2b=-5a^2b$
21. $a^x+3a^x+8a^x=12a^x$
22. $-5a^{x+1}-3a^{x+1}-5a^{x+1}=-13a^{x+1}$
23. $a+\frac{1}{2}a+\frac{2}{3}a=\frac{6+3+4}{6}a=\frac{13}{6}a$
24. $-x-\frac{2}{3}x-\frac{1}{6}x=\frac{-6-4-1}{6}x=-\frac{11}{6}x$
25. $\frac{1}{5}ax+\frac{3}{10}ax+ax=\frac{2+3+10}{10}ax=\frac{15}{10}ax=\frac{3}{2}ax$
26. $-\frac{3}{4}a^2x-\frac{5}{6}a^2x-a^2x=\frac{-9-10-12}{12}a^2x=-\frac{31}{12}a^2x$
27. $11a+8a+9a+11a=39a$
28. $m^{x+1}+3m^{x+1}+4m^{x+1}+6m^{x+1}=14m^{x+1}$
29. $-x^2y-8x^2y-9x^2y-20x^2y=-38x^2y$
30. $-3a^m-5a^m-6a^m-9a^m=-23a^m$
31. $\frac{1}{2}a+\frac{1}{4}a+\frac{1}{8}a+a=\frac{4+2+1+8}{8}a=\frac{15}{8}a$
32. $\frac{2}{5}ax+\frac{1}{2}ax+\frac{1}{10}ax+\frac{1}{20}ax=\frac{8+10+2+1}{20}ax=\frac{21}{20}ax$
33. $0{,}5m+0{,}6m+0{,}7m+0{,}8m=2{,}6m$
34. $-\frac{1}{7}ab-\frac{1}{14}ab-\frac{1}{28}ab-ab$
$=\frac{-4-2-1-28}{28}ab=-\frac{35}{28}ab=-\frac{5}{4}ab$
35. $-\frac{2}{3}x^3y-\frac{1}{6}x^3y-\frac{1}{9}x^3y-\frac{1}{12}x^3y$
$=\frac{-24-6-4-3}{36}x^3y=-\frac{37}{36}x^3y$
36. $ab^2+ab^2+7ab^2+9ab^2+21ab^2=39ab^2$
37. $-m-m-8m-7m-3m=-20m$
38. $-x^{a+1}-8x^{a+1}-4x^{a+1}-5x^{a+1}-x^{a+1}=-19x^{a+1}$
39. $\frac{1}{2}a+\frac{1}{3}a+\frac{1}{4}a+\frac{1}{5}a+\frac{1}{6}a$
$=\frac{30+20+15+12+10}{60}a=\frac{87}{60}a=\frac{29}{20}a$
40. $-\frac{1}{3}ab-\frac{1}{6}ab-\frac{1}{2}ab-\frac{1}{12}ab-\frac{1}{9}ab$
$=\frac{-12-6-18-3-4}{36}ab=-\frac{43}{36}ab$

EJERCICIO 8

1. $8a-6a=2a$
2. $6a-8a=-2a$
3. $9ab-15ab=-6ab$
4. $15ab-9ab=6ab$
5. $2a-2a=0$
6. $-7b+7b=0$
7. $-14xy+32xy=18xy$

8. $-25x^2y+32x^2y=7x^2y$

9. $40x^3y-51x^3y=-11x^3y$

10. $-m^2n+6m^2n=5m^2n$

11. $-15xy+40xy=25xy$

12. $55a^3b^2-81a^3b^2=-26a^3b^2$

13. $-x^2y+x^2y=0$

14. $-9ab^2+9ab^2=0$

15. $7x^2y-7x^2y=0$

16. $-101mn+118mn=17mn$

17. $502ab-405ab=97ab$

18. $-1024x+1018x=-6x$

19. $-15ab+15ab=0$

20. $\frac{1}{2}a-\frac{3}{4}a=\frac{2-3}{4}a=-\frac{1}{4}a$

21. $\frac{3}{4}a-\frac{1}{2}a=\frac{3-2}{4}a=\frac{1}{4}a$

22. $\frac{5}{6}a^2b-\frac{5}{12}a^2b=\frac{10-5}{12}a^2b=\frac{5}{12}a^2b$

23. $-\frac{4}{7}x^2y+\frac{9}{14}x^2y=\frac{-8+9}{14}x^2y=\frac{1}{14}x^2y$

24. $\frac{3}{8}am-\frac{5}{4}am=\frac{3-10}{8}am=-\frac{7}{8}am$

25. $-am+\frac{3}{5}am=\frac{-5+3}{5}am=-\frac{2}{5}am$

26. $\frac{5}{6}mn-\frac{7}{8}mn=\frac{20-21}{24}mn=-\frac{1}{24}mn$

27. $-a^2b+\frac{3}{11}a^2b=\frac{-11+3}{11}a^2b=-\frac{8}{11}a^2b$

28. $3{,}4a^4b^3-5{,}6a^4b^3=-2{,}2a^4b^3$

29. $-1{,}2yz+3{,}4yz=2{,}2yz$

30. $4a^x-2a^x=2a^x$

31. $-8a^{x+1}+8a^{x+1}=0$

32. $25m^{a-1}-32m^{a-1}=-7m^{a-1}$

33. $-x^{a+1}+x^{a+1}=0$

34. $-\frac{1}{4}a^{m-2}+\frac{1}{2}a^{m-2}=\frac{-1+2}{4}a^{m-2}=\frac{1}{4}a^{m-2}$

35. $\frac{5}{6}a^{m+1}-\frac{7}{12}a^{m+1}=\frac{10-7}{12}a^{m+1}=\frac{3}{12}a^{m+1}=\frac{1}{4}a^{m+1}$

36. $4a^2-\frac{1}{3}a^2=\frac{12-1}{3}a^2=\frac{11}{3}a^2$

37. $-5mn+\frac{3}{4}mn=\frac{-20+3}{4}mn=-\frac{17}{4}mn$

38. $8a^{x+2}b^{x+3}-25a^{x+2}b^{x+3}=-17a^{x+2}b^{x+3}$

39. $-\frac{7}{8}a^mb^n+a^mb^n=\frac{-7+8}{8}a^mb^n=\frac{1}{8}a^mb^n$

40. $0{,}85mxy-0{,}5mxy=0{,}35mxy$

EJERCICIO 9

1. $9a-3a+5a=11a$

2. $-8x+9x-x=0$

3. $12mn-23mn-5mn=-16mn$

4. $-x+19x-18x=0$

5. $19m-10m+6m=15m$

6. $-11ab-15ab+26ab=0$

7. $-5a^x+9a^x-35a^x=-31a^x$

8. $-24a^{x+2}-15a^{x+2}+39a^{x+2}=0$

9. $\frac{2}{3}y+\frac{1}{3}y-y=\frac{2+1-3}{3}y=\frac{0}{3}y=0$

10. $-\frac{3}{5}m+\frac{1}{4}m-\frac{1}{2}m=\frac{-12+5-10}{20}m=-\frac{17}{20}m$

11. $\frac{3}{8}a^2b+\frac{1}{4}a^2b-a^2b=\frac{3+2-8}{8}a^2b=-\frac{3}{8}a^2b$

12. $-a+8a+9a-15a=a$

13. $7ab-11ab+20ab-31ab=-15ab$

14. $25x^2-50x^2+11x^2+14x^2=0$

15. $-xy-8xy-19xy+40xy=12xy$

16. $7ab+21ab-ab-80ab=-53ab$

17. $-25xy^2+11xy^2+60xy^2-82xy^2=-36xy^2$

18. $-72ax+87ax-101ax+243ax=157ax$

19. $-82bx-71bx-53bx+206bx=0$

20. $105a^3-464a^3+58a^3+301a^3=0$

21. $\frac{1}{2}x-\frac{1}{3}x+\frac{1}{4}x-\frac{1}{5}x=\frac{30-20+15-12}{60}x=\frac{13}{60}x$

22. $\frac{1}{3}y-\frac{1}{3}y+\frac{1}{6}y-\frac{1}{12}y=\frac{4-4+2-1}{12}y=\frac{1}{12}y$

23. $\frac{3}{5}a^2b-\frac{1}{6}a^2b+\frac{1}{3}a^2b-a^2b$

$=\frac{18-5+10-30}{30}a^2b=-\frac{7}{30}a^2b$

24. $-\frac{5}{6}ab^2-\frac{1}{6}ab^2+ab^2-\frac{3}{8}ab^2$

$=\frac{-20-4+24-9}{24}ab^2=-\frac{9}{24}ab^2=-\frac{3}{8}ab^2$

25. $-a+8a-11a+15a-75a=-64a$

26. $-7c+21c+14c-30c+82c=80c$

27. $-mn+14mn-31mn-mn+20mn=mn$

28. $a^2y-7a^2y-93a^2y+51a^2y+48a^2y=0$

29. $-a+a-a+a-3a+6a=3a$

30. $\frac{1}{2}x+\frac{2}{3}x-\frac{7}{6}x+\frac{1}{2}x-x$

$=\frac{3+4-7+3-6}{6}x=-\frac{3}{6}x=-\frac{1}{2}x$

31. $-2x+\frac{3}{4}x+\frac{1}{4}x+x-\frac{5}{6}x$

$=\frac{-48+18+6+24-20}{24}x=-\frac{20}{24}x=-\frac{5}{6}x$

32. $7a^x-30a^x-41a^x-9a^x+73a^x=0$

33. $-a^{x+1}+7a^{x+1}-11a^{x+1}-20a^{x+1}+26a^{x+1}=a^{x+1}$

34. $a+6a-20a+150a-80a+31a=88a$

35. $-9b-11b-17b-81b-b+110b=-9b$

36. $-a^2b+15a^2b+a^2b-85a^2b-131a^2b+39a^2b=-162a^2b$

37. $84m^2x-501m^2x-604m^2x-715m^2x+231m^2x+165m^2x$

$=-1340m^2x$

38. $\frac{5}{6}a^3b^2+\frac{2}{3}a^3b^2-\frac{1}{4}a^3b^2-\frac{5}{8}a^3b^2+4a^3b^2$

$=\frac{20+16-6-15+96}{24}a^3b^2=\frac{111}{24}a^3b^2$

$=\frac{37}{8}a^3b^2=4\tfrac{5}{8}a^3b^2$

39. $40a-81a+130a+41a-83a-91a+16a=-28a$

40. $-21ab+52ab-60ab+84ab-31ab-ab-23ab=0$

EJERCICIO 10

1. $7a+6a-9b-4b$

$7a+6a=13a \quad -9b-4b=-13b$

$=13a-13b$

2. $a+b-c-b-c+2c-a$

$a-a=0 \quad b-b=0 \quad -c-c+2c=0$

$=0$

3. $5x-11y-9+20x-1-y$

$5x+20x=25x \quad -11y-y=-12y \quad -9-1=-10$

$=25x-12y-10$

4. $-6m+8n+5-m-n-6m-11$

$-6m-m-6m=-13m \quad 8n-n=7n \quad 5-11=-6$

$=-13m+7n-6$

5. $-a+b+2b-2c+3a+2c-3b$

$-a+3a=2a \quad b+2b-3b=0 \quad -2c+2c=0$

$=2a$

6. $-81x+19y-30z+6y+80x+x-25y$

$-81x+80x+x=0 \quad 19y+6y-25y=0 \quad -30z$

$=-30z$

7. $15a^2-6ab-8a^2+20-5ab-31+a^2-ab$

$15a^2-8a^2+a^2=8a^2$

$-6ab-5ab-ab=-12ab \quad 20-31=-11$

$=8a^2-12ab-11$

8. $-3a+4b-6a+81b-114b+31a-a-b$

$-3a-6a+31a-a=21a$

$4b+81b-114b-b=-30b$

$=21a-30b$

9. $-71a^3b-84a^4b^2+50a^3b+84a^4b^2-45a^3b+18a^3b$

$-71a^3b+50a^3b-45a^3b+18a^3b=-48a^3b$

$-84a^4b^2+84a^4b^2=0$

$=-48a^3b$

10. $-a+b-c+8+2a+2b-19-2c-3a-3-3b+3c$

$-a+2a-3a=-2a \quad b+2b-3b=0$

$-c-2c+3c=0 \quad 8-19-3=-14$

$=-2a-14$

11. $m^2+71mn-14m^2-65mn+m^3-m^2-115m^2+6m^3$

$m^3+6m^3=7m^3$ $\quad m^2-14m^2-m^2-115m^2=-129m^2$ $\quad 71mn-65mn=6mn$

$=7m^3-129m^2+6mn$

12. $x^4y-x^3y^2+x^2y-8x^4y-x^2y-10+x^3y^2-7x^3y^2-9+21x^4y-y^3+50$

$x^4y-8x^4y+21x^4y=14x^4y$ $\quad -x^3y^2+x^3y^2-7x^3y^2=-7x^3y^2$

$x^2y-x^2y=0$ $\quad -y^3$ $\quad -10-9+50=31$

$=14x^4y-7x^3y^2-y^3+31$

13. $5a^{x+1}-3b^{x+2}-8c^{x+3}-5a^{x+1}-50+4b^{x+2}-65-b^{x+2}+90+c^{x+3}+7c^{x+3}$

$-8c^{x+3}+c^{x+3}+7c^{x+3}=0$ $\quad -3b^{x+2}+4b^{x+2}-b^{x+2}=0$ $\quad 5a^{x+1}-5a^{x+1}=0$ $\quad -50-65+90=-25$

$=-25$

14. $a^{m+2}-x^{m+3}-5+8-3a^{m+2}+5x^{m+3}-6+a^{m+2}-5x^{m+3}$

$-x^{m+3}+5x^{m+3}-5x^{m+3}=-x^{m+3}$ $\quad a^{m+2}-3a^{m+2}+a^{m+2}=-a^{m+2}$ $\quad -5+8-6=-3$

$=-x^{m+3}-a^{m+2}-3$

15. $0{,}3a+0{,}4b+0{,}5c-0{,}6a-0{,}7b-0{,}9c+3a-3b-3c$

$0{,}3a-0{,}6a+3a=2{,}7a$ $\quad 0{,}4b-0{,}7b-3b=-3{,}3b$ $\quad 0{,}5c-0{,}9c-3c=-3{,}4c$

$=2{,}7a-3{,}3b-3{,}4c$

16. $\frac{1}{2}a+\frac{1}{3}b+2a-3b-\frac{3}{4}a-\frac{1}{6}b+\frac{3}{4}-\frac{1}{2}$

$\frac{1}{2}a+2a-\frac{3}{4}a=\frac{2+8-3}{4}a=\frac{7}{4}a$ $\quad \frac{1}{3}b-3b-\frac{1}{6}b=\frac{2-18-1}{6}b=-\frac{17}{6}b$ $\quad \frac{3}{4}-\frac{1}{2}=\frac{3-2}{4}=\frac{1}{4}$

$=\frac{7}{4}a-\frac{17}{6}b+\frac{1}{4}$

17. $\frac{3}{5}m^2-2mn+\frac{1}{10}m^2-\frac{1}{3}mn+2mn-2m^2$

$\frac{3}{5}m^2+\frac{1}{10}m^2-2m^2=\frac{6+1-20}{10}m^2=-\frac{13}{10}m^2$ $\quad -2mn-\frac{1}{3}mn+2mn=\frac{-6-1+6}{3}mn=-\frac{1}{3}mn$

$=-\frac{13}{10}m^2-\frac{1}{3}mn$

18. $-\frac{3}{4}a^2+\frac{1}{2}ab-\frac{5}{6}b^2+2\frac{1}{3}a^2-\frac{3}{4}ab+\frac{1}{6}b^2-\frac{1}{3}b^2-2ab$

$-\frac{3}{4}a^2+\frac{7}{3}a^2=\frac{-9+28}{12}a^2=\frac{19}{12}a^2$ $\quad \frac{1}{2}ab-\frac{3}{4}ab-2ab=\frac{2-3-8}{4}ab=-\frac{9}{4}ab$

$-\frac{5}{6}b^2+\frac{1}{6}b^2-\frac{1}{3}b^2=\frac{-5+1-2}{6}b^2=-\frac{6}{6}b^2=-b^2$

$=\frac{19}{12}a^2-\frac{9}{4}ab-b^2$

19. $0{,}4x^2y+31+\frac{3}{8}xy^2-0{,}6y^3-\frac{2}{5}x^2y-0{,}2xy^2+\frac{1}{4}y^3-6$

$0{,}4x^2y-\frac{2}{5}x^2y=\frac{2-2}{5}x^2y=0$

$\frac{3}{8}xy^2-0{,}2xy^2=\frac{3-1{,}6}{8}xy^2=\frac{1{,}4}{8}xy^2=0{,}175xy^2$

$-0{,}6y^3+\frac{1}{4}y^3=\frac{-2{,}4+1}{4}y^3=-\frac{1{,}4}{4}y^3=-0{,}35y^3$

$31-6=25$

$=0{,}175xy^2-0{,}35y^3+25$

20. $\frac{3}{25}a^{m-1}-\frac{7}{50}b^{m-2}+\frac{3}{5}a^{m-1}-\frac{1}{25}b^{m-2}-0{,}2a^{m-1}+\frac{1}{5}b^{m-2}$

$\frac{3}{25}a^{m-1}+\frac{3}{5}a^{m-1}-0{,}2a^{m-1}=\frac{3+15-5}{25}a^{m-1}=\frac{13}{25}a^{m-1}$

$-\frac{7}{50}b^{m-2}-\frac{1}{25}b^{m-2}+\frac{1}{5}b^{m-2}=\frac{-7-2+10}{50}b^{m-2}=\frac{1}{50}b^{m-2}$

$=\frac{13}{25}a^{m-1}+\frac{1}{50}b^{m-2}$

EJERCICIO 11

Para resolver los problemas del 1 al 18 las literales toman los siguientes valores:
a = 1 b = 2 c = 3 m = 1/2 n = 1/3 p = 1/4

1. $3ab=3\cdot1\cdot2=6$

2. $5a^2b^3c=5\cdot1^2\cdot2^3\cdot3=5\cdot8\cdot3=120$

3. $b^2mn=2^2\cdot\frac{1}{2}\cdot\frac{1}{3}=\frac{4}{6}=\frac{2}{3}$

4. $24m^2n^3p=24\left(\frac{1}{2}\right)^2\left(\frac{1}{3}\right)^3\cdot\frac{1}{4}=6\cdot\frac{1}{27}\cdot\frac{1}{4}=\frac{6}{108}=\frac{1}{18}$

5. $\frac{2}{3}a^4b^2m^3=\frac{2}{3}\cdot1^4\cdot2^2\cdot\left(\frac{1}{2}\right)^3=\frac{2}{3}\cdot4\cdot\frac{1}{8}=\frac{8}{24}=\frac{1}{3}$

6. $\frac{7}{12}c^3p^2m=\frac{7}{12}\cdot3^3\cdot\left(\frac{1}{4}\right)^2\cdot\frac{1}{2}=\frac{189}{12}\cdot\frac{1}{16}\cdot\frac{1}{2}=\frac{189}{384}=\frac{63}{128}$

7. $m^bn^cp^a=\left(\frac{1}{2}\right)^2\left(\frac{1}{3}\right)^3\cdot\frac{1}{4}=\frac{1}{4}\cdot\frac{1}{27}\cdot\frac{1}{4}=\frac{1}{432}$

8. $\frac{5}{6}a^{b-1}\cdot m^{c-2}=\frac{5}{6}\cdot1^{2-1}\cdot\left(\frac{1}{2}\right)^{3-2}=\frac{5}{6}\cdot\frac{1}{2}=\frac{5}{12}$

9. $\sqrt{2bc^2}=\sqrt{2\cdot2\cdot3^2}=\sqrt{4\cdot9}=\sqrt{36}=6$

10. $4m\cdot\sqrt[3]{12bc^2}=4\cdot\frac{1}{2}\cdot\sqrt[3]{12\cdot2\cdot3^2}=2\cdot\sqrt[3]{216}=2\cdot6=12$

11. $mn\cdot\sqrt{8a^4b^3}=\frac{1}{2}\cdot\frac{1}{3}\cdot\sqrt{8\cdot1^4\cdot2^3}=\frac{1}{6}\cdot\sqrt{64}=\frac{8}{6}=\frac{4}{3}$

12. $\frac{4a}{3bc}=\frac{4\cdot1}{3\cdot2\cdot3}=\frac{4}{18}=\frac{2}{9}$

13. $\frac{5b^2m^2}{np}=\frac{5\cdot2^2\cdot\left(\frac{1}{2}\right)^2}{\frac{1}{3}\cdot\frac{1}{4}}=\frac{20\cdot\frac{1}{4}}{\frac{1}{12}}=\frac{5}{\frac{1}{12}}=60$

14. $\frac{\frac{3}{4}b^3}{\frac{2}{3}c^2}=\frac{\frac{3}{4}\cdot2^3}{\frac{2}{3}\cdot3^2}=\frac{\frac{3\cdot8}{4}}{\frac{2\cdot9}{3}}=\frac{\frac{24}{4}}{\frac{18}{3}}=\frac{6}{6}=1$

15. $\frac{2m}{\sqrt{n^2}}=\frac{2\cdot\frac{1}{2}}{\sqrt{\left(\frac{1}{3}\right)^2}}=\frac{1}{\sqrt{\frac{1}{9}}}=\frac{1}{\frac{1}{3}}=3$

16. $\frac{24mn}{2\cdot\sqrt{n^2p^2}}=\frac{24\cdot\frac{1}{2}\cdot\frac{1}{3}}{2\cdot\sqrt{\left(\frac{1}{3}\right)^2\left(\frac{1}{4}\right)^2}}=\frac{\frac{24}{6}}{2\cdot\sqrt{\frac{1}{9}\cdot\frac{1}{16}}}=\frac{4}{2\cdot\sqrt{\frac{1}{144}}}$

$=\frac{4}{2\cdot\frac{1}{12}}=\frac{4}{\frac{1}{6}}=24$

17. $\dfrac{3\cdot\sqrt[3]{64b^3c^6}}{2m}=\dfrac{3\cdot\sqrt[3]{64\cdot 2^3\cdot 3^6}}{2\cdot\frac{1}{2}}=3\cdot\sqrt[3]{64\cdot 8\cdot 729}=3\cdot\sqrt[3]{373.248}=3\cdot 72=216$

18. $\dfrac{\frac{3}{5}\cdot\sqrt{apb^2}}{\frac{3}{2}\cdot\sqrt[3]{125bm}}=\dfrac{\frac{3}{5}\cdot\sqrt{1\cdot\frac{1}{4}\cdot 2^2}}{\frac{3}{2}\cdot\sqrt[3]{125\cdot 2\cdot\frac{1}{2}}}=\dfrac{\frac{3}{5}\cdot\sqrt{\frac{4}{4}}}{\frac{3}{2}\cdot\sqrt[3]{\frac{250}{2}}}=\dfrac{\frac{3}{5}\cdot\sqrt{1}}{\frac{3}{2}\cdot\sqrt[3]{125}}=\dfrac{\frac{3}{5}}{\frac{3}{2}\cdot 5}=\dfrac{\frac{3}{5}}{\frac{15}{2}}=\dfrac{6}{75}=\dfrac{2}{25}$

EJERCICIO 12

Para resolver los problemas del 1 al 18 las literales toman los siguientes valores:

a = 3 b = 4 c = 1/3 d = 1/2 m = 6 n = 1/4

1. $a^2-2ab+b^2=3^2-2\cdot 3\cdot 4+4^2=9-24+16=1$

2. $c^2+2cd+d^2=\left(\frac{1}{3}\right)^2+2\cdot\frac{1}{3}\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2=\frac{1}{9}+\frac{1}{3}+\frac{1}{4}=\frac{4+12+9}{36}=\frac{25}{36}$

3. $\dfrac{a}{c}+\dfrac{b}{d}=\dfrac{3}{\frac{1}{3}}+\dfrac{4}{\frac{1}{2}}=9+8=17$

4. $\dfrac{c}{d}-\dfrac{m}{n}+2=\dfrac{\frac{1}{3}}{\frac{1}{2}}-\dfrac{6}{\frac{1}{4}}+2=\dfrac{2}{3}-24+2=\dfrac{2-72+6}{3}=-\dfrac{64}{3}$

5. $\dfrac{a^2}{3}-\dfrac{b^2}{2}+\dfrac{m^2}{6}=\dfrac{3^2}{3}-\dfrac{4^2}{2}+\dfrac{6^2}{6}=\dfrac{9}{3}-\dfrac{16}{2}+\dfrac{36}{6}=3-8+6=1$

6. $\dfrac{3}{5}c-\dfrac{1}{2}b+2d=\dfrac{3}{5}\cdot\dfrac{1}{3}-\dfrac{1}{2}\cdot 4+2\cdot\dfrac{1}{2}=\dfrac{1}{5}-2+1=\dfrac{1-10+5}{5}=-\dfrac{4}{5}$

7. $\dfrac{ab}{n}+\dfrac{ac}{d}-\dfrac{bd}{m}=\dfrac{3\cdot 4}{\frac{1}{4}}+\dfrac{3\cdot\frac{1}{3}}{\frac{1}{2}}-\dfrac{4\cdot\frac{1}{2}}{6}=\dfrac{12}{\frac{1}{4}}+\dfrac{1}{\frac{1}{2}}-\dfrac{2}{6}=48+2-\dfrac{1}{3}=\dfrac{144+6-1}{3}=\dfrac{149}{3}=49\tfrac{2}{3}$

8. $\sqrt{b}+\sqrt{n}+\sqrt{6m}=\sqrt{4}+\sqrt{\frac{1}{4}}+\sqrt{6\cdot 6}=2+\frac{1}{2}+6=\frac{4+1+12}{2}=\frac{17}{2}=8\tfrac{1}{2}$

9. $c\sqrt{3a}-d\sqrt{16b^2}+n\sqrt{8d}=\frac{1}{3}\cdot\sqrt{3\cdot 3}-\frac{1}{2}\cdot\sqrt{16\cdot 4^2}+\frac{1}{4}\cdot\sqrt{8\cdot\frac{1}{2}}=\frac{1}{3}\cdot 3-\frac{1}{2}\cdot 16+\frac{1}{4}\cdot 2=1-8+\frac{1}{2}=\frac{2-16+1}{2}=-\frac{13}{2}=-6\tfrac{1}{2}$

10. $\dfrac{m^a}{d^b}=\dfrac{6^3}{\left(\frac{1}{2}\right)^4}=\dfrac{216}{\frac{1}{16}}=216\cdot 16=3.456$

11. $\frac{3}{4}c^2+\frac{4n^2}{m}=\frac{3\left(\frac{1}{3}\right)^2}{4}+\frac{4\left(\frac{1}{4}\right)^2}{6}=\frac{\frac{1}{3}}{4}+\frac{\frac{1}{4}}{6}=\frac{1}{12}+\frac{1}{24}=\frac{2+1}{24}=\frac{3}{24}=\frac{1}{8}$

12. $\frac{4d^2}{2}+\frac{16n^2}{2}-1=2\cdot\left(\frac{1}{2}\right)^2+8\cdot\left(\frac{1}{4}\right)^2-1=\frac{2}{4}+\frac{8}{16}-1=\frac{1}{2}+\frac{1}{2}-1=1-1=0$

13. $\frac{a+b}{c}-\frac{b+m}{d}=\frac{3+4}{\frac{1}{3}}-\frac{4+6}{\frac{1}{2}}=\frac{7}{\frac{1}{3}}-\frac{10}{\frac{1}{2}}=21-20=1$

14. $\frac{b-a}{n}+\frac{m-b}{d}+5a=\frac{4-3}{\frac{1}{4}}+\frac{6-4}{\frac{1}{2}}+5\cdot3=\frac{1}{\frac{1}{4}}+\frac{2}{\frac{1}{2}}+15=4+4+15=23$

15. $\frac{12c-a}{2b}-\frac{16n-a}{m}+\frac{1}{d}=\frac{12\cdot\frac{1}{3}-3}{2\cdot4}-\frac{16\cdot\frac{1}{4}-3}{6}+\frac{1}{\frac{1}{2}}=\frac{4-3}{8}-\frac{4-3}{6}+2=\frac{1}{8}-\frac{1}{6}+2=\frac{3-4+48}{24}=\frac{47}{24}=1\frac{23}{24}$

16. $\sqrt{4b}+\frac{\sqrt{3a}}{3}-\frac{\sqrt{6m}}{6}=\sqrt{4\cdot4}+\frac{\sqrt{3\cdot3}}{3}-\frac{\sqrt{6\cdot6}}{6}=\sqrt{16}+\frac{\sqrt{9}}{3}-\frac{\sqrt{36}}{6}=4+\frac{3}{3}-\frac{6}{6}=4+1-1=4$

17. $\frac{\sqrt{b}+\sqrt{2d}}{2}-\frac{\sqrt{3c}+\sqrt{8d}}{4}=\frac{\sqrt{4}+\sqrt{2\cdot\frac{1}{2}}}{2}-\frac{\sqrt{3\cdot\frac{1}{3}}+\sqrt{8\cdot\frac{1}{2}}}{4}=\frac{2+1}{2}-\frac{1+2}{4}=\frac{3}{2}-\frac{3}{4}=\frac{6-3}{4}=\frac{3}{4}$

18. $\frac{2\cdot\sqrt{a^2b^2}}{3}+\frac{3\cdot\sqrt{2+d^2}}{4}-a\cdot\sqrt{n}=\frac{2\cdot\sqrt{3^2\cdot4^2}}{3}+\frac{3\cdot\sqrt{2+\left(\frac{1}{2}\right)^2}}{4}-3\cdot\sqrt{\frac{1}{4}}=\frac{2\cdot\sqrt{144}}{3}+\frac{3\cdot\sqrt{\frac{9}{4}}}{4}-3\cdot\frac{1}{2}$

$=\frac{2\cdot12}{3}+\frac{3\cdot\frac{3}{2}}{4}-\frac{3}{2}=\frac{24}{3}+\frac{\frac{9}{2}}{4}-\frac{3}{2}=8+\frac{9}{8}-\frac{3}{2}=\frac{64+9-12}{8}=\frac{61}{8}=7\frac{5}{8}$

EJERCICIO 13

Para los problemas 1 al 24 las literales toman los siguientes valores:
a = 1   b = 2   c = 3   d = 4   m = 1/2   n = 2/3   p = 1/4   x = 0

1. $(a+b)\cdot c-d=(1+2)\cdot3-4=3\cdot3-4=9-4=5$

2. $(a+b)(b-a)=(1+2)(2-1)=3\cdot1=3$

3. $(b-m)(c-n)+4a^2=\left(2-\frac{1}{2}\right)\left(3-\frac{2}{3}\right)+4\cdot1^2=\left(\frac{4-1}{2}\right)\left(\frac{9-2}{3}\right)+4=\frac{3}{2}\cdot\frac{7}{3}+4=\frac{7}{2}+4=\frac{7+8}{2}=\frac{15}{2}=7\frac{1}{2}$

4. $(2m+3n)(4p+b^2)=\left(2\cdot\frac{1}{2}+3\cdot\frac{2}{3}\right)\left(4\cdot\frac{1}{4}+2^2\right)=(1+2)(1+4)=3\cdot5=15$

5. $(4m+8p)(a^2+b^2)(6n-d)=\left(4\cdot\frac{1}{2}+8\frac{1}{4}\right)(1^2+2^2)\left(6\cdot\frac{2}{3}-4\right)=(2+2)\cdot3\cdot0=0$

6. $(c-b)(d-c)(b-a)(m-p)=(3-2)(4-3)(2-1)\left(\frac{1}{2}-\frac{1}{4}\right)=1\cdot1\cdot1\left(\frac{2-1}{4}\right)=\frac{1}{4}$

7. $b^2(c+d)-a^2(m+n)+2x=2^2(3+4)-1^2\left(\frac{1}{2}+\frac{2}{3}\right)+2\cdot0=4\cdot7-\left(\frac{3+4}{6}\right)=28-\frac{7}{6}=\frac{168-7}{6}=\frac{161}{6}=26\tfrac{5}{6}$

8. $2mx+6(b^2+c^2)-4d^2=2\cdot\frac{1}{2}\cdot0+6(2^2+3^2)-4\cdot4^2=6(4+9)-4\cdot16=6\cdot13-64=78-64=14$

9. $\left(\frac{8m}{9n}+\frac{16p}{b}\right)a=\left(\frac{8\cdot\frac{1}{2}}{9\cdot\frac{2}{3}}+\frac{16\cdot\frac{1}{4}}{2}\right)\cdot1=\frac{4}{\frac{18}{3}}+\frac{4}{2}=\frac{4}{6}+2=\frac{2}{3}+2=\frac{2+6}{3}=\frac{8}{3}=2\tfrac{2}{3}$

10. $x+m(a^b+d^c-c^a)=0+\frac{1}{2}(1^2+4^3-3^1)=\frac{1}{2}(1+64-3)=\frac{1}{2}\cdot62=31$

11. $\frac{4(m+p)}{a}\div\frac{a^2+b^2}{c^2}=\frac{4\left(\frac{1}{2}+\frac{1}{4}\right)}{1}\div\frac{1^2+2^2}{3^2}=4\left(\frac{2+1}{4}\right)\div\frac{1+4}{9}=4\cdot\frac{3}{4}\div\frac{5}{9}=\frac{3}{\frac{5}{9}}=\frac{27}{5}=5\tfrac{2}{5}$

12. $(2m+3n+4p)(8p+6n-4m)(9n+20p)$

$=\left(2\cdot\frac{1}{2}+3\cdot\frac{2}{3}+4\cdot\frac{1}{4}\right)\left(8\cdot\frac{1}{4}+6\cdot\frac{2}{3}-4\cdot\frac{1}{2}\right)\left(9\cdot\frac{2}{3}+20\cdot\frac{1}{4}\right)=(1+2+1)(2+4-2)(6+5)=4\cdot4\cdot11=176$

13. $c^2(m+n)-d^2(m+p)+b^2(n+p)$

$=3^2\left(\frac{1}{2}+\frac{2}{3}\right)-4^2\left(\frac{1}{2}+\frac{1}{4}\right)+2^2\left(\frac{2}{3}+\frac{1}{4}\right)=9\cdot\frac{7}{6}-16\cdot\frac{3}{4}+4\cdot\frac{11}{12}=\frac{21}{2}-12+\frac{44}{12}=\frac{126-144+44}{12}=\frac{26}{12}=2\tfrac{2}{12}=2\tfrac{1}{6}$

14. $\left(\frac{\sqrt{c^2+d^2}}{a}\div\frac{2}{\sqrt{d}}\right)m=\left(\frac{\sqrt{3^2+4^2}}{1}\div\frac{2}{\sqrt{4}}\right)\cdot\frac{1}{2}=\left(\frac{\sqrt{9+16}}{1}\div\frac{2}{2}\right)\cdot\frac{1}{2}=\left(\frac{\sqrt{25}}{1}\div\frac{2}{2}\right)\cdot\frac{1}{2}=5\cdot\frac{1}{2}=\frac{5}{2}=2\tfrac{1}{2}$

15. $(4p+2b)(18n-24p)+2(8m+2)(40p+a)$

$=\left(4\cdot\frac{1}{4}+2\cdot2\right)\left(18\cdot\frac{2}{3}-24\cdot\frac{1}{4}\right)+2\left(8\cdot\frac{1}{2}+2\right)\left(40\cdot\frac{1}{4}+1\right)=(1+4)(12-6)+2(4+2)(11)=5\cdot6+2\cdot66=30+132=162$

16. $\frac{a+\frac{d}{b}}{d-b}\cdot\frac{5+\frac{2}{m^2}}{p^2}=\frac{1+\frac{4}{2}}{4-2}\cdot\frac{5+\frac{2}{\left(\frac{1}{2}\right)^2}}{\left(\frac{1}{4}\right)^2}=\frac{\frac{2+4}{2}}{2}\cdot\frac{5+\frac{2}{\frac{1}{4}}}{\frac{1}{16}}=\frac{\frac{6}{2}}{2}\cdot\frac{5+8}{\frac{1}{16}}=\frac{3}{2}\cdot\frac{5+8}{\frac{1}{16}}=\frac{3}{2}\cdot\frac{13}{\frac{1}{16}}=\frac{3}{2}\cdot208=\frac{624}{2}=312$

17. $(a+b)\cdot\sqrt{c^2+8b}-m\cdot\sqrt{n^2}=(1+2)\cdot\sqrt{3^2+8\cdot2}-\frac{1}{2}\cdot\sqrt{\left(\frac{2}{3}\right)^2}=3\cdot\sqrt{25}-\frac{1}{2}\cdot\frac{2}{3}=3\cdot5-\frac{2}{6}=15-\frac{2}{6}=\frac{90-2}{6}=\frac{88}{6}=14\tfrac{4}{6}=14\tfrac{2}{3}$

18. $\left(\frac{\sqrt{a+c}}{2}+\frac{\sqrt{6n}}{b}\right)\div(c+d)\cdot\sqrt{p}=\left(\frac{\sqrt{1+3}}{2}+\frac{\sqrt{6\cdot\frac{2}{3}}}{2}\right)\div(3+4)\cdot\sqrt{\frac{1}{4}}=\left(\frac{\sqrt{4}}{2}+\frac{\sqrt{4}}{2}\right)\div7\cdot\frac{1}{2}=\frac{2}{\frac{7}{2}}=\frac{4}{7}$

19. $3(c-b)\cdot\sqrt{32m}-2(d-a)\cdot\sqrt{16p}-\frac{2}{n}$

$$=3(3-2)\cdot\sqrt{32\cdot\frac{1}{2}}-2(4-1)\cdot\sqrt{16\cdot\frac{1}{4}}-\frac{2}{\frac{2}{3}}=3\cdot\sqrt{16}-6\cdot\sqrt{4}-\frac{6}{2}=3\cdot4-6\cdot2-3=12-12-3=-3$$

20. $\frac{\sqrt{6abc}}{2\cdot\sqrt{8b}}+\frac{\sqrt{3mn}}{2(b-a)}-\frac{cdnp}{abc}$

$$=\frac{\sqrt{6\cdot1\cdot2\cdot3}}{2\cdot\sqrt{8\cdot2}}+\frac{\sqrt{3\cdot\frac{1}{2}\cdot\frac{2}{3}}}{2(2-1)}-\frac{3\cdot4\cdot\frac{2}{3}\cdot\frac{1}{4}}{1\cdot2\cdot3}=\frac{\sqrt{36}}{2\cdot\sqrt{16}}+\frac{\sqrt{1}}{2}-\frac{2}{6}=\frac{6}{8}+\frac{1}{2}-\frac{1}{3}=\frac{18+12-8}{24}=\frac{22}{24}=\frac{11}{12}$$

21. $\frac{a^2+b^2}{b^2-a^2}+3(a+b)(2a+3b)=\frac{1^2+2^2}{2^2-1^2}+3(1+2)(2\cdot1+3\cdot2)=\frac{5}{3}+3\cdot3\cdot8=\frac{5}{3}+72=\frac{5+216}{3}=\frac{221}{3}=73\tfrac{2}{3}$

22. $b^2+\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{n}+\frac{1}{m}\right)^2$

$$=2^2+\left(\frac{1}{1}+\frac{1}{2}\right)\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{\frac{2}{3}}+\frac{1}{\frac{1}{2}}\right)^2=4+\left(\frac{3}{2}\cdot\frac{5}{6}\right)+\left(\frac{3}{2}+2\right)^2=4+\frac{5}{4}+\left(\frac{7}{2}\right)^2=\frac{21}{4}+\frac{49}{4}=\frac{70}{4}=\frac{35}{2}=17\tfrac{1}{2}$$

23. $(2m+3n)(4p+2c)-4m^2n^2$

$$=\left(2\cdot\frac{1}{2}+3\cdot\frac{2}{3}\right)\left(4\cdot\frac{1}{4}+2\cdot3\right)-4\left(\frac{1}{2}\right)^2\left(\frac{2}{3}\right)^2=(1+2)(1+6)-4\cdot\frac{1}{4}\cdot\frac{4}{9}=3\cdot7-\frac{4}{9}=21-\frac{4}{9}=\frac{189-4}{9}=\frac{185}{9}=20\tfrac{5}{9}$$

24. $\frac{b^2-\frac{c}{3}}{2ab-m}-\frac{n}{b-m}=\frac{2^2-\frac{3}{3}}{2\cdot1\cdot2-\frac{1}{2}}-\frac{\frac{2}{3}}{2-\frac{1}{2}}=\frac{3}{4-\frac{1}{2}}-\frac{\frac{2}{3}}{\frac{3}{2}}=\frac{3}{\frac{7}{2}}-\frac{4}{9}=\frac{6}{7}-\frac{4}{9}=\frac{54-28}{63}=\frac{26}{63}$

EJERCICIO 14

1. $a+b+m$

2. $m^2+b^3+x^4$

3. $a+1$; $a+2$

4. $x-1$; $x-2$

5. $y+2$; $y+4$; $y+6$

6. $\$(a+x+m)$

7. $m-n$

8. $bs.(x-6)$

9. $(x-m)Km$

10. $\$(x+a-m)$

11. $(m-a-b-c)Km$

12. $\$(n-300)$

13. $(365-x)$ *días*

14. $\$8a$; $\$15a$; $\$ma$

15. $2a+3b+\frac{c}{2}$

16. $a\cdot b\ m^2$

17. $23n\ m^2$

18. $x^2\ m^2$

19. $\$(3a+6b)$; $\$(ax+bm)$

20. $(a+b)(x+y)$

21. $\$(8x+48)$

22. $bs.(a-8)(x+4)$

23. $\frac{75}{x}$ *sucres*

24. $\$\frac{a}{m}$

25. $\frac{3.000}{n-1}$ *colones*

26. $\frac{x}{a-3}$ *soles*

27. $\frac{m}{14}$ *m*

28. $\frac{x+1}{a}$ km/h

29. $\$\frac{a+b}{m-2}$

30. $\left(x+2x+\frac{x}{2}\right)$ *hab.*

31. $\left[1.000-\left(a+\frac{a}{3}+\frac{2a}{4}\right)\right]$ *sucres*

EJERCICIO 15

1. $m, n \to m+n$
2. $m, -n \to m+(-n)=m-n$
3. $-3a, 4b \to -3a+4b$
4. $5b, 6a \to -6a+5b$
5. $7, -6 \to 7+(-6)=7-6=1$
6. $-6, 9 \to -6+9=3$
7. $-2x, 3y \to -2x+3y$
8. $5mn, -m \to 5mn+(-m)=5mn-m$
9. $5a, 7a \to 5a+7a=12a$
10. $-8x, -5x \to -8x+(-5x)=-8x-5x=-13x$
11. $-11m, 8m \to -11m+8m=-3m$
12. $9ab, -15ab \to 9ab-15ab=-6ab$
13. $-xy, -9xy \to -9xy-xy=-10xy$
14. $mn, -11mn \to mn+(-11mn)=mn-11mn=-10mn$
15. $\frac{1}{2}a, -\frac{2}{3}b \to \frac{1}{2}a-\frac{2}{3}b$
16. $\frac{3}{5}b, \frac{3}{4}c \to \frac{3}{5}b+\frac{3}{4}c$
17. $\frac{1}{3}b, \frac{2}{3}b \to \frac{1}{3}b+\frac{2}{3}b=\frac{3}{3}b=b$
18. $-\frac{1}{2}xy, -\frac{1}{2}xy \to -\frac{1}{2}xy-\frac{1}{2}xy=-\frac{2}{2}xy=-xy$
19. $-\frac{3}{5}abc, -\frac{2}{5}abc$
    $-\frac{3}{5}abc-\frac{2}{5}abc=-\frac{5}{5}abc=-abc$
20. $-4x^2y, \frac{3}{8}x^2y$
    $-4x^2y+\frac{3}{8}x^2y=\frac{-32+3}{8}x^2y=-\frac{29}{8}x^2y$
21. $\frac{3}{8}mn, -\frac{3}{4}mn$
    $\frac{3}{8}mn-\frac{3}{4}mn=\frac{3-6}{8}mn=-\frac{3}{8}mn$
22. $a, b, c \to a+b+c$
23. $a, -b, c \to a-b+c$
24. $a, -b, 2c \to a-b+2c$
25. $3m, -2n, 4p \to 3m-2n+4p$
26. $a^2, -7ab, -5b^2 \to a^2-7ab-5b^2$
27. $x^2, -3xy, -4y^2 \to x^2-3xy-4y^2$
28. $x^3, -x^2y, 6 \to x^3-x^2y+6$
29. $2a, -b, 3a \to 2a+3a-b=5a-b$
30. $-m, -8n, 4n \to -m-8n+4n=-m-4n$
31. $-7a, 8a, -b \to -7a+8a-b=a-b$
32. $\frac{1}{2}x, \frac{2}{3}y, -\frac{3}{4}x$
    $\frac{1}{2}x-\frac{3}{4}x+\frac{2}{3}y=\frac{4-6}{8}x+\frac{2}{3}y=-\frac{1}{4}x+\frac{2}{3}y$
33. $-\frac{3}{5}m, -m, -\frac{2}{3}mn$
    $-\frac{3}{5}m-m-\frac{2}{3}mn=\frac{-3-5}{5}m-\frac{2}{3}mn=-\frac{8}{5}m-\frac{2}{3}mn$
34. $-7a^2, 5ab, 3b^2, -a^2$
    $-7a^2-a^2+5ab+3b^2=-8a^2+5ab+3b^2$

35. $-7mn^2+17mn^2-5m-4m\;=10mn^2-9m$

36. $x^3-7x^3-8x^2y+4x^2y+5=-6x^3-4x^2y+5$

37. $5x^2-x^2+9xy-6xy+7y^2=4x^2+3xy+7y^2$

38. $-8a^2b-a^2b+5ab^2-11ab^2-7b^3$
$=-9a^2b-6ab^2-7b^3$

39. $m^3-8m^2n+7m^2n+7mn^2-n^3$
$=m^3-m^2n+7mn^2-n^3$

40. $\frac{1}{2}a-\frac{1}{4}a+\frac{2}{3}b+\frac{1}{5}b-6$
$=\frac{2-1}{4}a+\frac{10+3}{15}b-6=\frac{1}{4}a+\frac{13}{15}b-6$

41. $a-a-3b+4b-8c+8c=b$

42. $m^3+5m^3-5m^3-4m^2n-4m^2n-7mn^2$
$=m^3-8m^2n-7mn^2$

43. $9x-x-11y-6y+4z-6z=8x-17y-2z$

44. $6a^2+9a^2-5ab-7b^2-8b^2-11$
$=15a^2-5ab-15b^2-11$

45. $-x^2y^2+x^2y^2-5xy^3+7xy^3-4y^4-8$
$=2xy^3-4y^4-8$

46. $3a-\frac{1}{2}a+\frac{1}{2}b-b-4+6$
$=\frac{6-1}{2}a+\frac{1-2}{2}b+2=\frac{5}{2}a-\frac{1}{2}b+2$

47. $\frac{1}{2}x^2+\frac{3}{4}x^2+\frac{2}{3}xy-\frac{1}{3}xy+\frac{5}{6}y^2-\frac{5}{6}y^2$
$=\frac{2+3}{4}x^2+\frac{2-1}{3}xy=\frac{5}{4}x^2+\frac{1}{3}xy$

48. $5ax-5ax-6a^{x+1}+a^{x+1}+5a^{x+1}+8a^{x+2}=8a^{x+2}$

49. $\frac{3}{4}x^2+x^2-\frac{2}{3}xy-\frac{1}{3}xy+\frac{1}{3}y^2+5y^2$
$=\frac{3+4}{4}x^2-\frac{3}{3}xy+\frac{1+15}{3}y^2=\frac{7}{4}x^2-xy+\frac{16}{3}y^2$

50. $\frac{3}{4}a^2b-\frac{1}{4}a^2b+a^2b+\frac{1}{2}ab^2+\frac{1}{2}ab^2-\frac{5}{6}ab^2$
$=\frac{3-1+4}{4}a^2b+\frac{3+3-5}{6}ab^2=\frac{3}{2}a^2b+\frac{1}{6}ab^2$

EJERCICIO 16

1. $$\begin{array}{l}3a+2b-c\\2a+3b+c\\\hline 5a+5b\end{array}$$

2. $$\begin{array}{r}7a-4b+5c\\-7a+4b-6c\\\hline -c\end{array}$$

3. $$\begin{array}{c}m+n-p\\-m-n+p\\\hline 0\end{array}$$

4. $$\begin{array}{l}9x-3y+5\\-x-y+4\\-5x+4y-9\\\hline 3x\end{array}$$

5. $$\begin{array}{r}a+b-c\\2a+2b-2c\\-3a-b+3c\\\hline 2b\end{array}$$

6. $$\begin{array}{r}p+q+r\\-2p-6q+3r\\p+5q-8r\\\hline -4r\end{array}$$

7. $$\begin{array}{l}-7x-4y+6z\\10x-20y-8z\\-5x+24y+2z\\\hline -2x\end{array}$$

8. $$\begin{array}{l}-2m+3n-6\\3m-8n+8\\-5m+n-10\\\hline -4m-4n-8\end{array}$$

9. $$\begin{array}{l}-5a-2b-3c\\7a-3b+5c\\-8a+5b-3c\\\hline -6a\quad -c\end{array}$$

10. $$\begin{array}{r}ab+bc+cd\\-8ab-3bc-3cd\\5ab+2bc+2cd\\\hline -2ab\end{array}$$

11. $$\begin{array}{r}ax-ay-az\\-5ax-7ay-6az\\4ax+9ay+8az\\\hline ay+az\end{array}$$

12. $$\begin{array}{l}5x-7y+8\\-4x-y+6\\-3x+8y+9\\\hline -2x\quad +23\end{array}$$

13. $$\begin{array}{l}-am+6mn-4s\\-am-5mn+6s\\3am-5mn-2s\\\hline am-4mn\end{array}$$

14. $$\begin{array}{l}2a+3b\\6b-4c\\-a\quad +8c\\\hline a+9b+4c\end{array}$$

15. $$\begin{array}{l}6m-3n\\-4n+5p\\-m\quad -5p\\\hline 5m-7n\end{array}$$

16. $$\begin{array}{l}2a+3b\\+5c-4\\8a\quad +6\\+7c-9\\\hline 10a+3b+12c-7\end{array}$$

17. $$\begin{array}{l}2x-3y\\+5z+9\\6x\quad -4\\3y\quad -5\\\hline 8x\quad +5z\end{array}$$

18.
$$\begin{array}{l} 8a+3b-c \\ 5a-b+c \\ -a-b-c \\ 7a-b+4c \\ \hline 19a \quad +3c \end{array}$$

19.
$$\begin{array}{l} 7x+2y \quad -4 \\ +9y-6z+5 \\ -y+3z-6 \\ 8x-3y \quad -5 \\ \hline 15x+7y-3z-10 \end{array}$$

20.
$$\begin{array}{l} -m-n-p \\ m+2n \quad -5 \\ -6m \quad +3p+4 \\ 5m+2n \quad -8 \\ \hline -m+3n+2p-9 \end{array}$$

21.
$$\begin{array}{l} 5a^x-3a^m-7a^n \\ -8a^x+5a^m-9a^n \\ -11a^x+5a^m+16a^n \\ \hline -14a^x+7a^m \end{array}$$

22.
$$\begin{array}{l} 6m^{a+1}-7m^{a+2}-5m^{a+3} \\ 4m^{a+1}-7m^{a+2}-m^{a+3} \\ -5m^{a+1}+3m^{a+2}+12m^{a+3} \\ \hline 5m^{a+1}-11m^{a+2}+6m^{a+3} \end{array}$$

23.
$$\begin{array}{l} 8x+y+z+u \\ -3x-4y-2z+3u \\ 4x+5y+3z-4u \\ -9x-y+z+2u \\ \hline y+3z+2u \end{array}$$

24.
$$\begin{array}{l} a+b-c+d \\ a-b+c-d \\ -2a+3b-2c+d \\ -3a-3b+4c-d \\ \hline -3a \quad +2c \end{array}$$

25.
$$\begin{array}{l} 5ab-3bc+4cd \\ +2bc+2cd-3de \\ -2ab+4bc \quad +3de \\ -ab-3bc-6cd \\ \hline 2ab \end{array}$$

26.
$$\begin{array}{l} a-b \\ b-c \\ c+d \\ a-c \\ c-d \\ -a+d \\ a-d \\ \hline 2a \end{array}$$

EJERCICIO 17

1.
$$\begin{array}{l} x^2+4x \\ x^2-5x \\ \hline 2x^2-x \end{array}$$

2.
$$\begin{array}{l} a^2+ab \\ -2ab+b^2 \\ \hline a^2-ab+b^2 \end{array}$$

3.
$$\begin{array}{l} x^3 \quad +2x \\ -x^2 \quad +4 \\ \hline x^3-x^2+2x+4 \end{array}$$

4.
$$\begin{array}{l} a^4 \quad -3a^2 \\ +a^3 \quad +4a \\ \hline a^4+a^3-3a^2+4a \end{array}$$

5.
$$\begin{array}{l} -x^2+3x \\ x^3 \quad +6 \\ \hline x^3-x^2+3x+6 \end{array}$$

6.
$$\begin{array}{l} x^2-4x \\ -7x+6 \\ 3x^2 \quad -5 \\ \hline 4x^2-11x+1 \end{array}$$

7.
$$\begin{array}{l} m^2+n^2 \\ +4n^2-3mn \\ -5m^2-5n^2 \\ \hline -4m^2 \quad -3mn \end{array}$$

8.
$$\begin{array}{l} 3x+x^3 \\ -4x^2+5 \\ -x^3+4x^2-6 \\ \hline 3x \quad -1 \end{array}$$

9.
$$\begin{array}{l} x^2-3xy+y^2 \\ -x^2+3xy-2y^2 \\ x^2+3xy-y^2 \\ \hline x^2+3xy-2y^2 \end{array}$$

10.
$$\begin{array}{l} a^2-3ab+b^2 \\ +a^2-5ab-b^2 \\ -2a^2+8ab-b^2 \\ \hline -b^2 \end{array}$$

11.
$$\begin{array}{l} -7x^2+5x-6 \\ 4x^2+8x-9 \\ -x^2-7x+14 \\ \hline -4x^2+6x-1 \end{array}$$

12.
$$\begin{array}{l} a^3 \quad -4a+5 \\ a^3-2a^2 \quad +6 \\ +a^2-7a+4 \\ \hline 2a^3-a^2-11a+15 \end{array}$$

13.
$$\begin{array}{l} -x^2+x-6 \\ x^3-7x^2 \quad +5 \\ -x^3 \quad +8x-5 \\ \hline -8x^2+9x-6 \end{array}$$

14.
$$\begin{array}{l} a^3 \quad -b^3 \\ +5a^2b-4ab^2 \\ a^3 \quad -7ab^2-b^3 \\ \hline 2a^3+5a^2b-11ab^2-2b^3 \end{array}$$

15.
$$\begin{array}{l} x^3 \quad +xy^2+y^3 \\ x^3-5x^2y \quad -y^3 \\ 2x^3 \quad -4xy^2-5y^3 \\ \hline 4x^3-5x^2y-3xy^2-5y^3 \end{array}$$

16.
$$\begin{array}{l} -7m^2n \quad +4n^3 \\ +m^3 \quad +6mn^2-n^3 \\ -m^3+7m^2n \quad +5n^3 \\ \hline +6mn^2+8n^3 \end{array}$$

17. $$\begin{array}{l} x^4 - x^2 + x \\ +x^3 - 4x^2 + 5 \\ +7x^2 - 4x + 6 \\ \hline x^4 + x^3 + 2x^2 - 3x + 11 \end{array}$$

18. $$\begin{array}{l} +a^6 + a^4 + 6 \\ +a^5 - 3a^3 + 8 \\ + a^3 - a^2 - 14 \\ \hline +a^6 + a^5 + a^4 - 2a^3 - a^2 \end{array}$$

19. $$\begin{array}{l} x^5 + x - 9 \\ +3x^4 - 7x^2 + 6 \\ -3x^3 - 4x + 5 \\ \hline x^5 + 3x^4 - 3x^3 - 7x^2 - 3x + 2 \end{array}$$

20. $$\begin{array}{l} a^3 + a \\ + a^2 + 5 \\ +4a + 7a^2 \\ -8a^2 - 6 \\ \hline a^3 + 5a - 1 \end{array}$$

21. $$\begin{array}{l} x^4 - x^2y^2 \\ -5x^3y + 6xy^3 \\ -4xy^3 + y^4 \\ -4x^2y^2 - 6 \\ \hline x^4 - 5x^3y - 5x^2y^2 + 2xy^3 + y^4 - 6 \end{array}$$

22. $$\begin{array}{l} + x^2 + xy \\ - x^2 + 4xy - 7y^2 \\ - x^2 + 6xy + 5y^2 \\ -6x^2 - 4xy + y^2 \\ \hline -7x^2 + 7xy - y^2 \end{array}$$

23. $$\begin{array}{l} a^3 - 8ax^2 + x^3 \\ -6ax^2 - x^3 + 5a^2x \\ 3a^3 - x^3 - 5a^2x \\ a^3 + 14ax^2 - x^3 \\ \hline 5a^3 - 2x^3 \end{array}$$

24. $$\begin{array}{l} -8a^2m + 6am^2 - m^3 \\ + a^3 - 5am^2 + m^3 \\ -4a^3 + 4a^2m - 3am^2 \\ +7a^2m - 4am^2 - 6 \\ \hline -3a^3 + 3a^2m - 6am^2 - 6 \end{array}$$

25. $$\begin{array}{l} x^5 - x^3y^2 - xy^4 \\ +2x^4y + 3x^2y^3 - y^5 \\ +3x^3y^2 - 4xy^4 - y^5 \\ x^5 + 5xy^4 + 2y^5 \\ \hline 2x^5 + 2x^4y + 2x^3y^2 + 3x^2y^3 \end{array}$$

26. $$\begin{array}{l} a^5 + a^6 + a^2 \\ +a^4 + a^3 + 6 \\ +3a^2 - 8 + 5a \\ -a^5 - 4a^2 + 6 - 5a \\ \hline +a^6 + a^4 + a^3 + 4 \end{array}$$

27. $$\begin{array}{l} a^4 - b^4 \\ - a^3b + a^2b^2 - ab^3 \\ -3a^4 + 5a^3b - 4a^2b^2 \\ -4a^3b + 3a^2b^2 - 3b^4 \\ \hline -2a^4 - ab^3 - 4b^4 \end{array}$$

28. $$\begin{array}{l} m^3 - n^3 + 6m^2n \\ +n^3 - 4m^2n + 5mn^2 \\ m^3 - n^3 + 6mn^2 \\ -2m^3 + n^3 - 2m^2n \\ \hline 11mn^2 \end{array}$$

29. $$\begin{array}{l} a^x - 3a^{x-2} \\ +5a^{x-1} + 6a^{x-3} \\ + 7a^{x-3} + a^{x-4} \\ + a^{x-1} - 13a^{x-3} \\ \hline a^x + 6a^{x-1} - 3a^{x-2} + a^{x-4} \end{array}$$

30. $$\begin{array}{l} -a^x + a^{x+1} + a^{x+2} \\ -3a^{x+3} - a^{x-1} + a^{x-2} \\ -a^x + 4a^{x+3} - 5a^{x+2} \\ +a^{x-1} - a^{x-2} + a^{x+2} \\ \hline -2a^x + a^{x+1} + a^{x+3} - 3a^{x+2} \end{array}$$

$\Rightarrow a^{x+3} - 3a^{x+2} + a^{x+1} - 2a^x$

EJERCICIO 18

1.
$$\frac{1}{2}x^2+\frac{1}{3}xy$$
$$+\frac{1}{2}xy+\frac{1}{4}y^2$$
$$\frac{1}{2}x^2+\frac{2+3}{6}xy+\frac{1}{4}y^2=\frac{1}{2}x^2+\frac{5}{6}xy+\frac{1}{4}y^2$$

2.
$$a^2+\frac{1}{2}ab$$
$$-\frac{1}{4}ab+\frac{1}{2}b^2$$
$$-\frac{1}{4}ab-\frac{1}{5}b^2$$
$$a^2+\frac{2-1-1}{4}ab+\frac{5-2}{10}b^2=a^2+\frac{3}{10}b^2$$

3.
$$x^2+\frac{2}{3}xy$$
$$-\frac{1}{6}xy+y^2$$
$$-\frac{5}{6}xy+\frac{2}{3}y^2$$
$$x^2+\frac{4-1-5}{6}xy+\frac{3+2}{3}y^2$$
$$=x^2-\frac{2}{6}xy+\frac{5}{3}y^2=x^2-\frac{1}{3}xy+\frac{5}{3}y^2$$

4.
$$\frac{3}{4}x^2-\frac{1}{2}y^2$$
$$-\frac{2}{5}xy+\frac{1}{6}y^2$$
$$+\frac{1}{10}xy+\frac{1}{3}y^2$$
$$\frac{3}{4}x^2+\frac{-4+1}{10}xy+\frac{-3+1+2}{6}y^2=\frac{3}{4}x^2-\frac{3}{10}xy$$

5.
$$\frac{2}{3}a^2+\frac{1}{5}ab-\frac{1}{2}b^2$$
$$\frac{5}{6}a^2-\frac{1}{10}ab+\frac{1}{6}b^2$$
$$-\frac{1}{12}a^2+\frac{1}{20}ab-\frac{1}{3}b^2$$
$$\frac{8+10-1}{12}a^2+\frac{4-2+1}{20}ab+\frac{-3+1-2}{6}b^2$$
$$=\frac{17}{12}a^2+\frac{3}{20}ab-\frac{4}{6}b^2=\frac{17}{12}a^2+\frac{3}{20}ab-\frac{2}{3}b^2$$

6.
$$\frac{5}{6}x^2-\frac{2}{3}y^2+\frac{3}{4}xy$$
$$-\frac{1}{6}x^2+\frac{1}{8}y^2-\frac{1}{2}xy$$
$$-\frac{1}{3}x^2+\frac{1}{4}y^2+\frac{5}{6}xy$$
$$\frac{5-1-2}{6}x^2+\frac{-16+3+6}{24}y^2+\frac{9-6+10}{12}xy$$
$$=\frac{2}{6}x^2-\frac{7}{24}y^2+\frac{13}{12}xy\Rightarrow\frac{1}{3}x^2+\frac{13}{12}xy-\frac{7}{24}y^2$$

7.
$$a^3-\frac{1}{2}ab^2+b^3$$
$$-\frac{3}{8}ab^2-2b^3+\frac{5}{6}a^2b$$
$$\frac{1}{4}a^3-\frac{3}{5}b^3-\frac{1}{2}a^2b$$
$$\frac{4+1}{4}a^3+\frac{-4-3}{8}ab^2+\frac{5-10-3}{5}b^3+\frac{5-3}{6}a^2b$$
$$=\frac{5}{4}a^3-\frac{7}{8}ab^2-\frac{8}{5}b^3+\frac{2}{6}a^2b\Rightarrow\frac{5}{4}a^3+\frac{1}{3}a^2b-\frac{7}{8}ab^2-\frac{8}{5}b^3$$

8.
$$x^4-x^2+5$$
$$-3+\frac{2}{3}x^3-\frac{3}{8}x$$
$$-\frac{3}{5}x^4+\frac{5}{6}x^3-\frac{3}{4}x$$
$$\frac{5-3}{5}x^4-x^2+2+\frac{4+5}{6}x^3+\frac{-3-6}{8}x$$
$$=\frac{2}{5}x^4-x^2+2+\frac{9}{6}x^3-\frac{9}{8}x\Rightarrow\frac{2}{5}x^4+\frac{3}{2}x^3-x^2-\frac{9}{8}x+2$$

9.
$$\frac{2}{3}m^3-\frac{1}{4}mn^2+\frac{2}{5}n^3$$
$$+\frac{1}{8}mn^2-\frac{3}{5}n^3+\frac{1}{6}m^2n$$
$$m^3-n^3-\frac{1}{2}m^2n$$
$$\frac{2+3}{3}m^3+\frac{-2+1}{8}mn^2+\frac{2-3-5}{5}n^3+\frac{1-3}{6}m^2n$$
$$=\frac{5}{3}m^3-\frac{1}{8}mn^2-\frac{6}{5}n^3-\frac{2}{6}m^2n\Rightarrow\frac{5}{3}m^3-\frac{1}{3}m^2n-\frac{1}{8}mn^2-\frac{6}{5}n^3$$

10.
$$x^4+2x^2y^2+\frac{2}{7}y^4$$
$$-\frac{5}{6}x^4+\frac{3}{8}x^2y^2-\frac{1}{14}y^4-\frac{1}{6}xy^3$$
$$-\frac{1}{4}x^2y^2+\frac{1}{7}y^4-\frac{5}{6}x^3y$$
$$\frac{6-5}{6}x^4+\frac{16+3-2}{8}x^2y^2+\frac{4-1+2}{14}y^4-\frac{1}{6}xy^3-\frac{5}{6}x^3y$$
$$=\frac{1}{6}x^4+\frac{17}{8}x^2y^2+\frac{5}{14}y^4-\frac{1}{6}xy^3-\frac{5}{6}x^3y$$
$$\Rightarrow\frac{1}{6}x^4-\frac{5}{6}x^3y+\frac{17}{8}x^2y^2-\frac{1}{6}xy^3+\frac{5}{14}y^4$$

11.

$$\begin{array}{l} x^5 - \frac{2}{3}x^3 + \frac{4}{5}x \\ -3x^5 - \frac{1}{10}x + \frac{3}{8}x^2 \\ + \frac{1}{6}x^3 - \frac{1}{4}x^2 - \frac{2}{3}x^4 \\ - \frac{1}{12}x^3 + \frac{3}{5}x - 4 \\ \hline -2x^5 + \frac{-8+2-1}{12}x^3 + \frac{8-1+6}{10}x + \frac{3-2}{8}x^2 - \frac{2}{3}x^4 - 4 \end{array}$$

$$= -2x^5 - \frac{7}{12}x^3 + \frac{13}{10}x + \frac{1}{8}x^2 - \frac{2}{3}x^4 - 4$$

$$\Rightarrow -2x^5 - \frac{2}{3}x^4 - \frac{7}{12}x^3 + \frac{1}{8}x^2 + \frac{13}{10}x - 4$$

13.

$$\begin{array}{l} a^6 - a^4 + a^2 \\ + \frac{3}{5}a^5 - \frac{3}{8}a^3 - \frac{1}{2}a \\ - \frac{3}{7}a^4 - \frac{5}{8}a^2 + 6 \\ - \frac{3}{8}a - 6 \\ \hline a^6 + \frac{-7-3}{7}a^4 + \frac{8-5}{8}a^2 + \frac{3}{5}a^5 - \frac{3}{8}a^3 + \frac{-4-3}{8}a \end{array}$$

$$= a^6 - \frac{10}{7}a^4 + \frac{3}{8}a^2 + \frac{3}{5}a^5 - \frac{3}{8}a^3 - \frac{7}{8}a$$

$$\Rightarrow a^6 + \frac{3}{5}a^5 - \frac{10}{7}a^4 - \frac{3}{8}a^3 + \frac{3}{8}a^2 - \frac{7}{8}a$$

12.

$$\begin{array}{l} \frac{2}{9}a^3 + \frac{5}{6}ax^2 - \frac{1}{3}x^3 \\ - \frac{3}{7}a^2x - \frac{7}{8}ax^2 - \frac{1}{9}x^3 \\ -\frac{2}{3}a^3 + \frac{1}{2}a^2x - \frac{1}{4}ax^2 \\ \hline \frac{2-6}{9}a^3 + \frac{-6+7}{14}a^2x + \frac{20-21-6}{24}ax^2 + \frac{-3-1}{9}x^3 \end{array}$$

$$= -\frac{4}{9}a^3 + \frac{1}{14}a^2x - \frac{7}{24}ax^2 - \frac{4}{9}x^3$$

14.

$$\begin{array}{l} x^5 - y^5 \\ + \frac{1}{10}x^3y^2 - \frac{3}{4}xy^4 - \frac{1}{6}y^5 \\ - \frac{1}{9}y^5 + \frac{3}{5}x^4y - \frac{5}{6}x^2y^3 \\ - \frac{2}{5}x^3y^2 - \frac{1}{3}y^5 + 2x^4y \\ \hline x^5 + \frac{1-4}{10}x^3y^2 - \frac{3}{4}xy^4 + \frac{-18-3-2-6}{18}y^5 + \frac{3+10}{5}x^4y - \frac{5}{6}x^2y^3 \end{array}$$

$$= x^5 - \frac{3}{10}x^3y^2 - \frac{3}{4}xy^4 - \frac{29}{18}y^5 + \frac{13}{5}x^4y - \frac{5}{6}x^2y^3$$

$$\Rightarrow x^5 + \frac{13}{5}x^4y - \frac{3}{10}x^3y^2 - \frac{5}{6}x^2y^3 - \frac{3}{4}xy^4 - \frac{29}{18}y^5$$

EJERCICIO 19

Para los problemas 1 al 14 las literales toman los siguientes valores:
a = 2 b = 3 c = 10 x = 5 y = 4 m = 2/3 n = 1/5

1.

$$\begin{array}{l} 4x - 5y \\ -3x + 6y - 8 \\ -x + y \\ \hline 2y - 8 \end{array}$$

$$\Rightarrow 2 \cdot 4 - 8 = 8 - 8 = 0$$

2.

$$\begin{array}{l} x^2 - 5x + 8 \\ -x^2 + 10x - 30 \\ -6x^2 + 5x - 50 \\ \hline -6x^2 + 10x - 72 \end{array}$$

$$\Rightarrow -6 \cdot 5^2 + 10 \cdot 5 - 72 = -6 \cdot 25 + 50 - 72$$

$$= -150 + 50 - 72 = -172$$

3.

$$\begin{array}{l} x^4 - y^4 \\ 2x^4 - 5x^2y^2 - 8 \\ -4x^4 + 7x^3y + 10xy^3 \\ \hline -x^4 - y^4 - 5x^2y^2 - 8 + 7x^3y + 10xy^3 \end{array}$$

$$\Rightarrow -x^4 + 7x^3y - 5x^2y^2 + 10xy^3 - y^4 - 8$$

$$= -5^4 + 7 \cdot 5^3 \cdot 4 - 5 \cdot 5^2 \cdot 4^2 + 10 \cdot 5 \cdot 4^3 - 4^4 - 8$$

$$= -625 + 28 \cdot 125 - 5 \cdot 25 \cdot 16 + 50 \cdot 64 - 256 - 8$$

$$= -625 + 3.500 - 2.000 + 3.200 - 256 - 8 = 3.811$$

4.

$$\begin{array}{l} 3m - 5n + 6 \\ -6m - 20n + 8 \\ 12m - 20n - 12 \\ \hline 9m - 45n + 2 \end{array}$$

$$\Rightarrow 9 \cdot \frac{2}{3} - 45 \cdot \frac{1}{5} + 2 = \frac{18}{3} - \frac{45}{5} + 2 = 6 - 9 + 2 = -1$$

5.

$$\begin{array}{l} nx + cn - ab \\ 8nx - 2cn - ab \\ nx \qquad - ab - 5 \\ \hline 10nx - cn - 3ab - 5 \end{array}$$

$$\Rightarrow 10\cdot\frac{1}{5}\cdot 5 - 10\cdot\frac{1}{5} - 3\cdot 2\cdot 3 - 5$$

$$= \frac{50}{5} - \frac{10}{5} - 6\cdot 3 - 5$$

$$= 10 - 2 - 18 - 5 = -15$$

6.

$$\begin{array}{l} a^3 \qquad\qquad\qquad + b^3 \\ \quad -3a^2b + 8ab^2 - b^3 \\ -5a^3 \qquad -6ab^2 \qquad +8 \\ \quad +3a^2b \qquad -2b^3 \\ \hline -4a^3 \qquad +2ab^2 - 2b^3 + 8 \end{array}$$

$$\Rightarrow -4\cdot 2^3 + 2\cdot 2\cdot 3^2 - 2\cdot 3^3 + 8$$

$$= -4\cdot 8 + 4\cdot 9 - 2\cdot 27 + 8$$

$$= -32 + 36 - 54 + 8 = -42$$

7.

$$\begin{array}{l} 27m^3 \qquad\qquad\qquad + 125n^3 \\ \quad - 9m^2n + 25mn^2 \\ \qquad\qquad -14mn^2 \qquad\qquad -8 \\ \quad +10m^2n + 11mn^2 \\ \hline 27m^3 + m^2n + 22mn^2 + 125n^3 - 8 \end{array}$$

$$\Rightarrow 27\cdot\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2\cdot\frac{1}{5} + 22\cdot\frac{2}{3}\cdot\left(\frac{1}{5}\right)^2 + 125\cdot\left(\frac{1}{5}\right)^3 - 8$$

$$= 27\cdot\frac{8}{27} + \frac{4}{9}\cdot\frac{1}{5} + \frac{44}{3}\cdot\frac{1}{25} + 125\cdot\frac{1}{125} - 8$$

$$= 8 + \frac{4}{45} + \frac{44}{75} + 1 - 8 = \frac{4}{45} + \frac{44}{75} + 1$$

$$= \frac{20 + 132 + 225}{225} = \frac{377}{225} = 1\tfrac{152}{225}$$

8.

$$\begin{array}{l} x^{a-1} + y^{b-2} + m^{x-4} \\ 2x^{a-1} - 2y^{b-2} - 2m^{x-4} \\ \qquad + 3y^{b-2} - 2m^{x-4} \\ \hline 3x^{a-1} + 2y^{b-2} - 3m^{x-4} \end{array}$$

$$\Rightarrow 3\cdot 5^{2-1} + 2\cdot 4^{3-2} - 3\left(\frac{2}{3}\right)^{5-4} = 15 + 8 - 2 = 21$$

9.

$$\begin{array}{l} n^{b-1} - m^{x-3} + 8 \\ -5n^{b-1} - 3m^{x-3} + 10 \\ 4n^{b-1} + 5m^{x-3} - 18 \\ \hline \qquad m^{x-3} \end{array}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{5-3} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

10.

$$\begin{array}{l} x^3y - xy^3 + 5 \\ 5x^3y \qquad -6 + x^4 - x^2y^2 \\ \quad -6xy^3 + 2 \qquad + x^2y^2 \\ \quad +3xy^3 + 1 \qquad\qquad - y^4 \\ \hline 6x^3y - 4xy^3 + 2 + x^4 \qquad - y^4 \end{array}$$

$$\Rightarrow x^4 + 6x^3y - 4xy^3 - y^4 + 2$$

$$= 5^4 + 6\cdot 5^3\cdot 4 - 4\cdot 5\cdot 4^3 - 4^4 + 2$$

$$= 625 + 24\cdot 125 - 20\cdot 64 - 256 + 2$$

$$= 625 + 3.000 - 1280 - 256 + 2 = 2.091$$

11.

$$\begin{array}{l} \frac{3}{4}a^2 \qquad\qquad + \frac{2}{3}b^2 \\ \qquad - \frac{1}{3}ab + \frac{1}{9}b^2 \\ \qquad + \frac{1}{6}ab - \frac{1}{3}b^2 \\ \hline \frac{3}{4}a^2 - \frac{-2+1}{6}ab + \frac{6+1-3}{9}b^2 \end{array}$$

$$= \frac{3}{4}a^2 - \frac{1}{6}ab + \frac{4}{9}b^2$$

$$\Rightarrow \frac{3}{4}\cdot 2^2 - \frac{1}{6}\cdot 2\cdot 3 + \frac{4}{9}\cdot 3^2$$

$$= \frac{3}{4}\cdot 4 - \frac{1}{6}\cdot 6 + \frac{4}{9}\cdot 9 = 3 - 1 + 4 = 6$$

12.

$$\begin{array}{lllll} \frac{9}{17}m^2 & + \frac{25}{34}n^2 & - & \frac{1}{4} & \\ \frac{7}{34}m^2 & + \frac{5}{17}n^2 & + & \frac{1}{2} & -15mn \\ -\frac{7}{34}m^2 & & - & \frac{1}{4} & -30mn \\ & & + & 3 & \\ \hline \multicolumn{5}{l}{\frac{18+7-7}{34}m^2 + \frac{25+10}{34}n^2 + \frac{-1+2-1+12}{4} - 45mn} \end{array}$$

$$= \frac{18}{34}m^2 + \frac{35}{34}n^2 + \frac{12}{4} - 45mn$$

$$\Rightarrow \frac{9}{17}m^2 - 45mn + \frac{35}{34}n^2 + 3$$

$$= \frac{9}{17}\left(\frac{2}{3}\right)^2 - 45\cdot\frac{2}{3}\cdot\frac{1}{5} + \frac{35}{34}\left(\frac{1}{5}\right)^2 + 3$$

$$= \frac{9}{17}\cdot\frac{4}{9} - \frac{90}{15} + \frac{35}{34}\cdot\frac{1}{25} + 3$$

$$= \frac{4}{17} - 6 + \frac{35}{850} + 3 = \frac{4}{17} + \frac{7}{170} - 3$$

$$= \frac{40 + 7 - 510}{170} = -\frac{463}{170} = -2\tfrac{123}{170}$$

13.

$$\begin{array}{rrr}
\frac{1}{2}b^2m & -\frac{3}{5}cn & -2 \\
\frac{3}{4}b^2m & -\frac{1}{10}cn & +6 \\
-\frac{1}{4}b^2m & +\frac{1}{25}cn & +4 \\
-\frac{1}{8}b^2m & +2cn & +\frac{3}{5} \\
\hline
\frac{4+6-2-1}{8}b^2m & +\frac{-30-5+2+100}{50}cn & +\frac{-10+30+20+3}{5}
\end{array}$$

$$\Rightarrow \frac{7}{8}b^2m+\frac{67}{50}cn+\frac{43}{5}=\frac{7}{8}\cdot 3^2\cdot\frac{2}{3}+\frac{67}{50}\cdot 10\cdot\frac{1}{5}+\frac{43}{5}$$

$$=\frac{7\cdot 9\cdot 2}{24}+\frac{67}{50}+\frac{43}{5}=\frac{126}{24}+\frac{670}{250}+\frac{43}{5}=\frac{21}{4}+\frac{67}{25}+\frac{43}{5}$$

$$=\frac{525+268+860}{100}=\frac{1.653}{100}=16\tfrac{53}{100}$$

14.

$$\begin{array}{rrrr}
0{,}2a^3 & +0{,}4ab^2 & -0{,}5a^2b & \\
 & +0{,}6ab^2 & -0{,}3a^2b & -0{,}8b^3 \\
-0{,}4a^3 & & -0{,}8a^2b & +6 \\
0{,}2a^3 & & +1{,}5a^2b+0{,}9b^3 & \\
\hline
 & ab^2 & -0{,}1a^2b+0{,}1b^3 & +6
\end{array}$$

$$\Rightarrow -0{,}1a^2b+ab^2+0{,}1b^3+6$$

$$=-0{,}1\cdot 2^2\cdot 3+2\cdot 3^2+0{,}1\cdot 3^3+6$$

$$=-0{,}3\cdot 4+2\cdot 9+0{,}1\cdot 27+6$$

$$=-1{,}2+18+2{,}7+6$$

$$=25{,}5$$

EJERCICIO 20

1. $-8-5=-13$

2. $-7-4=-11$

3. $8-11=-3$

4. $-8-(-11)=-8+11=3$

5. $-1-(-9)=-1+9=8$

6. $2a-3b=2a-3b$

7. $3b-2=3b-2$

8. $4x-6b=4x-6b$

9. $-5a-6b=-5a-6b$

10. $-8x-(-3)=-8x+3$

11. $-9a^2-5b^2=-9a^2-5b^2$

12. $-7xy-(-5yz)=-7xy+5yz$

13. $3a-4a=-a$

14. $11m^2-25m^2=-14m^2$

15. $-6x^2y-(-x^2y)=-6x^2y+x^2y=-5x^2y$

16. $11a^3m^2-(-7a^3m^2)=11a^3m^2+7a^3m^2=18a^3m^2$

17. $-8ab^2-(-8ab^2)=-8ab^2+8ab^2=0$

18. $31x^2y-(-46x^2y)=31x^2y+46x^2y=77x^2y$

19. $-84a^2b-(-84a^2b)=-84a^2b+84a^2b=0$

20. $3a^{x+1}-5b^{x+2}=3a^{x+1}-5b^{x+2}$

21. $-8x^{a+2}-11=-8x^{a+2}-11$

22. $6a^n-(-5a^n)=6a^n+5a^n=11a^n$

23. $-45a^{x-1}-(-60a^{x-1})=-45a^{x-1}+60a^{x-1}=15a^{x-1}$

24. $54b^{n-1}-(-86b^{n-1})=54b^{n-1}+86b^{n-1}=140b^{n-1}$

25. $-35m^a-(-60m^a)=-35m^a+60m^a=25m^a$

26. $5-\left(-\frac{1}{2}\right)=5+\frac{1}{2}=\frac{10+1}{2}=\frac{11}{2}=5\tfrac{1}{2}$

27. $-\frac{2}{3}-\frac{3}{4}=\frac{-8-9}{12}=-\frac{17}{12}$

28. $\frac{1}{3}x^2-\left(-\frac{2}{3}x^2\right)=\frac{1}{3}x^2+\frac{2}{3}x^2=\frac{3}{3}x^2=x^2$

29. $\frac{4}{5}x^3y-\left(-\frac{5}{6}x^3y\right)=\frac{24+25}{30}x^3y=\frac{49}{30}x^3y$

30. $-\frac{1}{8}ab^2-\left(-\frac{3}{4}ab^2\right)=\frac{-1+6}{8}ab^2=\frac{5}{8}ab^2$

31. $-2-3=-5$

32. $7-(-1)=7+1=8$

33. $-8-(-5)=-8+5=-3$

34. $5-(-4)=5+4=9$

35. $-7-(-7)=-7+7=0$

36. $2a-(-5)=2a+5$

37. $-3x-b=-3x-b$

38. $-2n-5m=-2n-5m$

39. $3b-\left(-6a\right)=6a+3b$

40. $8b-\left(-5a^{3}\right)=5a^{3}+8b$

41. $-7a-\left(-9\right)=-7a+9$

42. $25ab-\left(-25\right)=25ab+25$

43. $3a-\left(-a\right)=3a+a=4a$

44. $-4b-\left(-3b\right)=-4b+3b=-b$

45. $54x^{3}-\left(-11x^{3}\right)=54x^{3}+11x^{3}=65x^{3}$

46. $78a^{2}b-14a^{2}b=64a^{2}b$

47. $-54a^{2}y-\left(-43a^{2}y\right)=-54a^{2}y+43a^{2}y=-11a^{2}y$

48. $-ab-9ab=-10ab$

49. $-31x^{2}y-\left(-31x^{2}y\right)=-31x^{2}y+31x^{2}y=0$

50. $-3a^{x}-a^{x}=-4a^{x}$

51. $311a^{x+1}-\left(-7a^{x+1}\right)=311a^{x+1}+7a^{x+1}=318a^{x+1}$

52. $105m^{x}-9m^{x}=96m^{x}$

53. $-31a^{x-1}-18a^{x-1}=-49a^{x-1}$

54. $-236m^{a}-\left(-19m^{a}\right)=-236m^{a}+19m^{a}=-217m^{a}$

55. $-85a^{x+2}-54a^{x+2}=-139a^{x+2}$

56. $\frac{1}{4}-\left(-6a\right)=\frac{1}{4}+6a$

57. $-\frac{2}{3}-(-5)=\frac{-2+15}{3}=\frac{13}{3}$

58. $-\frac{7}{10}m^{3}-\frac{3}{8}m^{3}$

$=\frac{-56-30}{80}m^{3}=-\frac{86}{80}m^{3}=-\frac{43}{40}m^{3}$

59. $\frac{5}{6}a^{2}b^{2}-\left(-\frac{11}{12}a^{2}b^{2}\right)$

$=\frac{10+11}{12}a^{2}b^{2}=\frac{21}{12}a^{2}b^{2}=\frac{7}{4}a^{2}b^{2}$

60. $-\frac{1}{9}a^{3}b^{2}-45a^{3}b^{2}$

$=\frac{-1-405}{9}a^{3}b^{2}=\frac{-406}{9}a^{3}b^{2}=-45\tfrac{1}{9}a^{3}b^{2}$

EJERCICIO 21

1.
$$\begin{array}{r} a+b \\ -a+b \\ \hline 2b \end{array}$$

2.
$$\begin{array}{l} 2x-3y \\ x-2y \\ \hline 3x-5y \end{array}$$

3.
$$\begin{array}{l} 8a+b \\ 3a-4 \\ \hline 11a+b-4 \end{array}$$

4.
$$\begin{array}{r} x^{2}-3x \\ 5x-6 \\ \hline x^{2}+2x-6 \end{array}$$

5.
$$\begin{array}{r} a^{3}-a^{2}b \\ -7a^{2}b-9ab^{2} \\ \hline a^{3}-8a^{2}b-9ab^{2} \end{array}$$

6.
$$\begin{array}{c} x-y+z \\ -x+y-z \\ \hline 0 \end{array}$$

7.
$$\begin{array}{l} x+y-z \\ x+y-z \\ \hline 2x+2y-2z \end{array}$$

8.
$$\begin{array}{l} x^{2}+y^{2}-3xy \\ -3x^{2}+y^{2}+4xy \\ \hline -2x^{2}+2y^{2}+xy \end{array}$$
$\Rightarrow -2x^{2}+xy+2y^{2}$

9.
$$\begin{array}{r} x^{3}-x^{2}+6 \\ -5x^{2}-6+4x \\ \hline x^{3}-6x^{2}+4x \end{array}$$

10.
$$\begin{array}{l} y^{2}+6y^{3}-8 \\ 3y^{2}-2y^{4}-6y \\ \hline 4y^{2}-2y^{4}+6y^{3}-6y-8 \end{array}$$
$\Rightarrow -2y^{4}+6y^{3}+4y^{2}-6y-8$

11.
$$\begin{array}{r} a^{3}-6ab^{2}+9a \\ +8a-15a^{2}b-5 \\ \hline a^{3}-6ab^{2}+17a-15a^{2}b-5 \end{array}$$
$\Rightarrow a^{3}-15a^{2}b-6ab^{2}+17a-5$

12.
$$\begin{array}{r} x^{4}+9xy^{3}-11y^{4} \\ -20y^{4}+8x^{3}y+6x^{2}y^{2} \\ \hline x^{4}+9xy^{3}-31y^{4}+8x^{3}y+6x^{2}y^{2} \end{array}$$
$\Rightarrow x^{4}+8x^{3}y+6x^{2}y^{2}+9xy^{3}-31y^{4}$

13.
$$\begin{array}{l} a+b+c-d \\ a+b-c+d \\ \hline 2a+2b \end{array}$$

14.
$$\begin{array}{r} ab+2ac-3cd-5de \\ -8ab+4ac+5cd-5de \\ \hline -7ab+6ac+2cd-10de \end{array}$$

15.
$$\begin{array}{r} x^{3}+6x^{2}-9x-19 \\ -6x^{3}+11x^{2}-21x+43 \\ \hline -5x^{3}+17x^{2}-30x+24 \end{array}$$

16. $$\begin{array}{l} y^5 - 9y^3 + 6y^2 \qquad -31 \\ \quad -31y^3 + 8y^2 + 11y^4 + 19y \\ \hline y^5 - 40y^3 + 14y^2 + 11y^4 + 19y - 31 \end{array} \Rightarrow y^5 + 11y^4 - 40y^3 + 14y^2 + 19y - 31$$

17. $$\begin{array}{l} \quad 5m^3 - 9n^3 + 6m^2n - 8mn^2 \\ -5m^3 \qquad + 21m^2n - 14mn^2 + 18 \\ \hline \qquad -9n^3 + 27m^2n - 22mn^2 + 18 \end{array} \Rightarrow 27m^2n - 22mn^2 - 9n^3 + 18$$

18. $$\begin{array}{l} 4x^3y - 19xy^3 + y^4 - 6x^2y^2 \\ 25x^3y + 51xy^3 \qquad - 32x^2y^2 + x^4 \\ \hline 29x^3y + 32xy^3 + y^4 - 38x^2y^2 + x^4 \end{array} \Rightarrow x^4 + 29x^3y - 38x^2y^2 + 32xy^3 + y^4$$

19. $$\begin{array}{l} m^6 + m^4n^2 - 9m^2n^4 \quad + 19 \\ \qquad\qquad + 30m^2n^4 + 61 + 13m^3n^3 - 16mn^5 \\ \hline m^6 + m^4n^2 + 21m^2n^4 + 80 + 13m^3n^3 - 16mn^5 \end{array} \Rightarrow m^6 + m^4n^2 + 13m^3n^3 + 21m^2n^4 - 16mn^5 + 80$$

20. $$\begin{array}{l} \quad -a^5b \qquad + 6a^3b^3 \qquad - 18ab^5 \qquad + 42 \\ 8a^6 \qquad + 11a^4b^2 \qquad + 11a^2b^4 \qquad - 9b^6 \\ \hline 8a^6 - a^5b + 11a^4b^2 + 6a^3b^3 + 11a^2b^4 - 18ab^5 - 9b^6 + 42 \end{array}$$

21. $$\begin{array}{l} 1 - x^2 + x^4 - x^3 + 3x - 6x^5 \\ 24 + 30x^2 - 8x^4 \qquad - 15x \qquad + x^6 \\ \hline 25 + 29x^2 - 7x^4 - x^3 - 12x - 6x^5 + x^6 \end{array} \Rightarrow x^6 - 6x^5 - 7x^4 - x^3 + 29x^2 - 12x + 25$$

22. $$\begin{array}{l} -6x^2y^3 + 8x^5 - 23x^4y + 80x^3y^2 - 18 \\ \qquad\qquad + 51x^4y + 21x^3y^2 - 80 + y^5 - 9xy^4 \\ \hline -6x^2y^3 + 8x^5 + 28x^4y + 101x^3y^2 - 98 + y^5 - 9xy^4 \end{array} \Rightarrow 8x^5 + 28x^4y + 101x^3y^2 - 6x^2y^3 - 9xy^4 + y^5 - 98$$

23. $$\begin{array}{l} m^6 \qquad - 8m^4n^2 \qquad + 21m^2n^4 - 6mn^5 \qquad + 8 \\ \quad 23m^5n \qquad - 14m^3n^3 \qquad + 24mn^5 - 8n^6 + 14 \\ \hline m^6 + 23m^5n - 8m^4n^2 - 14m^3n^3 + 21m^2n^4 + 18mn^5 - 8n^6 + 22 \end{array}$$

24. $$\begin{array}{l} x^7 - 8x + 16x^5 - 23x^2 - 15 \\ \quad - 51x \qquad\qquad + 18 + 8x^6 - 25x^4 + 30x^3 \\ \hline x^7 - 59x + 16x^5 - 23x^2 + 3 + 8x^6 - 25x^4 + 30x^3 \end{array} \Rightarrow x^7 + 8x^6 + 16x^5 - 25x^4 + 30x^3 - 23x^2 - 59x + 3$$

25. $$\begin{array}{l} 9a^6 - 15a^4b^2 + 31a^2b^4 - b^6 \qquad\qquad\qquad + 14 \\ \quad + 15a^4b^2 \qquad - 3b^6 - 25a^5b - 53a^3b^3 + 9ab^5 \\ \hline 9a^6 \qquad + 31a^2b^4 - 4b^6 - 25a^5b - 53a^3b^3 + 9ab^5 + 14 \end{array} \Rightarrow 9a^6 - 25a^5b - 53a^3b^3 + 31a^2b^4 + 9ab^5 - 4b^6 + 14$$

26. $$\begin{array}{l} \quad a^x + a^{x+1} - a^{x+2} \\ -5a^x + 6a^{x+1} + a^{x+2} \\ \hline -4a^x + 7a^{x+1} \end{array}$$

27.
$$\begin{array}{l} m^a - m^{a-1} + 3m^{a-2} \\ 4m^a \qquad - 5m^{a-2} - 3m^{a+1} - 8m^{a-3} \\ \hline 5m^a - m^{a-1} - 2m^{a-2} - 3m^{a+1} - 8m^{a-3} \end{array} \Rightarrow -3m^{a+1} + 5m^a - m^{a-1} - 2m^{a-2} - 8m^{a-3}$$

28.
$$\begin{array}{l} a^{m+4} - 7a^{m+2} - 8a^m + 6a^{m-1} \\ \qquad +14a^{m+2} \qquad + 8a^{m-1} + 5a^{m+3} + 11a^{m+1} \\ \hline a^{m+4} + 7a^{m+2} - 8a^m + 14a^{m-1} + 5a^{m+3} + 11a^{m+1} \end{array} \Rightarrow a^{m+4} + 5a^{m+3} + 7a^{m+2} + 11a^{m+1} - 8a^m + 14a^{m-1}$$

29.
$$\begin{array}{l} x^{a+2} - 7x^a + 9x^{a-1} + 25x^{a-2} \\ \qquad -19x^a - 45x^{a-1} \qquad + 11x^{a+1} - 60x^{a-3} \\ \hline x^{a+2} - 26x^a - 36x^{a-1} + 25x^{a-2} + 11x^{a+1} - 60x^{a-3} \end{array} \Rightarrow x^{a+2} + 11x^{a+1} - 26x^a - 36x^{a-1} + 25x^{a-2} - 60x^{a-3}$$

30.
$$\begin{array}{l} m^{n+1} - 6m^{n-2} + 8m^{n-3} - 19m^{n-5} \\ \qquad - 5m^{n-2} - 6m^{n-3} - 9m^{n-5} - 8m^n - m^{n-4} \\ \hline m^{n+1} - 11m^{n-2} + 2m^{n-3} - 28m^{n-5} - 8m^n - m^{n-4} \end{array} \Rightarrow m^{n+1} - 8m^n - 11m^{n-2} + 2m^{n-3} - m^{n-4} - 28m^{n-5}$$

EJERCICIO 22

1.
$$\begin{array}{l} b - a \\ b - a \\ \hline 2b - 2a \end{array} \Rightarrow -2a + 2b$$

2.
$$\begin{array}{l} 2x + 3y \\ -x + y \\ \hline x + 4y \end{array}$$

3.
$$\begin{array}{l} -7a + 5 \\ 5a \qquad - b \\ \hline -2a + 5 - b \end{array} \Rightarrow -2a - b + 5$$

4.
$$\begin{array}{l} -x^2 + 6 \\ -x^2 \qquad + 5x \\ \hline -2x^2 + 6 + 5x \end{array} \Rightarrow -2x^2 + 5x + 6$$

5.
$$\begin{array}{l} x^2y + 5xy^2 \\ \quad + xy^2 - x^3 \\ \hline x^2y + 6xy^2 - x^3 \end{array} \Rightarrow -x^3 + x^2y + 6xy^2$$

6.
$$\begin{array}{l} 7a^2b + 5ab^2 \\ -6a^2b \qquad + 8a^3 \\ \hline a^2b + 5ab^2 + 8a^3 \end{array} \Rightarrow 8a^3 + a^2b + 5ab^2$$

7.
$$\begin{array}{l} -a + 2b - 3c \\ -a + b - 2c \\ \hline -2a + 3b - 5c \end{array}$$

8.
$$\begin{array}{l} -3n + 4m + 5p \\ +n - m - p \\ \hline -2n + 3m + 4p \end{array} \Rightarrow 3m - 2n + 4p$$

9.
$$\begin{array}{l} x + 3y - 6z \\ x - y + z \\ \hline 2x + 2y - 5z \end{array}$$

10.
$$\begin{array}{l} -5b^2 + 8ab + a^2 \\ 6b^2 - ab - 3a^2 \\ \hline b^2 + 7ab - 2a^2 \end{array} \Rightarrow -2a^2 + 7ab + b^2$$

11.
$$\begin{array}{l} -5m^2 - n^2 + 6mn \\ -m^2 + n^2 + 3mn \\ \hline -6m^2 \qquad + 9mn \end{array}$$

12.
$$\begin{array}{l} -8x^2 + 5x - 4 \\ \qquad + x - 6 + x^3 \\ \hline -8x^2 + 6x - 10 + x^3 \end{array} \Rightarrow x^3 - 8x^2 + 6x - 10$$

13.
$$\begin{array}{l} 14m^2-8n+16 \\ -14m^2 \quad -9-m^3 \\ \hline -8n+7-m^3 \Rightarrow -m^3-8n+7 \end{array}$$

14.
$$\begin{array}{l} 8ab+5bc+6cd \\ -ab+bc-6cd \\ \hline 7ab+6bc \end{array}$$

15.
$$\begin{array}{l} a^3-9a^2b-b^3 \\ -25a^2b+b^3+8ab^2 \\ \hline a^3-34a^2b+8ab^2 \end{array}$$

16.
$$\begin{array}{l} 6x^3-8x^2y-6xy^2 \\ -xy^2+6y^3-4 \\ \hline 6x^3-8x^2y-7xy^2+6y^3-4 \end{array}$$

17.
$$\begin{array}{l} m^2-9n+11c+14 \\ -m^2-7n+8c-d \\ \hline -16n+19c+14-d \Rightarrow 19c-d-16n+14 \end{array}$$

18.
$$\begin{array}{l} 5a^4+9a^3b-40ab^3+6b^4 \\ -7a^3b+8a^2b^2-5ab^3-b^4 \\ \hline 5a^4+2a^3b+8a^2b^2-45ab^3+5b^4 \end{array}$$

19.
$$\begin{array}{l} x^5-8x^4+25x^2+15 \\ -6x^2+7-6x^3+9x \\ \hline x^5-8x^4+19x^2+22-6x^3+9x \\ \Rightarrow x^5-8x^4-6x^3+19x^2+9x+22 \end{array}$$

20.
$$\begin{array}{l} -3xy^4-8x^3y^2-19y^5+18 \\ -6xy^4-25y^5-x^5+x^2y^3 \\ \hline -9xy^4-8x^3y^2-44y^5+18-x^5+x^2y^3 \\ \Rightarrow -x^5-8x^3y^2+x^2y^3-9xy^4-44y^5+18 \end{array}$$

21.
$$\begin{array}{l} x^3-6x^4+8x^2-9+15x \\ -25x^3+18x^2+46-25x+11x^5 \\ \hline -24x^3-6x^4+26x^2+37-10x+11x^5 \\ \Rightarrow 11x^5-6x^4-24x^3+26x^2-10x+37 \end{array}$$

22.
$$\begin{array}{l} a^5-26a^3b^2+8ab^4-b^5+6 \\ -8a^4b-a^3b^2+15a^2b^3+45ab^4+8 \\ \hline a^5-8a^4b-27a^3b^2+15a^2b^3+53ab^4-b^5+14 \end{array}$$

23.
$$\begin{array}{l} y^6+y^3+y^2+9 \\ -23y^3+5-8y^4+15y^5+8y \\ \hline y^6-22y^3+y^2+14-8y^4+15y^5+8y \\ \Rightarrow y^6+15y^5-8y^4-22y^3+y^2+8y+14 \end{array}$$

24.
$$\begin{array}{l} x^8-x^6+3x^4-5x^2-9 \\ -36-7x^7-5x^5+23x^3-51x \\ \hline x^8-x^6+3x^4-5x^2-45-7x^7-5x^5+23x^3-51x \\ \Rightarrow x^8-7x^7-x^6-5x^5+3x^4+23x^3-5x^2-51x-45 \end{array}$$

25.
$$\begin{array}{l} x^7-3x^5y^2+35x^4y^3-8x^2y^5+60 \\ +60x^4y^3+x^2y^5-y^7-90x^3y^4+50xy^6 \\ \hline x^7-3x^5y^2+95x^4y^3-7x^2y^5+60-y^7-90x^3y^4+50xy^6 \\ \Rightarrow x^7-3x^5y^2+95x^4y^3-90x^3y^4-7x^2y^5+50xy^6-y^7+60 \end{array}$$

26.
$$\begin{array}{l} a^{x+3}-8a^{x+1}-5 \\ +5a^{x+1}-a^{x+2}+6a^x \\ \hline a^{x+3}-3a^{x+1}-5-a^{x+2}+6a^x \\ \Rightarrow a^{x+3}-a^{x+2}-3a^{x+1}+6a^x-5 \end{array}$$

27.
$$\begin{array}{l} -8a^n+16a^{n-4}+15a^{n-2}+a^{n-3} \\ -7a^n-5a^{n-2}-a^{n-3}-8a^{n-1} \\ \hline -15a^n+16a^{n-4}+10a^{n-2}-8a^{n-1} \\ \Rightarrow -15a^n-8a^{n-1}+10a^{n-2}+16a^{n-4} \end{array}$$

28.
$$\begin{array}{l} 15x^{a+3}+5x^{a+2}-6x^a+41x^{a-1} \\ +9x^{a+2}+18x^{a-1}-31x^{a+1}+x^{a+4} \\ \hline 15x^{a+3}+14x^{a+2}-6x^a+59x^{a-1}-31x^{a+1}+x^{a+4} \\ \Rightarrow x^{a+4}+15x^{a+3}+14x^{a+2}-31x^{a+1}-6x^a+59x^{a-1} \end{array}$$

29.
$$\begin{array}{l} 9a^{m-1}-21a^{m-2}+26^{m-3}+14a^{m-5} \\ 5a^{m-1}-12a^{m-2}+a^m+8a^{m-4} \\ \hline 14a^{m-1}-33a^{m-2}+26^{m-3}+14a^{m-5}+a^m+8a^{m-4} \\ \Rightarrow a^m+14a^{m-1}-33a^{m-2}+26^{m-3}+8a^{m-4}+14a^{m-5} \end{array}$$

30.
$$\begin{array}{l} -15m^{x+3}+50m^{x+1}-14m^x-6m^{x-1}+8m^{x-2} \\ m^{x+4}+23m^{x+2}+6m^{x+1}+m^{x-1} \\ \hline m^{x+4}-15m^{x+3}+23m^{x+2}+56m^{x+1}-14m^x-5m^{x-1}+8m^{x-2} \end{array}$$

EJERCICIO 23

1.
$$\begin{array}{l} \quad 1 \\ -a+1 \\ \hline -a+2 \end{array}$$

2.
$$\begin{array}{l} \quad 0 \\ -a+8 \\ \hline -a+8 \end{array}$$

3.
$$\begin{array}{l} \qquad -9 \\ -3a-a^2+5 \\ \hline -3a-a^2-4 \Rightarrow -a^2-3a-4 \end{array}$$

4.
$$\begin{array}{l} \qquad\quad +16 \\ -5xy+x^2-16 \\ \hline -5xy+x^2 \qquad \Rightarrow x^2-5xy \end{array}$$

5.
$$\begin{array}{l} 1 \\ \quad -a^3+a^2b-ab^2 \\ \hline 1-a^3+a^2b-ab^2 \Rightarrow -a^3+a^2b-ab^2+1 \end{array}$$

6.
$$\begin{array}{l} x^3 \\ x^3+8x^2y+6xy^2 \\ \hline 2x^3+8x^2y+6xy^2 \end{array}$$

7.
$$\begin{array}{l} a^3 \\ \quad +8a^2b-6ab^2+b^3 \\ \hline a^3+8a^2b-6ab^2+b^3 \end{array}$$

8.
$$\begin{array}{l} \qquad\qquad\qquad\qquad +y^4 \\ 5x^3y-7x^2y^2+8xy^3 \\ \hline 5x^3y-7x^2y^2+8xy^3+y^4 \end{array}$$

9.
$$\begin{array}{l} m^4 \\ -5m^4+18am^3-7a^2m^2-a^3m+a^4 \\ \hline -4m^4+18am^3-7a^2m^2-a^3m+a^4 \end{array}$$

10.
$$\begin{array}{l} 16 \\ 14-b+a-c-d \\ \hline 30-b+a-c-d \Rightarrow a-b-c-d+30 \end{array}$$

11.
$$\begin{array}{l} x^2-1 \\ \qquad -xy-y^2 \\ \hline x^2-1-xy-y^2 \Rightarrow x^2-xy-y^2-1 \end{array}$$

12.
$$\begin{array}{l} a^3 \qquad\qquad\qquad\qquad +6 \\ \quad -5a^2b+8ab^2-b^3 \\ \hline a^3-5a^2b+8ab^2-b^3+6 \end{array}$$

13.
$$\begin{array}{l} x^3 \qquad\qquad\qquad +y^3 \\ \quad +5x^2y-17xy^2 \qquad +5 \\ \hline x^3+5x^2y-17xy^2+y^3+5 \end{array}$$

14.
$$\begin{array}{l} x^4 \qquad\qquad\qquad\qquad\qquad -1 \\ \quad -9x^3y+8x^2y^2+15xy^3 \\ \hline x^4-9x^3y+8x^2y^2+15xy^3-1 \end{array}$$

15.
$$\begin{array}{l} a^5 \qquad\qquad\qquad\qquad\qquad\qquad +b^5 \\ \quad +11a^4b-2a^2b^3-8a^3b^2+4ab^4 \\ \hline a^5+11a^4b-2a^2b^3-8a^3b^2+4ab^4+b^5 \\ \Rightarrow a^5+11a^4b-8a^3b^2-2a^2b^3+4ab^4+b^5 \end{array}$$

16.
$$\begin{array}{l} x^4 \qquad +x^2 \qquad +50 \\ \quad -5x^3 \qquad +25x \\ \hline x^4-5x^3+x^2+25x+50 \end{array}$$

17.
$$\begin{array}{l} y^6+y-41 \\ \qquad\qquad -9y^5-17y^4+y^3-18y^2 \\ \hline y^6+y-41-9y^5-17y^4+y^3-18y^2 \\ \Rightarrow y^6-9y^5-17y^4+y^3-18y^2+y-41 \end{array}$$

18.
$$\begin{array}{l} a^6 \qquad\quad +9a^4b^2 \qquad\qquad +a^2b^4 \\ \quad +15a^5b \qquad\quad -17a^3b^3 \qquad +14ab^5+b^6 \\ \hline a^6+15a^5b+9a^4b^2-17a^3b^3+a^2b^4+14ab^5+b^6 \end{array}$$

19.
$$\begin{array}{l} x^4+x^3-11x \\ \qquad\quad -5x+x^2+34 \\ \hline x^4+x^3-16x+x^2+34 \Rightarrow x^4+x^3+x^2-16x+34 \end{array}$$

20.
$$\begin{array}{l} m^3 \qquad\qquad\qquad\qquad -1 \\ \quad -m^2n-7mn^2+3n^3 \\ \hline m^3-m^2n-7mn^2+3n^3-1 \end{array}$$

EJERCICIO 24

1.

$$\begin{array}{l} \frac{1}{2}a^2 \\ \frac{1}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2 \\ \hline \frac{2+1}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2 \\ = \frac{3}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2 \end{array}$$

2.

$$\begin{array}{l} \qquad\qquad\qquad 15\frac{5}{9} \\ -\frac{4}{5}xy - \frac{2}{3}yz + \\ \hline -\frac{4}{5}xy - \frac{2}{3}yz + \frac{135+5}{9} \\ = -\frac{4}{5}xy - \frac{2}{3}yz + \frac{140}{9} \\ = -\frac{4}{5}xy - \frac{2}{3}yz + 15\tfrac{5}{9} \end{array}$$

3.

$$\begin{array}{l} \frac{3}{5}bc \\ -\frac{1}{6}bc + \frac{3}{4}ab + \frac{2}{9}cd \\ \hline \frac{18-5}{30}bc + \frac{3}{4}ab + \frac{2}{9}cd \\ \Rightarrow \frac{3}{4}ab + \frac{13}{30}bc + \frac{2}{9}cd \end{array}$$

4.

$$\begin{array}{l} \frac{1}{2}a - \frac{2}{3}b \\ -\frac{4}{5}a - \frac{2}{9}b + \frac{1}{2} \\ \hline \frac{5-8}{10}a - \frac{6+2}{9}b + \frac{1}{2} \\ = -\frac{3}{10}a - \frac{8}{9}b + \frac{1}{2} \end{array}$$

5.

$$\begin{array}{l} \frac{5}{9}x^2 - \frac{3}{8}y^2 \\ \quad - \frac{1}{10}y^2 - \frac{5}{7}xy + \frac{3}{11} \\ \hline \frac{5}{9}x^2 - \frac{30+8}{80}y^2 - \frac{5}{7}xy + \frac{3}{11} \\ = \frac{5}{9}x^2 - \frac{38}{80}y^2 - \frac{5}{7}xy + \frac{3}{11} \\ \Rightarrow \frac{5}{9}x^2 - \frac{5}{7}xy - \frac{19}{40}y^2 + \frac{3}{11} \end{array}$$

6.

$$\begin{array}{l} \frac{5}{6}m^3 + \frac{2}{9}n^3 \\ \quad + \frac{1}{5}n^3 + \frac{1}{2}m^2n - \frac{3}{8}mn^2 \\ \hline \frac{5}{6}m^3 + \frac{10+9}{45}n^3 + \frac{1}{2}m^2n - \frac{3}{8}mn^2 \\ \Rightarrow \frac{5}{6}m^3 + \frac{1}{2}m^2n - \frac{3}{8}mn^2 + \frac{19}{45}n^3 \end{array}$$

7.

$$\begin{array}{l} \frac{3}{7}a^2 + \frac{1}{3}ab - \frac{3}{5}b^2 \\ -\frac{5}{14}a^2 - \frac{1}{2}ab + \frac{1}{8} \\ \hline \frac{6-5}{14}a^2 + \frac{2-3}{6}ab \\ \frac{1}{14}a^2 - \frac{1}{6}ab - \frac{3}{5}b^2 + \frac{1}{8} \end{array}$$

8.

$$\begin{array}{l} \frac{3}{8}x^2 + \frac{5}{6}xy - \frac{1}{10}y^2 \\ \frac{3}{5}x^2 + \frac{3}{10}xy - 2y^2 \\ \hline \frac{15+24}{40}x^2 + \frac{50+18}{60}xy - \frac{1+20}{10}y^2 \\ = \frac{39}{40}x^2 + \frac{68}{60}xy - \frac{21}{10}y^2 = \frac{39}{40}x^2 + \frac{17}{15}xy - \frac{21}{10}y^2 \end{array}$$

9.

$$\begin{array}{l} a^3 + a^2 - a + \frac{5}{6} \\ \quad + \frac{7}{8}a^2 - \frac{9}{10}a - \frac{7}{8} \\ \hline a^3 + \frac{8+7}{8}a^2 - \frac{10+9}{10}a + \frac{20-21}{24} \\ = a^3 + \frac{15}{8}a^2 - \frac{19}{10}a - \frac{1}{24} \end{array}$$

10 .

$$\begin{array}{l} m^3 + \frac{7}{12}mn^2 - \frac{4}{7}n^3 \\ \quad - \frac{5}{9}mn^2 - n^3 + \frac{5}{21}m^2n + \frac{1}{8} \\ \hline m^3 + \frac{21-20}{36}mn^2 - \frac{4+7}{7}n^3 + \frac{5}{21}m^2n + \frac{1}{8} \\ \Rightarrow m^3 + \frac{5}{21}m^2n + \frac{1}{36}mn^2 - \frac{11}{7}n^3 + \frac{1}{8} \end{array}$$

11.

$$\frac{3}{5}x^4 + \frac{3}{4}x^3y \qquad - \frac{5}{7}xy^3 + \frac{2}{3}y^4$$

$$-x^4 \qquad -\frac{5}{8}x^2y^2 + \frac{1}{3}xy^3 - \frac{5}{6}y^4$$

$$\frac{3-5}{5}x^4 + \frac{3}{4}x^3y - \frac{5}{8}x^2y^2 - \frac{15-7}{21}xy^3 + \frac{4-5}{6}y^4$$

$$= -\frac{2}{5}x^4 + \frac{3}{4}x^3y - \frac{5}{8}x^2y^2 - \frac{8}{21}xy^3 - \frac{1}{6}y^4$$

12.

$$\frac{1}{2}a + \frac{3}{5}b - \frac{7}{8}c + \frac{8}{9}d$$

$$+\frac{7}{20}b - \frac{1}{8}c + \frac{1}{9}d - \frac{7}{8}$$

$$\frac{1}{2}a + \frac{12+7}{20}b - \frac{7+1}{8}c + \frac{8+1}{9}d - \frac{7}{8}$$

$$= \frac{1}{2}a + \frac{19}{20}b - \frac{8}{8}c + \frac{9}{9}d - \frac{7}{8}$$

$$= \frac{1}{2}a + \frac{19}{20}b - c + d - \frac{7}{8}$$

EJERCICIO 25

1.

$$\frac{3}{8}a^2 - \frac{5}{6}a$$

$$-\frac{5}{6}a^2$$

$$\frac{9-20}{24}a^2 - \frac{5}{6}a = -\frac{11}{24}a^2 - \frac{5}{6}a$$

2.

$$8a + 6b - 5$$

$$-\frac{1}{2}a + \frac{3}{5}b$$

$$\frac{16-1}{2}a + \frac{30+3}{5}b - 5 = \frac{15}{2}a + \frac{33}{5}b - 5$$

3.

$$x^3 + \frac{2}{3}x^2y - 6$$

$$-\frac{7}{9}x^2y$$

$$x^3 + \frac{6-7}{9}x^2y - 6 = x^3 - \frac{1}{9}x^2y - 6$$

4.

$$a + b - c$$

$$-\frac{1}{2}a + \frac{3}{4}b - \frac{2}{3}c$$

$$\frac{2-1}{2}a + \frac{4+3}{4}b - \frac{3+2}{3}c = \frac{1}{2}a + \frac{7}{4}b - \frac{5}{3}c$$

5.

$$\frac{2}{3}m + \frac{5}{6}n + \frac{1}{2}p$$

$$-m - n + p$$

$$\frac{2-3}{3}m + \frac{5-6}{6}n + \frac{1+2}{2}p = -\frac{1}{3}m - \frac{1}{6}n + \frac{3}{2}p$$

6.

$$\frac{5}{8}a^2b + \frac{1}{4}ab^2 - \frac{1}{3}$$

$$+\frac{7}{8}ab^2 - 6 \qquad -\frac{5}{6}a^3$$

$$\frac{5}{8}a^2b + \frac{2+7}{8}ab^2 - \frac{1+18}{3} - \frac{5}{6}a^3$$

$$= \frac{5}{8}a^2b + \frac{9}{8}ab^2 - \frac{19}{3} - \frac{5}{6}a^3$$

$$\Rightarrow -\frac{5}{6}a^3 + \frac{5}{8}a^2b + \frac{9}{8}ab^2 - \frac{19}{3}$$

7.

$$\frac{2}{11}m^3n + \frac{5}{14}m^2n^2 + \frac{1}{3}mn^3 - 6$$

$$m^4 \qquad - \frac{7}{8}m^2n^2 + \frac{2}{9}mn^3$$

$$m^4 + \frac{2}{11}m^3n + \frac{20-49}{56}m^2n^2 + \frac{3+2}{9}mn^3 - 6$$

$$= m^4 + \frac{2}{11}m^3n - \frac{29}{56}m^2n^2 + \frac{5}{9}mn^3 - 6$$

8.

$$-\frac{7}{8}x^4y + \frac{1}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{1}{3}xy^4 - 7$$

$$\frac{1}{2}x^5 \qquad - \frac{3}{7}x^3y^2 \qquad + \frac{1}{8}xy^4 - \frac{2}{9}$$

$$\frac{1}{2}x^5 - \frac{7}{8}x^4y + \frac{1-6}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{8+3}{24}xy^4 - \frac{63+2}{9}$$

$$= \frac{1}{2}x^5 - \frac{7}{8}x^4y - \frac{5}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{11}{24}xy^4 - \frac{65}{9}$$

$$= \frac{1}{2}x^5 - \frac{7}{8}x^4y - \frac{5}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{11}{24}xy^4 - 7\tfrac{2}{9}$$

9.

$$\frac{7}{9}x^5y+\frac{2}{3}x^4y^2-\frac{1}{8}x^3y^3-x^2y^4+xy^5+\frac{2}{13}y^6$$

$$-x^6+\frac{7}{9}x^4y^2-\frac{1}{11}x^2y^4-xy^5+y^6$$

$$-x^6+\frac{7}{9}x^5y+\frac{6+7}{9}x^4y^2-\frac{1}{8}x^3y^3-\frac{11+1}{11}x^2y^4+\frac{2+13}{13}y^6=-x^6+\frac{7}{9}x^5y+\frac{13}{9}x^4y^2-\frac{1}{8}x^3y^3-\frac{12}{11}x^2y^4+\frac{15}{13}y^6$$

10.

$$\frac{1}{3}x^3-\frac{7}{9}x^2y+\frac{5}{8}xy^2-\frac{7}{11}y^3-\frac{2}{5}$$

$$\frac{2}{3}x^3+\frac{1}{6}x^2y-\frac{3}{4}xy^2-6$$

$$\frac{1+2}{3}x^3-\frac{14-3}{18}x^2y+\frac{5-6}{8}xy^2-\frac{7}{11}y^3-\frac{2+30}{5}=x^3-\frac{11}{18}x^2y-\frac{1}{8}xy^2-\frac{7}{11}y^3-\frac{32}{5}$$

11.

$$\frac{3}{10}m^4n^2-\frac{3}{7}m^2n^4+\frac{5}{9}n^6$$

$$\frac{2}{13}m^6+\frac{7}{20}m^4n^2-\frac{5}{14}m^2n^4-\frac{1}{3}n^6+\frac{3}{5}$$

$$\frac{2}{13}m^6+\frac{6+7}{20}m^4n^2-\frac{6+5}{14}m^2n^4+\frac{5-3}{9}n^6+\frac{3}{5}=\frac{2}{13}m^6+\frac{13}{20}m^4n^2-\frac{11}{14}m^2n^4+\frac{2}{9}n^6+\frac{3}{5}$$

12.

$$\frac{3}{8}c^5+\frac{7}{22}c^4d+\frac{7}{12}c^3d^2+\frac{1}{2}c^2d^3-\frac{1}{3}d^5-35$$

$$+\frac{5}{11}c^4d+\frac{5}{6}c^3d^2-\frac{3}{4}cd^4-\frac{3}{13}d^5$$

$$\frac{3}{8}c^5+\frac{7+10}{22}c^4d+\frac{7+10}{12}c^3d^2+\frac{1}{2}c^2d^3-\frac{3}{4}cd^4-\frac{13+9}{39}d^5-35$$

$$=\frac{3}{8}c^5+\frac{17}{22}c^4d+\frac{17}{12}c^3d^2+\frac{1}{2}c^2d^3-\frac{3}{4}cd^4-\frac{22}{39}d^5-35$$

EJERCICIO 26

Para los problemas 1 al 14 las literales toman los siguientes valores:
a = 1   b = 2   c = 3   x = 4   y = 5   m = 3/2   n = 2/5

1. a^2-ab

$$-3ab-b^2$$

$$a^2-4ab-b^2$$

$$=1^2-4\cdot1\cdot2-2^2=1-8-4=-11$$

2. a^3+b^3

$$+2b^3+5a^2b-6ab^2$$

$$a^3+3b^3+5a^2b-6ab^2\Rightarrow a^3+5a^2b-6ab^2+3b^3$$

$$=1^3+5\cdot1^2\cdot2-6\cdot1\cdot2^2+3\cdot2^3=1+5\cdot2-6\cdot4+3\cdot8$$

$$=1+10-24+24=11$$

3.
$$\frac{1}{2}a$$
$$-a \quad -\frac{1}{2}b+\frac{5}{3}c$$
$$\frac{1-2}{2}a-\frac{1}{2}b+\frac{5}{3}c=-\frac{1}{2}a-\frac{1}{2}b+\frac{5}{3}c$$
$$\Rightarrow -\frac{1}{2}\cdot 1-\frac{1}{2}\cdot 2+\frac{5}{3}\cdot 3=-\frac{1}{2}-1+5$$
$$=\frac{-1-2+10}{2}=\frac{7}{2}=3\tfrac{1}{2}$$

4.
$$3m^2-5n^2$$
$$-m^2-10n^2-8mn$$
$$2m^2-15n^2-8mn \Rightarrow 2m^2-8mn-15n^2$$
$$=2\left(\frac{3}{2}\right)^2-8\cdot\frac{3}{2}\cdot\frac{2}{5}-15\left(\frac{2}{5}\right)^2=2\cdot\frac{9}{4}-\frac{48}{10}-3\cdot\frac{4}{5}$$
$$=\frac{9}{2}-\frac{24}{5}-\frac{12}{5}==\frac{45-48-24}{10}=-\frac{27}{10}$$

5.
$$x^4 \quad -18x^2y^2 \quad +15y^4$$
$$16x^3y \quad +6xy^3-9y^4$$
$$x^4+16x^3y-18x^2y^2+6xy^3+6y^4$$
$$=4^4+16\cdot 4^3\cdot 5-18\cdot 4^2\cdot 5^2+6\cdot 4\cdot 5^3+6\cdot 5^4$$
$$=256+80\cdot 64-18\cdot 16\cdot 25+24\cdot 125+6\cdot 625$$
$$=256+5.120-7.200+3.000+3.750=4.926$$

6.
$$a^3-7am^2+m^3$$
$$+5am^2+5m^3-8a^2m$$
$$a^3-2am^2+6m^3-8a^2m \Rightarrow a^3-8a^2m-2am^2+6m^3$$
$$=1^3-8\cdot 1^2\cdot\frac{3}{2}-2\cdot 1\left(\frac{3}{2}\right)^2+6\left(\frac{3}{2}\right)^3=1-\frac{24}{2}-2\cdot\frac{9}{4}+6\cdot\frac{27}{8}$$
$$=1-12-\frac{9}{2}+\frac{81}{4}=\frac{4-48-18+81}{4}=\frac{19}{4}$$

7.
$$\frac{2}{3}a^2+\frac{7}{8}ab-\frac{1}{5}b^2$$
$$-\frac{1}{6}a^2-ab+\frac{1}{10}b^2$$
$$\frac{4-1}{6}a^2+\frac{7-8}{8}ab-\frac{2-1}{10}b^2=\frac{1}{2}a^2-\frac{1}{8}ab-\frac{1}{10}b^2$$
$$\Rightarrow \frac{1}{2}\cdot 1^2-\frac{1}{8}\cdot 1\cdot 2-\frac{1}{10}\cdot 2^2=\frac{1}{2}-\frac{2}{8}-\frac{4}{10}$$
$$=\frac{1}{2}-\frac{1}{4}-\frac{2}{5}=\frac{10-5-8}{20}=-\frac{3}{20}$$

8.
$$\frac{2}{3}m^2n+\frac{3}{4}mn^2-\frac{1}{2}n^3$$
$$\frac{1}{6}m^2n+\frac{1}{4}mn^2+\frac{1}{2}n^3+m^3$$
$$\frac{4+1}{6}m^2n+\frac{3+1}{4}mn^2+m^3=m^3+\frac{5}{6}m^2n+mn^2$$
$$=\left(\frac{3}{2}\right)^3+\frac{5}{6}\left(\frac{3}{2}\right)^2\cdot\frac{2}{5}+\frac{3}{2}\left(\frac{2}{5}\right)^2=\frac{27}{8}+\frac{5}{6}\cdot\frac{18}{20}+\frac{3}{2}\cdot\frac{4}{25}$$
$$=\frac{27}{8}+\frac{3}{4}+\frac{12}{50}=\frac{675+150+48}{200}=\frac{873}{200}$$

9.
$$a^5 \quad -3a^2b^4+b^5$$
$$-a^4b^2+5a^3b^3$$
$$a^5-a^4b^2+5a^3b^3-3a^2b^4+b^5$$
$$=1^5-1^4\cdot 2^2+5\cdot 1^3\cdot 2^3-3\cdot 1^2\,2^4+2^5$$
$$=1-4+5\cdot 8-3\cdot 16+32=1-4+40-48+32=21$$

10.
$$-ab \quad +10mn-8mx$$
$$-15ab$$
$$-16ab+10mn-8mx$$
$$=-16\cdot 1\cdot 2+10\cdot\frac{3}{2}\cdot\frac{2}{5}-8\cdot\frac{3}{2}\cdot 4$$
$$=-32+6-48=-74$$

11.
$$a^3$$
$$-11a^2b+9ab^2-b^3$$
$$a^3-11a^2b+9ab^2-b^3$$
$$=1^3-11\cdot 1^2\cdot 2+9\cdot 1\cdot 2^2-2^3$$
$$=1-22+36-8=7$$

12.
$$\frac{1}{64}x^4$$
$$-\frac{2}{3}x^2-\frac{5}{6}x+\frac{3}{8}$$
$$\frac{1}{64}x^4-\frac{2}{3}x^2-\frac{5}{6}x+\frac{3}{8}$$
$$=\frac{1}{64}\cdot 4^4-\frac{2}{3}\cdot 4^2-\frac{5}{6}\cdot 4+\frac{3}{8}$$
$$=4-\frac{32}{3}-\frac{20}{6}+\frac{3}{8}=\frac{96-256-80+9}{24}$$
$$=-\frac{231}{24}=-\frac{77}{8}=-9\tfrac{5}{8}$$

13.
$$\begin{array}{l}
x^3 + \frac{3}{16}x^2y - \frac{2}{5}xy^2 \\
-\frac{3}{4}x^3 + \frac{3}{5}xy^2 + \frac{1}{25}y^3 \\
\hline
\frac{4-3}{4}x^3 + \frac{3}{16}x^2y - \frac{2-3}{5}xy^2 + \frac{1}{25}y^3 \\
\Rightarrow \frac{1}{4}x^3 + \frac{3}{16}x^2y + \frac{1}{5}xy^2 + \frac{1}{25}y^3 \\
= \frac{1}{4}\cdot 4^3 + \frac{3}{16}\cdot 4^2\cdot 5 + \frac{1}{5}\cdot 4\cdot 5^2 + \frac{1}{25}\cdot 5^3 \\
= \frac{1}{4}\cdot 64 + \frac{15}{16}\cdot 16 + \frac{4}{5}\cdot 25 + \frac{125}{25} \\
= 16 + 15 + 20 + 5 = 56
\end{array}$$

14.
$$\begin{array}{l}
\frac{2}{5}a^{x-1} + a^x - \frac{5}{6}a^{x-3} + a^{x-2} \\
-a^{x-1} + 9a^{x-3} - a^{x-2} \\
\hline
\frac{2-5}{5}a^{x-1} + a^x - \frac{5-54}{6}a^{x-3} \\
\Rightarrow a^x - \frac{3}{5}a^{x-1} + \frac{49}{6}a^{x-3} \\
= 1^4 - \frac{3}{5}\cdot 1^{4-1} + \frac{49}{6}\cdot 1^{4-3} \\
= 1 - \frac{3}{5} + \frac{49}{6} = \frac{+30 - 18 + 245}{30} = \frac{257}{30} = 8\tfrac{17}{30}
\end{array}$$

EJERCICIO 27

1.
$$\begin{array}{r} +ab + b^2 \\ a^2 \quad -5b^2 \\ \hline a^2 + ab - 4b^2 \end{array} \qquad \begin{array}{r} a^2 \\ -a^2 - ab + 4b^2 \\ \hline -ab + 4b^2 \end{array}$$

2.
$$\begin{array}{r} a + 8 \\ -a + 6 \\ \hline 14 \end{array} \qquad \begin{array}{r} 1 \\ -14 \\ \hline -13 \end{array}$$

3.
$$\begin{array}{l} -x^3 + 4xy^2 \\ +5x^2y + y^3 \\ \hline -x^3 + 5x^2y + 4xy^2 + y^3 \end{array}$$

$$\begin{array}{l} -7x^2y \\ x^3 - 5x^2y - 4xy^2 - y^3 \\ \hline x^3 - 12x^2y - 4xy^2 - y^3 \end{array}$$

4.
$$\begin{array}{r} -3m^3n + 4mn^2 - n^3 \\ 3m^3n - 4mn^2 + 5n^3 \\ \hline +4n^3 \end{array} \qquad \begin{array}{r} 5m^4 \\ -4n^3 \\ \hline 5m^4 - 4n^3 \end{array}$$

5.
$$\begin{array}{r} 8a + 9b - 3c \\ -7a - 9b + 3c \\ \hline a \end{array} \qquad \begin{array}{r} 6a \\ -a \\ \hline 5a \end{array}$$

6.
$$\begin{array}{r} a - b + c \\ -2a + b - c \\ \hline -a \end{array} \qquad \begin{array}{r} a + b - c \\ a \\ \hline 2a + b - c \end{array}$$

7.
$$\begin{array}{r} -m + n - p \\ 2m - 2n + 2p \\ \hline m - n + p \end{array} \qquad \begin{array}{r} m - n + p \\ -m + n - p \\ \hline 0 \end{array}$$

8.
$$\begin{array}{r} 9ax - a^2 \\ -9ax + 7a^2 + 25x^2 \\ \hline 6a^2 + 25x^2 \end{array} \qquad \begin{array}{r} x^2 - 5ax + 3a^2 \\ -25x^2 - 6a^2 \\ \hline -24x^2 - 5ax - 3a^2 \end{array}$$

9.
$$\begin{array}{r} 5a^2 + 6a - 4 \\ 2a^3 - 8a + 6 \\ \hline 2a^3 + 5a^2 - 2a + 2 \end{array} \qquad \begin{array}{r} a^3 \quad -1 \\ -2a^3 - 5a^2 + 2a - 2 \\ \hline -a^3 - 5a^2 + 2a - 3 \end{array}$$

10.
$$\begin{array}{r} 5x^3 - 9x^2 + 4 \\ -11x^4 - 7x^3 - 6x \\ \hline -11x^4 - 2x^3 - 9x^2 - 6x + 4 \end{array} \qquad \begin{array}{r} x^4 \quad -1 \\ 11x^4 + 2x^3 + 9x^2 + 6x - 4 \\ \hline 12x^4 + 2x^3 + 9x^2 + 6x - 5 \end{array}$$

11.
$$\begin{array}{r} 35a^2b - 7ab^2 - 11 \\ -7a^3 - 35a^2b + 8ab^2 + 6 \\ \hline -7a^3 + ab^2 - 5 \end{array} \qquad \begin{array}{r} a^3 + b^3 \\ 7a^3 - ab^2 + 5 \\ \hline 8a^3 - ab^2 + b^3 + 5 \end{array}$$

12.
$$\begin{array}{r} -11n^4 + 14n^2 - 25n + 8 \\ 19n^3 - 6n^2 + 9n - 4 \\ \hline -11n^4 + 19n^3 + 8n^2 - 16n + 4 \end{array}$$

$$\begin{array}{r} n^5 - 7n^3 + 4n \\ 11n^4 - 19n^3 - 8n^2 + 16n - 4 \\ \hline n^5 + 11n^4 - 26n^3 - 8n^2 + 20n - 4 \end{array}$$

13.

$$\begin{array}{r} -6a^3m \quad +5am^3 \quad -6 \\ 7a^4-5a^3m-11a^2m^2 \quad -6m^4 \\ \hline 7a^4-11a^3m-11a^2m^2+5am^3-6m^4-6 \end{array}$$

$$\begin{array}{r} a^4 \quad -8a^2m^2 \quad +m^4 \\ -7a^4+11a^3m+11a^2m^2-5am^3+6m^4+6 \\ \hline -6a^4+11a^3m+3a^2m^2-5am^3+7m^4+6 \end{array}$$

14.

$$\begin{array}{r} -4x^4y \quad +13x^2y^3-9xy^4 \\ -6x^5 \quad +8x^3y^2 \quad +xy^4-2y^5 \\ \hline -6x^5-4x^4y+8x^3y^2+13x^2y^3-8xy^4-2y^5 \end{array}$$

$$\begin{array}{r} x^5 \quad -30x^3y^2 \quad +40xy^4+y^5 \\ 6x^5+4x^4y-8x^3y^2-13x^2y^3+8xy^4+2y^5 \\ \hline 7x^5+4x^4y-38x^3y^2-13x^2y^3+48xy^4+3y^5 \end{array}$$

15.

$$\begin{array}{r} a+b \\ a-b \\ \hline 2a \end{array} \qquad \begin{array}{r} 2a \\ -2a+b \\ \hline b \end{array}$$

16.

$$\begin{array}{r} 8x \quad +9 \\ 6y-5 \\ \hline 8x+6y+4 \end{array} \qquad \begin{array}{r} 8x+6y+4 \\ 2 \\ \hline 8x+6y+6 \end{array}$$

17.

$$\begin{array}{r} x^2 \quad -6y^2 \\ -7xy+40y^2 \\ \hline x^2-7xy+34y^2 \end{array} \qquad \begin{array}{r} x^2-7xy+34y^2 \\ 9y^2-16 \\ \hline x^2-7xy+43y^2-16 \end{array}$$

18.

$$\begin{array}{r} 4a^2+8ab-5b^2 \\ a^2-7ab+6b^2 \\ \hline 5a^2+ab+b^2 \end{array} \qquad \begin{array}{r} 5a^2+ab+b^2 \\ -4a^2-ab+b^2 \\ \hline a^2 \quad +2b^2 \end{array}$$

19.

$$\begin{array}{r} x^3 \quad -y^3 \\ -14x^2y+5xy^2 \\ \hline x^3-14x^2y+5xy^2-y^3 \end{array}$$

$$\begin{array}{r} x^3-14x^2y+5xy^2-y^3 \\ 3x^3 \quad -19y^3 \\ \hline 4x^3-14x^2y+5xy^2-20y^3 \end{array}$$

20.

$$\begin{array}{r} x^4-6x^2y^2+y^4 \\ +8x^2y^2+31y^4 \\ \hline x^4+2x^2y^2+32y^4 \end{array} \qquad \begin{array}{r} x^4-2x^2y^2+32y^4 \\ -x^4-2x^2y^2-32y^4 \\ \hline 0 \end{array}$$

21.

$$\begin{array}{r} -6n^5+n^4 \quad +n^2 \\ +7n^3-n^2-8n-6 \\ \hline -6n^5+n^4+7n^3 \quad -8n-6 \end{array}$$

$$\begin{array}{r} -6n^5+n^4+7n^3-8n-6 \\ n^6 \quad +3n^4+8n^3 \quad -19 \\ \hline n^6-6n^5+4n^4+15n^3-8n-25 \end{array}$$

22.

$$\begin{array}{r} a^5 \quad -3a^3b^2+6ab^4 \\ +22a^4b+10a^3b^2-11ab^4-b^5 \\ \hline a^5+22a^4b+7a^3b^2-5ab^4-b^5 \end{array}$$

$$\begin{array}{r} a^5+22a^4b+7a^3b^2 \quad -5ab^4-b^5 \\ -5a^4b \quad +7a^2b^3 \quad -b^5 \\ \hline a^5+17a^4b+7a^3b^2+7a^2b^3-5ab^4-2b^5 \end{array}$$

23.

$$\begin{array}{r} 4m^3-5m^2-2m \\ -7m^3 \quad +8m+4 \\ \hline -3m^3-5m^2+6m+4 \end{array} \qquad \begin{array}{r} -3m^3-5m^2+6m+4 \\ m^4 \quad -5 \\ \hline m^4-3m^3-5m^2+6m-1 \end{array}$$

24.

$$\begin{array}{r} 7a^2-11ab+b^2 \\ -7a^2+11ab+b^2-8 \\ \hline 2b^2-8 \end{array} \qquad \begin{array}{r} 2b^2-8 \\ +4 \\ \hline 2b^2-4 \end{array}$$

25.

$$\begin{array}{r} 3a-4b+5c \\ -7a+8b \quad -11 \\ -a+2b-7c \\ \hline -5a+6b-2c-11 \end{array} \qquad \begin{array}{r} -5a+6b-2c-11 \\ -a+b+2c \\ \hline -6a+7b \quad -11 \end{array}$$

26.

$$\begin{array}{r} 5a^3+14a^2-19a+8 \\ a^5 \quad +9a-1 \\ -a^4 \quad +3a^2 \quad -1 \\ \hline a^5-a^4+5a^3+17a^2-10a+6 \\ -a^4+3a^3 \quad -5 \\ \hline a^5-2a^4+8a^3+17a^2-10a+1 \end{array}$$

27.

$$\begin{array}{l} m^4 + 10m^2n^2 + 15n^4 \\ -11m^3n - 14m^2n^2 - 3mn^3 + n^4 \\ \hline m^4 - 11m^3n - 4m^2n^2 - 3mn^3 + 16n^4 \end{array}$$

$$\begin{array}{l} 6m^4 + 7m^2n^2 + 8mn^3 - n^4 \\ -m^4 + 11m^3n + 4m^2n^2 + 3mn^3 - 16n^4 \\ \hline 5m^4 + 11m^3n + 11m^2n^2 + 11mn^3 - 17n^4 \end{array}$$

28.

$$\begin{array}{l} a^5 + 4a^3b^2 + 8ab^4 - b^5 \\ -7a^4b + 15a^2b^3 - 25ab^4 + 3b^5 \\ -a^3b^2 + 3a^2b^3 - 5ab^4 \\ \hline a^5 - 7a^4b + 3a^3b^2 + 18a^2b^3 - 22ab^4 + 2b^5 \end{array}$$

$$\begin{array}{l} 3a^5 - 6a^2b^3 - 21ab^4 - 6 \\ -a^5 + 7a^4b - 3a^3b^2 - 18a^2b^3 + 22ab^4 - 2b^5 \\ \hline 2a^5 + 7a^4b - 3a^3b^2 - 24a^2b^3 + ab^4 - 2b^5 - 6 \end{array}$$

29.

$$\begin{array}{l} x^5 + y^5 \\ 3x^4y + 21x^3y^2 + 18x^2y^3 - y^5 \\ \hline x^5 + 3x^4y + 21x^3y^2 + 18x^2y^3 \end{array}$$

$$\begin{array}{l} x^5 + 32x^4y - 26x^3y^2 + 18x^2y^3 - 2xy^4 + y^5 \\ -x^5 - 3x^4y - 21x^3y^2 - 18x^2y^3 \\ \hline 29x^4y - 47x^3y^2 - 2xy^4 + y^5 \end{array}$$

30.

$$\begin{array}{l} 3a^x + 6a^{x-1} \\ a^x - 7a^{x-1} + a^{x-2} \\ \hline 4a^x - a^{x-1} + a^{x-2} \end{array}$$

$$\begin{array}{l} 8a^{x+2} - 7a^{x+1} - a^x + 12a^{x-1} \\ -4a^x + a^{x-1} - a^{x-2} \\ \hline 8a^{x+2} - 7a^{x+1} - 5a^x + 13a^{x-1} - a^{x-2} \end{array}$$

EJERCICIO 28

1.

$$\begin{array}{l} x^2 + 5 \\ 2x - 6 \\ \hline x^2 + 2x - 1 \end{array} \qquad \begin{array}{l} x - 4 \\ -x + 6 \\ \hline 2 \end{array} \qquad \begin{array}{l} x^2 + 2x - 1 \\ -2 \\ \hline x^2 + 2x - 3 \end{array}$$

2.

$$\begin{array}{l} 3a - 5b + c \\ a - b - 3c \\ \hline 4a - 6b - 2c \end{array} \qquad \begin{array}{l} 7a + b \\ -8b - 3c \\ \hline 7a - 7b - 3c \end{array} \qquad \begin{array}{l} 4a - 6b - 2c \\ -7a + 7b + 3c \\ \hline -3a + b + c \end{array}$$

3.

$$\begin{array}{l} x^3 + 1 \\ 5x^3 - x^2 + 7 \\ \hline 6x^3 - x^2 + 8 \end{array} \qquad \begin{array}{l} 9x + 4 \\ -3x^2 - x + 1 \\ \hline -3x^2 + 8x + 5 \end{array} \qquad \begin{array}{l} 6x^3 - x^2 + 8 \\ 3x^2 - 8x - 5 \\ \hline 6x^3 + 2x^2 - 8x + 3 \end{array}$$

4.

$$\begin{array}{l} a^2 + 1 \\ a^3 - 1 \\ \hline a^3 + a^2 \end{array} \qquad \begin{array}{l} a^4 + 2 \\ a - 2 \\ \hline a^4 + a \end{array} \qquad \begin{array}{l} a^3 + a^2 \\ -a^4 - a \\ \hline -a^4 + a^3 + a^2 - a \end{array}$$

5.

$$\begin{array}{l} ab + bc + ac \\ -7bc + 8ac - 9 \\ \hline ab - 6bc + 9ac - 9 \end{array} \qquad \begin{array}{l} 5ab - 3bc + 4ac \\ -ab + 3bc + 5ac \\ \hline 4ab + 9ac \end{array} \qquad \begin{array}{l} ab - 6bc + 9ac - 9 \\ -4ab - 9ac \\ \hline -3ab - 6bc - 9 \end{array}$$

6.

$$\begin{array}{l} a^2x - 3x^3 \\ a^3 + 3ax^2 \\ \hline a^3 + a^2x + 3ax^2 - 3x^3 \end{array} \qquad \begin{array}{l} -5a^2x + 11ax^2 - 11x^3 \\ a^3 - 4a^2x + 6ax^2 + 8x^3 \\ \hline a^3 - 9a^2x + 17ax^2 - 3x^3 \end{array}$$

$$\begin{array}{l} a^3 + a^2x + 3ax^2 - 3x^3 \\ -a^3 + 9a^2x - 17ax^2 + 3x^3 \\ \hline 10a^2x - 14ax^2 \end{array}$$

7.

$$\begin{array}{l} x^4 + x^2 - 3 \\ -x^3 - 3x + 5 \\ x^4 - 5x^2 + 4x \\ \hline 2x^4 - x^3 - 4x^2 + x + 2 \end{array} \qquad \begin{array}{l} -7x^3 + 8x^2 - 3x + 4 \\ x^4 - 3 \\ \hline x^4 - 7x^3 + 8x^2 - 3x + 1 \end{array}$$

$$\begin{array}{l} 2x^4 - x^3 - 4x^2 + x + 2 \\ -x^4 + 7x^3 - 8x^2 + 3x - 1 \\ \hline x^4 + 6x^3 - 12x^2 + 4x + 1 \end{array}$$

8. $m^4 - n^4$

$$\begin{array}{r} 17m^3n - 4m^2n^2 - 7mn^3 \\ -m^4 + 6m^2n^2 - 80n^4 \\ \hline 17m^3n + 2m^2n^2 - 7mn^3 - 81n^4 \end{array} \qquad \begin{array}{r} -m^4 + 6 \\ -m^2n^2 + mn^3 - 4 \\ \hline -m^4 - m^2n^2 + mn^3 + 2 \end{array} \qquad \begin{array}{r} 17m^3n + 2m^2n^2 - 7mn^3 - 81n^4 \\ m^4 + m^2n^2 - mn^3 - 2 \\ \hline m^4 + 17m^3n + 3m^2n^2 - 8mn^3 - 81n^4 - 2 \end{array}$$

9. $+a^3 + a - 7$

$$\begin{array}{r} a^5 - a^4 - 6a^2 + 8 \\ -5a^2 - 11a + 26 \\ \hline a^5 - a^4 + a^3 - 11a^2 - 10a + 27 \end{array} \qquad \begin{array}{r} -a^4 - 4a^3 + a^2 \\ 16a^3 - 8a^2 - 7a - 15 \\ \hline -a^4 + 12a^3 - 7a^2 - 7a - 15 \end{array} \qquad \begin{array}{r} a^5 - a^4 + a^3 - 11a^2 - 10a + 27 \\ +a^4 - 12a^3 + 7a^2 + 7a + 15 \\ \hline a^5 - 11a^3 - 4a^2 - 3a + 42 \end{array}$$

10.

$$\begin{array}{r} 3x^2 - y^2 \\ -11xy + 9y^2 - 14 \\ \hline 3x^2 - 11xy + 8y^2 - 14 \end{array} \qquad \begin{array}{r} x^2 - 3xy - y^2 \\ 19x^2 - 8xy + 9y^2 \\ \hline 20x^2 - 11xy + 8y^2 \end{array} \qquad \begin{array}{r} 20x^2 - 11xy + 8y^2 \\ -3x^2 + 11xy - 8y^2 + 14 \\ \hline 17x^2 + 14 \end{array}$$

11.

$$\begin{array}{r} a - 1 \\ -a + 1 \\ \hline 0 \end{array} \qquad \begin{array}{r} a^2 - 3 \\ a - 4 \\ -3a + 8 \\ \hline a^2 - 2a + 1 \end{array}$$

12.

$$\begin{array}{r} a^2 - ab + b^2 \\ 3a^2 - 8ab + 7b^2 \\ -5a^2 + 11ab - 17b^2 \\ \hline -a^2 + 2ab - 9b^2 \end{array} \qquad \begin{array}{r} -a^2 + 9ab + 3b^2 \\ -8ab - 7b^2 \\ \hline -a^2 + ab - 4b^2 \end{array} \qquad \begin{array}{r} -a^2 + ab - 4b^2 \\ a^2 - 2ab + 9b^2 \\ \hline -ab + 5b^2 \end{array}$$

13.

$$\begin{array}{r} m^4 - 1 \\ -m^3 + 8m^2 - 6m + 5 \\ -m^2 - 7m + 1 \\ \hline m^4 - m^3 + 7m^2 - 13m + 5 \end{array} \qquad \begin{array}{r} m^5 - 16 \\ -16m^4 + 7m^2 - 3 \\ \hline m^5 - 16m^4 + 7m^2 - 19 \end{array} \qquad \begin{array}{r} m^5 - 16m^4 + 7m^2 - 19 \\ -m^4 + m^3 - 7m^2 + 13m - 5 \\ \hline m^5 - 17m^4 + m^3 + 13m - 24 \end{array}$$

14.

$$\begin{array}{r} x^5 - y^5 \\ -2x^4y + 5x^3y^2 - 7x^2y^3 - 3y^5 \\ -7x^3y^2 + 6xy^4 - 8 \\ \hline x^5 - 2x^4y - 2x^3y^2 - 7x^2y^3 + 6xy^4 - 4y^5 - 8 \end{array} \qquad \begin{array}{r} 7x^4y - x^3y^2 + 11xy^4 \\ -xy^4 - 1 \\ \hline 7x^4y - x^3y^2 + 10xy^4 - 1 \end{array}$$

$$\begin{array}{r} -x^5 + 2x^4y + 2x^3y^2 + 7x^2y^3 - 6xy^4 + 4y^5 + 8 \\ 7x^4y - x^3y^2 + 10xy^4 - 1 \\ \hline -x^5 + 9x^4y + x^3y^2 + 7x^2y^3 + 4xy^4 + 4y^5 + 7 \end{array}$$

15.

$$\begin{array}{r} -a^6 + 7a^4 - 8a \\ -3a^5 + 11a^3 - a^2 + 4 \\ -6a^4 - 11a^3 - 2a + 8 \\ -5a^3 + 5a^2 - 4a + 1 \\ \hline -a^6 - 3a^5 + a^4 - 5a^3 + 4a^2 - 14a + 13 \end{array} \qquad \begin{array}{r} -3a^4 + 7a^2 - 8a + 5 \\ 5a^5 - 7a^3 + 41a^2 - 50a + 8 \\ \hline 5a^5 - 3a^4 - 7a^3 + 48a^2 - 58a + 13 \end{array}$$

$$\begin{array}{r} 5a^5 - 3a^4 - 7a^3 + 48a^2 - 58a + 13 \\ a^6 + 3a^5 - a^4 + 5a^3 - 4a^2 + 14a - 13 \\ \hline a^6 + 8a^5 - 4a^4 - 2a^3 + 44a^2 - 44a \end{array}$$

16.

$$\begin{array}{r} a^5 - 7a^3x^2 + 9 \\ -20a^4x + 21a^2x^3 - 19ax^4 \\ +9a^3x^2 - 7ax^4 + x^5 - 80 \\ \hline a^5 - 20a^4x + 2a^3x^2 + 21a^2x^3 - 26ax^4 + x^5 - 71 \end{array} \qquad \begin{array}{r} +18a^3x^2 - 4x^5 - 8 \\ -9a^4x - 17a^3x^2 + 11a^2x^3 \\ a^5 + 36 \\ \hline a^5 - 9a^4x + a^3x^2 + 11a^2x^3 - 4x^5 + 28 \end{array}$$

$$\begin{array}{r} a^5 - 9a^4x + a^3x^2 + 11a^2x^3 - 4x^5 + 28 \\ -a^5 + 20a^4x - 2a^3x^2 - 21a^2x^3 + 26ax^4 - x^5 + 71 \\ \hline 11a^4x - a^3x^2 - 10a^2x^3 + 26ax^4 - 5x^5 + 99 \end{array}$$

EJERCICIO 29

1.

$$\begin{array}{l} a+\frac{1}{2}b \\ -\frac{2}{3}a+\frac{3}{4}b \\ \hline \frac{3-2}{3}a+\frac{2+3}{4}b \\ =\frac{1}{3}a+\frac{5}{4}b \end{array} \qquad \begin{array}{l} \frac{3}{4}a \\ -\frac{1}{3}a-\frac{5}{4}b \\ \hline \frac{9-4}{12}a-\frac{5}{4}b \\ =\frac{5}{12}a-\frac{5}{4}b \end{array}$$

2.

$$\begin{array}{l} \frac{3}{8}a-6 \\ -\frac{5}{6}a^3+\frac{3}{5}a^2 \\ \hline -\frac{5}{6}a^3+\frac{3}{5}a^2+\frac{3}{8}a-6 \end{array} \qquad \begin{array}{l} \frac{1}{2}a^3+\frac{3}{5}a^2 \\ \frac{5}{6}a^3-\frac{3}{5}a^2-\frac{3}{8}a+6 \\ \hline \frac{3+5}{6}a^3-\frac{3}{8}a+6 \\ =\frac{4}{3}a^3-\frac{3}{8}a+6 \end{array}$$

3.

$$\begin{array}{l} a+3b \\ -\frac{2}{5}a-\frac{2}{3}b+6 \\ \hline \frac{5-2}{5}a+\frac{9-2}{3}b+6 \\ =\frac{3}{5}a+\frac{7}{3}b+6 \end{array} \qquad \begin{array}{l} \frac{3}{5}a+\frac{7}{3}b+6 \\ -\frac{1}{5}a+\frac{1}{6}b \\ \hline \frac{3-1}{5}a+\frac{14+1}{6}b+6 \\ =\frac{2}{5}a+\frac{5}{2}b+6 \end{array}$$

4.

$$\begin{array}{l} \frac{1}{3}x^3-\frac{3}{7}x^2+\frac{1}{5} \\ \frac{1}{14}x^2-\frac{2}{9}x+6 \\ \hline \frac{1}{3}x^3-\frac{6-1}{14}x^2-\frac{2}{9}x+\frac{1+30}{5} \\ =\frac{1}{3}x^3-\frac{5}{14}x^2-\frac{2}{9}x+\frac{31}{5} \end{array}$$

$$\begin{array}{l} -\frac{5}{6}x^3 \\ -\frac{1}{3}x^3+\frac{5}{14}x^2+\frac{2}{9}x-\frac{31}{5} \\ \hline -\frac{5+2}{6}x^3+\frac{5}{14}x^2+\frac{2}{9}x-\frac{31}{5} \\ =-\frac{7}{6}x^3+\frac{5}{14}x^2+\frac{2}{9}x-6\frac{1}{5} \end{array}$$

5.

$$\begin{array}{l} \frac{7}{12}a^4 \\ -\frac{3}{7}a^3+\frac{2}{5}a^2-6 \\ \hline \frac{7}{12}a^4-\frac{3}{7}a^3+\frac{2}{5}a^2-6 \end{array} \qquad \begin{array}{l} \frac{7}{12}a^4-\frac{3}{7}a^3+\frac{2}{5}a^2-6 \\ \frac{3}{4}a^4-\frac{1}{5}a+\frac{1}{3} \\ \hline \frac{7+9}{12}a^4-\frac{3}{7}a^3+\frac{2}{5}a^2-\frac{1}{5}a-\frac{18-1}{3} \\ =\frac{4}{3}a^4-\frac{3}{7}a^3+\frac{2}{5}a^2-\frac{1}{5}a-\frac{17}{3} \end{array}$$

6.

$$\begin{array}{l} -\frac{1}{2}x+\frac{2}{3}y-\frac{1}{4}z \\ -\frac{1}{9}y-\frac{2}{5}z+3 \\ \hline -\frac{1}{2}x+\frac{6-1}{9}y-\frac{5+8}{20}z+3 \\ =-\frac{1}{2}x+\frac{5}{9}y-\frac{13}{20}z+3 \end{array} \qquad \begin{array}{l} \frac{5}{9}y-\frac{2}{5} \\ \frac{1}{2}x-\frac{5}{9}y+\frac{13}{20}z-3 \\ \hline \frac{1}{2}x+\frac{13}{20}z-\frac{2+15}{5} \\ =\frac{1}{2}x+\frac{13}{20}z-\frac{17}{5} \end{array}$$

7.

$$\begin{array}{l} -\frac{3}{2}a^2b+\frac{3}{4}ab^2-b^3 \\ \frac{1}{8}a^2b-\frac{5}{6}ab^2+\frac{2}{3}b^3 \\ \hline -\frac{12-1}{8}a^2b+\frac{9-10}{12}ab^2-\frac{3-2}{3}b^3 \\ =-\frac{11}{8}a^2b-\frac{1}{12}ab^2-\frac{1}{3}b^3 \end{array}$$

$$\begin{array}{l} \frac{1}{2}a^3-\frac{1}{3}b^3 \\ \frac{11}{8}a^2b+\frac{1}{12}ab^2+\frac{1}{3}b^3 \\ \hline \frac{1}{2}a^3+\frac{11}{8}a^2b+\frac{1}{12}ab^2 \end{array}$$

8.

$$\begin{array}{l} \frac{1}{2}a-\frac{2}{9}b \\ \frac{1}{3}b-\frac{3}{5}c \\ \hline \frac{1}{2}a-\frac{2-3}{9}b-\frac{3}{5}c \\ =\frac{1}{2}a+\frac{1}{9}b-\frac{3}{5}c \end{array} \qquad \begin{array}{l} \frac{2}{3}b+\frac{1}{5}c \\ -\frac{5}{9}b-\frac{1}{10}c \\ \hline \frac{6-5}{9}b+\frac{2-1}{10}c \\ =\frac{1}{9}b+\frac{1}{10}c \end{array} \qquad \begin{array}{l} \frac{1}{2}a+\frac{1}{9}b-\frac{3}{5}c \\ -\frac{1}{9}b-\frac{1}{10}c \\ \hline \frac{1}{2}a-\frac{6+1}{10}c \\ =\frac{1}{2}a-\frac{7}{10}c \end{array}$$

9.

$$\begin{array}{l}\frac{1}{3}a^3+\frac{1}{8}a^2+\frac{1}{5}\\ -\frac{3}{5}a^2-\frac{3}{4}a-\frac{1}{10}\\ \hline \frac{1}{3}a^3+\frac{5-24}{40}a^2-\frac{3}{4}a+\frac{2-1}{10}\\ =\frac{1}{3}a^3-\frac{19}{40}a^2-\frac{3}{4}a+\frac{1}{10}\end{array}$$

$$\begin{array}{l}\frac{1}{4}a^2-\frac{2}{3}a+\frac{1}{4}\\ \frac{1}{3}a^3-\frac{29}{40}a^2-\frac{1}{8}\\ \hline \frac{1}{3}a^3+\frac{10-29}{40}a^2-\frac{2}{3}a+\frac{2-1}{8}\\ =\frac{1}{3}a^3-\frac{19}{40}a^2-\frac{2}{3}a+\frac{1}{8}\end{array}$$

$$\begin{array}{l}\frac{1}{3}a^3-\frac{19}{40}a^2-\frac{2}{3}a+\frac{1}{8}\\ -\frac{1}{3}a^3+\frac{19}{40}a^2+\frac{3}{4}a-\frac{1}{10}\\ \hline -\frac{8-9}{12}a+\frac{10-8}{80}\\ =\frac{1}{12}a+\frac{2}{80}=\frac{1}{12}a+\frac{1}{40}\end{array}$$

10.

$$\begin{array}{l}\frac{3}{5}x^2-\frac{5}{6}xy+\frac{2}{9}y^2\\ -\frac{3}{2}xy-\frac{1}{3}y^2+\frac{1}{4}\\ \hline \frac{3}{5}x^2-\frac{10+18}{12}xy+\frac{2-3}{9}y^2+\frac{1}{4}\\ =\frac{3}{5}x^2-\frac{28}{12}xy-\frac{1}{9}y^2+\frac{1}{4}\\ =\frac{3}{5}x^2-\frac{7}{3}xy-\frac{1}{9}y^2+\frac{1}{4}\end{array}$$

$$\begin{array}{l}\frac{2}{9}x^2+\frac{1}{9}xy-\frac{2}{3}y^2\\ \frac{17}{45}x^2-\frac{22}{9}xy-\frac{3}{2}y^2-\frac{1}{2}\\ \hline \frac{10+17}{45}x^2+\frac{1-22}{9}xy-\frac{4+9}{6}y^2-\frac{1}{2}\\ =\frac{27}{45}x^2-\frac{21}{9}xy-\frac{13}{6}y^2-\frac{1}{2}\\ =\frac{3}{5}x^2-\frac{7}{3}xy-\frac{13}{6}y^2-\frac{1}{2}\end{array}$$

$$\begin{array}{l}-\frac{3}{5}x^2+\frac{7}{3}xy+\frac{13}{6}y^2+\frac{1}{2}\\ \frac{3}{5}x^2-\frac{7}{3}xy-\frac{1}{9}y^2+\frac{1}{4}\\ \hline \frac{39-2}{18}y^2+\frac{2+1}{4}\\ =\frac{37}{18}y^2+\frac{3}{4}\end{array}$$

11.

$$\begin{array}{l}\frac{2}{7}a^3-\frac{1}{5}b^3\\ -\frac{3}{4}a^2b+\frac{3}{8}ab^2+\frac{1}{10}b^3\\ \hline \frac{2}{7}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2-\frac{2-1}{10}b^3\\ =\frac{2}{7}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2-\frac{1}{10}b^3\end{array}$$

$$\begin{array}{l}\frac{1}{2}a^2b+\frac{1}{4}ab^2-\frac{1}{5}\\ -\frac{5}{4}a^2b+\frac{1}{8}ab^2-\frac{3}{2}b^3-\frac{1}{2}\\ \hline \frac{2-5}{4}a^2b+\frac{2+1}{8}ab^2-\frac{3}{2}b^3-\frac{2+5}{10}\\ =-\frac{3}{4}a^2b+\frac{3}{8}ab^2-\frac{3}{2}b^3-\frac{7}{10}\end{array}$$

$$\begin{array}{l}-\frac{3}{4}a^2b+\frac{3}{8}ab^2-\frac{3}{2}b^3-\frac{7}{10}\\ -\frac{2}{7}a^3+\frac{3}{4}a^2b-\frac{3}{8}ab^2+\frac{1}{10}b^3\\ \hline -\frac{2}{7}a^3-\frac{15-1}{10}b^3-\frac{7}{10}\\ =-\frac{2}{7}a^3-\frac{14}{10}b^3-\frac{7}{10}=-\frac{2}{7}a^3-\frac{7}{5}b^3-\frac{7}{10}\end{array}$$

12.

$$\begin{array}{l}\frac{1}{3}m^2n^2-\frac{1}{4}mn^3-n^4\\ \frac{2}{7}m^4+\frac{3}{5}m^3n-\frac{2}{5}m^2n^2+\frac{5}{3}n^4\\ \frac{1}{14}m^4-\frac{7}{20}m^3n+\frac{1}{4}m^2n^2-\frac{2}{3}n^4\\ \hline \frac{4+1}{14}m^4+\frac{12-7}{20}m^3n+\frac{20-24+15}{60}m^2n^2-\frac{1}{4}mn^3-\frac{3-5+2}{3}n^4\\ =\frac{5}{14}m^4+\frac{5}{20}m^3n+\frac{11}{60}m^2n^2-\frac{1}{4}mn^3-\frac{0}{3}n^4\\ =\frac{5}{14}m^4+\frac{1}{4}m^3n+\frac{11}{60}m^2n^2-\frac{1}{4}mn^3\end{array}$$

$$\begin{array}{l}\frac{5}{14}m^4-\frac{2}{5}n^4\\ -\frac{5}{14}m^4-\frac{1}{4}m^3n-\frac{11}{60}m^2n^2+\frac{1}{4}mn^3\\ \hline -\frac{1}{4}m^3n-\frac{11}{60}m^2n^2+\frac{1}{4}mn^3-\frac{2}{5}n^4\end{array}$$

13.

$$\begin{array}{l}
\frac{1}{2}x+\frac{1}{3}y \\
\qquad \frac{3}{4}y-\frac{1}{6}z \\
\qquad\qquad \frac{2}{5}z+\frac{1}{4}m \\
\qquad\qquad\qquad -\frac{1}{2}m+\frac{1}{3}n+\frac{3}{8} \\
\hline
\frac{1}{2}x+\frac{4+9}{12}y-\frac{5-12}{30}z+\frac{1-2}{4}m+\frac{1}{3}n+\frac{3}{8} \\
=\frac{1}{2}x+\frac{13}{12}y+\frac{7}{30}z-\frac{1}{4}m+\frac{1}{3}n+\frac{3}{8}
\end{array}$$

$$\begin{array}{r}
5 \\
-\frac{1}{2}x-\frac{13}{12}y-\frac{7}{30}z+\frac{1}{4}m-\frac{1}{3}n-\frac{3}{8} \\
\hline
-\frac{1}{2}x-\frac{13}{12}y-\frac{7}{30}z+\frac{1}{4}m-\frac{1}{3}n+\frac{40-3}{8} \\
=-\frac{1}{2}x-\frac{13}{12}y-\frac{7}{30}z+\frac{1}{4}m-\frac{1}{3}n+\frac{37}{8}
\end{array}$$

14.

$$\begin{array}{l}
\frac{5}{6}a^4+\frac{1}{2}a^3\qquad -\frac{3}{5}a \\
\qquad -\frac{2}{3}a^2-\frac{3}{8}a+5 \\
-\frac{3}{8}a^4-\frac{3}{4}a^3+\frac{1}{6}a^2\qquad -\frac{2}{3} \\
\qquad +\frac{1}{6}a^3\qquad +\frac{39}{40}a+\frac{3}{11} \\
\hline
\frac{20-9}{24}a^4+\frac{6-9+2}{12}a^3-\frac{4-1}{6}a^2-\frac{24+15-39}{40}a+\frac{165-22+9}{33} \\
=\frac{11}{24}a^4-\frac{1}{12}a^3-\frac{3}{6}a^2-\frac{0}{40}a+\frac{152}{33}=\frac{11}{24}a^4-\frac{1}{12}a^3-\frac{1}{2}a^2+\frac{152}{33}
\end{array}$$

$$\begin{array}{l}
\frac{11}{24}a^4-\frac{1}{12}a^3-\frac{1}{2}a^2+\frac{152}{33} \\
-a^4+\frac{1}{12}a^3\qquad -\frac{3}{8} \\
\hline
\frac{11-24}{24}a^4\qquad -\frac{1}{2}a^2+\frac{1216-99}{264} \\
=-\frac{13}{24}a^4-\frac{1}{2}a^2+\frac{1117}{264}
\end{array}$$

EJERCICIO 30

1.

$$\begin{array}{r}
x^3-x^2\qquad +5 \\
-x^3+x^2+3x-5-6 \\
\hline
3x\quad -6
\end{array}$$

Rta. $-x^3+x^2+3x-11$

2.

$$\begin{array}{r}
-5a+9b-6c \\
5a-9b+6c+8x+9 \\
\hline
8x+9
\end{array}$$

Rta. $5a-9b+6c+8x+9$

3.

$$\begin{array}{r}
a^3\qquad\qquad -b^3 \\
-a^3-8a^2b+5ab^2-3b^3 \\
\hline
-8a^2b+5ab^2-4b^3
\end{array}$$

Rta. $-a^3-8a^2b+5ab^2-3b^3$

4.

$$\begin{array}{r}
x^3-4x^2\qquad +8 \\
-x^3+4x^2+x-13 \\
\hline
x-5
\end{array}$$

Rta. x^3-4x^2-x+13

5.

$$\begin{array}{r}
m^4\qquad\qquad -3mn^3+6n^4 \\
-m^4+4m^2n^2+3mn^3-6n^4-8 \\
\hline
4m^2n^2\qquad\qquad -8
\end{array}$$

Rta. $m^4-4m^2n^2-3mn^3+6n^4+8$

6.

$$\begin{array}{r}
4x^3+5x^2-5x-2 \\
-5x^2-4x+8 \\
\hline
4x^3\qquad -9x+6
\end{array}$$

Rta. $4x^3+5x^2-5x-2$

7.

$$\begin{array}{l}
5a^3+8ab^2-b^3-11 \\
-a^3\qquad +b^3 \\
\hline
4a^3+8ab^2\qquad -11
\end{array}$$

Rta. $5a^3+8ab^2-b^3-11$

8.

$$\begin{array}{r}
\frac{1}{2}x-\frac{1}{3}y-4 \\
-\frac{1}{2}x+\frac{1}{3}y \\
\hline
-4
\end{array}$$

Rta. $\frac{1}{2}x-\frac{1}{3}y-4$

9. $5x^2-7xy-8y^2$

$-5x^2+7xy+8y^2+1$

1

$Rta. -5x^2+7xy+8y^2+1$

10. n^3

$-9m^3+8m^2n-5mn^2+n^3$

$-9m^3+8m^2n-5mn^2+2n^3$

$10m^3-8m^2n+5mn^2-2n^3$

m^3

$Rta.\ 10m^3-8m^2n+5mn^2-2n^3$

11. 0

$-a^3+5a-8$

$-a^3+5a-8$

$Rta. 0$

EJERCICIO 31

1. $x-(x-y)$
$=x-x+y$
$=y$

2. $x^2+(-3x-x^2+5)$
$=x^2-3x-x^2+5$
$=-3x+5$

3. $a+b-(-2a+3)$
$=a+b+2a-3$
$=3a+b-3$

4. $4m-(-2m-n)$
$=4m+2m+n$
$=6m+n$

5. $2x+3y-(4x+3y)$
$=2x+3y-4x-3y$
$=-2x$

6. $a+(a-b)+(-a+b)$
$=a+a-b-a+b$
$=2a-a=a$

7. $a^2+[-b^2+2a^2]-[a^2-b^2]$
$=a^2-b^2+2a^2-a^2+b^2$
$=3a^2-a^2=2a^2$

8. $2a-\{-x+a-1\}-\{a+x-3\}$
$=2a+x-a+1-a-x+3$
$=1+3=4$

9. $x^2+y^2-(x^2+2xy+y^2)+[-x^2+y^2]$
$=x^2+y^2-x^2-2xy-y^2-x^2+y^2$
$=-x^2-2xy+y^2$

10. $(-5m+6)+(-m+5)-6$
$=-5m+6-m+5-6$
$=-6m+5$

11. $x+y+(x-y+z)-(x+y-z)$
$=x+y+x-y+z-x-y+z$
$=x-y+2z$

12. $a-(b+a)+(-a+b)-(-a+2b)$
$=a-b-a-a+b+a-2b$
$=-2b$

13. $-(x^2-y^2)+xy+(-2x^2+3xy)-[-y^2+xy]$
$=-x^2+y^2+xy-2x^2+3xy+y^2-xy$
$=-3x^2+3xy+2y^2$

14. $8x^2+[-2xy+y^2]-\{-x^2+xy-3y^2\}-(x^2-3xy)$
$=8x^2-2xy+y^2+x^2-xy+3y^2-x^2+3xy$
$=8x^2+4y^2$

15. $-(a+b)+(-a-b)-(-b+a)+(3a+b)$
$=-a-b-a-b+b-a+3a+b$
$=0$

EJERCICIO 32

1. $2a+[a-(a+b)]$
$=2a+[a-a-b]$
$=2a+a-a-b$
$=2a-b$

2. $3x-[x+y-(2x+y)]$
$=3x-[x+y-2x-y]$
$=3x-x-y+2x+y$
$=4x$

3. $2m-[(m-n)-(m+n)]$
$=2m-[m-n-m-n]$
$=2m-m+n+m+n$
$=2m+2n$

4. $4x^2+\left[-\left(x^2-xy\right)+\left(-3y^2+2xy\right)-\left(-3x^2+y^2\right)\right]$
$= 4x^2+\left[-x^2+xy-3y^2+2xy+3x^2-y^2\right]$
$=4x^2-x^2+xy-3y^2+2xy+3x^2-y^2$
$=6x^2+3xy-4y^2$

5. $a+\left\{\left(-2a+b\right)-\left(-a+b-c\right)+a\right\}$
$=a+\left\{-2a+b+a-b+c+a\right\}$
$=a-2a+b+a-b+c+a$
$=a+c$

6. $4m-\left[2m+\left(n-3\right)\right]+\left[-4n-\left(2m+1\right)\right]$
$=4m-\left[2m+n-3\right]+\left[-4n-2m-1\right]$
$=4m-2m-n+3-4n-2m-1$
$=-5n+2$

7. $2x+\left[-5x-\left(-2y+\left\{-x+y\right\}\right)\right]$
$=2x+\left[-5x-\left(-2y-x+y\right)\right]$
$=2x+\left[-5x+2y+x-y\right]$
$=2x-5x+2y+x-y$
$=-2x+y$

8. $x^2-\left\{-7xy+\left[-y^2+\left(-x^2+3xy-2y^2\right)\right]\right\}$
$=x^2-\left\{-7xy+\left[-y^2-x^2+3xy-2y^2\right]\right\}$
$=x^2-\left\{-7xy-y^2-x^2+3xy-2y^2\right\}$
$=x^2+7xy+y^2+x^2-3xy+2y^2$
$=2x^2+4xy+3y^2$

9. $-\left(a+b\right)+\left[-3a+b-\left\{-2a+b-\left(a-b\right)\right\}+2a\right]$
$=-a-b+\left[-a+b-\left\{-2a+b-a+b\right\}\right]$
$=-a-b+\left[-a+b+2a-b+a-b\right]$
$=-a-b-a+b+2a-b+a-b$
$=a-2b$

10. $\left(-x+y\right)-\left\{4x+2y+\left[-x-y-\left(x+y\right)\right]\right\}$
$=-x+y-\left\{4x+2y+\left[-x-y-x-y\right]\right\}$
$=-x+y-\left\{4x+2y-x-y-x-y\right\}$
$=-x+y-4x-2y+x+y+x+y$
$=-3x+y$

11. $-\left(-a+b\right)+\left[-\left(a+b\right)-\left(-2a+3b\right)+\left(-b+a-b\right)\right]$
$=a-b+\left[-a-b+2a-3b-b+a-b\right]$
$=a-b-a-b+2a-3b-b+a-b$
$=3a-7b$

12. $7m^2-\left\{-\left[m^2+3n-\left(5-n\right)-\left(-3+m^2\right)\right]\right\}-\left(2n+3\right)$
$=7m^2-\left\{-\left[m^2+3n-5+n+3-m^2\right]\right\}-2n-3$
$=7m^2-\left\{-m^2-3n+5-n-3+m^2\right\}-2n-3$
$=7m^2+m^2+3n-5+n+3-m^2-2n-3$
$=7m^2+2n-5$

13. $2a-\left(-4a+b\right)-\left\{-\left[-4a+\left(b-a\right)-\left(-b+a\right)\right]\right\}$
$=2a+4a-b-\left\{-\left[-4a+b-a+b-a\right]\right\}$
$=6a-b-\left\{4a-b+a-b+a\right\}$
$=6a-b-4a+b-a+b-a$
$=b$

14. $3x-\left(5y+\left[-2x+\left\{y-\left(6+x\right)\right\}-\left(-x+y\right)\right]\right)$
$=3x-\left(5y+\left[-2x+\left\{y-6-x\right\}+x-y\right]\right)$
$=3x-\left(5y+\left[-2x+y-6-x+x-y\right]\right)$
$=3x-\left(5y-2x+y-6-x+x-y\right)$
$=3x-5y+2x-y+6+x-x+y$
$=5x-5y+6$

15. $6c-\left[-\left(2a+c\right)+\left\{-\left(a+c\right)-2a-\left(a+c\right)\right\}+2c\right]$
$=6c-\left[-2a-c+\left\{-a-c-2a-a-c\right\}+2c\right]$
$=6c-\left[-2a-c-a-c-2a-a-c+2c\right]$
$=6c+2a+c+a+c+2a+a+c-2c$
$=6a+7c$

16. $-\left(3m+n\right)-\left[2m+\left\{-m+\left(2m-\left(2n-5\right)\right)\right\}-\left(n+6\right)\right]$
$=-3m-n-\left[2m+\left\{-m+\left(2m-2n+5\right)\right\}-n-6\right]$
$=-3m-n-\left[2m+\left\{-m+2m-2n+5\right\}-n-6\right]$
$=-3m-n-\left[2m-m+2m-2n+5-n-6\right]$
$=-3m-n-2m+m-2m+2n-5+n+6$
$=-6m+2n+1$

17. $2a+\left\{-\left[5b+\left(3a-c\right)+2-\left(-a+b-\left(c+4\right)\right)\right]-\left(-a+b\right)\right\}$

$=2a+\left\{-\left[5b+3a-c+2-\left(-a+b-c-4\right)\right]+a-b\right\}$

$=2a+\left\{-\left[5b+3a-c+2+a-b+c+4\right]+a-b\right\}$

$=2a+\left\{-5b-3a+c-2-a+b-c-4+a-b\right\}$

$=2a-5b-3a+c-2-a+b-c-4+a-b$

$=-a-5b-6$

18. $-\left[-3x+\left(-x-\left(2y-3\right)\right)\right]+\left\{-\left(2x+y\right)+\left(-x-3\right)+2-\left(x+y\right)\right\}$

$=-\left[-3x+\left(-x-2y+3\right)\right]+\left\{-2x-y-x-3+2-x-y\right\}$

$=-\left[-3x-x-2y+3\right]-2x-y-x-3+2-x-y$

$=3x+x+2y-3-4x-2y-1$

$=-4$

19. $-\left[-\left(-a\right)\right]-\left[+\left(-a\right)\right]+\left\{-\left[-b+c\right]-\left[+\left(-c\right)\right]\right\}$

$=-\left[a\right]-\left[-a\right]+\left\{b-c-\left[-c\right]\right\}$

$=-a+a+\left\{b-c+c\right\}$

$=b-c+c$

$=b$

20. $-\left\{-\left[-\left(a+b\right)\right]\right\}-\left\{+\left[-\left(-b-a\right)\right]\right\}-\left(a+b\right)$

$=-\left\{-\left[-a-b\right]\right\}-\left\{+\left[b+a\right]\right\}-a-b$

$=-\left\{a+b\right\}-\left\{b+a\right\}-a-b$

$=-a-b-b-a-a-b$

$=-3a-3b$

21. $-\left\{-\left[-\left(a+b-c\right)\right]\right\}-\left\{+\left[-\left(c-a+b\right)\right]\right\}+\left[-\left\{-a+\left(-b\right)\right\}\right]$

$=-\left\{-\left[-a-b+c\right]\right\}-\left\{+\left[-c+a-b\right]\right\}+\left[-\left\{-a-b\right\}\right]$

$=-\left\{a+b-c\right\}-\left\{-c+a-b\right\}+\left[a+b\right]$

$=-a-b+c+c-a+b+a+b$

$=-a+b+2c$

22. $-\left[3m+\left\{-m-\left(n-\left(m+4\right)\right)\right\}+\left\{-\left(m+n\right)+\left(-2n+3\right)\right\}\right]$

$=-\left[3m+\left\{-m-\left(n-m-4\right)\right\}+\left\{-m-n-2n+3\right\}\right]$

$=-\left[3m+\left\{-m-n+m+4\right\}-m-n-2n+3\right]$

$=-\left[3m-m-n+m+4-m-n-2n+3\right]$

$=-3m+m+n-m-4+m+n+2n-3$

$=-2m+4n-7$

23. $-\left[x+\left\{-\left(x+y\right)-\left[-x+\left(y-z\right)-\left(-x+y\right)\right]-y\right\}\right]$

$=-\left[x+\left\{-x-y-\left[-x+y-z+x-y\right]-y\right\}\right]$

$=-\left[x+\left\{-x-y+x-y+z-x+y-y\right\}\right]$

$=-\left[x-x-2y+z\right]$

$=-x+x+2y-z=2y-z$

24. $-\left[-a+\left\{-a+\left(a-b\right)-\left(a-b+c\right)-\left[-\left(-a\right)+b\right]\right\}\right]$

$=-\left[-a+\left\{-a+a-b-a+b-c-\left[a+b\right]\right\}\right]$

$=-\left[-a+\left\{-a-c-a-b\right\}\right]$

$=-\left[-a-a-c-a-b\right]=3a+b+c$

EJERCICIO 33

1. $a-b+c-d$

$=a+\left(-b+c-d\right)$

2. $x^2-3xy-y^2+6$

$=x^2+\left(-3xy-y^2+6\right)$

3. x^3+4x^2-3x+1

$=x^3+\left(4x^2-3x+1\right)$

4. $a^3-5a^2b+3ab^2-b^3$

$=a^3+\left(-5a^2b+3ab^2-b^3\right)$

5. $x^4-x^3+2x^2-2x+1$

$=x^4-x^3+\left(2x^2-2x+1\right)$

6. $2a+b-c+d$

$=2a-\left(-b+c-d\right)$

7. x^3+x^2+3x-4

$=x^3-\left(-x^2-3x+4\right)$

8. $x^3-5x^2y+3xy^2-y^3$

$=x^3-\left(5x^2y-3xy^2+y^3\right)$

9. $a^2-x^2-2xy-y^2$

$=a^2-\left(x^2+2xy+y^2\right)$

10. $a^2+b^2-2bc-c^2$

$=a^2-\left(-b^2+2bc+c^2\right)$

EJERCICIO 34

1. $x+2y+(x-y)$
$=x-[-2y-(x-y)]$

2. $4m-2n+3-(-m+n)+(2m-n)$
$=4m-[2n-3+(-m+n)-(2m-n)]$

3. $x^2-3xy+[(x^2-xy)+y^2]$
$=x^2-\{3xy-[(x^2-xy)+y^2]\}$

4. $x^3-3x^2+[-4x+2]-3x-(2x+3)$
$=x^3-[3x^2-[-4x+2]+3x+(2x+3)]$

5. $2a+3b-\{-2a+[a+(b-a)]\}$
$=2a-[-3b+\{-2a+[a+(b-a)]\}]$

6. $-2a+(-3a+b)$
$=-[2a-(-3a+b)]$

7. $2x^2+3xy-(y^2+xy)+(-x^2+y^2)$
$=-[-2x^2-3xy+(y^2+xy)-(-x^2+y^2)]$

8. $x^3-[-3x^2+4x-2]$
$=-\{x^3+[-3x^2+4x-2]\}$

9. $[m^4-(3m^2+2m+3)]+(-2m+3)$
$=-\{-[m^4-(3m^2+2m+3)]-(-2m+3)\}$

EJERCICIO 35

1. $2\cdot -3=-6$

2. $-4\cdot -8=32$

3. $-15\cdot 16=-240$

4. $ab\cdot -ab=-a^{1+1}b^{1+1}=-a^2b^2$

5. $2x^2\cdot -3x=-6x^{2+1}=-6x^3$

6. $-4a^2b\cdot -ab^2=4a^{2+1}b^{1+2}=4a^3b^3$

7. $-5x^3y\cdot xy^2=-5x^{3+1}y^{1+2}=-5x^4y^3$

8. $a^2b^3\cdot 3a^2x=3a^{2+2}b^3x=3a^4b^3x$

9. $-4m^2\cdot -5mn^2p=20m^{2+1}n^2p=20m^3n^2p$

10. $5a^2y\cdot -6x^2=-30a^2x^2y$

11. $-x^2y^3\cdot -4y^3z^4=4x^2y^{3+3}z^4=4x^2y^6z^4$

12. $abc\cdot cd=abc^{1+1}d=abc^2d$

13. $-15x^4y^3\cdot -16a^2x^3=240a^2x^{4+3}y^3=240a^2x^7y^3$

14. $3a^2b^3\cdot -4x^2y=-12a^2b^3x^2y$

15. $3a^2bx\cdot 7b^3x^5=21a^2b^{1+3}x^{1+5}=21a^2b^4x^6$

16. $-8m^2n^3\cdot -9a^2mx^4=72a^2m^{2+1}n^3x^4=72a^2m^3n^3x^4$

17. $a^mb^n\cdot -ab=-a^{m+1}b^{n+1}$

18. $-5a^mb^n\cdot -6a^2b^3x=30a^{m+2}b^{n+3}x$

19. $x^my^nc\cdot -x^my^nc^x=-x^{m+m}y^{n+n}c^{1+x}=-x^{2m}y^{2n}c^{1+x}$

20. $-m^xn^a\cdot -6m^2n=6m^{x+2}n^{a+1}$

EJERCICIO 36

1. $a^m\cdot a^{m+1}=a^{m+m+1}=a^{2m+1}$

2. $-x^a\cdot -x^{a+2}=x^{a+a+2}=x^{2a+2}$

3. $4a^nb^x\cdot -ab^{x+1}=-4a^{n+1}b^{x+x+1}=-4a^{n+1}b^{2x+1}$

4. $-a^{n+1}b^{n+2}\cdot a^{n+2}b^n=-a^{n+1+n+2}b^{n+2+n}=-a^{2n+3}b^{2n+2}$

5. $-3a^{n+4}b^{n+1}\cdot -4a^{n+2}b^{n+3}=12a^{2n+6}b^{2n+4}$

6. $3x^2y^3\cdot 4x^{m+1}y^{m+2}=12x^{2+m+1}y^{3+m+2}=12x^{m+3}y^{m+5}$

7. $4x^{a+2}b^{a+4}\cdot -5x^{a+5}b^{a+1}=-20x^{2a+7}b^{2a+5}$

8. $a^mb^nc\cdot -a^mb^{2n}=-a^{m+m}b^{n+2n}c=-a^{2m}b^{3n}c$

9. $-x^{m+1}y^{a+2}\cdot -4x^{m-3}y^{a-5}c^2=4x^{2m-2}y^{2a-3}c^2$

10. $-5m^an^{b-1}c\cdot -7m^{2a-3}n^{b-4}=35m^{3a-3}n^{2b-5}c$

EJERCICIO 37

1. $\frac{1}{2}a^2\cdot\frac{4}{5}a^3b=\frac{1}{2}\cdot\frac{4}{5}a^5b=\frac{4}{10}a^5b=\frac{2}{5}a^5b$

2. $-\frac{3}{7}m^2n\cdot-\frac{7}{14}a^2m^3$

$=-\frac{3}{7}\cdot-\frac{7}{14}a^2m^5n=\frac{21}{98}a^2m^5n=\frac{3}{14}a^2m^5n$

3. $\frac{2}{3}x^2y^3\cdot-\frac{3}{5}a^2x^4y=-\frac{6}{15}a^2x^6y^4=-\frac{2}{5}a^2x^6y^4$

4. $-\frac{1}{8}m^3n^4\cdot-\frac{4}{5}a^3m^2n=\frac{4}{40}a^3m^5n^5=\frac{1}{10}a^3m^5n^5$

5. $-\frac{7}{8}abc\cdot\frac{2}{7}a^3=-\frac{14}{56}a^4bc=-\frac{1}{4}a^4bc$

6. $-\frac{3}{5}x^3y^4\cdot-\frac{5}{6}a^2by^5=\frac{3}{6}a^2bx^3y^9=\frac{1}{2}a^2bx^3y^9$

7. $\frac{1}{3}a\cdot\frac{3}{5}a^m=\frac{3}{15}a^{m+1}=\frac{1}{5}a^{m+1}$

8. $-\frac{3}{4}a^m\cdot-\frac{2}{5}ab^3=\frac{3}{2}\cdot\frac{1}{5}a^{m+1}b^3=\frac{3}{10}a^{m+1}b^3$

9. $\frac{5}{6}a^mb^n\cdot-\frac{3}{10}ab^2c$

$=-\frac{1}{2}\cdot\frac{1}{2}a^{m+1}b^{n+2}c=-\frac{1}{4}a^{m+1}b^{n+2}c$

10. $-\frac{2}{9}a^xb^{m+1}\cdot-\frac{3}{5}a^{x-1}b^m$

$=\frac{2}{3}\cdot\frac{1}{5}a^{2x-1}b^{2m+1}=\frac{2}{15}a^{2x-1}b^{2m+1}$

11. $\frac{3}{8}a^mb^n\cdot-\frac{4}{5}a^{2m}b^n=-\frac{12}{40}a^{3m}b^{2n}=-\frac{3}{10}a^{3m}b^{2n}$

12. $-\frac{2}{11}a^{x+1}b^{x-3}c^2\cdot-\frac{44}{7}a^{x-3}b^2$

$=2\cdot\frac{4}{7}a^{2x-2}b^{x-1}c^2=\frac{8}{7}a^{2x-2}b^{x-1}c^2$

EJERCICIO 38

1. $a\cdot-3a\cdot a^2=-3a^{1+1+2}=-3a^4$

2. $3x^2\cdot-x^3y\cdot-a^2x=3a^2x^{2+3+1}y=3a^2x^6y$

3. $-m^2n\cdot-3m^2\cdot-5mn^3=-15m^{2+2+1}n^{1+3}=-15m^5n^4$

4. $4a^2\cdot-5a^3x^2\cdot-ay^2=20a^{2+3+1}x^2y^2=20a^6x^2y^2$

5. $-a^m\cdot-2ab\cdot-3a^2b^x=-6a^{m+1+2}b^{1+x}=-6a^{m+3}b^{x+1}$

6. $\frac{1}{2}x^3\cdot-\frac{2}{3}a^2x\cdot-\frac{3}{5}a^4m=\frac{6}{30}a^{2+4}x^{3+1}m=\frac{1}{5}a^6x^4m$

7. $\frac{2}{3}a^m\cdot\frac{3}{4}a^2b^4\cdot-3a^4b^{x+1}$

$=-\frac{18}{12}a^{m+2+4}b^{4+x+1}=-\frac{3}{2}a^{m+6}b^{x+5}$

8. $-\frac{3}{5}m^3\cdot-5a^2m\cdot-\frac{1}{10}a^xm^a$

$=-\frac{15}{50}a^{2+x}m^{3+1+a}=-\frac{3}{10}a^{x+2}m^{a+4}$

9. $2a\cdot-a^2\cdot-3a^3\cdot4a=24a^{1+2+3+1}=24a^7$

10. $-3b^2\cdot-4a^3b\cdot ab\cdot-5a^2x$

$=-60a^{3+1+2}b^{2+1+1}x=-60a^6b^4x$

11. $a^mb^x\cdot-a^2\cdot-2ab\cdot-3a^2x$

$=-6a^{m+2+1+2}b^{x+1}x=-6a^{m+5}b^{x+1}x$

12. $-\frac{1}{2}x^2y\cdot-\frac{3}{5}xy^2\cdot-\frac{10}{3}x^3\cdot-\frac{3}{4}x^2y$

$=\frac{6}{8}x^{2+1+3+2}y^{1+2+1}=\frac{3}{4}x^8y^4$

EJERCICIO 39

1. $$\begin{array}{r} 3x^3-x^2 \\ -2x \\ \hline -6x^4+2x^3 \end{array}$$

2. $$\begin{array}{r} 8x^2y-3y^2 \\ 2ax^3 \\ \hline 16ax^5y-6ax^3y^2 \end{array}$$

3. $$\begin{array}{r} x^2-4x+3 \\ -2x \\ \hline -2x^3+8x^2-6x \end{array}$$

4. $$\begin{array}{r} a^3-4a^2+6a \\ 3ab \\ \hline 3a^4b-12a^3b+18a^2b \end{array}$$

5. $$\begin{array}{l} a^2-2ab+b^2 \\ -ab \\ \hline -a^3b+2a^2b^2-ab^3 \end{array}$$

6. $$\begin{array}{l} x^5-6x^3-8x \\ 3a^2x^2 \\ \hline 3a^2x^7-18a^2x^5-24a^2x^3 \end{array}$$

7. $$\begin{array}{l} m^4-3m^2n^2+7n^4 \\ -4m^3x \\ \hline -4m^7x+12m^5n^2x-28m^3n^4x \end{array}$$

8. $$\begin{array}{l} x^3-4x^2y+6xy^2 \\ ax^3y \\ \hline ax^6y-4ax^5y^2+6ax^4y^3 \end{array}$$

9. $$\begin{array}{l} a^3-5a^2b-8ab^2 \\ -4a^4m^2 \\ \hline -4a^7m^2+20a^6bm^2+32a^5b^2m^2 \end{array}$$

10. $$\begin{array}{l} a^m-a^{m-1}+a^{m-2} \\ -2a \\ \hline -2a^{m+1}+2a^{m-1+1}-2a^{m-2+1} \\ =-2a^{m+1}+2a^m-2a^{m-1} \end{array}$$

11. $$\begin{array}{l} x^{m+1}+3x^m-x^{m-1} \\ 3x^{2m} \\ \hline 3x^{3m+1}+9x^{3m}-3x^{3m-1} \end{array}$$

12. $$\begin{array}{l} a^mb^n+a^{m-1}b^{n+1}-a^{m-2}b^{n+2} \\ 3a^2b \\ \hline 3a^{m+2}b^{n+1}+3a^{m+1}b^{n+2}-3a^mb^{n+3} \end{array}$$

13. $$\begin{array}{l} x^3-3x^2+5x-6 \\ -4x^2 \\ \hline -4x^5+12x^4-20x^3+24x^2 \end{array}$$

14. $$\begin{array}{l} a^4-6a^3x+9a^2x^2-8 \\ 3bx^3 \\ \hline 3a^4bx^3-18a^3bx^4+27a^2bx^5-24bx^3 \end{array}$$

15. $$\begin{array}{l} a^{n+3}-3a^{n+2}-4a^{n+1}-a^n \\ -a^nx^2 \\ \hline -a^{2n+3}x^2+3a^{2n+2}x^2+4a^{2n+1}x^2+a^{2n}x^2 \end{array}$$

16. $$\begin{array}{l} x^4-6x^3+8x^2-7x+5 \\ -3a^2x^3 \\ \hline -3a^2x^7+18a^2x^6-24a^2x^5+21a^2x^4-15a^2x^3 \end{array}$$

17. $$\begin{array}{l} -3x^3+5x^2y-7xy^2-4y^3 \\ 5a^2xy^2 \\ \hline -15a^2x^4y^2+25a^2x^3y^3-35a^2x^2y^4-20a^2xy^5 \end{array}$$

18. $$\begin{array}{l} x^{a+5}-3x^{a+4}+x^{a+3}-5x^{a+1} \\ -2x^2 \\ \hline -2x^{a+7}+6x^{a+6}-2x^{a+5}+10x^{a+3} \end{array}$$

19. $$\begin{array}{l} a^8-3a^6b^2+a^4b^4-3a^2b^6+b^8 \\ -5a^3y^2 \\ \hline -5a^{11}y^2+15a^9b^2y^2-5a^7b^4y^2+15a^5b^6y^2-5a^3b^8y^2 \end{array}$$

20. $$\begin{array}{l} a^mb^n+3a^{m-1}b^{n+2}-a^{m-2}b^{n+4}+a^{m-3}b^{n+6} \\ 4a^mb^3 \\ \hline 4a^{2m}b^{n+3}+12a^{2m-1}b^{n+5}-4a^{2m-2}b^{n+7}+4a^{2m-3}b^{n+9} \end{array}$$

EJERCICIO 40

1. $$\begin{array}{l} \frac{1}{2}a-\frac{2}{3}b \\ \frac{2}{5}a^2 \\ \hline \frac{2}{10}a^{1+2}-\frac{4}{15}a^2b \\ =\frac{1}{5}a^3-\frac{4}{15}a^2b \end{array}$$

2. $$\begin{array}{l} \frac{2}{3}a-\frac{3}{4}b \\ -\frac{2}{3}a^3b \\ \hline -\frac{4}{9}a^{3+1}b+\frac{6}{12}a^3b^{1+1} \\ =-\frac{4}{9}a^4b+\frac{1}{2}a^3b^2 \end{array}$$

3. $$\begin{array}{l} \frac{3}{5}a-\frac{1}{6}b+\frac{2}{5}c \\ -\frac{5}{3}ac^2 \\ \hline -\frac{15}{15}a^{1+1}c^2+\frac{5}{18}abc^2-\frac{10}{15}ac^{2+1} \\ =-a^2c^2+\frac{5}{18}abc^2-\frac{2}{3}ac^3 \end{array}$$

4. $$\begin{array}{l} \frac{2}{5}a^2+\frac{1}{3}ab-\frac{2}{9}b^2 \\ 3a^2x \\ \hline \frac{6}{5}a^{2+2}x+\frac{3}{3}a^{2+1}bx-\frac{6}{9}a^2b^2x \\ =\frac{6}{5}a^4x+a^3bx-\frac{2}{3}a^2b^2x \end{array}$$

5.
$$\begin{array}{l}
\frac{1}{3}x^2-\frac{2}{5}xy-\frac{1}{4}y^2\\
\frac{3}{2}y^3\\
\hline
\frac{3}{6}x^2y^3-\frac{6}{10}xy^{3+1}-\frac{3}{8}y^{2+3}\\
=\frac{1}{2}x^2y^3-\frac{3}{5}xy^4-\frac{3}{8}y^5
\end{array}$$

6.
$$\begin{array}{l}
3a-5b+6c\\
-\frac{3}{10}a^2x^3\\
\hline
-\frac{9}{10}a^{2+1}x^3+\frac{15}{10}a^2bx^3-\frac{18}{10}a^2cx^3\\
=-\frac{9}{10}a^3x^3+\frac{3}{2}a^2bx^3-\frac{9}{5}a^2cx^3
\end{array}$$

7.
$$\begin{array}{l}
\frac{2}{9}x^4-x^2y^2+\frac{1}{3}y^4\\
\frac{3}{7}x^3y^4\\
\hline
\frac{6}{63}x^{4+3}y^4-\frac{3}{7}x^{2+3}y^{2+4}+\frac{3}{21}x^3y^{4+4}\\
=\frac{2}{21}x^7y^4-\frac{3}{7}x^5y^6+\frac{1}{7}x^3y^8
\end{array}$$

8.
$$\begin{array}{l}
\frac{1}{2}a^2-\frac{1}{3}b^2+\frac{1}{4}x^2-\frac{1}{5}y^2\\
-\frac{5}{8}a^2m\\
\hline
-\frac{5}{16}a^{2+2}m+\frac{5}{24}a^2b^2m-\frac{5}{32}a^2mx^2+\frac{5}{40}a^2my^2\\
=-\frac{5}{16}a^4m+\frac{5}{24}a^2b^2m-\frac{5}{32}a^2mx^2+\frac{1}{8}a^2my^2
\end{array}$$

9.
$$\begin{array}{l}
\frac{2}{3}m^3+\frac{1}{2}m^2n-\frac{5}{6}mn^2-\frac{1}{9}n^3\\
\frac{3}{4}m^2n^3\\
\hline
\frac{6}{12}m^{3+2}n^3+\frac{3}{8}m^{2+2}n^{1+3}-\frac{15}{24}m^{1+2}n^{2+3}-\frac{3}{36}m^2n^{3+3}\\
=\frac{1}{2}m^5n^3+\frac{3}{8}m^4n^4-\frac{5}{8}m^3n^5-\frac{1}{12}m^2n^6
\end{array}$$

10.
$$\begin{array}{l}
\frac{2}{5}x^6-\frac{1}{3}x^4y^2+\frac{3}{5}x^2y^4-\frac{1}{10}y^6\\
-\frac{5}{7}a^3x^4y^3\\
\hline
-\frac{10}{35}a^3x^{6+4}y^3+\frac{5}{21}a^3x^{4+4}y^{2+3}-\frac{15}{35}a^3x^{2+4}y^{4+3}+\frac{5}{70}a^3x^4y^{6+3}\\
=-\frac{2}{7}a^3x^{10}y^3+\frac{5}{21}a^3x^8y^5-\frac{3}{7}a^3x^6y^7+\frac{1}{14}a^3x^4y^9
\end{array}$$

EJERCICIO 41

1.
$$\begin{array}{r}
a+3\\
a-1\\
\hline
a^2+3a\\
-a-3\\
\hline
a^2+2a-3
\end{array}$$

2.
$$\begin{array}{r}
a-3\\
a+1\\
\hline
a^2-3a\\
a-3\\
\hline
a^2-2a-3
\end{array}$$

3.
$$\begin{array}{r}
x+5\\
x-4\\
\hline
x^2+5x\\
-4x-20\\
\hline
x^2+x-20
\end{array}$$

4.
$$\begin{array}{r}
m-6\\
m-5\\
\hline
m^2-6m\\
-5m+30\\
\hline
m^2-11m+30
\end{array}$$

5.
$$\begin{array}{r}
-x+3\\
-x+5\\
\hline
x^2-3x\\
-5x+15\\
\hline
x^2-8x+15
\end{array}$$

6.
$$\begin{array}{r}
-a-2\\
-a-3\\
\hline
a^2+2a\\
+3a+6\\
\hline
a^2+5a+6
\end{array}$$

7.
$$\begin{array}{r}
3x-2y\\
2x+y\\
\hline
6x^2-4xy\\
+3xy-2y^2\\
\hline
6x^2-xy-2y^2
\end{array}$$

8.
$$\begin{array}{r}
5x-4y\\
-3x+2y\\
\hline
-15x^2+12xy\\
+10xy-8y^2\\
\hline
-15x^2+22xy-8y^2
\end{array}$$

9.
$$\begin{array}{l} 5a-7b \\ \;\; a+3b \\ \hline 5a^2-7ab \\ \qquad +15ab-21b^2 \\ \hline 5a^2+8ab-21b^2 \end{array}$$

10.
$$\begin{array}{l} 7x-3 \\ 2x+4 \\ \hline 14x^2-6x \\ \qquad +28x-12 \\ \hline 14x^2+22x-12 \end{array}$$

11.
$$\begin{array}{l} -a+b \\ 8a-4b \\ \hline -8a^2+8ab \\ \qquad +4ab-4b^2 \\ \hline -8a^2+12ab-4b^2 \end{array}$$

12.
$$\begin{array}{l} 6m-5n \\ m-n \\ \hline 6m^2-5mn \\ \qquad -6mn+5n^2 \\ \hline 6m^2-11mn+5n^2 \end{array}$$

13.
$$\begin{array}{l} -9m+8n \\ \;\; 6m+4n \\ \hline -54m^2+48mn \\ \qquad -36mn+32n^2 \\ \hline -54m^2+12mn+32n^2 \end{array}$$

14.
$$\begin{array}{l} -7y-3 \\ \;\; 2y-11 \\ \hline -14y^2-6y \\ \qquad +77y+33 \\ \hline -14y^2+71y+33 \end{array}$$

EJERCICIO 42

1.
$$\begin{array}{l} x^2+xy+y^2 \\ x-y \\ \hline x^3+x^2y+xy^2 \\ \quad -x^2y-xy^2-y^3 \\ \hline x^3 \qquad\qquad -y^3 \end{array}$$

2.
$$\begin{array}{l} a^2-2ab+b^2 \\ a-b \\ \hline a^3-2a^2b+ab^2 \\ \quad -a^2b+2ab^2-b^3 \\ \hline a^3-3a^2b+3ab^2-b^3 \end{array}$$

3.
$$\begin{array}{l} a^2+2ab+b^2 \\ a+b \\ \hline a^3+2a^2b+ab^2 \\ \quad +a^2b+2ab^2+b^3 \\ \hline a^3+3a^2b+3ab^2+b^3 \end{array}$$

4.
$$\begin{array}{l} x^3-3x^2+1 \\ x+3 \\ \hline x^4-3x^3 \qquad +x \\ \quad +3x^3-9x^2 \quad +3 \\ \hline x^4 \qquad -9x^2+x+3 \end{array}$$

5.
$$\begin{array}{l} a^3+a^2-a \\ a-1 \\ \hline a^4+a^3-a^2 \\ \quad -a^3-a^2+a \\ \hline a^4 \quad -2a^2+a \end{array}$$

6.
$$\begin{array}{l} m^4+m^2n^2+n^4 \\ m^2-n^2 \\ \hline m^6+m^4n^2+m^2n^4 \\ \quad -m^4n^2-m^2n^4-n^6 \\ \hline m^6 \qquad\qquad -n^6 \end{array}$$

7.
$$\begin{array}{l} x^3-2x^2+3x-1 \\ 2x+3 \\ \hline 2x^4-4x^3+6x^2-2x \\ \quad +3x^3-6x^2+9x-3 \\ \hline 2x^4-x^3 \qquad +7x-3 \end{array}$$

9.
$$\begin{array}{l} 3y^3-6y+5 \\ y^2+2 \\ \hline 3y^5-6y^3+5y^2 \\ \quad +6y^3 \qquad -12y+10 \\ \hline 3y^5 \qquad +5y^2-12y+10 \end{array}$$

9.
$$\begin{array}{l} m^3-m^2+m-2 \\ am+a \\ \hline am^4-am^3+am^2-2am \\ \quad +am^3-am^2+am-2a \\ \hline am^4 \qquad\qquad -am-2a \end{array}$$

10.
$$\begin{array}{l} 3a^2-5ab+2b^2 \\ 4a-5b \\ \hline 12a^3-20a^2b+8ab^2 \\ \quad -15a^2b+25ab^2-10b^3 \\ \hline 12a^3-35a^2b+33ab^2-10b^3 \end{array}$$

11.
$$\begin{array}{l} 5m^4-3m^2n^2+n^4 \\ 3m-n \\ \hline 15m^5 \qquad -9m^3n^2 \qquad +3mn^4 \\ \quad -5m^4n \qquad +3m^2n^3 \qquad -n^5 \\ \hline 15m^5-5m^4n-9m^3n^2+3m^2n^3+3mn^4-n^5 \end{array}$$

12.
$$\begin{array}{l} a^2+a+1 \\ a^2-a-1 \\ \hline a^4+a^3+a^2 \\ \quad -a^3-a^2-a \\ \qquad -a^2-a-1 \\ \hline a^4 \quad -a^2-2a-1 \end{array}$$

13.
$$\begin{array}{l}
x^3+2x^2-x \\
x^2-2x+5 \\ \hline
x^5+2x^4-x^3 \\
\quad -2x^4-4x^3+2x^2 \\
\qquad\qquad +5x^3+10x^2-5x \\ \hline
x^5 \qquad\qquad\qquad +12x^2-5x
\end{array}$$

14.
$$\begin{array}{l}
m^3-3m^2n+2mn^2 \\
m^2-2mn-8n^2 \\ \hline
m^5-3m^4n+2m^3n^2 \\
\quad -2m^4n+6m^3n^2-4m^2n^3 \\
\qquad\qquad -8m^3n^2+24m^2n^3-16mn^4 \\ \hline
m^5-5m^4n \qquad\qquad +20m^2n^3-16mn^4
\end{array}$$

15.
$$\begin{array}{l}
x^2+x+1 \\
x^2-x-1 \\ \hline
x^4+x^3+x^2 \\
\quad -x^3-x^2-x \\
\qquad -x^2-x-1 \\ \hline
x^4 \quad -x^2-2x-1
\end{array}$$

16.
$$\begin{array}{l}
x^4-3x^2+2 \\
x^2-2x+3 \\ \hline
x^6 \quad -3x^4 \quad +2x^2 \\
\quad -2x^5 \quad +6x^3 \quad -4x \\
\qquad +3x^4 \quad -9x^2 \quad +6 \\ \hline
x^6-2x^5 \quad +6x^3-7x^2-4x+6
\end{array}$$

17.
$$\begin{array}{l}
m^3+m^2-4m-1 \\
m^3+1 \\ \hline
m^6+m^5-4m^4-m^3 \\
\qquad\qquad +m^3+m^2-4m-1 \\ \hline
m^6+m^5-4m^4 \qquad +m^2-4m-1
\end{array}$$

18.
$$\begin{array}{l}
a^3-5a+2 \\
a^2-a+5 \\ \hline
a^5 \quad -5a^3+2a^2 \\
\quad -a^4 \quad +5a^2-2a \\
\qquad +5a^3 \quad -25a+10 \\ \hline
a^5-a^4 \quad +7a^2-27a+10
\end{array}$$

19.
$$\begin{array}{l}
x^2-2xy+y^2 \\
-x^2+xy+3y^2 \\ \hline
-x^4+2x^3y-x^2y^2 \\
\quad +x^3y-2x^2y^2+xy^3 \\
\qquad +3x^2y^2-6xy^3+3y^4 \\ \hline
-x^4+3x^3y \qquad -5xy^3+3y^4
\end{array}$$

20.
$$\begin{array}{l}
n^2-2n+1 \\
n^2-1 \\ \hline
n^4-2n^3+n^2 \\
\qquad -n^2+2n-1 \\ \hline
n^4-2n^3 \quad +2n-1
\end{array}$$

21.
$$\begin{array}{l}
a^3-3a^2b+4ab^2 \\
a^2b-2ab^2-10b^3 \\ \hline
a^5b-3a^4b^2+4a^3b^3 \\
\quad -2a^4b^2+6a^3b^3-8a^2b^4 \\
\qquad -10a^3b^3+30a^2b^4-40ab^5 \\ \hline
a^5b-5a^4b^2 \qquad +22a^2b^4-40ab^5
\end{array}$$

22.
$$\begin{array}{l}
8x^3-12x^2y+6xy^2-9y^3 \\
2x+3y \\ \hline
16x^4-24x^3y+12x^2y^2-18xy^3 \\
\quad +24x^3y-36x^2y^2+18xy^3-27y^4 \\ \hline
16x^4 \quad -24x^2y^2 \quad -27y^4
\end{array}$$

23.
$$\begin{array}{l}
2y^3-3y^2+y-4 \\
2y+5 \\ \hline
4y^4-6y^3+2y^2-8y \\
\quad +10y^3-15y^2+5y-20 \\ \hline
4y^4+4y^3-13y^2-3y-20
\end{array}$$

24.
$$\begin{array}{l}
-a^3+2ax^2+3x^3 \\
2a^2-3ax-x^2 \\ \hline
-2a^5 \quad +4a^3x^2+6a^2x^3 \\
\quad +3a^4x \quad -6a^2x^3-9ax^4 \\
\qquad +a^3x^2 \quad -2ax^4-3x^5 \\ \hline
-2a^5+3a^4x+5a^3x^2 \quad -11ax^4-3x^5
\end{array}$$

25.
$$\begin{array}{l}
x^4-3x^3y+2x^2y^2+xy^3 \\
-x^2-xy-y^2 \\ \hline
-x^6+3x^5y-2x^4y^2-x^2y^3 \\
\quad -x^5y+3x^4y^2-2x^3y^3-x^2y^4 \\
\qquad -x^4y^2+3x^3y^3-2x^2y^4-xy^5 \\ \hline
-x^6+2x^5y \qquad\qquad -3x^2y^4-xy^5
\end{array}$$

26.
$$\begin{array}{l}
a^3-5a^2+2a-3 \\
a^3-2a-7 \\ \hline
a^6-5a^5+2a^4-3a^3 \\
\quad -2a^4+10a^3-4a^2+6a \\
\qquad -7a^3+35a^2-14a+21 \\ \hline
a^6-5a^5 \qquad +31a^2-8a+21
\end{array}$$

27.
$$\begin{array}{l}
m^4+m^3-m^2+3 \\
m^2-2m+3 \\ \hline
m^6+m^5-m^4 \quad +3m^2 \\
\quad -2m^5-2m^4+2m^3 \quad -6m \\
\qquad +3m^4+3m^3-3m^2 \quad +9 \\ \hline
m^6-m^5 \quad +5m^3 \quad -6m+9
\end{array}$$

28.
$$\begin{array}{l}
a^4+a^3b-3a^2b^2-ab^3+b^4 \\
a^2-2ab+b^2 \\ \hline
a^6+a^5b-3a^4b^2-a^3b^3+a^2b^4 \\
\quad -2a^5b-2a^4b^2+6a^3b^3+2a^2b^4-2ab^5 \\
\qquad +a^4b^2+a^3b^3-3a^2b^4-ab^5+b^6 \\ \hline
a^6-a^5b-4a^4b^2+6a^3b^3 \quad -3ab^5+b^6
\end{array}$$

29.
$$\begin{array}{l}
x^4-x^3y+x^2y^2-xy^3+y^4 \\
x^2+xy-2y^2 \\ \hline
x^6-x^5y+x^4y^2-x^3y^3+x^2y^4 \\
\quad +x^5y-x^4y^2+x^3y^3-x^2y^4+xy^5 \\
\qquad -2x^4y^2+2x^3y^3-2x^2y^4+2xy^5-2y^6 \\ \hline
x^6 \quad -2x^4y^2+2x^3y^3-2x^2y^4+3xy^5-2y^6
\end{array}$$

30.
$$\begin{array}{l}
y^2-2y+1 \\
y^4-2y^2+2 \\ \hline
y^6-2y^5+y^4 \\
\quad -2y^4+4y^3-2y^2 \\
\qquad +2y^2-4y+2 \\ \hline
y^6-2y^5-y^4+4y^3 \quad -4y+2
\end{array}$$

31.
$$\begin{array}{l}
m^4-3m^2+4 \\
3m^3-2m+1 \\ \hline
3m^7-9m^5 \quad +12m^3 \\
\quad -2m^5 \quad +6m^3 \quad -8m \\
\qquad +m^4 \quad -3m^2 \quad +4 \\ \hline
3m^7-11m^5+m^4+18m^3-3m^2-8m+4
\end{array}$$

32.
$$\begin{array}{l}
a^3+a^2-a+1 \\
a^3+a^2-2a-1 \\ \hline
a^6+a^5-a^4+a^3 \\
\quad +a^5+a^4-a^3+a^2 \\
\qquad -2a^4-2a^3+2a^2-2a \\
\qquad\quad -a^3-a^2+a-1 \\ \hline
a^6+2a^5-2a^4-3a^3+2a^2-a-1
\end{array}$$

33.
$$\begin{array}{l}
8x^3-12x^2y-6xy^2+y^3 \\
3x^2-2xy+4y^2 \\ \hline
24x^5-36x^4y-18x^3y^2+3x^2y^3 \\
\quad -16x^4y+24x^3y^2+12x^2y^3-2xy^4 \\
\qquad +32x^3y^2-48x^2y^3-24xy^4+4y^5 \\ \hline
24x^5-52x^4y+38x^3y^2-33x^2y^3-26xy^4+4y^5
\end{array}$$

34.
$$\begin{array}{l}
5a^4-4a^3+2a^2-3a-1 \\
a^4-2a^2+2 \\ \hline
5a^8-4a^7+2a^6-3a^5-a^4 \\
\quad -10a^6+8a^5-4a^4+6a^3+2a^2 \\
\qquad +10a^4-8a^3+4a^2-6a-2 \\ \hline
5a^8-4a^7-8a^6+5a^5+5a^4-2a^3+6a^2-6a-2
\end{array}$$

35.
$$\begin{array}{l}
x^4-x^3+x^2-x+1\\
x^3-2x^2+3x+6\\ \hline
x^7-x^6+x^5-x^4+x^3\\
\quad -2x^6+2x^5-2x^4+2x^3-2x^2\\
\qquad +3x^5-3x^4+3x^3-3x^2+3x\\
\qquad\quad +6x^4-6x^3+6x^2-6x+6\\ \hline
x^7-3x^6+6x^5+x^2-3x+6
\end{array}$$

36.
$$\begin{array}{l}
3a^3+2a^2-5a-4\\
a^3+a^2-2a+1\\ \hline
3a^6+2a^5-5a^4-4a^3\\
\quad +3a^5+2a^4-5a^3-4a^2\\
\qquad -6a^4-4a^3+10a^2+8a\\
\qquad\quad +3a^3+2a^2-5a-4\\ \hline
3a^6+5a^5-9a^4-10a^3+8a^2+3a-4
\end{array}$$

37.
$$\begin{array}{l}
5y^4-3y^3+4y^2+2y\\
y^4-3y^2-1\\ \hline
5y^8-3y^7+4y^6+2y^5\\
\quad -15y^6+9y^5-12y^4-6y^3\\
\qquad -5y^4+3y^3-4y^2-2y\\ \hline
5y^8-3y^7-11y^6+11y^5-17y^4-3y^3-4y^2-2y
\end{array}$$

38.
$$\begin{array}{l}
m^4-2m^3n+3m^2n^2-4n^4\\
-m^3+3m^2n-5mn^2+n^3\\ \hline
-m^7+2m^6n-3m^5n^2+4m^3n^4\\
\quad +3m^6n-6m^5n^2+9m^4n^3-12m^2n^5\\
\qquad -5m^5n^2+10m^4n^3-15m^3n^4+20mn^6\\
\qquad\quad +m^4n^3-2m^3n^4+3m^2n^5-4n^7\\ \hline
-m^7+5m^6n-14m^5n^2+20m^4n^3-13m^3n^4-9m^2n^5+20mn^6-4n^7
\end{array}$$

39.
$$\begin{array}{l}
x^6-3x^4y^2-x^2y^4+y^6\\
x^5-2x^3y^2+3xy^4\\ \hline
x^{11}-3x^9y^2-x^7y^4+x^5y^6\\
\quad -2x^9y^2+6x^7y^4+2x^5y^6-2x^3y^8\\
\qquad +3x^7y^4-9x^5y^6-3x^3y^8+3xy^{10}\\ \hline
x^{11}-5x^9y^2+8x^7y^4-6x^5y^6-5x^3y^8+3xy^{10}
\end{array}$$

40.
$$\begin{array}{l}
3a^5-6a^3+2a^2-3a+2\\
a^4-3a^2+4a-5\\ \hline
3a^9-6a^7+2a^6-3a^5+2a^4\\
\quad -9a^7+18a^5-6a^4+9a^3-6a^2\\
\qquad +12a^6-24a^4+8a^3-12a^2+8a\\
\qquad\quad -15a^5+30a^3-10a^2+15a-10\\ \hline
3a^9-15a^7+14a^6-28a^4+47a^3-28a^2+23a-10
\end{array}$$

41.
$$\begin{array}{l}
a+b-c\\
a-b+c\\ \hline
a^2+ab-ac\\
\quad -ab-b^2+bc\\
\qquad +ac+bc-c^2\\ \hline
a^2-b^2+2bc-c^2
\end{array}$$

42.
$$\begin{array}{l}
x+2y-z\\
x-y+z\\ \hline
x^2+2xy-xz\\
\quad -xy-2y^2+yz\\
\qquad +xz+2yz-z^2\\ \hline
x^2+xy-2y^2+3yz-z^2
\end{array}$$

43.
$$\begin{array}{l}
2x-3y+5z\\
-x+y+2z\\ \hline
-2x^2+3xy-5xz\\
\quad +2xy-3y^2+5yz\\
\qquad +4xz-6yz+10z^2\\ \hline
-2x^2+5xy-xz-3y^2-yz+10z^2
\end{array}$$

44.
$$\begin{array}{l}
x^2-xy-xz+y^2-yz+z^2\\
x+y+z\\ \hline
x^3-x^2y-x^2z+xy^2-xyz+xz^2\\
\quad +x^2y-xy^2-xyz+y^3-y^2z+yz^2\\
\qquad +x^2z-xyz-xz^2+y^2z-yz^2+z^3\\ \hline
x^3-3xyz+y^3+z^3
\end{array}$$

EJERCICIO 43

1.
$$\begin{array}{l} a^{x+2}-a^{x+1}+a^{x} \\ a+1 \\ \hline a^{x+3}-a^{x+2}+a^{x+1} \\ \qquad +a^{x+2}-a^{x+1}+a^{x} \\ \hline a^{x+3} \qquad\qquad\quad +a^{x} \end{array}$$

2.
$$\begin{array}{l} -x^{n+3}+2x^{n+2}+x^{n+1} \\ x^{2}+x \\ \hline -x^{n+5}+2x^{n+4}+x^{n+3} \\ \qquad -x^{n+4}+2x^{n+3}+x^{n+2} \\ \hline -x^{n+5}+x^{n+4}+3x^{n+3}+x^{n+2} \end{array}$$

3.
$$\begin{array}{l} m^{a+2}+m^{a+1}-m^{a}+m^{a-1} \\ m^{2}-2m+3 \\ \hline m^{a+4}+m^{a+3}-m^{a+2}+m^{a+1} \\ \qquad -2m^{a+3}-2m^{a+2}+2m^{a+1}-2m^{a} \\ \qquad\qquad +3m^{a+2}+3m^{a+1}-3m^{a}+3m^{a-1} \\ \hline m^{a+4}-m^{a+3} \qquad\quad +6m^{a+1}-5m^{a}+3m^{a-1} \end{array}$$

4.
$$\begin{array}{l} a^{n+2}+3a^{n+1}-2a^{n} \\ a^{n+1}+a^{n} \\ \hline a^{2n+3}+3a^{2n+2}-2a^{2n+1} \\ \qquad +a^{2n+2}+3a^{2n+1}-2a^{2n} \\ \hline a^{2n+3}+4a^{2n+2}+a^{2n+1}-2a^{2n} \end{array}$$

5.
$$\begin{array}{l} x^{a+2}+2x^{a+1}-x^{a} \\ x^{a+3}-2x^{a+1} \\ \hline x^{2a+5}+2x^{2a+4}-x^{2a+3} \\ \qquad\qquad -2x^{2a+3}-4x^{2a+2}+2x^{2a+1} \\ \hline x^{2a+5}+2x^{2a+4}-3x^{2a+3}-4x^{2a+2}+2x^{2a+1} \end{array}$$

6.
$$\begin{array}{l} a^{x}-2a^{x-1}+3a^{x-2} \\ a^{2}+2a-1 \\ \hline a^{x+2}-2a^{x+1}+3a^{x} \\ \qquad +2a^{x+1}-4a^{x}+6a^{x-1} \\ \qquad\qquad -a^{x}+2a^{x-1}-3a^{x-2} \\ \hline a^{x+2} \qquad\quad -2a^{x}+8a^{x-1}-3a^{x-2} \end{array}$$

7.
$$\begin{array}{l} a^{x}+3a^{x-1}-2a^{x-2} \\ a^{x}-a^{x-1}+a^{x-2} \\ \hline a^{2x}+3a^{2x-1}-2a^{2x-2} \\ \qquad -a^{2x-1}-3a^{2x-2}+2a^{2x-3} \\ \qquad\qquad +a^{2x-2}+3a^{2x-3}-2a^{2x-4} \\ \hline a^{2x}+2a^{2x-1}-4a^{2x-2}+5a^{2x-3}-2a^{2x-4} \end{array}$$

8.
$$\begin{array}{l} m^{a+4}-m^{a+3}-2m^{a+2}+m^{a+1} \\ -m^{a-1}+m^{a-2}+m^{a-3} \\ \hline -m^{2a+3}+m^{2a+2}+2m^{2a+1}-m^{2a} \\ \qquad +m^{2a+2}-m^{2a+1}-2m^{2a}+m^{2a-1} \\ \qquad\qquad +m^{2a+1}-m^{2a}-2m^{2a-1}+m^{2a-2} \\ \hline -m^{2a+3}+2m^{2a+2}+2m^{2a+1}-4m^{2a}-m^{2a-1}+m^{2a-2} \end{array}$$

9.
$$\begin{array}{l} x^{a-1}+2x^{a-2}-x^{a-3}+x^{a-4} \\ x^{a-1}-x^{a-2}-x^{a-3} \\ \hline x^{2a-2}+2x^{2a-3}-x^{2a-4}+x^{2a-5} \\ \qquad -x^{2a-3}-2x^{2a-4}+x^{2a-5}-x^{2a-6} \\ \qquad\qquad -x^{2a-4}-2x^{2a-5}+x^{2a-6}-x^{2a-7} \\ \hline x^{2a-2}+x^{2a-3}-4x^{2a-4} \qquad\qquad -x^{2a-7} \end{array}$$

10.
$$\begin{array}{l} a^{n}b-a^{n-1}b^{2}+2a^{n-2}b^{3}-a^{n-3}b^{4} \\ a^{n}b^{2}-a^{n-2}b^{4} \\ \hline a^{2n}b^{3}-a^{2n-1}b^{4}+2a^{2n-2}b^{5}-a^{2n-3}b^{6} \\ \qquad\qquad -a^{2n-2}b^{5}+a^{2n-3}b^{6}-2a^{2n-4}b^{7}+a^{2n-5}b^{8} \\ \hline a^{2n}b^{3}-a^{2n-1}b^{4}+a^{2n-2}b^{5} \qquad -2a^{2n-4}b^{7}+a^{2n-5}b^{8} \end{array}$$

11.
$$\begin{array}{l} a^{x}+b^{x} \\ a^{m}+b^{m} \\ \hline a^{m+x}+a^{m}b^{x} \\ \qquad +a^{x}b^{m}+b^{m+x} \\ \hline a^{m+x}+a^{m}b^{x}+a^{x}b^{m}+b^{m+x} \end{array}$$

12.
$$\begin{array}{l} a^{x-1}-b^{n-1} \\ a-b \\ \hline a^{x}-ab^{n-1} \\ \qquad -a^{x-1}b+b^{n} \\ \hline a^{x}-ab^{n-1}-a^{x-1}b+b^{n} \end{array}$$

13.

$$
\begin{array}{l}
-5a^{2m+2}+a^{2m+1}+3a^{2m} \\
6a^{3m-1}-8a^{3m-2}+a^{3m-3} \\
\hline
-30a^{5m+1}+6a^{5m}+18a^{5m-1} \\
\qquad +40a^{5m}-8a^{5m-1}-24a^{5m-2} \\
\qquad\qquad -5a^{5m-1}+a^{5m-2}+3a^{5m-3} \\
\hline
-30a^{5m+1}+46a^{5m}+5a^{5m-1}-23a^{5m-2}+3a^{5m-3}
\end{array}
$$

14.

$$
\begin{array}{l}
x^{a+2}y^{x-1}-4x^{a+1}y^{x}+3x^{a}y^{x+1} \\
-2x^{2a-1}y^{x-2}-4x^{2a-2}y^{x-1}-10x^{2a-3}y^{x} \\
\hline
-2x^{3a+1}y^{2x-3}+8x^{3a}y^{2x-2}-6x^{3a-1}y^{2x-1} \\
\qquad -4x^{3a}y^{2x-2}+16x^{3a-1}y^{2x-1}-12x^{3a-2}y^{2x} \\
\qquad\qquad -10x^{3a-1}y^{2x-1}+40x^{3a-2}y^{2x}-30x^{3a-3}y^{2x+1} \\
\hline
-2x^{3a+1}y^{2x-3}+4x^{3a}y^{2x-2}+28x^{3a-2}y^{2x}-30x^{3a-3}y^{2x+1}
\end{array}
$$

EJERCICIO 44

1.

$$
\begin{array}{l}
\frac{1}{2}a-\frac{1}{3}b \\
\frac{1}{3}a+\frac{1}{2}b \\
\hline
\frac{1}{6}a^2-\frac{1}{9}ab \\
\qquad +\frac{1}{4}ab-\frac{1}{6}b^2 \\
\hline
\frac{1}{6}a^2-\frac{4-9}{36}ab-\frac{1}{6}b^2 \\
=\frac{1}{6}a^2+\frac{5}{36}ab-\frac{1}{6}b^2
\end{array}
$$

2.

$$
\begin{array}{l}
x-\frac{2}{5}y \\
\frac{1}{3}x+\frac{5}{6}y \\
\hline
\frac{1}{3}x^2-\frac{2}{15}xy \\
\qquad +\frac{5}{6}xy-\frac{10}{30}y^2 \\
\hline
\frac{1}{3}x^2-\frac{4-25}{30}xy-\frac{10}{30}y^2 \\
=\frac{1}{3}x^2+\frac{21}{30}xy-\frac{1}{3}y^2 \\
=\frac{1}{3}x^2+\frac{7}{10}xy-\frac{1}{3}y^2
\end{array}
$$

3.

$$
\begin{array}{l}
\frac{1}{2}x^2-\frac{1}{3}xy+\frac{1}{4}y^2 \\
\frac{2}{3}x-\frac{3}{2}y \\
\hline
\frac{2}{6}x^3-\frac{2}{9}x^2y+\frac{2}{12}xy^2 \\
\qquad -\frac{3}{4}x^2y+\frac{3}{6}xy^2-\frac{3}{8}y^3 \\
\hline
\frac{2}{6}x^3-\frac{8+27}{36}x^2y+\frac{2+6}{12}xy^2-\frac{3}{8}y^3 \\
=\frac{1}{3}x^3-\frac{35}{36}x^2y+\frac{8}{12}xy^2-\frac{3}{8}y^3 \\
=\frac{1}{3}x^3-\frac{35}{36}x^2y+\frac{2}{3}xy^2-\frac{3}{8}y^3
\end{array}
$$

4.

$$
\begin{array}{l}
\frac{1}{4}a^2-ab+\frac{2}{3}b^2 \\
\frac{1}{4}a-\frac{3}{2}b \\
\hline
\frac{1}{16}a^3-\frac{1}{4}a^2b+\frac{2}{12}ab^2 \\
\qquad -\frac{3}{8}a^2b+\frac{3}{2}ab^2-\frac{6}{6}b^3 \\
\hline
\frac{1}{16}a^3-\frac{2+3}{8}a^2b+\frac{2+18}{12}ab^2-b^3 \\
=\frac{1}{16}a^3-\frac{5}{8}a^2b+\frac{20}{12}ab^2-b^3 \\
=\frac{1}{16}a^3-\frac{5}{8}a^2b+\frac{5}{3}ab^2-b
\end{array}
$$

5.

$$\frac{2}{5}m^2+\frac{1}{3}mn-\frac{1}{2}n^2$$
$$\frac{3}{2}m^2-mn+2n^2$$
$$\frac{6}{10}m^4+\frac{3}{6}m^3n-\frac{3}{4}m^2n^2$$
$$-\frac{2}{5}m^3n-\frac{1}{3}m^2n^2+\frac{1}{2}mn^3$$
$$+\frac{4}{5}m^2n^2+\frac{2}{3}mn^3-\frac{2}{2}n^4$$
$$\frac{6}{10}m^4+\frac{15-12}{30}m^3n-\frac{45+20-48}{60}m^2n^2+\frac{3+4}{6}mn^3-\frac{2}{2}n^4$$
$$=\frac{6}{10}m^4+\frac{3}{30}m^3n-\frac{17}{60}m^2n^2+\frac{7}{6}mn^3-n^4$$
$$=\frac{3}{5}m^4+\frac{1}{10}m^3n-\frac{17}{60}m^2n^2+\frac{7}{6}mn^3-n^4$$

6.

$$\frac{3}{8}x^2+\frac{1}{4}x-\frac{2}{5}$$
$$2x^3-\frac{1}{3}x+2$$
$$\frac{6}{8}x^5+\frac{2}{4}x^4-\frac{4}{5}x^3$$
$$-\frac{3}{24}x^3-\frac{1}{12}x^2+\frac{2}{15}x$$
$$+\frac{6}{8}x^2+\frac{2}{4}x-\frac{4}{5}$$
$$\frac{6}{8}x^5+\frac{2}{4}x^4-\frac{96+15}{120}x^3-\frac{2-18}{24}x^2+\frac{8+30}{60}x-\frac{4}{5}$$
$$=\frac{3}{4}x^5+\frac{1}{2}x^4-\frac{111}{120}x^3+\frac{16}{24}x^2+\frac{38}{60}x-\frac{4}{5}$$
$$=\frac{3}{4}x^5+\frac{1}{2}x^4-\frac{37}{40}x^3+\frac{2}{3}x^2+\frac{19}{30}x-\frac{4}{5}$$

7.

$$-\frac{1}{2}x^2+\frac{1}{3}ax+\frac{3}{2}a^2$$
$$\frac{3}{2}x^2-ax+\frac{2}{3}a^2$$
$$-\frac{3}{4}x^4+\frac{3}{6}ax^3+\frac{9}{4}a^2x^2$$
$$+\frac{1}{2}ax^3-\frac{1}{3}a^2x^2-\frac{3}{2}a^3x$$
$$-\frac{2}{6}a^2x^2+\frac{2}{9}a^3x+\frac{6}{6}a^4$$
$$-\frac{3}{4}x^4+\frac{3+3}{6}ax^3+\frac{27-4-4}{12}a^2x^2-\frac{27-4}{18}a^3x+\frac{6}{6}a^4$$
$$=-\frac{3}{4}x^4+\frac{6}{6}ax^3+\frac{19}{12}a^2x^2-\frac{23}{18}a^3x+\frac{6}{6}a^4$$
$$=-\frac{3}{4}x^4+ax^3+\frac{19}{12}a^2x^2-\frac{23}{18}a^3x+a^4$$

8.

$$\frac{2}{7}x^3-\frac{1}{5}x^2y+\frac{1}{2}xy^2$$
$$\frac{1}{4}x^2-\frac{2}{3}xy+\frac{5}{6}y^2$$
$$\frac{2}{28}x^5-\frac{1}{20}x^4y+\frac{1}{8}x^3y^2$$
$$-\frac{4}{21}x^4y+\frac{2}{15}x^3y^2-\frac{2}{6}x^2y^3$$
$$+\frac{10}{42}x^3y^2-\frac{5}{30}x^2y^3+\frac{5}{12}xy^4$$
$$\frac{2}{28}x^5-\frac{21+80}{420}x^4y+\frac{105+112+200}{840}x^3y^2-\frac{10+5}{30}x^2y^3+\frac{5}{12}xy^4$$
$$=\frac{2}{28}x^5-\frac{101}{420}x^4y+\frac{417}{840}x^3y^2-\frac{15}{30}x^2y^3+\frac{5}{12}xy^4$$
$$=\frac{1}{14}x^5-\frac{101}{420}x^4y+\frac{139}{280}x^3y^2-\frac{1}{2}x^2y^3+\frac{5}{12}xy^4$$

9.

$$\frac{1}{4}x^3+\frac{1}{3}x^2-\frac{1}{4}x+\frac{1}{2}$$
$$\frac{3}{2}x^2+\frac{1}{10}x-\frac{1}{5}$$
$$\frac{3}{8}x^5+\frac{3}{6}x^4-\frac{3}{8}x^3+\frac{3}{4}x^2$$
$$+\frac{1}{40}x^4+\frac{1}{30}x^3-\frac{1}{40}x^2+\frac{1}{20}x$$
$$-\frac{1}{20}x^3-\frac{1}{15}x^2+\frac{1}{20}x-\frac{1}{10}$$
$$\frac{3}{8}x^5+\frac{60+3}{120}x^4-\frac{45-4+6}{120}x^3+\frac{90-3-8}{120}x^2+\frac{1+1}{20}x-\frac{1}{10}$$
$$=\frac{3}{8}x^5+\frac{63}{120}x^4-\frac{47}{120}x^3+\frac{79}{120}x^2+\frac{2}{20}x-\frac{1}{10}=\frac{3}{8}x^5+\frac{21}{40}x^4-\frac{47}{120}x^3+\frac{79}{120}x^2+\frac{1}{10}x-\frac{1}{10}$$

10. $\frac{3}{4}m^3-\frac{1}{2}m^2n+\frac{2}{5}mn^2-\frac{1}{4}n^3$

$\frac{2}{3}m^2-\frac{2}{3}mn+\frac{5}{2}n^2$

$\frac{6}{12}m^5-\frac{2}{6}m^4n+\frac{4}{15}m^3n^2-\frac{2}{12}m^2n^3$

$-\frac{6}{12}m^4n+\frac{2}{6}m^3n^2-\frac{4}{15}m^2n^3+\frac{2}{12}mn^4$

$+\frac{15}{8}m^3n^2-\frac{5}{4}m^2n^3+\frac{10}{10}mn^4-\frac{5}{8}n^5$

$\frac{6}{12}m^5-\frac{4+6}{12}m^4n+\frac{32+40+225}{120}m^3n^2-\frac{10+16+75}{60}m^2n^3+\frac{20+120}{120}mn^4-\frac{5}{8}n^5$

$=\frac{1}{2}m^5-\frac{10}{12}m^4n+\frac{297}{120}m^3n^2-\frac{101}{60}m^2n^3+\frac{140}{120}mn^4-\frac{5}{8}n^5=\frac{1}{2}m^5-\frac{5}{6}m^4n+\frac{99}{40}m^3n^2-\frac{101}{60}m^2n^3+\frac{7}{6}mn^4-\frac{5}{8}n^5$

EJERCICIO 45

1. x^3-x^2+x *por* x^2-1

$1-1+1$

$1+0-1$

$1-1+1$

$-1+1-1$

$1-1+0+1-1$

$=x^5-x^4+x^2-x$

2. $x^4+3x^3-5x^2+8$ *por* x^3-2x^2-7

$1+3-5+0+8$

$1-2+0-7$

$1+3-5+0+8$

$-2-6+10+0-16$

$-7-21+35+0-56$

$1+1-11+3-13+19+0-56$

$=x^7+x^6-11x^5+3x^4-13x^3+19x^2-56$

3. $a^4+3a^3b-2a^2b^2+5ab^3-b^4$ *por* $a^2-2ab+b^2$

$1+3-2+5-1$

$1-2+1$

$1+3-2+5-1$

$-2-6+4-10+2$

$+1+3-2+5-1$

$1+1-7+12-13+7-1$

$=a^6+a^5b-7a^4b^2+12a^3b^3-13a^2b^4+7ab^5-b^6$

4. $m^3-5m^2n+6mn^2+n^3$ *por* $m^3-4mn^2-n^3$

$1-5+6+1$

$1+0-4-1$

$1-5+6+1$

$-4+20-24-4$

$-1+5-6-1$

$1-5+2+20-19-10-1$

$=m^6-5m^5n+2m^4n^2+20m^3n^3-19m^2n^4-10mn^5-n^6$

5. x^4-8x^2+3 *por* x^4+6x^2-5

$1+0-8+0+3$

$1+0+6+0-5$

$1+0-8+0+3$

$+6+0-48+0+18$

$-5+0+40+0-15$

$1+0-2+0-50+0+58+0-15$

$=x^8-2x^6-50x^4+58x^2-15$

6. $a^6-3a^4-6a^2+10$ *por* $a^8-4a^6+3a^4-2a^2$

$1+0-3+0-6+0+10$

$1+0-4+0+3+0-2$

$1+0-3+0-6+0+10$

$-4+0+12+0+24+0-40$

$+3+0-9+0-18+0+30$

$-2+0+6+0+12+0-20$

$1+0-7+0+9+0+23+0-52+0+42+0-20$

$=a^{14}-7a^{12}+9a^{10}+23a^8-52a^6+42a^4-20a^2$

7. $x^9-4x^6+3x^3-2$ *por* $3x^6-8x^3+10$

$1+0+0-4+0+0+3+0+0-2$
$3+0+0-8+0+0+10$
$3+0+0-12+0+0+9+0+0-6$
$-8+0+0+32+0+0-24+0+0+16$
$+10+0+0-40+0+0+30+0+0-20$
$3+0+0-20+0+0+51+0+0-70+0+0+46+0+0-20$
$=3x^{15}-20x^{12}+51x^9-70x^6+46x^3-20$

8. $m^{12}-7m^8+9m^4-15$ *por* $m^{16}-5m^{12}+9m^8-4m^4+3$

$1-7+9-15$
$1-5+9-4+3$
$1-7+9-15$
$-5+35-45+75$
$+9-63+81-135$
$-4+28-36+60$
$+3-21+27-45$
$1-12+53-127+187-192+87-45$
$=m^{28}-12m^{24}+53m^{20}-127m^{16}+187m^{12}-192m^8+87m^4-45$

9. $x^5-3x^4y-6x^3y^2-4x^2y^3-y^5$ *por* $2x^2+4y^2$

$1-3-6-4+0-1$
$2+0+4$
$2-6-12-8+0-2$
$+4-12-24-16+0-4$
$2-6-8-20-24-18+0-4$
$=2x^7-6x^6y-8x^5y^2-20x^4y^3-24x^3y^4-18x^2y^5-4y^7$

10. $6a^5-4a^2+6a-2$ *por* a^4-2a^2+a-7

$6+0+0-4+6-2$
$1+0-2+1-7$
$6+0+0-4+6-2$
$-12+0+0+8-12+4$
$+6+0+0-4+6-2$
$-42+0+0+28-42+14$
$6+0-12+2-36+6-16+38-44+14$
$=6a^9-12a^7+2a^6-36a^5+6a^4-16a^3+38a^2-44a+14$

11. $n^6-3n^4+5n^3-8n+4$ *por* n^4-3n^2+4

$1+0-3+5+0-8+4$
$1+0-3+0+4$
$1+0-3+5+0-8+4$
$-3+0+9-15+0+24-12$
$+4+0-12+20+0-32+16$
$1+0-6+5+13-23-8+44-12-32+16$
$=n^{10}-6n^8+5n^7+13n^6-23n^5-8n^4+44n^3-12n^2-32n+16$

12. $3x^4-4x^3y-y^4$ *por* $x^3-5xy^2+3y^3$

$3-4+0+0-1$
$1+0-5+3$
$3-4+0+0-1$
$-15+20+0+0+5$
$+9-12+0+0-3$
$3-4-15+29-13+0+5-3$
$=3x^7-4x^6y-15x^5y^2+29x^4y^3-13x^3y^4+5xy^6-3y^7$

13. $x^{10}-5x^6y^4+3x^2y^8-6y^{10}$ *por* $x^6-4x^4y^2-5x^2y^4+y^6$

$1+0+0+0-5+0+0+0+3+0-6$
$1+0-4+0-5+0+1$
$1+0+0+0-5+0+0+0+3+0-6$
$-4+0+0+0+20+0+0+0-12+0+24$
$-5+0+0+0+25+0+0+0-15+0+30$
$+1+0+0+0-5+0+0+0+3+0-6$
$1+0-4+0-10+0+21+0+28+0-23+0+9+0+33+0-6$
$x^{16}-4x^{14}y^2-10x^{12}y^4+21x^{10}y^6+28x^8y^8-23x^6y^{10}+9x^4y^{12}+33x^2y^{14}-6y^{16}$

14. $a^m-3a^{m-1}+5a^{m-3}$ *por* a^2-5

$1-3+0+5$
$1+0-5$
$1-3+0+5$
$-5+15+0-25$
$1-3-5+20+0-25$
$=a^{m+2}-3a^{m+1}-5a^m+20a^{m-1}-25a^{m-3}$

15. $a^{x+2}-5a^{x+1}-7a^{x-1}$ *por* $7a^{x+3}+6a^{x+1}+a^x$

$1-5+0-7$
$7+0+6+1$
$7-35+0-49$
$+6-30+0-42$
$+1-5+0-7$
$7-35+6-78-5-42-7$
$=7a^{2x+5}-35a^{2x+4}+6a^{2x+3}$
$-78a^{2x+2}-5a^{2x+1}-42a^{2x}-7a^{2x-1}$

16. $x^{a+2}-5x^{a}-6x^{a-2}$ *por* $6x^{a+1}-4x^{a}+2x^{a-1}+x^{a-2}$

$$\begin{array}{l} 1+0-5+0-6 \\ 6-4+2+1 \\ \hline 6+0-30+0-36 \\ \quad -4+0+20+0+24 \\ \qquad +2+0-10+0-12 \\ \qquad\quad +1+0-5+0-6 \\ \hline 6-4-28+21-46+19-12-6 \end{array}$$

$= 6x^{2a+3}-4x^{2a+2}-28x^{2a+1}+21x^{2a}$
$-46x^{2a-1}+19x^{2a-2}-12x^{2a-3}-6x^{2a-4}$

17. $a^{2x+2}-3a^{2x+1}-a^{2x}-5a^{2x-1}$ *por* $6a^{3x+1}-5a^{3x}+3a^{3x-1}$

$$\begin{array}{l} 1-3-1-5 \\ 6-5+3 \\ \hline 6-18-6-30 \\ \quad -5+15+5+25 \\ \qquad +3-9-3-15 \\ \hline 6-23+12-34+22-15 \end{array}$$

$=6a^{5x+3}-23a^{5x+2}+12a^{5x+1}-34a^{5x}+22a^{5x-1}-15a^{5x-2}$

EJERCICIO 46

1. $4(a+5)(a-3)$

$$\begin{array}{l} 4a+20 \\ \quad a-3 \\ \hline 4a^2+20a \\ \qquad -12a-60 \\ \hline 4a^2+8a-60 \end{array}$$

2. $3a^2(x+1)(x-1)$

$$\begin{array}{l} 3a^2x+3a^2 \\ x-1 \\ \hline 3a^2x^2+3a^2x \\ \qquad -3a^2x-3a^2 \\ \hline 3a^2x^2 \qquad -3a^2 \end{array}$$

3. $2(a-3)(a-1)(a+4)$

$$\begin{array}{l} 2a-6 \\ a-1 \\ \hline 2a^2-6a \\ \qquad -2a+6 \\ \hline 2a^2-8a+6 \\ a+4 \\ \hline 2a^3-8a^2+6a \\ \qquad +8a^2-32a+24 \\ \hline 2a^3 \qquad -26a+24 \end{array}$$

4. $(x^2+1)(x^2-1)(x^2+1)$

$$\begin{array}{l} x^2+1 \\ x^2+1 \\ \hline x^4+x^2 \\ \qquad +x^2+1 \\ \hline x^4+2x^2+1 \\ x^2-1 \\ \hline x^6+2x^4+x^2 \\ \qquad -x^4-2x^2-1 \\ \hline x^6+x^4-x^2-1 \end{array}$$

5. $m(m-4)(m-6)(3m+2)$

$$\begin{array}{l} m^2-4m \\ m-6 \\ \hline m^3-4m^2 \\ \qquad -6m^2+24m \\ \hline m^3-10m^2+24m \\ 3m+2 \\ \hline 3m^4-30m^3+72m^2 \\ \qquad +2m^3-20m^2+48m \\ \hline 3m^4-28m^3+52m^2+48m \end{array}$$

6. $(a-b)(a^2-2ab+b^2)(a+b)$

$$\begin{array}{l} a^2-2ab+b^2 \\ a+b \\ \hline a^3-2a^2b+ab^2 \\ \qquad +a^2b-2ab^2+b^3 \\ \hline a^3-a^2b-ab^2+b^3 \\ a-b \\ \hline a^4-a^3b-a^2b^2+ab^3 \\ \qquad -a^3b+a^2b^2+ab^3-b^4 \\ \hline a^4-2a^3b \qquad +2ab^3-b^4 \end{array}$$

7. $3x(x^2-2x+1)(x-1)(x+1)$

$$\begin{array}{l} 3x^3-6x^2+3x \\ x-1 \\ \hline 3x^4-6x^3+3x^2 \\ \qquad -3x^3+6x^2-3x \\ \hline 3x^4-9x^3+9x^2-3x \\ x+1 \\ \hline 3x^5-9x^4+9x^3-3x^2 \\ \qquad +3x^4-9x^3+9x^2-3x \\ \hline 3x^5-6x^4 \qquad +6x^2-3x \end{array}$$

8. $\left(x^2-x+1\right)\left(x^2+x-1\right)\left(x-2\right)$

$$\begin{array}{l}
x^2-x+1\\
x^2+x-1\\
\hline
x^4-x^3+x^2\\
\quad +x^3-x^2\ \ +x\\
\qquad\qquad -x^2\ \ +x\ \ -1\\
\hline
x^4\qquad\quad -x^2+2x-1\\
x-2\\
\hline
x^5\qquad\quad -x^3\ +2x^2-x\\
\quad -2x^4\qquad +2x^2-4x+2\\
\hline
x^5\ -2x^4-x^3+4x^2-5x+2
\end{array}$$

9. $\left(a^m-3\right)\left(a^{m-1}+2\right)\left(a^{m-1}-1\right)$

$$\begin{array}{l}
a^m-3\\
a^{m-1}+2\\
\hline
\qquad +a^{2m-1}-3a^{m-1}\\
2a^m\qquad\qquad\qquad\qquad -6\\
\hline
2a^m\ +a^{2m-1}-3a^{m-1}-6\\
a^{m-1}-1\\
\hline
2a^{2m-1}+a^{3m-2}-3a^{2m-2}-6a^{m-1}\\
-\ a^{2m-1}\qquad\qquad\qquad +3a^{m-1}-2a^m\ +6\\
\hline
\ a^{2m-1}+a^{3m-2}-3a^{2m-2}-3a^{m-1}-2a^m\ +6\\
\Rightarrow\quad a^{3m-2}+a^{2m-1}\ -3a^{2m-2}-2a^m\ -3a^{m-1}+6
\end{array}$$

10. $a\left(a-1\right)\left(a-2\right)\left(a-3\right)$

$$\begin{array}{l}
a^2-a\\
a-2\\
\hline
a^3-a^2\\
\quad -2a^2+2a\\
\hline
a^3-3a^2+2a\\
a-3\\
\hline
a^4-3a^3+2a^2\\
\quad -3a^3+9a^2\ \ -6a\\
\hline
a^4-6a^3+11a^2-6a
\end{array}$$

11. $\left(x-3\right)\left(x+4\right)\left(x-5\right)\left(x+1\right)$

$$\begin{array}{ll}
x-3 & x-5\\
x+4 & x+1\\
\hline
x^2-3x & x^2-5x\\
\quad +4x-12 & \quad +\ \ x\ -5\\
\hline
x^2+\ \ x\ -12 & x^2-4x-5
\end{array}$$

$$\begin{array}{l}
x^2-4x-5\\
\hline
x^4+x^3\ \ -12x^2\\
\quad -4x^3-\ \ 4x^2+48x\\
\qquad\quad -\ \ 5x^2-5x\ \ +60\\
\hline
x^4-3x^3-21x^2+43x+60
\end{array}$$

12. $\left(x^2-3\right)\left(x^2+2x+1\right)\left(x-1\right)\left(x^2+3\right)$

$$\begin{array}{ll}
x^2+2x+1 & x^2-3\\
x^2+3 & x-1\\
\hline
x^4+2x^3+x^2 & x^3\qquad -3x\\
\qquad +3x^2+6x+3 & \quad -x^2\qquad +3\\
\hline
x^4+2x^3+4x^2+6x+3 & x^3-x^2-3x+3
\end{array}$$

$$\begin{array}{l}
x^3-x^2-3x+3\\
\hline
x^7+2x^6+4x^5+6x^4+3x^3\\
\quad -x^6\ -2x^5\ -4x^4-\ \ 6x^3-3x^2\\
\qquad\quad -3x^5\ -6x^4-12x^3-18x^2-9x\\
\qquad\qquad\qquad +3x^4+\ \ 6x^3+12x^2+18x+9\\
\hline
x^7\ +x^6\ \ -x^5\ \ -\ \ x^4-\ \ 9x^3-\ \ 9x^2\ +\ \ 9x\ +9
\end{array}$$

13. $9a^2\left(3a-2\right)\left(2a+1\right)\left(a-1\right)\left(2a-1\right)$

$$\begin{array}{ll}
27a^3-18a^2 & 2a-1\\
2a+1 & \quad a-1\\
\hline
54a^4-36a^3 & 2a^2\ -a\\
\qquad +27a^3-18a^2 & \quad -2a+1\\
\hline
54a^4-\ \ 9a^3\ -18a^2 & 2a^2-3a+1
\end{array}$$

$$\begin{array}{l}
2a^2-3a+1\\
\hline
108a^6-\ 18a^5\ -36a^4\\
\qquad -162a^5+27a^4+54a^3\\
\qquad\qquad\qquad +54a^4-\ \ 9a^3-18a^2\\
\hline
108a^6-180a^5+45a^4-45a^3-18a^2
\end{array}$$

14. $a^x\left(a^{x+1}+b^{x+2}\ \right)\left(a^{x+1}-b^{x+2}\right)b^x$

$$\begin{array}{l}
a^{2x+1}+a^xb^{x+2}\\
a^{x+1}b^x-\ b^{2x+2}\\
\hline
a^{3x+2}b^x+a^{2x+1}b^{2x+2}\\
\qquad\quad -a^{2x+1}b^{2x+2}\ -\ a^xb^{3x+4}\\
\hline
a^{3x+2}b^x\qquad\qquad\qquad -\ a^xb^{3x+4}
\end{array}$$

EJERCICIO 47

1. $4(x+3)+5(x+2)$

$=4x+12+5x+10$

$=9x+22$

2. $6(x^2+4)-3(x^2+1)+5(x^2+2)$

$=6x^2+24-3x^2-3+5x^2+10$

$=8x^2+31$

3. $a(a-x)+3a(x+2a)-a(x-3a)$

$=a^2-ax+3ax+6a^2-ax+3a^2$

$=10a^2+ax$

4. $x^2(y^2+1)+y^2(x^2+1)-3x^2y^2$

$=x^2y^2+x^2+x^2y^2+y^2-3x^2y^2$

$=-x^2y^2+x^2+y^2$

5. $4m^3-5mn^2+3m^2(m^2+n^2)-3m(m^2-n^2)$

$=4m^3-5mn^2+3m^4+3m^2n^2-3m^3+3mn^2$

$=3m^4+m^3+3m^2n^2-2mn^2$

6. $y^2+x^2y^3-y^3(x^2+1)+y^2(x^2+1)-y^2(x^2-1)$

$=y^2+x^2y^3-x^2y^3-y^3+x^2y^2+y^2-x^2y^2+y^2$

$=-y^3+3y^2$

7. $5(x+2)-(x+1)(x+4)-6x$

$=5x+10-(x^2+4x+x+4)-6x$

$=-x+10-x^2-4x-x-4$

$=-x^2-6x+6$

8. $(a+5)(a-5)-3(a+2)(a-2)+5(a+4)$

$=a^2-5a+5a-25-3(a^2-2a+2a-4)+5a+20$

$=a^2+5a-5-3(a^2-4)$

$=a^2+5a-5-3a^2+12$

$=-2a^2+5a+7$

9. $(a+b)(4a-3b)-(5a-2b)(3a+b)-(a+b)(3a-6b)$

$=4a^2+ab-3b^2-(15a^2-ab-2b^2)-(3a^2-3ab-6b^2)$

$=4a^2+ab-3b^2-15a^2+ab+2b^2-3a^2+3ab+6b^2$

$=-14a^2+5ab+5b^2$

10. $(a+c)^2-(a-c)^2$

$=(a+c)(a+c)-(a-c)(a-c)$

$=a^2+ac+ac+c^2-(a^2-ac-ac+c^2)$

$=a^2+2ac+c^2-a^2+ac+ac-c^2$

$=4ac$

11. $3(x+y)^2-4(x-y)^2+3x^2-3y^2$

$=3(x+y)(x+y)-4(x-y)(x-y)+3x^2-3y^2$

$=3(x^2+xy+xy+y^2)-4(x^2-xy-xy+y^2)+3x^2-3y^2$

$=3(x^2+2xy+y^2)-4(x^2-2xy+y^2)+3x^2-3y^2$

$=3x^2+6xy+3y^2-4x^2+8xy-4y^2+3x^2-3y^2$

$=2x^2+14xy-4y^2$

12. $(m+n)^2-(2m+n)^2+(m-4n)^2$

$=(m+n)(m+n)-(2m+n)(2m+n)+(m-4n)(m-4n)$

$=m^2+2mn+n^2-(4m^2+4mn+n^2)+m^2-8mn+16n^2$

$=2m^2-6mn+17n^2-4m^2-4mn-n^2$

$=-2m^2-10mn+16n^2$

13. $x(a+x)+3x(a+1)-(x+1)(a+2x)-(a-x)^2$

$=ax+x^2+3ax+3x-(ax+2x^2+a+2x)-(a-x)(a-x)$

$=4ax+x^2+3x-ax-2x^2-a-2x-(a^2-ax-ax+x^2)$

$=3ax-x^2+x-a-a^2+ax+ax-x^2$

$=-2x^2+x+5ax-a-a^2$

14. $(a+b-c)^2+(a-b+c)^2-(a+b+c)^2$

$$\begin{array}{l} a+b-c \\ a+b-c \\ \hline a^2+ab-ac \\ \quad +ab \quad +b^2-bc \\ \quad\quad -ac \quad -bc+c^2 \\ \hline a^2+2ab-2ac+b^2-2bc+c^2 \end{array}$$

$$\begin{array}{l} a-b+c \\ a-b+c \\ \hline a^2-ab+ac \\ \quad -ab \quad +b^2-bc \\ \quad\quad +ac \quad -bc+c^2 \\ \hline a^2-2ab+2ac+b^2-2bc+c^2 \end{array}$$

$$\begin{array}{l} a+b+c \\ a+b+c \\ \hline a^2+ab+ac \\ \quad +ab \quad +b^2+bc \\ \quad\quad +ac \quad +bc+c^2 \\ \hline a^2+2ab+2ac+b^2+2bc+c^2 \end{array}$$

$$\begin{array}{r} a^2+2ab-2ac+b^2-2bc+c^2 \\ a^2-2ab+2ac+b^2-2bc+c^2 \\ -a^2-2ab-2ac-b^2-2bc-c^2 \\ \hline a^2-2ab-2ac+b^2-6bc+c^2 \end{array}$$

$\Rightarrow a^2+b^2+c^2-2ab-2ac-6bc$

15. $\left(x^2+x-3\right)^2-\left(x^2-2+x\right)^2+\left(x^2-x-3\right)^2$

$$\begin{array}{r} x^2+x-3 \\ x^2+x-3 \\ \hline x^4+x^3-3x^2 \\ +x^3+x^2-3x \\ -3x^2-3x+9 \\ \hline x^4+2x^3-5x^2-6x+9 \end{array} \qquad \begin{array}{r} x^2+x-2 \\ x^2+x-2 \\ \hline x^4+x^3-2x^2 \\ +x^3+x^2-2x \\ -2x^2-2x+4 \\ \hline x^4+2x^3-3x^2-4x+4 \end{array}$$

$$\begin{array}{r} x^2-x-3 \\ x^2-x-3 \\ \hline x^4-x^3-3x^2 \\ -x^3+x^2+3x \\ -3x^2+3x+9 \\ \hline x^4-2x^3-5x^2+6x+9 \end{array} \qquad \begin{array}{r} x^4+2x^3-5x^2-6x+9 \\ -x^4-2x^3+3x^2+4x-4 \\ x^4-2x^3-5x^2+6x+9 \\ \hline x^4-2x^3-7x^2+4x+14 \end{array}$$

16. $\left(x+y+z\right)^2-\left(x+y\right)\left(x-y\right)+3\left(x^2+xy+y^2\right)$

$$\begin{array}{l} x+y+z \\ x+y+z \\ \hline x^2+xy+xz \\ \quad +xy \qquad +y^2+yz \\ \qquad\quad +xz \qquad +yz+z^2 \\ \hline x^2+2xy+2xz+y^2+2yz+z^2 \end{array} \qquad \begin{array}{r} x+y \\ x-y \\ \hline x^2+xy \\ -xy-y^2 \\ \hline x^2 \qquad -y^2 \end{array}$$

$$\begin{array}{l} x^2+2xy+2xz+y^2+2yz+z^2 \\ -x^2 \qquad\qquad +y^2 \\ 3x^2+3xy \qquad +3y^2 \\ \hline 3x^2+5xy+2xz+5y^2+2yz+z^2 \end{array}$$

$\Rightarrow 3x^2+5y^2+z^2+5xy+2xz+2yz$

17. $\left[x+\left(2x-3\right)\right]\left[3x-\left(x+1\right)\right]+4x-x^2$

$=\left[x+2x-3\right]\left[3x-x-1\right]+4x-x^2$

$=\left[3x-3\right]\left[2x-1\right]+4x-x^2$

$=6x^2-3x-6x+3+4x-x^2$

$=5x^2-5x+3$

18. $\left[3\left(x+2\right)-4\left(x+1\right)\right]\left[3\left(x+4\right)-2\left(x+2\right)\right]$

$=\left[3x+6-4x-4\right]\left[3x+12-2x-4\right]$

$=\left[-x+2\right]\left[x+8\right]$

$=-x^2-8x+2x+16$

$=-x^2-6x+16$

19. $\left[\left(m+n\right)\left(m-n\right)-\left(m+n\right)\left(m+n\right)\right]\left[2\left(m+n\right)-3\left(m-n\right)\right]$

$=\left[m^2-n^2-\left(m^2+2mn+n^2\right)\right]\left[2m+2n-3m+3n\right]$

$=\left[m^2-n^2-m^2-2mn-n^2\right]\left[-m+5n\right]$

$=\left[-2mn-2n^2\right]\left[-m+5n\right]$

$=2m^2n-10mn^2+2mn^2-10n^3$

$=2m^2n-8mn^2-10n^3$

20. $\left[\left(x+y\right)^2-3\left(x-y\right)^2\right]\left[\left(x+y\right)\left(x-y\right)+x\left(y-x\right)\right]$

$=\left[x^2+2xy+y^2-3\left(x^2-2xy+y^2\right)\right]\left[x^2-y^2+xy-x^2\right]$

$=\left[x^2+2xy+y^2-3x^2+6xy-3y^2\right]\left[xy-y^2\right]$

$=\left[-2x^2+8xy-2y^2\right]\left[xy-y^2\right]$

$=-2x^3y+2x^2y^2+8x^2y^2-8xy^3-2xy^3+2y^4$

$=-2x^3y+10x^2y^2-10xy^3+2y^4$

EJERCICIO 48

1. $x-\left[3a+2\left(-x+1\right)\right]$

$=x-\left[3a-2x+2\right]$

$=x-3a+2x-2$

$=3x-3a-2$

2. $-\left(a+b\right)-3\left[2a+b\left(-a+2\right)\right]$

$=-a-b-3\left[2a-ab+2b\right]$

$=-a-b-6a+3ab-6b$

$=-7a-7b+3ab \Rightarrow -7a+3ab-7b$

3. $-\left[3x-2y+\left(x-2y\right)-2\left(x+y\right)-3\left(2x+1\right)\right]$

$=-\left[3x-2y+x-2y-2x-2y-6x-3\right]$

$=-\left[-4x-6y-3\right]$

$=4x+6y+3$

4. $4x^2-\left\{-3x+5-\left[-x+x\left(2-x\right)\right]\right\}$

$=4x^2-\left\{-3x+5-\left[-x+2x-x^2\right]\right\}$

$=4x^2-\left\{-3x+5+x-2x+x^2\right\}$

$=4x^2-\left\{-4x+5+x^2\right\}$

$=4x^2+4x-5-x^2$

$=3x^2+4x-5$

5. $2a-\{-3x+2[-a+3x-2(-a+b-(2+a))]\}$

$=2a-\{-3x+2[-a+3x-2(-a+b-2-a)]\}$

$=2a-\{-3x+2[-a+3x+2a-2b+4+2a]\}$

$=2a-\{-3x+2[3a+3x-2b+4]\}$

$=2a-\{-3x+6a+6x-4b+8\}$

$=2a+3x-6a-6x+4b-8$

$=-4a-3x+4b-8 \Rightarrow -4a+4b-3x-8$

6. $a-(x+y)-3(x-y)+2[-(x-2y)-2(-x-y)]$

$=a-x-y-3x+3y+2[-x+2y+2x+2y]$

$=a-4x+2y+2[x+4y]$

$=a-4x+2y+2x+8y$

$=a-2x+10y$

7. $m-(m+n)-3\{-2m+[-2m+n+2(-1+n)-(m+n-1)]\}$

$=m-m-n-3\{-2m+[-2m+n-2+2n-m-n+1]\}$

$=-n-3\{-2m+[-3m+2n-1]\}$

$=-n-3\{-2m-3m+2n-1\}$

$=-n-3\{-5m+2n-1\}$

$=-n+15m-6n+3$

$=-7n+15m+3 \Rightarrow 15m-7n+3$

8. $-2(a-b)-3(a+2b)-4\{a-2b+2[-a+b-1+2(a-b)]\}$

$=-2a+2b-3a-6b-4\{a-2b+2[-a+b-1+2a-2b]\}$

$=-5a-4b-4\{a-2b+2[a-b-1]\}$

$=-5a-4b-4\{a-2b+2a-2b-2\}$

$=-5a-4b-4\{3a-4b-2\}$

$=-5a-4b-12a+16b+8$

$=-17a+12b+8$

9. $-5(x+y)-[2x-y+2\{-x+y-3-(x-y-1)\}]+2x$

$=-5x-5y-[2x-y+2\{-x+y-3-x+y+1\}]+2x$

$=-3x-5y-[2x-y+2\{-2x+2y-2\}]$

$=-3x-5y-[2x-y-4x+4y-4]$

$=-3x-5y-[-2x+3y-4]$

$=-3x-5y+2x-3y+4$

$=-x-8y+4$

10. $m-3(m+n)+[-\{-(-2m+n-2-3[m-n+1])+m\}]$

$=m-3m-3n+[-\{-(-2m+n-2-3m+3n-3)+m\}]$

$=-2m-3n+[-\{-(-5m+4n-5)+m\}]$

$=-2m-3n+[-\{5m-4n+5+m\}]$

$=-2m-3n+[-\{6m-4n+5\}]$

$=-2m-3n+[-6m+4n-5]$

$=-2m-3n-6m+4n-5$

$=-8m+n-5$

11. $-3(x-2y)+2\{-4[-2x-3(x+y)]\}-\{-[-(x+y)]\}$

$=-3x+6y+2\{-4[-2x-3x-3y]\}-\{-[-x-y]\}$

$=-3x+6y+2\{-4[-5x-3y]\}-\{x+y\}$

$=-3x+6y+2\{20x+12y\}-x-y$

$=-4x+5y+40x+24y$

$=36x+29y$

12. $5\{-(a+b)-3[-2a+3b-(a+b)+(-a-b)+2(-a+b)]-a\}$

$=5\{-a-b-3[-2a+3b-a-b-a-b-2a+2b]-a\}$

$=5\{-a-b-3[-6a+3b]-a\}$

$=5\{-2a-b+18a-9b\}$

$=5\{16a-10b\}$

$=80a-50b$

13. $-3\{-[+(-a+b)]\}-4\{-[-(-a-b)]\}$

$=-3\{-[-a+b]\}-4\{-[a+b]\}$

$=-3\{a-b\}-4\{-a-b\}$

$=-3a+3b+4a+4b$

$=a+7b$

14. $-\{a+b-2(a-b)+3\{-[2a+b-3(a+b-1)]\}-3[-a+2(-1+a)]\}$

$=-\{a+b-2a+2b+3\{-[2a+b-3a-3b+3]\}-3[-a-2+2a]\}$

$=-\{-a+3b+3\{a+2b-3\}-3a+6\}$

$=-\{-4a+3b+6+3a+6b-9\}$

$=-\{-a+9b-3\}$

$=a-9b+3$

EJERCICIO 49

1. $-24 \div 8 = -3$

2. $-63 \div -7 = 9$

3. $-5a^2 \div -a = 5a^{2-1} = 5a$

4. $14a^3b^4 \div -2ab^2 = -7a^{3-1}b^{4-2} = -7a^2b^2$

5. $-a^3b^4c \div a^3b^4 = -a^{3-3}b^{4-4}c = -c$

6. $-a^2b \div -ab = a^{2-1}b^{1-1} = a$

7. $54x^2y^2z^3 \div -6xy^2z^3 = -9x^{2-1}y^{2-2}z^{3-3} = -9x$

8. $-5m^2n \div m^2n = -5m^{2-2}n^{1-1} = -5$

9. $-8a^2x^3 \div -8a^2x^3 = a^{2-2}x^{3-3} = 1$

10. $-xy^2 \div 2y = -\frac{xy^{2-1}}{2} = -\frac{xy}{2}$

11. $5x^4y^5 \div -6x^4y = -\frac{5}{6}x^{4-4}y^{5-1} = -\frac{5}{6}y^4$

12. $-a^8b^9c^4 \div 8c^4 = -\frac{1}{8}a^8b^9c^{4-4} = -\frac{1}{8}a^8b^9$

13. $16m^6n^4 \div -5n^3 = -\frac{16}{5}m^6n^{4-3} = -\frac{16}{5}m^6n$

14. $-108a^7b^6c^8 \div -20b^6c^8 = \frac{108}{20}a^7b^{6-6}c^{8-8} = \frac{27}{5}a^7$

15. $-2m^2n^6 \div -3mn^6 = \frac{2}{3}m^{2-1}n^{6-6} = \frac{2}{3}m$

16. $a^x \div a^2 = a^{x-2}$

17. $-3a^xb^m \div ab^2 = -3a^{x-1}b^{m-2}$

18. $5a^mb^nc \div -6a^3b^4c = -\frac{5}{6}a^{m-3}b^{n-4}c^{1-1} = -\frac{5}{6}a^{m-3}b^{n-4}$

19. $a^xb^m \div -4a^mb^n = -\frac{1}{4}a^{x-m}b^{m-n}$

20. $-3m^an^xx^3 \div -5m^xn^2x^3 = \frac{3}{5}m^{a-x}n^{x-2}x^{3-3} = \frac{3}{5}m^{a-x}n^{x-2}$

EJERCICIO 50

1. $a^{m+3} \div a^{m+2} = a^{m+3-(m+2)} = a^{m+3-m-2} = a$

2. $2x^{a+4} \div -x^{a+2} = -2x^{a+4-(a+2)} = -2x^{a+4-a-2} = -2x^2$

3. $-3a^{m-2} \div -5a^{m-5} = \frac{3}{5}a^{m-2-(m-5)} = \frac{3}{5}a^{m-2-m+5} = \frac{3}{5}a^3$

4. $x^{2n+3} \div -4x^{n+4} = -\frac{1}{4}x^{2n+3-(n+4)} = -\frac{1}{4}x^{2n+3-n-4} = -\frac{1}{4}x^{n-1}$

5. $-4a^{x-2}b^n \div -5a^3b^2 = \frac{4}{5}a^{x-2-3}b^{n-2} = \frac{4}{5}a^{x-5}b^{n-2}$

6. $-7x^{m+3}y^{m-1} \div -8x^4y^2 = \frac{7}{8}x^{m+3-4}y^{m-1-2} = \frac{7}{8}x^{m-1}y^{m-3}$

7. $5a^{2m-1}b^{x-3} \div -6a^{2m-2}b^{x-4} = -\frac{5}{6}a^{2m-1-2m+2}b^{x-3-x+4} = -\frac{5}{6}ab$

8. $-4x^{n-1}y^{n+1} \div 5x^{n-1}y^{n+1} = -\frac{4}{5}x^{n-1-n+1}y^{n+1-n-1} = -\frac{4}{5}$

9. $a^{m+n}b^{x+a} \div a^mb^a = a^{m+n-m}b^{x+a-a} = a^nb^x$

10. $-5ab^2c^3 \div 6a^mb^nc^x = -\frac{5}{6}a^{1-m}b^{2-n}c^{3-x}$

EJERCICIO 51

1. $\frac{1}{2}x^2 \div \frac{2}{3} = \frac{\frac{1}{2}}{\frac{2}{3}}x^2 = \frac{3}{4}x^2$

2. $-\frac{3}{5}a^3b \div -\frac{4}{5}a^2b = \frac{\frac{3}{5}}{\frac{4}{5}}a^{3-2}b^{1-1} = \frac{15}{20}a = \frac{3}{4}a$

3. $\frac{2}{3}xy^5z^3 \div -\frac{1}{6}z^3 = -\frac{\frac{2}{3}}{\frac{1}{6}}xy^5z^{3-3} = -\frac{12}{3}xy^5 = -4xy^5$

4. $-\frac{7}{8}a^mb^n \div -\frac{3}{4}ab^2 = \frac{\frac{7}{8}}{\frac{3}{4}}a^{m-1}b^{n-2} = \frac{28}{24}a^{m-1}b^{n-2} = \frac{7}{6}a^{m-1}b^{n-2}$

5. $-\frac{2}{9}x^4y^5 \div -2 = \frac{\frac{2}{9}}{2}x^4y^5 = \frac{2}{18}x^4y^5 = \frac{1}{9}x^4y^5$

6. $3m^4n^5p^6 \div -\frac{1}{3}m^4np^5 = -\frac{3}{\frac{1}{3}}m^{4-4}n^{5-1}p^{6-5} = -9n^4p$

7. $-\frac{7}{8}a^2b^5c^6 \div -\frac{5}{2}ab^5c^6 = \frac{\frac{7}{8}}{\frac{5}{2}}a^{2-1}b^{5-5}c^{6-6} = \frac{14}{40}a = \frac{7}{20}a$

8. $\frac{2}{3}a^xb^m \div -\frac{3}{5}ab^2 = -\frac{\frac{2}{3}}{\frac{3}{5}}a^{x-1}b^{m-2} = -\frac{10}{9}a^{x-1}b^{m-2}$

9. $-\frac{3}{8}c^3d^{5} \div \frac{3}{4}d^{x} = -\frac{\frac{3}{8}}{\frac{3}{4}}c^3d^{5-x} = -\frac{12}{24}c^3d^{5-x} = -\frac{1}{2}c^3d^{5-x}$

10. $\frac{3}{4}a^mb^n \div -\frac{3}{2}b^3 = -\frac{\frac{3}{4}}{\frac{3}{2}}a^mb^{n-3} = -\frac{6}{12}a^mb^{n-3} = -\frac{1}{2}a^mb^{n-3}$

11. $-2a^{x+4}b^{m-3} \div -\frac{1}{2}a^4b^3 = \frac{2}{\frac{1}{2}}a^{x+4-4}b^{m-3-3} = 4a^xb^{m-6}$

12. $-\frac{1}{15}a^{x-3}b^{m+5}c^2 \div \frac{3}{5}a^{x-4}b^{m-1}$

$$= -\frac{\frac{1}{15}}{\frac{3}{5}}a^{x-3-x+4}b^{m+5-m+1}c^2 = -\frac{5}{45}ab^6c^2 = -\frac{1}{9}ab^6c^2$$

EJERCICIO 52

1. $a^2 - ab \div a = \frac{a^2 - ab}{a} = \frac{a^2}{a} - \frac{ab}{a} = a - b$

2. $3x^2y^3 - 5a^2x^4 \div -3x^2$

$$= \frac{3x^2y^3 - 5a^2x^4}{-3x^2} = \frac{3x^2y^3}{-3x^2} - \frac{5a^2x^4}{-3x^2} = -y^3 + \frac{5}{3}a^2x^2$$

3. $\frac{3a^3 - 5ab^2 - 6a^2b^3}{-2a}$

$$= \frac{3a^3}{-2a} - \frac{5ab^2}{-2a} - \frac{6a^2b^3}{-2a} = -\frac{3}{2}a^2 + \frac{5}{2}b^2 + 3ab^3$$

4. $\frac{x^3 - 4x^2 + x}{x} = \frac{x^3}{x} - \frac{4x^2}{x} + \frac{x}{x} = x^2 - 4x + 1$

5. $\frac{4x^8 - 10x^6 - 5x^4}{2x^3}$

$$= \frac{4x^8}{2x^3} - \frac{10x^6}{2x^3} - \frac{5x^4}{2x^3} = 2x^5 - 5x^3 - \frac{5}{2}x$$

6. $\frac{6m^3 - 8m^2n + 20mn^2}{-2m}$

$$= \frac{6m^3}{-2m} - \frac{8m^2n}{-2m} + \frac{20mn^2}{-2m} = -3m^2 + 4mn - 10n^2$$

7. $\frac{6a^8b^8 - 3a^6b^6 - a^2b^3}{3a^2b^3}$

$$= \frac{6a^8b^8}{3a^2b^3} - \frac{3a^6b^6}{3a^2b^3} - \frac{a^2b^3}{3a^2b^3} = 2a^6b^5 - a^4b^3 - \frac{1}{3}$$

8. $\frac{x^4 - 5x^3 - 10x^2 + 15x}{-5x}$

$$= \frac{x^4}{-5x} - \frac{5x^3}{-5x} - \frac{10x^2}{-5x} + \frac{15x}{-5x} = -\frac{1}{5}x^3 + x^2 + 2x - 3$$

9. $\frac{8m^9n^2 - 10m^7n^4 - 20m^5n^6 + 12m^3n^8}{2m^2}$

$$= \frac{8m^9n^2}{2m^2} - \frac{10m^7n^4}{2m^2} - \frac{20m^5n^6}{2m^2} + \frac{12m^3n^8}{2m^2}$$

$$= 4m^7n^2 - 5m^5n^4 - 10m^3n^6 + 6mn^8$$

10. $\frac{a^x + a^{m-1}}{a^2} = \frac{a^x}{a^2} + \frac{a^{m-1}}{a^2} = a^{x-2} + a^{m-3}$

11. $\frac{2a^m - 3a^{m+2} + 6a^{m+4}}{-3a^3}$

$$= \frac{2a^m}{-3a^3} - \frac{3a^{m+2}}{-3a^3} + \frac{6a^{m+4}}{-3a^3} = -\frac{2}{3}a^{m-3} + a^{m-1} - 2a^{m+1}$$

12. $\frac{a^mb^n + a^{m-1}b^{n+2} - a^{m-2}b^{n+4}}{a^2b^3}$

$$= \frac{a^mb^n}{a^2b^3} + \frac{a^{m-1}b^{n+2}}{a^2b^3} - \frac{a^{m-2}b^{n+4}}{a^2b^3}$$

$$= a^{m-2}b^{n-3} + a^{m-3}b^{n-1} - a^{m-4}b^{n+1}$$

13. $\frac{x^{m+2} - 5x^m + 6x^{m+1} - x^{m-1}}{x^{m-2}}$

$$= \frac{x^{m+2}}{x^{m-2}} - \frac{5x^m}{x^{m-2}} + \frac{6x^{m+1}}{x^{m-2}} - \frac{x^{m-1}}{x^{m-2}}$$

$$= x^4 - 5x^2 + 6x^3 - x \Rightarrow x^4 + 6x^3 - 5x^2 - x$$

14. $\frac{4a^{x+4}b^{m-1} - 6a^{x+3}b^{m-2} + 8a^{x+2}b^{m-3}}{-2a^{x+2}b^{m-4}}$

$$= \frac{4a^{x+4}b^{m-1}}{-2a^{x+2}b^{m-4}} - \frac{6a^{x+3}b^{m-2}}{-2a^{x+2}b^{m-4}} + \frac{8a^{x+2}b^{m-3}}{-2a^{x+2}b^{m-4}}$$

$$= -2a^2b^3 + 3ab^2 - 4b$$

EJERCICIO 53

1. $\frac{\frac{1}{2}x^2}{\frac{2}{3}x} - \frac{\frac{2}{3}x}{\frac{2}{3}x} = \frac{3}{4}x - \frac{6}{6} = \frac{3}{4}x - 1$

2. $\frac{\frac{1}{3}a^3}{-\frac{3}{5}} - \frac{\frac{3}{5}a^2}{-\frac{3}{5}} + \frac{\frac{1}{4}a}{-\frac{3}{5}}$

$$= -\frac{5}{9}a^3 + \frac{15}{15}a^2 - \frac{5}{12}a = -\frac{5}{9}a^3 + a^2 - \frac{5}{12}a$$

3. $\dfrac{\frac{1}{4}m^4}{\frac{1}{4}m^2} - \dfrac{\frac{2}{3}m^3n}{\frac{1}{4}m^2} + \dfrac{\frac{3}{8}m^2n^2}{\frac{1}{4}m^2}$

$= \frac{4}{4}m^2 - \frac{8}{3}mn + \frac{12}{8}n^2 = m^2 - \frac{8}{3}mn + \frac{3}{2}n^2$

4. $\dfrac{\frac{2}{3}x^4y^3}{-\frac{1}{5}xy^3} - \dfrac{\frac{1}{5}x^3y^4}{-\frac{1}{5}xy^3} + \dfrac{\frac{1}{4}x^2y^5}{-\frac{1}{5}xy^3} - \dfrac{xy^6}{-\frac{1}{5}xy^3}$

$= -\frac{10}{3}x^3 + \frac{5}{5}x^2y - \frac{5}{4}xy^2 + 5y^3$

$= -\frac{10}{3}x^3 + x^2y - \frac{5}{4}xy^2 + 5y^3$

5. $\dfrac{\frac{2}{5}a^5}{5a} - \dfrac{\frac{1}{3}a^3b^3}{5a} - \dfrac{ab^5}{5a} = \frac{2}{25}a^4 - \frac{1}{15}a^2b^3 - \frac{1}{5}b^5$

6. $\dfrac{\frac{1}{3}a^m}{\frac{1}{2}a} + \dfrac{\frac{1}{4}a^{m-1}}{\frac{1}{2}a} = \frac{2}{3}a^{m-1} + \frac{2}{4}a^{m-2} = \frac{2}{3}a^{m-1} + \frac{1}{2}a^{m-2}$

7. $\dfrac{\frac{2}{3}a^{x+1}}{\frac{1}{6}a^{x-2}} - \dfrac{\frac{1}{4}a^{x-1}}{\frac{1}{6}a^{x-2}} - \dfrac{\frac{2}{5}a^{x}}{\frac{1}{6}a^{x-2}}$

$= \frac{12}{3}a^3 - \frac{6}{4}a - \frac{12}{5}a^2 \Rightarrow 4a^3 - \frac{12}{5}a^2 - \frac{3}{2}a$

8. $\dfrac{-\frac{3}{4}a^{n-1}x^{m+2}}{-\frac{2}{5}a^3x^2} + \dfrac{\frac{1}{8}a^nx^{m+1}}{-\frac{2}{5}a^3x^2} - \dfrac{\frac{2}{3}a^{n+1}x^m}{-\frac{2}{5}a^3x^2}$

$= \frac{15}{8}a^{n-4}x^m - \frac{5}{16}a^{n-3}x^{m-1} + \frac{10}{6}a^{n-2}x^{m-2}$

$= \frac{15}{8}a^{n-4}x^m - \frac{5}{16}a^{n-3}x^{m-1} + \frac{5}{3}a^{n-2}x^{m-2}$

EJERCICIO 54

1. $$\begin{array}{r|l} a^2+2a-3 & a+3 \\ \cline{2-2} -a^2-3a & a-1 \\ \hline -a-3 & \\ +a+3 & \\ \hline \end{array}$$

2. $$\begin{array}{r|l} a^2-2a-3 & a+1 \\ \cline{2-2} -a^2-a & a-3 \\ \hline -3a-3 & \\ +3a+3 & \\ \hline \end{array}$$

3. $$\begin{array}{r|l} x^2+x-20 & x+5 \\ \cline{2-2} -x^2-5x & x-4 \\ \hline -4x-20 & \\ +4x+20 & \\ \hline \end{array}$$

4. $$\begin{array}{r|l} m^2-11m+30 & m-6 \\ \cline{2-2} -m^2+6m & m-5 \\ \hline -5m+30 & \\ +5m-30 & \\ \hline \end{array}$$

5. $$\begin{array}{r|l} x^2-8x+15 & -x+3 \\ \cline{2-2} -x^2+3x & -x+5 \\ \hline -5x+15 & \\ +5x-15 & \\ \hline \end{array}$$

6. $$\begin{array}{r|l} a^2+5a+6 & a+2 \\ \cline{2-2} -a^2-2a & a+3 \\ \hline 3a+6 & \\ -3a-6 & \\ \hline \end{array}$$

7. $$\begin{array}{r|l} 6x^2-xy-2y^2 & 2x+y \\ \cline{2-2} -6x^2-3xy & 3x-2y \\ \hline -4xy-2y^2 & \\ +4xy+2y^2 & \\ \hline \end{array}$$

8. $$\begin{array}{r|l} -15x^2+22xy-8y^2 & -3x+2y \\ \cline{2-2} +15x^2-10xy & 5x-4y \\ \hline 12xy-8y^2 & \\ -12xy+8y^2 & \\ \hline \end{array}$$

9. $$\begin{array}{r|l} 5a^2+8ab-21b^2 & a+3b \\ \cline{2-2} -5a^2-15ab & 5a-7b \\ \hline -7ab-21b^2 & \\ +7ab+21b^2 & \\ \hline \end{array}$$

10. $$\begin{array}{r|l} 14x^2+22x-12 & 7x-3 \\ \cline{2-2} -14x^2+6x & 2x+4 \\ \hline 28x-12 & \\ -28x+12 & \\ \hline \end{array}$$

11. $$\begin{array}{r|l} -8a^2+12ab-4b^2 & -a+b \\ \cline{2-2} +8a^2-8ab & 8a-4b \\ \hline 4ab-4b^2 & \\ -4ab+4b^2 & \\ \hline \end{array}$$

12. $$\begin{array}{r|l} 6m^2-11mn+5n^2 & m-n \\ \cline{2-2} -6m^2+6mn & 6m-5n \\ \hline -5mn+5n^2 & \\ +5mn-5n^2 & \\ \hline \end{array}$$

13. $$\begin{array}{r|l} -54m^2+12mn+32n^2 & -9m+8n \\ \cline{2-2} +54m^2-48mn & 6m+4n \\ \hline -36mn+32n^2 & \\ +36mn-32n^2 & \\ \hline \end{array}$$

14. $$\begin{array}{r|l} -14y^2+71y+33 & -7y-3 \\ \cline{2-2} +14y^2+6y & 2y-11 \\ \hline +77y+33 & \\ -77y-33 & \\ \hline \end{array}$$

15. $x^3 \qquad -y^3 \;\big|\; x-y$

$-x^3+x^2y \qquad\qquad x^2+xy+y^2$

x^2y

$-x^2y+xy^2$

xy^2-y^3

$-xy^2+y^3$

16. $a^3-3a^2b+3ab^2-b^3 \;\big|\; a-b$

$-a^3+a^2b \qquad\qquad a^2-2ab+b^2$

$-2a^2b+3ab^2$

$+2a^2b-2ab^2$

ab^2-b^3

$-ab^2+b^3$

17. $x^4 \qquad -9x^2 \quad +x+3 \;\big|\; x+3$

$-x^4-3x^3 \qquad\qquad x^3-3x^2+1$

$-3x^3-9x^2$

$+3x^3+9x^2$

$x+3$

$-x-3$

18. $a^4 \qquad +a \;\big|\; a+1$

$-a^4-a^3 \qquad\qquad a^3-a^2+a$

$-a^3$

$+a^3+a^2$

a^2+a

$-a^2-a$

19. $m^6 \qquad -n^6 \;\big|\; m^2-n^2$

$-m^6+m^4n^2 \qquad\qquad m^4+m^2n^2+n^4$

m^4n^2

$-m^4n^2+m^2n^4$

$m^2n^4-n^6$

$-m^2n^4+n^6$

20. $2x^4-x^3 \qquad +7x-3 \;\big|\; 2x+3$

$-2x^4-3x^3 \qquad\qquad x^3-2x^2+3x-1$

$-4x^3$

$+4x^3+6x^2$

$6x^2+7x$

$-6x^2-9x$

$-2x-3$

$2x+3$

21. $3y^5 \qquad -12y+5y^2+10 \;\big|\; y^2+2$

$-3y^5-6y^3 \qquad\qquad 3y^3-6y+5$

$-6y^3-12y$

$+6y^3+12y$

$5y^2+10$

$-5y^2-10$

22. $am^4 \qquad -am-2a \;\big|\; am+a$

$-am^4-am^3 \qquad\qquad m^3-m^2+m-2$

$-am^3$

$+am^3+am^2$

am^2-am

$-am^2-am$

$-2am-2a$

$+\;2am+2a$

23. $12a^3-35a^2b+33ab^2-10b^3 \;\big|\; 4a-5b$

$-12a^3+15a^2b \qquad\qquad 3a^2-5ab+2b^2$

$-20a^2b+33ab^2$

$+20a^2b-25ab^2$

$8ab^2-10b^3$

$-8ab^2+10b^3$

24. $15m^5-5m^4n-9m^3n^2+3m^2n^3+3mn^4-n^5 \;\big|\; 3m-n$

$-15m^5+5m^4n \qquad\qquad 5m^4-3m^2n^2+n^4$

$-9m^3n^2+3m^2n^3$

$+9m^3n^2-3m^2n^3$

$3mn^4-n^5$

$-3mn^4+n^5$

EJERCICIO 55

1. $a^4 \qquad -a^2-2a-1 \;\big|\; a^2+a+1$

$-a^4-a^3-a^2 \qquad\qquad a^2-a-1$

$-a^3-2a^2-2a$

$+a^3+a^2+a$

$-a^2-a-1$

$+a^2+a+1$

2.

$$\begin{array}{l|l}
x^5 \qquad +12x^2-5x & \underline{x^2-2x+5} \\
\underline{-x^5+2x^4-5x^3} & x^3+2x^2-x \\
2x^4-5x^3+12x^2 & \\
\underline{-2x^4+4x^3-10x^2} & \\
-x^3+2x^2-5x & \\
\underline{+x^3-2x^2+5x} &
\end{array}$$

3.

$$\begin{array}{l|l}
m^5-5m^4n \qquad +20m^2n^3-16mn^4 & \underline{m^2-2mn-8n^2} \\
\underline{-m^5+2m^4n+8m^3n^2} & m^3-3m^2n+2mn^2 \\
-3m^4n+8m^3n^2+20m^2n^3 & \\
\underline{+3m^4n-6m^3n^2-24m^2n^3} & \\
2m^3n^2-4m^2n^3-16mn^4 & \\
\underline{-2m^3n^2+4m^2n^3+16mn^4} &
\end{array}$$

4.

$$\begin{array}{l|l}
x^4 \qquad -x^2-2x-1 & \underline{x^2-x-1} \\
\underline{-x^4+x^3+x^2} & x^2+x+1 \\
+x^3 \qquad -2x & \\
\underline{-x^3+x^2+x} & \\
x^2-x-1 & \\
\underline{-x^2+x+1} &
\end{array}$$

5.

$$\begin{array}{l|l}
x^6-2x^5 \qquad +6x^3-7x^2-4x+6 & \underline{x^4-3x^2+2} \\
\underline{-x^6 \qquad +3x^4 \qquad -2x^2} & x^2-2x+3 \\
-2x^5+3x^4+6x^3-9x^2-4x & \\
\underline{+2x^5 \qquad -6x^3 \qquad +4x} & \\
3x^4 \qquad -9x^2 \qquad +6 & \\
\underline{-3x^4 \qquad +9x^2 \qquad -6} &
\end{array}$$

6.

$$\begin{array}{l|l}
m^6+m^5-4m^4 \qquad +m^2-4m-1 & \underline{m^3+m^2-4m-1} \\
\underline{-m^6-m^5+4m^4+m^3} & m^3+1 \\
m^3+m^2-4m-1 & \\
\underline{-m^3-m^2+4m+1} &
\end{array}$$

7.

$$\begin{array}{l|l}
a^5-a^4 \qquad +7a^2-27a+10 & \underline{a^2-a+5} \\
\underline{-a^5+a^4-5a^3} & a^3-5a+2 \\
-5a^3+7a^2-27a & \\
\underline{+5a^3-5a^2+25a} & \\
2a^2-2a+10 & \\
\underline{-2a^2+2a-10} &
\end{array}$$

8.

$$\begin{array}{l|l}
-x^4+3x^3y \qquad -5xy^3+3y^4 & \underline{x^2-2xy+y^2} \\
\underline{+x^4-2x^3y+x^2y^2} & -x^2+xy+3y^2 \\
x^3y+x^2y^2-5xy^3 & \\
\underline{-x^3y+2x^2y^2-xy^3} & \\
3x^2y^2-6xy^3+3y^4 & \\
\underline{-3x^2y^2+6xy^3-3y^4} &
\end{array}$$

9.

$$\begin{array}{l|l}
n^4-2n^3 \qquad +2n-1 & \underline{n^2-2n+1} \\
\underline{-n^4+2n^3-n^2} & n^2-1 \\
-n^2+2n-1 & \\
\underline{+n^2-2n+1} &
\end{array}$$

10.

$$\begin{array}{l|l}
a^5b-5a^4b^2 \qquad +22a^2b^4-40ab^5 & \underline{a^2b-2ab^2-10b^3} \\
\underline{-a^5b+2a^4b^2+10a^3b^3} & a^3-3a^2b+4ab^2 \\
-3a^4b^2+10a^3b^3+22a^2b^4 & \\
\underline{+3a^4b^2-6a^3b^3-30a^2b^4} & \\
4a^3b^3-8a^2b^4-40ab^5 & \\
\underline{-4a^3b^3+8a^2b^4+40ab^5} &
\end{array}$$

11.

$$\begin{array}{l|l}
16x^4 \qquad -24x^2y^2 \qquad -27y^4 & \underline{8x^3-12x^2y+6xy^2-9y^3} \\
\underline{-16x^4+24x^3y-12x^2y^2+18xy^3} & 2x+3y \\
24x^3y-36x^2y^2+18xy^3-27y^4 & \\
\underline{-24x^3y+36x^2y^2-18xy^3+27y^4} &
\end{array}$$

12.

$$\begin{array}{l|l}
4y^4+4y^3-13y^2-3y-20 & \underline{2y+5} \\
\underline{-4y^4-10y^3} & 2y^3-3y^2+y-4 \\
-6y^3-13y^2 & \\
\underline{+6y^3+15y^2} & \\
2y^2-3y & \\
\underline{-2y^2-5y-20} & \\
-8y-20 & \\
\underline{8y+20} &
\end{array}$$

13.

$$\begin{array}{l|l}
-2a^5-3a^4x+5a^3x^2 \qquad -11ax^4-3x^5 & \underline{-a^3+2ax^2+3x^3} \\
\underline{+2a^5 \qquad -4a^3x^2-6a^2x^3} & 2a^2-3ax-x^2 \\
3a^4x+a^3x^2-6a^2x^3-11ax^4 & \\
\underline{-3a^4x \qquad +6a^2x^3+9ax^4} & \\
a^3x^2 \qquad -2ax^4-3x^5 & \\
\underline{-a^3x^2 \qquad +2ax^4+3x^5} &
\end{array}$$

14. $(-x^6+2x^5y-3x^2y^4-xy^5) : (x^4-3x^3y+2x^2y^2+xy^3) = -x^2-xy-y^2$

$$\begin{array}{l}
-x^6+2x^5y-3x^2y^4-xy^5 \\
\underline{+x^6-3x^5y+2x^4y^2+x^3y^3} \\
-x^5y+2x^4y^2+x^3y^3-3x^2y^4 \\
\underline{+x^5y-3x^4y^2+2x^3y^3+x^2y^4} \\
-x^4y^2+3x^3y^3-2x^2y^4-xy^5 \\
\underline{+x^4y^2-3x^3y^3+2x^2y^4+xy^5}
\end{array}$$

15. $(a^6-5a^5+31a^2-8a+21) : (a^3-2a-7) = a^3-5a^2+2a-3$

$$\begin{array}{l}
a^6-5a^5+31a^2-8a+21 \\
\underline{-a^6+2a^4+7a^3} \\
-5a^5+2a^4+7a^3+31a^2 \\
\underline{+5a^5-10a^3-35a^2} \\
2a^4-3a^3-4a^2-8a \\
\underline{-2a^4+4a^2+14a} \\
-3a^3+6a+21 \\
\underline{+3a^3-6a-21}
\end{array}$$

16. $(m^6-m^5+5m^3-6m+9) : (m^4+m^3-m^2+3) = m^2-2m+3$

$$\begin{array}{l}
m^6-m^5+5m^3-6m+9 \\
\underline{-m^6-m^5+m^4-3m^2} \\
-2m^5+m^4+5m^3-3m^2-6m \\
\underline{+2m^5+2m^4-2m^3+6m} \\
3m^4+3m^3-3m^2+9 \\
\underline{-3m^4-3m^3+3m^2-9}
\end{array}$$

17. $(a^6-a^5b-4a^4b^2+6a^3b^3-3ab^5+b^6) : (a^2-2ab+b^2) = a^4+a^3b-3a^2b^2-ab^3+b^4$

$$\begin{array}{l}
a^6-a^5b-4a^4b^2+6a^3b^3-3ab^5+b^6 \\
\underline{-a^6+2a^5b-a^4b^2} \\
a^5b-5a^4b^2+6a^3b^3 \\
\underline{-a^5b+2a^4b^2-a^3b^3} \\
-3a^4b^2+5a^3b^3 \\
\underline{+3a^4b^2-6a^3b^3+3a^2b^4} \\
-a^3b^3+3a^2b^4-3ab^5 \\
\underline{+a^3b^3-2a^2b^4+ab^5} \\
a^2b^4-2ab^5+b^6 \\
\underline{-a^2b^4+2ab^5-b^6}
\end{array}$$

18. $(x^6-2x^4y^2+2x^3y^3-2x^2y^4+3xy^5-2y^6) : (x^2+xy-2y^2) = x^4-x^3y+x^2y^2-xy^3+y^4$

$$\begin{array}{l}
x^6-2x^4y^2+2x^3y^3-2x^2y^4+3xy^5-2y^6 \\
\underline{-x^6-x^5y+2x^4y^2} \\
-x^5y+2x^3y^3 \\
\underline{x^5y+x^4y^2-2x^3y^3} \\
+x^4y^2-2x^2y^4 \\
\underline{-x^4y^2-x^3y^3+2x^2y^4} \\
-x^3y^3+3xy^5 \\
\underline{+x^3y^3+x^2y^4-2xy^5} \\
x^2y^4+xy^5-2y^6 \\
\underline{-x^2y^4-xy^5+2y^6}
\end{array}$$

19. $(y^6-2y^5-y^4+4y^3-4y+2) : (y^4-2y^2+2) = y^2-2y+1$

$$\begin{array}{l}
y^6-2y^5-y^4+4y^3-4y+2 \\
\underline{-y^6+2y^4-2y^2} \\
-2y^5+y^4+4y^3-2y^2-4y \\
\underline{+2y^5-4y^3+4y} \\
y^4-2y^2+2 \\
\underline{-y^4+2y^2-2}
\end{array}$$

20. $(3m^7-11m^5+m^4+18m^3-3m^2-8m+4) : (m^4-3m^2+4) = 3m^3-2m+1$

$$\begin{array}{l}
3m^7-11m^5+m^4+18m^3-3m^2-8m+4 \\
\underline{-3m^7+9m^5-12m^3} \\
-2m^5+m^4+6m^3-8m \\
\underline{+2m^5-6m^3+8m} \\
m^4-3m^2+4 \\
\underline{-m^4+3m^2-4}
\end{array}$$

21. $(a^6+2a^5-2a^4-3a^3+2a^2-a-1) : (a^3+a^2-a+1) = a^3+a^2-2a-1$

$$\begin{array}{l}
a^6+2a^5-2a^4-3a^3+2a^2-a-1 \\
\underline{-a^6-a^5+a^4-a^3} \\
a^5-a^4-4a^3+2a^2 \\
\underline{-a^5-a^4+a^3-a^2} \\
-2a^4-3a^3+a^2-a \\
\underline{+2a^4+2a^3-2a^2+2a-1} \\
-a^3-a^2+a-1 \\
\underline{+a^3+a^2-a+1}
\end{array}$$

22. $(24x^5-52x^4y+38x^3y^2-33x^2y^3-26xy^4+4y^5) : (8x^3-12x^2y-6xy^2+y^3) = 3x^2-2xy+4y^2$

$$\begin{array}{l}
24x^5-52x^4y+38x^3y^2-33x^2y^3-26xy^4+4y^5 \\
\underline{-24x^5+36x^4y+18x^3y^2-3x^2y^3} \\
-16x^4y+56x^3y^2-36x^2y^3-26xy^4 \\
\underline{+16x^4y-24x^3y^2-12x^2y^3+2xy^4} \\
32x^3y^2-48x^2y^3-24xy^4+4y^5 \\
\underline{-32x^3y^2-48x^2y^3+24xy^4-4y^5}
\end{array}$$

23. $(5a^8-4a^7-8a^6+5a^5+6a^4-2a^3+4a^2-6a) : (a^4-2a^2+2) = 5a^4-4a^3+2a^2-3a$

$$\begin{array}{l}
5a^8-4a^7-8a^6+5a^5+6a^4-2a^3+4a^2-6a \\
\underline{-5a^8+10a^6-10a^4} \\
-4a^7+2a^6+5a^5-4a^4-2a^3 \\
\underline{+4a^7-8a^5+8a^3} \\
+2a^6-3a^5-4a^4+6a^3+4a^2 \\
\underline{-2a^6+4a^4-4a^2} \\
-3a^5+6a^3-6a \\
\underline{3a^5-6a^3+6a}
\end{array}$$

24.

$$\begin{array}{l|l}
x^7-3x^6+6x^5+x^2-3x+6 & x^3-2x^2+3x+6 \\ \cline{2-2}
\underline{-x^7+2x^6-3x^5-6x^4} & x^4-x^3+x^2-x+1 \\
-x^6+3x^5-6x^4 & \\
\underline{+x^6-2x^5+3x^4+6x^3} & \\
x^5-3x^4+6x^3+x^2 & \\
\underline{-x^5+2x^4-3x^3-6x^2-3x} & \\
-x^4+3x^3-5x^2-3x & \\
\underline{+x^4-2x^3+3x^2+6x+6} & \\
x^3-2x^2+3x+6 & \\
\underline{-x^3+2x^2-3x-6} &
\end{array}$$

25.

$$\begin{array}{l|l}
3a^6+5a^5-9a^4-10a^3+8a^2+3a-4 & 3a^3+2a^2-5a-4 \\ \cline{2-2}
\underline{-3a^6-2a^5+5a^4+4a^3} & a^3+a^2-2a+1 \\
+3a^5-4a^4-6a^3+8a^2 & \\
\underline{-3a^5-2a^4+5a^3+4a^2} & \\
-6a^4-a^3+12a^2+3a & \\
\underline{+6a^4+4a^3-10a^2-8a} & \\
3a^3+2a^2-5a-4 & \\
\underline{-3a^3-2a^2+5a+4} &
\end{array}$$

26.

$$\begin{array}{l|l}
5y^8-3y^7-11y^6+11y^5-17y^4-3y^3-4y^2-2y & 5y^4-3y^3+4y^2+2y \\ \cline{2-2}
\underline{-5y^8+3y^7-4y^6-2y^5} & y^4-3y^2-1 \\
-15y^6+9y^5-17y^4-3y^3 & \\
\underline{+15y^6-9y^5+12y^4+6y^3} & \\
-5y^4+3y^3-4y^2-2y & \\
\underline{+5y^4-3y^3+4y^2+2y} &
\end{array}$$

27.

$$\begin{array}{l|l}
-m^7+5m^6n-14m^5n^2+20m^4n^3-13m^3n^4-9m^2n^5+20mn^6-4n^7 & -m^3+3m^2n-5mn^2+n^3 \\ \cline{2-2}
\underline{+m^7-3m^6n+5m^5n^2-m^4n^3} & m^4-2m^3n+3m^2n^2-4n^4 \\
2m^6n-9m^5n^2+19m^4n^3-13m^3n^4 & \\
\underline{-2m^6n+6m^5n^2-10m^4n^3+2m^3n^4} & \\
-3m^5n^2+9m^4n^3-11m^3n^4-9m^2n^5 & \\
\underline{+3m^5n^2-9m^4n^3+15m^3n^4-3m^2n^5} & \\
4m^3n^4-12m^2n^5+20mn^6-4n^7 & \\
\underline{-4m^3n^4+12m^2n^5-20mn^6+4n^7} &
\end{array}$$

28.

$$\begin{array}{l|l}
x^{11}-5x^9y^2+8x^7y^4-6x^5y^6-5x^3y^8+3xy^{10} & x^5-2x^3y^2+3xy^4 \\ \cline{2-2}
\underline{-x^{11}+2x^9y^2-3x^7y^4} & x^6-3x^4y^2-x^2y^4+y^6 \\
-3x^9y^2+5x^7y^4-6x^5y^6 & \\
\underline{+3x^9y^2-6x^7y^4+9x^5y^6} & \\
-x^7y^4+3x^5y^6-5x^3y^8 & \\
\underline{+x^7y^4-2x^5y^6+3x^3y^8} & \\
+x^5y^6-2x^3y^8+3xy^{10} & \\
\underline{-x^5y^6+2x^3y^8-3xy^{10}} &
\end{array}$$

29.

$$\begin{array}{l|l}
3a^9-15a^7+14a^6-28a^4+47a^3-28a^2+23a-10 & 3a^5-6a^3+2a^2-3a+2 \\ \cline{2-2}
\underline{-3a^9+6a^7-2a^6+3a^5-2a^4} & a^4-3a^2+4a-5 \\
-9a^7+12a^6+3a^5-30a^4+47a^3-28a^2 & \\
\underline{+9a^7-18a^5+6a^4-9a^3+6a^2} & \\
+12a^6-15a^5-24a^4+38a^3-22a^2+23a & \\
\underline{-12a^6+24a^4-8a^3+12a^2-8a} & \\
-15a^5+30a^3-10a^2+15a-10 & \\
\underline{+15a^5-30a^3+10a^2-15a+10} &
\end{array}$$

30.

$$\begin{array}{l|l}
a^2-b^2+2bc-c^2 & a+b-c \\ \cline{2-2}
\underline{-a^2-ab+ac} & a-b+c \\
-ab-b^2+ac+2bc & \\
\underline{+ab+b^2-bc} & \\
ac+bc-c^2 & \\
\underline{-ac-bc+c^2} &
\end{array}$$

31.

$$\begin{array}{l|l}
-2x^2+5xy-xz-3y^2-yz+10z^2 & 2x-3y+5z \\ \cline{2-2}
\underline{+2x^2-3xy+5xz} & -x+y+2z \\
2xy+4xz-3y^2-yz & \\
\underline{-2xy+3y^2-5yz} & \\
4xz-6yz+10z^2 & \\
\underline{-4xz+6yz-10z^2} &
\end{array}$$

32.

$$\begin{array}{l|l}
x^3+y^3-3xyz+z^3 & x^2-xy-xz+y^2-yz+z^2 \\ \cline{2-2}
\underline{-x^3+x^2y+x^2z-xy^2+xyz-xz^2} & x+y+z \\
x^2y+x^2z+y^3-xy^2-2xyz+z^3-xz^2 & \\
\underline{-x^2y-y^3+xy^2+xyz+y^2z-yz^2} & \\
x^2z-xyz+z^3-xz^2+y^2z-yz^2 & \\
\underline{-x^2z+xyz-z^3+xz^2-y^2z+yz^2} &
\end{array}$$

33.

$$a^5 \qquad +b^5 \;\big|\; a+b$$
$$\text{Quotient: } a^4-a^3b+a^2b^2-ab^3+b^4$$
$$\begin{array}{l}
\underline{-a^5-a^4b} \\
-a^4b \\
\underline{+a^4b+a^3b^2} \\
a^3b^2 \\
\underline{-a^3b^2-a^2b^3} \\
-a^2b^3 \\
\underline{+a^2b^3+ab^4} \\
ab^4+b^5 \\
\underline{-ab^4-b^5}
\end{array}$$

34.

$$21x^5 \qquad -21y^5 \;\big|\; 3x-3y$$
$$\text{Quotient: } 7x^4+7x^3y+7x^2y^2+7xy^3+7y^4$$
$$\begin{array}{l}
\underline{-21x^5+21x^4y} \\
21x^4y \\
\underline{-21x^4y+21x^3y^2} \\
21x^3y^2 \\
\underline{-21x^3y^2+21x^2y^3} \\
21x^2y^3 \\
\underline{-21x^2y^3+21xy^4} \\
21xy^4-21y^5 \\
\underline{-21xy^4+21y^5}
\end{array}$$

35.

$$16x^8 \qquad -16y^8 \;\big|\; 2x^2+2y^2$$
$$\text{Quotient: } 8x^6-8x^4y^2+8x^2y^4-8y^6$$
$$\begin{array}{l}
\underline{-16x^8-16x^6y^2} \\
-16x^6y^2 \\
\underline{+16x^6y^2+16x^4y^4} \\
16x^4y^4 \\
\underline{-16x^4y^4-16x^2y^6} \\
-16x^2y^6-16y^8 \\
\underline{+16x^2y^6+16y^8}
\end{array}$$

36.

$$x^{10} \qquad -y^{10} \;\big|\; x^2-y^2$$
$$\text{Quotient: } x^8+x^6y^2+x^4y^4+x^2y^6+y^8$$
$$\begin{array}{l}
\underline{-x^{10}+x^8y^2} \\
x^8y^2 \\
\underline{-x^8y^2+x^6y^4} \\
x^6y^4 \\
\underline{-x^6y^4+x^4y^6} \\
x^4y^6 \\
\underline{-x^4y^6+x^2y^8} \\
x^2y^8-y^{10} \\
\underline{-x^2y^8+y^{10}}
\end{array}$$

37.

$$x^{15} \qquad +y^{15} \;\big|\; x^3+y^3$$
$$\text{Quotient: } x^{12}-x^9y^3+x^6y^6-x^3y^9+y^{12}$$
$$\begin{array}{l}
\underline{-x^{15}-x^{12}y^3} \\
-x^{12}y^3 \\
\underline{x^{12}y^3+x^9y^6} \\
x^9y^6 \\
\underline{-x^9y^6-x^6y^9} \\
-x^6y^9 \\
\underline{x^6y^9+x^3y^{12}} \\
x^3y^{12}+y^{15} \\
\underline{-x^3y^{12}-y^{15}}
\end{array}$$

38.

$$x^3+3x^2y+3xy^2+y^3-1 \;\big|\; x^2+2xy+x+y+y^2+1$$
$$\text{Quotient: } x+y-1$$
$$\begin{array}{l}
\underline{-x^3-2x^2y-xy^2 \quad -x^2-xy-x} \\
x^2y+2xy^2+y^3-1-x^2-xy-x \\
\underline{-x^2y-2xy^2-y^3 \quad -xy \quad -y^2-y} \\
-1-x^2-2xy-x-y^2-y \\
\underline{+1+x^2+2xy+x+y^2+y}
\end{array}$$

39.

$$x^5 \qquad +y^5 \;\big|\; x^4-x^3y+x^2y^2-xy^3+y^4$$
$$\text{Quotient: } x+y$$
$$\begin{array}{l}
\underline{-x^5+x^4y-x^3y^2+x^2y^3-xy^4} \\
x^4y-x^3y^2+x^2y^3-xy^4+y^5 \\
\underline{-x^4y+x^3y^2-x^2y^3+xy^4-y^5}
\end{array}$$

EJERCICIO 56

1.

$$a^{x+3} \qquad +a^x \;\big|\; a+1$$
$$\text{Quotient: } a^{x+2}-a^{x+1}+a^x$$
$$\begin{array}{l}
\underline{-a^{x+3}-a^{x+2}} \\
-a^{x+2} \\
\underline{+a^{x+2}+a^{x+1}} \\
a^{x+1}+a^x \\
\underline{-a^{x+1}-a^x}
\end{array}$$

2. $-x^{n+5}+x^{n+4}+3x^{n+3}+x^{n+2} \;\big|\; x^2+x$

$$-x^{n+3}+2x^{n+2}+x^{n+1}$$

$$\begin{array}{l}
+x^{n+5}+x^{n+4} \\ \hline
+2x^{n+4}+3x^{n+3} \\
-2x^{n+4}-2x^{n+3} \\ \hline
x^{n+3}+x^{n+2} \\
-x^{n+3}-x^{n+2} \\ \hline
\end{array}$$

8. $-m^{2a+3}+2m^{2a+2}+2m^{2a+1}-4m^{2a}-m^{2a-1}+m^{2a-2} \;\big|\; -m^{a-1}+m^{a-2}+m^{a-3}$

$$m^{a+4}-m^{a+3}-2m^{a+2}+m^{a+1}$$

$$\begin{array}{l}
+m^{2a+3}-m^{2a+2}-m^{2a+1} \\ \hline
+m^{2a+2}+m^{2a+1}-4m^{2a} \\
-m^{2a+2}+m^{2a+1}+m^{2a} \\ \hline
2m^{2a+1}-3m^{2a}-m^{2a-1} \\
-2m^{2a+1}+2m^{2a}+2m^{2a-1} \\ \hline
-m^{2a}+m^{2a-1}+m^{2a-2} \\
+m^{2a}-m^{2a-1}-m^{2a-2} \\ \hline
\end{array}$$

3. $m^{a+4}-m^{a+3}+6m^{a+1}-5m^{a}+3m^{a-1} \;\big|\; m^2-2m+3$

$$m^{a+2}+m^{a+1}-m^{a}+m^{a-1}$$

$$\begin{array}{l}
-m^{a+4}+2m^{a+3}-3m^{a+2} \\ \hline
m^{a+3}-3m^{a+2}+6m^{a+1} \\
-m^{a+3}+2m^{a+2}-3m^{a+1} \\ \hline
-m^{a+2}+3m^{a+1}-5m^{a} \\
+m^{a+2}-2m^{a+1}+3m^{a} \\ \hline
m^{a+1}-2m^{a}+3m^{a-1} \\
-m^{a+1}+2m^{a}-3m^{a-1} \\ \hline
\end{array}$$

9. $x^{2a-2}+x^{2a-3}-4x^{2a-4}-x^{2a-7} \;\big|\; x^{a-1}-x^{a-2}-x^{a-3}$

$$x^{a-1}+2x^{2a-2}-x^{a-3}+x^{a-4}$$

$$\begin{array}{l}
-x^{2a-2}+x^{2a-3}+x^{2a-4} \\ \hline
+2x^{2a-3}-3x^{2a-4} \\
-2x^{2a-3}+2x^{2a-4}+2x^{2a-5} \\ \hline
-x^{2a-4}+2x^{2a-5} \\
+x^{2a-4}-x^{2a-5}-x^{2a-6} \\ \hline
+x^{2a-5}-x^{2a-6}-x^{2a-7} \\
-x^{2a-5}+x^{2a-6}+x^{2a-7} \\ \hline
\end{array}$$

4. $a^{2n+3}+4a^{2n+2}+a^{2n+1}-2a^{2n} \;\big|\; a^{n+1}+a^{n}$

$$a^{n+2}+3a^{n+1}-2a^{n}$$

$$\begin{array}{l}
-a^{2n+3}-a^{2n+2} \\ \hline
3a^{2n+2}+a^{2n+1} \\
-3a^{2n+2}-3a^{2n+1} \\ \hline
-2a^{2n+1}-2a^{2n} \\
2a^{2n+1}+2a^{2n} \\ \hline
\end{array}$$

10. $a^{2n}b^3-a^{2n-1}b^4+a^{2n-2}b^5-2a^{2n-4}b^7+a^{2n-5}b^8 \;\big|\; a^nb-a^{n-1}b^2+2a^{n-2}b^3-a^{n-3}b^4$

$$a^nb^2-a^{n-2}b^4$$

$$\begin{array}{l}
-a^{2n}b^3+a^{2n-1}b^4-2a^{2n-2}b^5+a^{2n-3}b^6 \\ \hline
-a^{2n-2}b^5+a^{2n-3}b^6-2a^{2n-4}b^7+a^{2n-5}b^8 \\
+a^{2n-2}b^5-a^{2n-3}b^6+2a^{2n-4}b^7-a^{2n-5}b^8 \\ \hline
\end{array}$$

5. $x^{2a+5}+2x^{2a+4}-3x^{2a+3}-4x^{2a+2}+2x^{2a+1} \;\big|\; x^{a+3}-2x^{a+1}$

$$x^{a+2}+2x^{a+1}-x^{a}$$

$$\begin{array}{l}
-x^{2a+5}+2x^{2a+3} \\ \hline
+2x^{2a+4}-x^{2a+3} \\
-2x^{2a+4}+4x^{2a+2} \\ \hline
-x^{2a+3}+2x^{2a+1} \\
+x^{2a+3}-2x^{2a+1} \\ \hline
\end{array}$$

11. $a^{m+x}+a^mb^x+a^xb^m+b^{m+x} \;\big|\; a^x+b^x$

$$a^m+b^m$$

$$\begin{array}{l}
-a^{m+x}-a^mb^x \\ \hline
a^xb^m+b^{m+x} \\
-a^xb^m-b^{m+x} \\ \hline
\end{array}$$

6. $a^{x+2}-2a^{x}+8a^{x-1}-3a^{x-2} \;\big|\; a^x-2a^{x-1}+3a^{x-2}$

$$a^2+2a-1$$

$$\begin{array}{l}
-a^{x+2}+2a^{x+1}-3a^{x} \\ \hline
2a^{x+1}-5a^{x}+8a^{x-1} \\
-2a^{x+1}+4a^{x}-6a^{x-1} \\ \hline
-a^{x}+2a^{x-1}-3a^{x-2} \\
+a^{x}-2a^{x-1}+3a^{x-2} \\ \hline
\end{array}$$

12. $a^x-a^{x-1}b+b^n-ab^{n-1} \;\big|\; a-b$

$$a^{x-1}-b^{n-1}$$

$$\begin{array}{l}
-a^x+a^{x-1}b \\ \hline
b^n-ab^{n-1} \\
-b^n+ab^{n-1} \\ \hline
\end{array}$$

7. $a^{2x}+2a^{2x-1}-4a^{2x-2}+5a^{2x-3}-2a^{2x-4} \;\big|\; a^x-a^{x-1}+a^{x-2}$

$$a^x+3a^{x-1}-2a^{x-2}$$

$$\begin{array}{l}
-a^{2x}+a^{2x-1}-a^{2x-2} \\ \hline
3a^{2x-1}-5a^{2x-2}+5a^{2x-3} \\
-3a^{2x-1}+3a^{2x-2}-3a^{2x-3} \\ \hline
-2a^{2x-2}+2a^{2x-3}-2a^{2x-4} \\
+2a^{2x-2}-2a^{2x-3}+2a^{2x-4} \\ \hline
\end{array}$$

13. $3a^{5m-3}-23a^{5m-2}+5a^{5m-1}+46a^{5m}-30a^{5m+1} \;\big|\; a^{3m-3}-8a^{3m-2}+6a^{3m-1}$

$$3a^{2m}+a^{2m+1}-5a^{2m+2}$$

$$\begin{array}{l}
-3a^{5m-3}+24a^{5m-2}-18a^{5m-1} \\ \hline
+a^{5m-2}-13a^{5m-1}+46a^{5m} \\
-a^{5m-2}+8a^{5m-1}-6a^{5m} \\ \hline
-5a^{5m-1}+40a^{5m}-30a^{5m+1} \\
5a^{5m-1}-40a^{5m}+30a^{5m+1} \\ \hline
\end{array}$$

14.

$$2x^{3a+1}y^{2x-3}-4x^{3a}y^{2x-2}-28x^{3a-2}y^{2x}+30x^{3a-3}y^{2x+1} \quad \Big| \quad -x^{a+2}y^{x-1}+4x^{a+1}y^{x}-3x^{a}y^{x+1}$$

$$\text{Cociente: } -2x^{2a-1}y^{x-2}-4x^{2a-2}y^{x-1}-10x^{2a-3}y^{x}$$

$$\underline{-2x^{3a+1}y^{2x-3}+8x^{3a}y^{2x-2}-6x^{3a-1}y^{2x-1}}$$

$$+4x^{3a}y^{2x-2}-6x^{3a-1}y^{2x-1}-28x^{3a-2}y^{2x}$$

$$\underline{-4x^{3a}y^{2x-2}+16x^{3a-1}y^{2x-1}-12x^{3a-2}y^{2x}}$$

$$10x^{3a-1}y^{2x-1}-40x^{3a-2}y^{2x}+30x^{3a-3}y^{2x+1}$$

$$\underline{-10x^{3a-1}y^{2x-1}+40x^{3a-2}y^{2x}-30x^{3a-3}y^{2x+1}}$$

EJERCICIO 57

1.

$$\frac{1}{6}a^2+\frac{5}{36}ab-\frac{1}{6}b^2 \quad \Big| \quad \frac{1}{3}a+\frac{1}{2}b$$

$$\text{Cociente: } \frac{1}{2}a-\frac{1}{3}b$$

$$\underline{-\frac{1}{6}a^2-\frac{1}{4}ab}$$

$$-\frac{1}{9}ab-\frac{1}{6}b^2$$

$$\underline{\frac{1}{9}ab+\frac{1}{6}b^2}$$

2.

$$\frac{1}{3}x^2+\frac{7}{10}xy-\frac{1}{3}y^2 \quad \Big| \quad x-\frac{2}{5}y$$

$$\text{Cociente: } \frac{1}{3}x+\frac{5}{6}y$$

$$\underline{-\frac{1}{3}x^2+\frac{2}{15}xy}$$

$$\frac{5}{6}xy-\frac{1}{3}y^2$$

$$\underline{-\frac{5}{6}xy+\frac{1}{3}y^2}$$

3.

$$\frac{1}{3}x^3-\frac{35}{36}x^2y+\frac{2}{3}xy^2-\frac{3}{8}y^3 \quad \Big| \quad \frac{1}{2}x^2-\frac{1}{3}xy+\frac{1}{4}y^2$$

$$\text{Cociente: } \frac{2}{3}x-\frac{3}{2}y$$

$$\underline{-\frac{1}{3}x^3+\frac{2}{9}x^2y-\frac{1}{6}xy^2}$$

$$-\frac{3}{4}x^2y+\frac{1}{2}xy^2-\frac{3}{8}y^3$$

$$\underline{\frac{3}{4}x^2y-\frac{1}{2}xy^2+\frac{3}{8}y^3}$$

4.

$$\frac{1}{16}a^3-\frac{5}{8}a^2b+\frac{5}{3}ab^2-b^3 \quad \Big| \quad \frac{1}{4}a-\frac{3}{2}b$$

$$\text{Cociente: } \frac{1}{4}a^2-ab+\frac{2}{3}b^2$$

$$\underline{-\frac{1}{16}a^3+\frac{3}{8}a^2b}$$

$$-\frac{1}{4}a^2b+\frac{5}{3}ab^2$$

$$\underline{\frac{1}{4}a^2b-\frac{3}{2}ab^2}$$

$$\frac{1}{6}ab^2-b^3$$

$$\underline{-\frac{1}{6}ab^2+b^3}$$

5.

$$\frac{3}{5}m^4+\frac{1}{10}m^3n-\frac{17}{60}m^2n^2+\frac{7}{6}mn^3-n^4 \quad \Big| \quad \frac{3}{2}m^2-mn+2n^2$$

$$\text{Cociente: } \frac{2}{5}m^2+\frac{1}{3}mn-\frac{1}{2}n^2$$

$$\underline{-\frac{3}{5}m^4+\frac{2}{5}m^3n-\frac{4}{5}m^2n^2}$$

$$\frac{1}{2}m^3n-\frac{65}{60}m^2n^2+\frac{7}{6}mn^3$$

$$\underline{-\frac{1}{2}m^3n+\frac{1}{3}m^2n^2-\frac{2}{3}mn^3}$$

$$-\frac{3}{4}m^2n^2+\frac{1}{2}mn^3-n^4$$

$$\underline{\frac{3}{4}m^2n^2-\frac{1}{2}mn^3+n^4}$$

6.

$$\frac{3}{4}x^5+\frac{1}{2}x^4-\frac{37}{40}x^3+\frac{2}{3}x^2+\frac{19}{30}x-\frac{4}{5} \quad \Big| \quad 2x^3-\frac{1}{3}x+2$$

$$\text{Cociente: } \frac{3}{8}x^2+\frac{1}{4}x-\frac{2}{5}$$

$$\underline{-\frac{3}{4}x^5+\frac{1}{8}x^3-\frac{3}{4}x^2}$$

$$\frac{1}{2}x^4-\frac{4}{5}x^3-\frac{1}{12}x^2+\frac{19}{30}x$$

$$\underline{-\frac{1}{2}x^4+\frac{1}{12}x^2-\frac{1}{2}x}$$

$$-\frac{4}{5}x^3+\frac{2}{15}x-\frac{4}{5}$$

$$\underline{\frac{4}{5}x^3-\frac{2}{15}x+\frac{4}{5}}$$

7.

$$\frac{9}{4}a^4-a^3x-\frac{1}{12}a^2x^2+\frac{13}{18}ax^3-\frac{1}{3}x^4 \quad \Big| \quad \frac{3}{2}a^2-ax+\frac{2}{3}x^2$$

$$\text{Cociente: } \frac{3}{2}a^2+\frac{1}{3}ax-\frac{1}{2}x^2$$

$$\underline{-\frac{9}{4}a^4+\frac{3}{2}a^3x-a^2x^2}$$

$$\frac{1}{2}a^3x-\frac{13}{12}a^2x^2+\frac{13}{18}ax^3$$

$$\underline{-\frac{1}{2}a^3x+\frac{1}{3}a^2x^2-\frac{2}{9}ax^3}$$

$$-\frac{3}{4}a^2x^2+\frac{1}{2}ax^3-\frac{1}{3}x^4$$

$$\underline{\frac{3}{4}a^2x^2-\frac{1}{2}ax^3+\frac{1}{3}x^4}$$

8. $\frac{1}{14}x^5-\frac{101}{420}x^4y+\frac{139}{280}x^3y^2-\frac{1}{2}x^2y^3+\frac{5}{12}xy^4 \div \frac{2}{7}x^3-\frac{1}{5}x^2y+\frac{1}{2}xy^2$

Cociente: $\frac{1}{4}x^2-\frac{2}{3}xy+\frac{5}{6}y^2$

$-\frac{1}{14}x^5+\frac{1}{20}x^4y-\frac{1}{8}x^3y^2$

$-\frac{4}{21}x^4y+\frac{13}{35}x^3y^2-\frac{1}{2}x^2y^3$

$+\frac{4}{21}x^4y-\frac{2}{15}x^3y^2+\frac{1}{3}x^2y^3$

$\frac{5}{21}x^3y^2-\frac{1}{6}x^2y^3+\frac{5}{12}xy^4$

$-\frac{5}{21}x^3y^2+\frac{1}{6}x^2y^3-\frac{5}{12}xy^4$

9. $\frac{3}{8}x^5+\frac{21}{40}x^4-\frac{47}{120}x^3+\frac{79}{120}x^2+\frac{1}{10}x-\frac{1}{10} \div \frac{1}{4}x^3+\frac{1}{3}x^2-\frac{1}{4}x+\frac{1}{2}$

Cociente: $\frac{3}{2}x^2+\frac{1}{10}x-\frac{1}{5}x$

$-\frac{3}{8}x^5-\frac{1}{2}x^4+\frac{3}{8}x^3-\frac{3}{4}x^2$

$\frac{1}{40}x^4-\frac{1}{60}x^3-\frac{11}{120}x^2+\frac{1}{10}x$

$-\frac{1}{40}x^4-\frac{1}{30}x^3+\frac{1}{40}x^2-\frac{1}{20}x$

$-\frac{1}{20}x^3-\frac{1}{15}x^2+\frac{1}{20}x-\frac{1}{10}$

$\frac{1}{20}x^3+\frac{1}{15}x^2-\frac{1}{20}x+\frac{1}{10}$

10. $\frac{1}{2}m^5-\frac{5}{6}m^4n+\frac{99}{40}m^3n^2-\frac{101}{60}m^2n^3+\frac{7}{6}mn^4-\frac{5}{8}n^5 \div \frac{3}{4}m^3-\frac{1}{2}m^2n+\frac{2}{5}mn^2-\frac{1}{4}n^3$

Cociente: $\frac{2}{3}m^2-\frac{2}{3}mn+\frac{5}{2}n^2$

$-\frac{1}{2}m^5+\frac{1}{3}m^4n-\frac{4}{15}m^3n^2+\frac{1}{6}m^2n^3$

$-\frac{1}{2}m^4n+\frac{53}{24}m^3n^2-\frac{91}{60}m^2n^3+\frac{7}{6}mn^4$

$\frac{1}{2}m^4n-\frac{1}{3}m^3n^2+\frac{4}{15}m^2n^3-\frac{1}{6}mn^4$

$\frac{15}{8}m^3n^2-\frac{5}{4}m^2n^3+mn^4-\frac{5}{8}n^5$

$-\frac{15}{8}m^3n^2+\frac{5}{4}m^2n^3-mn^4+\frac{5}{8}n^5$

EJERCICIO 58

1. $x^5-x^4+x^2-x \div x^3-x^2+x$

$1-1+0+1-1 \mid 1-1+1$

$-1+1-1$ $1+0-1$

$-1+1-1$ $\Rightarrow x^2-1$

$+1-1+1$

2. $x^7+x^6-11x^5+3x^4-13x^3+19x^2-56 \div x^3-2x^2-7$

$1+1-11+3-13+19+0-56 \mid 1-2+0-7$

$-1+2-0+7$ $1+3-5+0+8$

$3-11+10-13$ $\Rightarrow x^4+3x^3-5x^2+8$

$-3+6-0+21$

$-5+10+8+19$

$+5-10+0-35$

$8-16+0-56$

$-8+16-0+56$

3. $a^6+a^5b-7a^4b^2+12a^3b^2-13a^2b^4+7ab^5-b^6 \div a^2-2ab+b^2$

$1+1-7+12-13+7-1 \mid 1-2+1$

$-1+2-1$ $1+3-2+5-1$

$3-8+12$ $\Rightarrow a^4+3a^3b-2a^2b^2+5ab^3-b^4$

$-3+6-3$

$-2+9-13$

$2-4+2$

$5-11+7$

$-5+10-5$

$-1+2-1$

$+1-2+1$

4. $m^6-5m^5n+2m^4n^2+20m^3n^3-19m^2n^4-10mn^5-n^6 \div m^3-4mn^2-n^3$

$1-5+2+20-19-10-1 \quad | \underline{\;1+0-4-1\;}$

$\underline{-1-0+4+1} \qquad 1-5+6+1$

$-5+6+21-19 \qquad \Rightarrow m^3-5m^2n+6mn^2+n^3$

$\underline{5+0-20-5}$

$6+1-24-10$

$\underline{-6-0+24+6}$

$1+0-4-1$

$\underline{-1-0+4+1}$

5. $x^8-2x^6-50x^4+58x^2-15 \div x^4+6x^2-5$

$1-2-50+58-15 \quad | \underline{\;1+6-5\;}$

$\underline{-1-6+5} \qquad 1-8+3$

$-8-45+58 \qquad \Rightarrow x^4-8x^2+3$

$\underline{+8+48-40}$

$3+18-15$

$\underline{-3-18+15}$

6. $a^{14}-7a^{12}+9a^{10}+23a^8-52a^6+42a^4-20a^2 \div a^8-4a^6+3a^4-2a^2$

$1-7+9+23-52+42-20 \quad | \underline{\;1-4+3-2\;}$

$\underline{-1+4-3+2} \qquad 1-3-6+10$

$-3+6+25-52 \qquad \Rightarrow a^6-3a^4-6a^2+10$

$\underline{3-12+9-6}$

$-6+34-58+42$

$\underline{+6-24+18-12}$

$10-40+30-20$

$\underline{-10+40-30+20}$

7. $3x^{15}-20x^{12}+51x^9-70x^6+46x^3-20 \div 3x^6-8x^3+10$

$3-20+51-70+46-20 \quad | \underline{\;3-8+10\;}$

$\underline{-3+8-10} \qquad 1-4+3-2$

$-12+41-70 \qquad \Rightarrow x^9-4x^6+3x^3-2$

$\underline{12-32+40}$

$9-30+46$

$\underline{-9+24-30}$

$-6+16-20$

$\underline{6-16+20}$

8. $m^{28}-12m^{24}+53m^{20}-127m^{16}+187m^{12}-192m^8+87m^4-45 \div m^{12}-7m^8+9m^4-15$

$1-12+53-127+187-192+87-45 \quad | \underline{\;1-7+9-15\;}$

$\underline{-1+7-9+15} \qquad 1-5+9-4+3$

$-5+44-112+187 \qquad \Rightarrow m^{16}-5m^{12}+9m^8-4m^4+3$

$\underline{+5-35+45-75}$

$9-67+112-192$

$\underline{-9+63-81+135}$

$-4+31-57+87$

$\underline{+4-28+36-60}$

$3-21+27-45$

$\underline{-3+21-27+45}$

9. $2x^7-6x^6y-8x^5y^2-20x^4y^3-24x^3y^4-18x^2y^5-4y^7 \div 2x^2+4y^2$

$2-6-8-20-24-18+0-4 \quad | \underline{\;2+0+4\;}$

$\underline{-2-0-4} \qquad 1-3-6-4+0-1$

$-6-12-20 \qquad \Rightarrow x^5-3x^4y-6x^3y^2-4x^2y^3-y^5$

$\underline{6+0+12}$

$-12-8-24$

$\underline{+12+0+24}$

$-8+0-18$

$\underline{+8+0+16}$

$-2+0-4$

$\underline{2+0+4}$

10. $6a^9-12a^7+2a^6-36a^5+6a^4-16a^3+38a^2-44a+14 \div a^4-2a^2+a-7$

$6+0-12+2-36+6-16+38-44+14 \quad | \underline{\;1+0-2+1-7\;}$

$\underline{-6-0+12-6+42} \qquad 6+0+0-4+6-2$

$-4+6+6-16+38 \qquad \Rightarrow 6a^5-4a^2+6a-2$

$\underline{+4+0-8+4-28}$

$6-2-12+10-44$

$\underline{-6-0+12-6+42}$

$-2+0+4-2+14$

$\underline{+2-0-4+2-14}$

11. $n^{10}-6n^{8}+5n^{7}+13n^{6}-23n^{5}-8n^{4}+44n^{3}-12n^{2}-32n+16 \div n^{6}-3n^{4}+5n^{3}-8n+4$

$1+0-6+5+13-23-8+44-12-32+16 \quad | \ 1+0-3+5+0-8+4$

$-1-0+3-5-0+8-4 \qquad 1+0-3+0+4$

$-3+0+13-15-12+44-12 \qquad \Rightarrow n^{4}-3n^{2}+4$

$+3+0-9+15+0-24+12$

$4+0-12+20+0-36+16$

$-4+0+12-20-0+36-16$

12. $3x^{7}-4x^{6}y-15x^{5}y^{2}+29x^{4}y^{3}-13x^{3}y^{4}+5xy^{6}-3y^{7} \div x^{3}-5xy^{2}+3y^{3}$

$3-4-15+29-13+0+5-3 \quad | \ 1+0-5+3$

$-3-0+15-9 \qquad 3-4+0+0-1$

$-4+0+20-13 \qquad \Rightarrow 3x^{4}-4x^{3}y-y^{4}$

$+4+0-20+12$

$-1+0+5-3$

$1-0-5+3$

13. $x^{16}-4x^{14}y^{2}-10x^{12}y^{4}+21x^{10}y^{6}+28x^{8}y^{8}-23x^{6}y^{10}+9x^{4}y^{12}+33x^{2}y^{14}-6y^{16} \div x^{6}-4x^{4}y^{2}-5x^{2}y^{4}+y^{6}$

$1-4-10+21+28-23+9+33-6 \quad | \ 1-4-5+1$

$-1+4+5-1 \qquad 1+0-5+0+3-6$

$-5+20+28-23 \qquad \Rightarrow x^{10}-5x^{6}y^{4}+3x^{2}y^{8}-6y^{10}$

$+5-20-25+5$

$3-18+9+33$

$-3+12+15-3$

$-6+24+30-6$

$+6-24-30+6$

14. $a^{m+2}-3a^{m+1}-5a^{m}+20a^{m-1}-25a^{m-3} \div a^{2}-5$

$1-3-5+20+0-25 \quad | \ 1+0-5$

$-1-0+5 \qquad 1-3+0+5$

$-3+0+20 \qquad \Rightarrow a^{m}-3a^{m-1}+5a^{m-3}$

$+3+0-15$

$+5+0-25$

$-5-0+25$

15. $7a^{2x+5}-35a^{2x+4}+6a^{2x+3}-78a^{2x+2}-5a^{2x+1}-42a^{2x}-7a^{2x-1} \div 7a^{x+3}+6a^{x+1}+a^{x}$

$7-35+6-78-5-42-7 \quad | \ 7+0+6+1$

$-7-0-6-1 \qquad 1-5+0-7$

$-35+0-79-5 \qquad \Rightarrow a^{x+2}-5a^{x+1}-7a^{x-1}$

$+35+0+30+5$

$-49+0-42-7$

$49-0+42+7$

16. $6x^{2a+3}-4x^{2a+2}-28x^{2a+1}+21x^{2a}-46x^{2a-1}+19x^{2a-2}-12x^{2a-3}-6x^{2a-4} \div 6x^{a+1}-4x^{a}+2x^{a-1}+x^{a-2}$

$6-4-28+21-46+19-12-6 \quad | \ 6-4+2+1$

$-6+4-2-1 \qquad 1+0-5+0-6$

$-30+20-46+19 \qquad \Rightarrow x^{a+2}-5x^{a}-6x^{a-2}$

$+30-20+10+5$

$-36+24-12-6$

$36-24+12+6$

17. $6a^{5x+3}-23a^{5x+2}+12a^{5x+1}-34a^{5x}+22a^{5x-1}-15a^{5x-2} \div a^{2x+2}-3a^{2x+1}-a^{2x}-5a^{2x-1}$

$6-23+12-34+22-15 \quad | \ 1-3-1-5$

$-6+18+6+30 \qquad 6-5+3$

$-5+18-4+22 \qquad \Rightarrow 6a^{3x+1}-5a^{3x}+3a^{3x-1}$

$+5-15-5-25$

$3-9-3-15$

$-3+9+3+15$

EJERCICIO 59

1. $a^2+b^2 \;\Big|\; a^2$

$\underline{-a^2} \qquad 1+\dfrac{b^2}{a^2}$

b^2

2. $a^4+2 \;\Big|\; a^3$

$\underline{-a^4} \qquad a+\dfrac{2}{a^3}$

$+2$

3. $9x^3+6x^2+7 \;\Big|\; 3x^2$

$\underline{-9x^3} \qquad 3x+2+\dfrac{7}{3x^2}$

$+6x^2$

$\underline{-6x^2}$

$+7$

4. $16a^4-20a^3b+8a^2b^2+7ab^3 \;\Big|\; 4a^2$

$\underline{-16a^4} \qquad 4a^2-5ab+2b^2+\dfrac{7b^3}{4a}$

$-20a^3b$

$\underline{+20a^3b}$

$8a^2b^2$

$\underline{-8a^2b^2}$

$+7ab^3$

5. $x^2+7x+10 \;\Big|\; x+6$

$\underline{-x^2-6x} \qquad x+1+\dfrac{4}{x+6}$

$x+10$

$\underline{-x-6}$

$+4$

6. $x^2-5x+7 \;\Big|\; x-4$

$\underline{-x^2+4x} \qquad x-1+\dfrac{3}{x-4}$

$-x+7$

$\underline{x-4}$

$+3$

7. $m^4-11m^2+34 \;\Big|\; m^2-3$

$\underline{-m^4+3m^2} \qquad m^2-8+\dfrac{10}{m^2-3}$

$-8m^2+34$

$\underline{8m^2-24}$

$+10$

8. $x^2-6xy+y^2 \;\Big|\; x+y$

$\underline{-x^2-xy} \qquad x-7y+\dfrac{8y^2}{x+y}$

$-7xy+y^2$

$\underline{+7xy+7y^2}$

$+8y^2$

9. $x^3-x^2+3x+2 \;\Big|\; x^2-x+1$

$\underline{-x^3+x^2-x} \qquad x+\dfrac{2x+2}{x^2-x+1}$

$+2x+2$

10. $x^3 \qquad +y^3 \;\Big|\; x-y$

$\underline{-x^3+x^2y} \qquad x^2+xy+y^2+\dfrac{2y^3}{x-y}$

x^2y

$\underline{-x^2y+xy^2}$

xy^2+y^3

$\underline{-xy^2+y^3}$

$2y^3$

11. $x^5 \qquad +y^5 \;\Big|\; x-y$

$\underline{-x^4+x^4y} \qquad x^4+x^3y+x^2y^2+xy^3+y^4+\dfrac{2y^5}{x-y}$

x^4y

$\underline{-x^4y+x^3y^2}$

x^3y^2

$\underline{-x^3y^2+x^2y^3}$

x^2y^3

$\underline{-x^2y^3+xy^4}$

$+xy^4+y^5$

$\underline{-xy^4+y^5}$

$2y^5$

12. $x^3+4x^2-5x+8 \;\Big|\; x^2-2x+1$

$\underline{-x^3+2x^2-x} \qquad x+6+\dfrac{6x+2}{x^2-2x+1}$

$6x^2-6x+8$

$\underline{-6x^2+12x-6}$

$6x+2$

13.

$$
\begin{array}{l|l}
8a^3-6a^2b+5ab^2-9b^3 & 2a-3b \\ \hline
\underline{-8a^3+12a^2b} & 4a^2+3ab+7b^2+\dfrac{12b^3}{2a-3b} \\
\quad 6a^2b+5ab^2 & \\
\quad \underline{-6a^2b+9ab^2} & \\
\qquad 14ab^2-9b^3 & \\
\qquad \underline{-14ab^2+21b^3} & \\
\qquad\qquad 12b^3 &
\end{array}
$$

14.

$$
\begin{array}{l|l}
x^5-3x^4\quad +9x^2+7x-4 & x^2-3x+2 \\ \hline
\underline{-x^5+3x^4-2x^3} & x^3-2x+3+\dfrac{20x-10}{x^2-3x+2} \\
\quad -2x^3+9x^2+7x & \\
\quad \underline{+2x^3-6x^2+4x} & \\
\qquad 3x^2+11x-4 & \\
\qquad \underline{-3x^2+9x-6} & \\
\qquad\qquad 20x-10 &
\end{array}
$$

EJERCICIO 60

Para los problemas del 1 al 9 las literales toman los siguentes valores: a = - 1 b = - 2 c = - 1/2

1. $a^2-2ab+b^2$

$$=(-1)^2-2(-1)\cdot 2+2^2$$
$$=1+2\cdot 2+4=1+4+4=9$$

2. $3a^3-4a^2b+3ab^2-b^3$

$$=3(-1)^3-4(-1)^2\cdot 2+3(-1)2^2-2^3$$
$$=-3-4\cdot 2-3\cdot 4-8=-3-8-12-8=-31$$

3. $a^4-3a^3+2ac-3bc$

$$=(-1)^4-3(-1)^3+2\,(-1)\left(-\frac{1}{2}\right)-3\cdot 2\left(-\frac{1}{2}\right)$$
$$=1+3+2\cdot\frac{1}{2}+6\cdot\frac{1}{2}=1+3+1+3=8$$

4. $a^5-8a^4c+16a^3c^2-20a^2c^3+40ac^4-c^5$

$$=(-1)^5-8(-1)^4\left(-\frac{1}{2}\right)+16\,(-1)^3\left(-\frac{1}{2}\right)^2-20(-1)^2\left(-\frac{1}{2}\right)^3+40(-1)\left(-\frac{1}{2}\right)^4-\left(-\frac{1}{2}\right)^5$$
$$=-1+8\cdot\frac{1}{2}-16\cdot\frac{1}{4}+20\cdot\frac{1}{8}-40\cdot\frac{1}{16}+\frac{1}{32}=-1+4-4+\frac{5}{2}-\frac{5}{2}+\frac{1}{32}=-1+\frac{1}{32}=\frac{-32+1}{32}=-\frac{31}{32}$$

5. $(a-b)^2+(b-c)^2-(a-c)^2$

$$=(-1-2)^2+\left[2-\left(-\frac{1}{2}\right)\right]^2-\left[-1-\left(-\frac{1}{2}\right)\right]^2$$
$$=(-3)^2+\left[2+\frac{1}{2}\right]^2-\left[-1+\frac{1}{2}\right]^2=9+\left(\frac{5}{2}\right)^2-\left(-\frac{1}{2}\right)^2$$
$$=9+\frac{25}{4}-\frac{1}{4}=9+\frac{24}{4}=9+6=15$$

6. $(b+a)^3-(b-c)^3-(a-c)^3$

$$=\left[2+(-1)\right]^3-\left[2-\left(-\frac{1}{2}\right)\right]^3-\left[-1-\left(-\frac{1}{2}\right)\right]^3$$
$$=\left[2-1\right]^3-\left[2+\frac{1}{2}\right]^3-\left[-1+\frac{1}{2}\right]^3=1^3-\left[\frac{5}{2}\right]^3-\left[-\frac{1}{2}\right]^3$$
$$=1-\frac{125}{8}+\frac{1}{8}=1-\frac{124}{8}=1-\frac{31}{2}=\frac{2-31}{2}=-\frac{29}{2}=-14\tfrac{1}{2}$$

7. $\dfrac{ab}{c}+\dfrac{ac}{b}-\dfrac{bc}{a}$

$$=\frac{-1\cdot 2}{-\frac{1}{2}}+\frac{-1\left(-\frac{1}{2}\right)}{2}-\frac{2\left(-\frac{1}{2}\right)}{-1}=\frac{2}{\frac{1}{2}}+\frac{\frac{1}{2}}{2}-\frac{2}{2}$$
$$=4+\frac{1}{4}-1=3+\frac{1}{4}=\frac{12+1}{4}=\frac{13}{4}=3\tfrac{1}{4}$$

8. $(a+b+c)^2-(a-b-c)^2+c$

$$=\left[-1+2+\left(-\frac{1}{2}\right)\right]^2-\left[-1-2-\left(-\frac{1}{2}\right)\right]^2+\left(-\frac{1}{2}\right)$$
$$=\left[1-\frac{1}{2}\right]^2-\left[-3+\frac{1}{2}\right]^2-\frac{1}{2}=\left[\frac{1}{2}\right]^2-\left[-\frac{5}{2}\right]^2-\frac{1}{2}$$
$$=\frac{1}{4}-\frac{25}{4}-\frac{1}{2}=\frac{1-25-2}{4}=-\frac{26}{4}=-6\tfrac{2}{4}=-6\tfrac{1}{2}$$

9. $3(2a+b)-4a(b+c)-2c(a-b)$

$$=3\left[2(-1)+2\right]-4(-1)\left[2+\left(-\frac{1}{2}\right)\right]-2\left(-\frac{1}{2}\right)(-1-2)=3\left[-2+2\right]+4\left[2-\frac{1}{2}\right]+\frac{2}{2}(-3)=3\cdot 0+4\left[\frac{3}{2}\right]-\frac{6}{2}=6-3=3$$

Para los problemas 10 al 16 las literales toman los siguientes valores:
a = 2   b = 1/3   x = - 2   y = - 1   m= 3   n = 1/2

10. $\frac{x^4}{8}-\frac{x^2y}{2}+\frac{3xy^2}{2}-y^3$

$=\frac{(-2)^4}{8}-\frac{(-2)^2(-1)}{2}+\frac{3(-2)(-1)^2}{2}-(-1)^3$

$=\frac{16}{8}-\frac{-4}{2}+\frac{-6}{2}+1=2+2-3+1=2$

11. $(a-x)^2+(x-y)^2+(x^2-y^2)(m+x-n)$

$=[2-(-2)]^2+[-2-(-1)]^2+[(-2)^2-(-1)^2]\left[3+(-2)-\frac{1}{2}\right]$

$=[2+2]^2+[-2+1]^2+[4-1]\left[3-2-\frac{1}{2}\right]=[4]^2+[-1]^2+3\left[1-\frac{1}{2}\right]$

$=16+1+3\left[\frac{1}{2}\right]=17+\frac{3}{2}=\frac{34+3}{2}=\frac{37}{2}=18\tfrac{1}{2}$

12. $-(x-y)+(x^2+y^2)(x-y-m)+3b(x+y+n)$

$=-[-2-(-1)]+[(-2)^2+(-1)^2][-2-(-1)-3]+3\cdot\frac{1}{3}\left[-2+(-1)+\frac{1}{2}\right]=-[-2+1]+[4+1][-5+1]+\left[-2-1+\frac{1}{2}\right]$

$=1+5[-4]+\left[-3+\frac{1}{2}\right]=1-20+\left[-\frac{5}{2}\right]=-19-\frac{5}{2}=-\frac{38+5}{2}=-\frac{43}{2}=-21\tfrac{1}{2}$

13. $(3x-2y)(2n-4m)+4x^2y^2-\frac{x-y}{2}$

$=[3(-2)-2(-1)]\left[2\cdot\frac{1}{2}-4\cdot 3\right]+4(-2)^2(-1)^2-\frac{(-2)-(-1)}{2}$

$=[-6+2][1-12]+4\cdot 4\cdot 1-\frac{-2+1}{2}=-4\,[-11]+16-\frac{-1}{2}$

$=44+16+\frac{1}{2}=60+\frac{1}{2}=60\tfrac{1}{2}$

14. $\frac{4x}{3y}-\frac{x^3}{2+y^3}+\left(\frac{1}{n}-\frac{1}{b}\right)x+x^4-m$

$=\frac{4(-2)}{3(-1)}-\frac{(-2)^3}{2+(-1)^3}+\left(\frac{1}{\frac{1}{2}}-\frac{1}{\frac{1}{3}}\right)(-2)+(-2)^4-3$

$=\frac{-8}{-3}-\frac{-8}{2-1}+(2-3)(-2)+16-3=\frac{8}{3}+8+(-1)(-2)+13$

$=\frac{8}{3}+21+2=\frac{8}{3}+23=\frac{8+69}{3}=\frac{77}{3}=25\tfrac{2}{3}$

15. $x^2(x-y+m)-(x-y)(x^2+y^2-n)+(x+y)^2(m^2-2n)$

$=(-2)^2[-2-(-1)+3]-[-2-(-1)]\left[(-2)^2+(-1)^2-\frac{1}{2}\right]+[-2+(-1)]^2\left(3^2-2\cdot\frac{1}{2}\right)$

$=4[1+1]-[-2+1]\left[4+1-\frac{1}{2}\right]+[-2-1]^2(9-1)=4\cdot 2-[-1]\left[5-\frac{1}{2}\right]+[-3]^2(8)=8+\frac{9}{2}+9\cdot 8=80+\frac{9}{2}=\frac{160+9}{2}=\frac{169}{2}=84\tfrac{1}{2}$

16. $\frac{3a}{x}+\frac{2y}{m}+\frac{3n}{y}-\frac{m}{n}+2(x^3-y^2+4)=\frac{3\cdot 2}{-2}+\frac{2(-1)}{3}+\frac{3\cdot\frac{1}{2}}{-1}-\frac{3}{\frac{1}{2}}+2[(-2)^3-(-1)^2+4]=-3-\frac{2}{3}-\frac{3}{2}-6+2[-8-1+4]$

$=-9-\frac{4+9}{6}+2[-5]=-9-\frac{13}{6}-10=-19-\frac{13}{6}=-\frac{114+13}{6}=-\frac{127}{6}=-21\tfrac{1}{6}$

EJERCICIO 61

1.

$$\begin{array}{cccc} 7am & 8am & 9am & 10am \\ \downarrow & \downarrow & \downarrow & \downarrow \\ +5^\circ & +2^\circ & -1^\circ & -4^\circ \end{array}$$

3.

$$\begin{array}{ll} x^2-3xy & x^2 \\ \underline{\quad 3xy-y^2} & \underline{-x^2+y^2} \\ x^2 \qquad -y^2 & \qquad y^2 \quad Rta \end{array}$$

4.

$$\begin{array}{l} 3x^2-5x+6 \\ \underline{-3x^2+8x-6} \quad Rta \\ \qquad 3x \end{array}$$

5.

$$\begin{array}{ll} \qquad 3 & 2a^2-3a+8 \\ \underline{2a^2-3a+5} & \underline{\qquad 8a+5} \\ 2a^2-3a+8 & 2a^2+5a+13 \quad Rta \end{array}$$

6. $-3x^2-\left\{-\left[4x^2+5x-\left(x^2-(x+6)\right)\right]\right\}$

$=-3x^2-\left\{-\left[4x^2+5x-\left(x^2-x-6\right)\right]\right\}$

$=-3x^2-\left\{-\left[4x^2+5x-x^2+x+6\right]\right\}$

$=-3x^2-\left\{-3x^2-6x-6\right\}=-3x^2+3x^2+6x+6=6x+6$

7. $(x+y)(x-y)-(x+y)^2$

$$= x^2-xy+xy-y^2-(x+y)(x+y)$$
$$= x^2-y^2-(x^2+2xy+y^2)$$
$$= x^2-y^2-x^2-2xy-y^2$$
$$= -2xy-2y^2$$

8. $a=2 \quad b=3 \quad c=1$

$$3(a+b)-4(c-b)+\sqrt{\frac{c-b}{-a}}$$
$$=3(2+3)-4(1-3)+\sqrt{\frac{1-3}{-2}}$$
$$=3\cdot 5-4(-2)+\sqrt{\frac{-2}{-2}}$$
$$=15+8+\sqrt{1}=23+1=24$$

9.

$$\begin{array}{l} 3x^2 \qquad -5y^2 \\ \underline{-x^2+3xy-y^2} \\ 2x^2+3xy-6y^2 \end{array} \qquad \begin{array}{l} 2x^2+5xy+6y^2 \\ \underline{-x^2-5xy \qquad\quad} \\ x^2 \qquad\quad +6y^2 \end{array}$$

$$\begin{array}{l} 2x^2+3xy-6y^2 \\ \underline{x^2 \qquad\quad +6y^2} \\ 3x^2+3xy \qquad Rta \end{array}$$

10.

$$\frac{2}{3}a^2-\frac{1}{2}ab+\frac{1}{5}b^2$$
$$\underline{\frac{1}{2}a^2+\frac{3}{4}ab-2b^2}$$
$$\frac{1}{3}a^4-\frac{1}{4}a^3b+\frac{1}{10}a^2b^2$$
$$+\frac{1}{2}a^3b-\frac{3}{8}a^2b^2+\frac{3}{20}ab^3$$
$$\underline{-\frac{4}{3}a^2b^2+ab^3-\frac{2}{5}b^4}$$
$$\frac{1}{3}a^4+\frac{1}{4}a^3b-\frac{193}{120}a^2b^2+\frac{23}{20}ab^3-\frac{2}{5}b^4$$

11.

$$x^5 \qquad -x^3+5x^2$$
$$-2x^4 \qquad +2x^2-10x$$
$$\underline{+6x^3 \qquad -6x+30}$$
$$x^5-2x^4+5x^3+7x^2-16x+30 \quad \big| \; x^2-2x+6$$
$$\underline{-x^5+2x^4-6x^3} \qquad x^3-x+5$$
$$-x^3+7x^2-16x$$
$$\underline{+x^3-2x^2+6x}$$
$$5x^2-10x+30$$
$$\underline{-5x^2+10x-30}$$

12.

$$\frac{1}{4}a^3 \qquad -\frac{1}{90}ab^2+\frac{1}{15}b^3 \quad \Big| \; \frac{1}{2}a+\frac{1}{3}b$$
$$\underline{-\frac{1}{4}a^3-\frac{1}{6}a^2b} \qquad \frac{1}{2}a^2-\frac{1}{3}ab+\frac{1}{5}b^2$$
$$-\frac{1}{6}a^2b-\frac{1}{90}ab^2$$
$$\underline{\frac{1}{6}a^2b+\frac{1}{9}ab^2}$$
$$\frac{1}{10}ab^2+\frac{1}{15}b^3$$
$$\underline{-\frac{1}{10}ab^2-\frac{1}{15}b^3}$$

$$\frac{1}{2}a^2+ab+\frac{1}{5}b^2$$
$$\underline{-\frac{1}{2}a^2+\frac{1}{3}ab-\frac{1}{5}b^2}$$
$$\frac{3+1}{3}ab=\frac{4}{3}ab$$

13.

$$\begin{array}{l} \qquad -3ab^2-b^3 \\ \underline{2a^2b+3ab^2-b^3} \\ 2a^2b \qquad -2b^3 \end{array} \qquad \begin{array}{l} a^3-a^2b+b^3 \\ \underline{\quad -2a^2b+2b^3} \\ a^3-3a^2b+3b^3 \end{array}$$

$$a^3-3a^2b+3b^3$$
$$\underline{a^2-ab+b^2}$$
$$a^5-3a^4b \qquad +3a^2b^3$$
$$-a^4b+3a^3b^2 \qquad -3ab^4$$
$$\underline{+a^3b^2-3a^2b^3 \qquad +3b^5}$$
$$a^5-4a^4b+4a^3b^2 \qquad -3ab^4+3b^5$$

14.

$$\begin{array}{l} x^3-5x^2+4x \\ \quad -6x^2-6x+3 \\ \underline{\quad -8x^2+8x-3} \\ x^3-19x^2+6x \end{array} \qquad \begin{array}{l} 2x^3-16x^2+5x+12 \\ \underline{-x^3+19x^2-6x} \\ x^3+3x^2-x+12 \end{array}$$

$$x^3+3x^2-x+12 \quad \big| \; x^2-x+3$$
$$\underline{-x^3+x^2-3x} \qquad x+4$$
$$4x^2-4x+12$$
$$\underline{-4x^2+4x-12}$$

15.

$$(2+x)^2(1+x^2)-(x^2-2)(x^2+x-3)=x^2(3x+10)+2(3x-1)$$
$$(4+4x+x^2)(1+x^2)-(x^4+x^3-5x^2-2x+6)=3x^3+10x^2+6x-2$$
$$4+5x^2+4x+4x^3-x^3+5x^2+2x-6=3x^3+10x^2+6x-2$$
$$3x^3+10x^2+6x-2=3x^3+10x^2+6x-2$$

16. $x=-2 \quad y=1$

$$(x+y)^2(x-y)^2+2(x+y)(x-y)$$
$$=(-2+1)^2(-2-1)^2+2(-2+1)(-2-1)$$
$$=(-1)^2(-3)^2+2(-1)(-3)$$
$$=1\cdot 9+2\cdot 3=9+6=15$$

17. x^2+2x+8

$$\begin{array}{r} x+4 \\ x-6 \\ \hline x^2+4x+6 \end{array} \qquad \begin{array}{r} x^2+4x+6 \\ 4x^2-8x-3 \\ \hline 5x^2-4x+3 \end{array} \quad Rta$$

18. $-\{3a+(-b+a)-2(a+b)\}$ $\qquad -2[(a+b)-(a-b)]$

$$=-\{3a-b+a-2a-2b\} \qquad =-2[a+b-a+b]$$
$$=-\{2a-3b\} \qquad =-2[2b]$$
$$=-2a+3b \qquad =-4b$$

$$\begin{array}{r} -4b \\ 2a-3b \\ \hline 2a-7b \end{array} \quad Rta$$

19. $5x+[-(3x-(x-y))]$ $\qquad 8x+[-2x+(-x+y)]$

$$=5x+[-(3x-x+y)] \qquad =8x+[-2x-x+y]$$
$$=5x+[-2x-y] \qquad =8x-3x+y$$
$$=5x-2x-y \qquad =5x+y$$
$$=3x-y$$

$$\begin{array}{r} 5x+y \\ 3x-y \\ \hline 15x^2+3xy \\ -5xy-y^2 \\ \hline 15x^2-2xy-y^2 \end{array} \quad Rta$$

20.

$$\begin{array}{l|l} \frac{1}{4}x^3+\frac{1}{24}x^2y+\frac{5}{12}xy^2+\frac{1}{3}y^3 & \frac{1}{2}x^2-\frac{1}{4}xy+y^2 \\ \cline{2-2} -\frac{1}{4}x^3+\frac{1}{8}x^2y-\frac{1}{2}xy^2 & \frac{1}{2}x+\frac{1}{3}y \\ \cline{1-1} \qquad \frac{1}{6}x^2y-\frac{1}{12}xy^2+\frac{1}{3}y^3 & \\ \qquad -\frac{1}{6}x^2y+\frac{1}{12}xy^2-\frac{1}{3}y^3 & \\ \cline{1-1} \end{array}$$

$$2x+[-5x-(x-y)] \qquad \begin{array}{r} -\quad 4x \quad + \quad y \\ -\quad \frac{1}{2}x \quad - \quad \frac{1}{3}y \\ \hline -\frac{8+1}{2}x+\frac{3-1}{3}y \end{array}$$
$$=2x+[-5x-x+y]$$
$$=2x+[-6x+y]$$
$$=2x-6x+y \qquad -\frac{9}{2}x+\frac{2}{3}y \quad Rta$$
$$=-4x+y$$

21.

$$[x^2-(3x+2)][x^2+(-x+3)] \qquad =x^2(x^2-4x+4)-(7x+6)$$
$$[x^2-3x-2][x^2-x+3] \qquad =x^4-4x^3+4x^2-7x-6$$
$$x^4-x^3+3x^2-3x^3+3x^2-9x-2x^2+2x-6=x^4-4x^3+4x^2-7x-6$$
$$x^4-4x^3+4x^2-7x-6=x^4-4x^3+4x^2-7x-6$$

22. $[x(x+y)-x(x-y)]$ $\qquad [2(x^2+y^2)-3(x^2-y^2)]$

$$=x^2+xy-x^2+xy \qquad =2x^2+2y^2-3x^2+3y^2$$
$$=2xy \qquad =-x^2+5y^2$$

$$\begin{array}{r} -x^2+5y^2 \\ 2xy \\ \hline -2x^3y+10xy^3 \end{array} \qquad \begin{array}{r} -2x^3y+10xy^3 \\ 4x^3y-7xy^3 \\ \hline 2x^3y+3xy^3 \end{array} \quad Rta$$

23.

$$\begin{array}{l|l} x^3 \qquad\qquad -y^3 & x-y \\ \cline{2-2} -x^3+x^2y & x^2+xy+y^2 \\ \cline{1-1} \quad x^2y & \\ \quad -x^2y+xy^2 & \\ \cline{1-1} \qquad xy^2 & \\ \qquad -xy^2+y^3 & \\ \cline{1-1} \end{array}$$

$$\begin{array}{r} x^2+3xy-y^2 \\ x^2+xy+y^2 \\ \hline x^4+3x^3y-x^2y^2 \qquad\qquad\qquad \\ +x^3y+3x^2y^2-xy^3 \qquad \\ +x^2y^2+3xy^3-y^4 \\ \hline x^4+4x^3y+3x^2y^2+2xy^3-y^4 \end{array} \quad Rta$$

24. $(x-y)(x^2+xy+y^2)-(x+y)(x^2-xy+y^2)$

$$=x^3+x^2y+xy^2-x^2y-xy^2-y^3-[x^3-x^2y+xy^2+x^2y-xy^2+y^3]$$
$$=x^3-y^3-[x^3+y^3]$$
$$=x^3-y^3-x^3-y^3$$
$$=-2y^3$$

25. $a=4 \quad b=9 \quad c=25$

$$\sqrt{\frac{ab}{c}}+2(b-a)\sqrt{\frac{9b}{a^2}}-3(c-b)\sqrt{\frac{c}{b}}$$
$$=\sqrt{\frac{4\cdot 9}{25}}+2(9-4)\sqrt{\frac{9\cdot 9}{4^2}}-3(25-9)\sqrt{\frac{25}{9}}$$
$$=\sqrt{\frac{36}{25}}+2\cdot 5\sqrt{\frac{81}{16}}-3\cdot 16\cdot\frac{5}{3}=\frac{6}{5}+10\cdot\frac{9}{4}-48\cdot\frac{5}{3}$$
$$=\frac{6}{5}+\frac{45}{2}-80=\frac{12+225-800}{10}=\frac{-563}{10}=-56\tfrac{3}{10}$$

26.

$$\begin{array}{l|l} x^3+3x^2-4x-12 & x+3 \\ \cline{2-2} -x^3-3x^2 & x^2-4 \\ \cline{1-1} \qquad\quad -4x-12 & \\ \qquad\quad +4x+12 & \\ \cline{1-1} \end{array} \qquad \frac{x^2-4}{?}=x-2$$

$$\begin{array}{l|l} x^2 \qquad -4 & x-2 \\ \cline{2-2} x^2+2x & x+2 \quad Rta \\ \cline{1-1} \quad 2x-4 & \\ \quad -2x+4 & \\ \cline{1-1} \end{array}$$

27. $4x^2-\{3x-(x^2-(4+x))\}+[x^2-\{x+(-3)\}]$

$=4x^2-\{3x-(x^2-4-x)\}+[x^2-\{x-3\}]$

$=4x^2-\{3x-x^2+4+x\}+[x^2-x+3]$

$=4x^2-4x+x^2-4+x^2-x+3$

$=6x^2-5x-1$

Si $x=-2$

$6x^2-5x-1$

$=6(-2)^2-5(-2)-1$

$=6(4)+10-1$

$=24+10-1=33$

28.

$$\begin{array}{r} x^2+7x-5 \\ \underline{x^2-9} \\ x^4+7x^3-5x^2 \\ \underline{-9x^2-63x+45} \\ x^4+7x^3-14x^2-63x+45 \end{array}$$

$$\begin{array}{r} x^4-11x^3 \qquad +21x \;\; Rta \\ \underline{18x^3-14x^2-84x+45} \\ x^4+7x^3-14x^2-63x+45 \end{array}$$

29.

$(a^2+b^2)(a+b)(a-b) \quad =a^4-[3a+2(a+2)-4(a+1)-a+b^4]$

$(a^2+b^2)(a^2-ab+ab-b^2) \quad =a^4-[3a+2a+4-4a-4-a+b^4]$

$(a^2+b^2)(a^2-b^2) \quad =a^4-[b^4]$

$a^4-a^2b^2+a^2b^2-b^4 \quad =a^4-b^4$

$a^4-b^4 \quad =a^4-b^4$

30.

$$\begin{array}{r} 3 \\ \underline{x^3+5x^2-6} \\ x^3+5x^2-3 \end{array}$$

$-[x^2+(-3x+4)-(-x+3)]$

$=-[x^2-3x+4+x-3]$

$=-[x^2-2x+1]$

$=-x^2+2x-1$

$$\begin{array}{r} x^3+5x^2 \quad -3 \\ \underline{x+1} \\ x^3+5x^2+x-2 \;\; Rta \end{array}$$

$$\begin{array}{r} x^2-x+2 \\ \underline{-x^2+2x-1} \\ x+1 \end{array}$$

EJERCICIO 62

1. $(m+3)^2=m^2+6m+9$

2. $(5+x)^2=25+10x+x^2$

3. $(6a+b)^2=36a^2+12ab+b^2$

4. $(9+4m)^2=81+72m+16m^2$

5. $(7x+11)^2=49x^2+154x+121$

6. $(x+y)^2=x^2+2xy+y^2$

7. $(1+3x^2)^2=1+6x^2+9x^4$

8. $(2x+3y)^2=4x^2+12xy+9y^2$

9. $(a^2x+by^2)^2=a^4x^2+2a^2xby^2+b^2y^4$

10. $(3a^3+8b^4)^2=9a^6+48a^3b^4+64b^8$

11. $(4m^5+5n^6)^2=16m^{10}+40m^5n^6+25n^{12}$

12. $(7a^2b^3+5x^4)^2=49a^4b^6+70a^2b^3x^4+25x^8$

13. $(4ab^2+5xy^3)^2=16a^2b^4+40ab^2xy^3+25x^2y^6$

14. $(8x^2y+9m^3)^2=64x^4y^2+144x^2ym^3+81m^6$

15. $(x^{10}+10y^{12})^2=x^{20}+20x^{10}y^{12}+100y^{24}$

16. $(a^m+a^n)^2=a^{2m}+2a^{m+n}+a^{2n}$

17. $(a^x+b^{x+1})^2=a^{2x}+2a^xb^{x+1}+b^{2x+2}$

18. $(x^{a+1}+y^{x-2})^2=x^{2a+2}+2x^{a+1}y^{x-2}+y^{2x-4}$

EJERCICIO 63

1. $(a-3)^2=a^2-6a+9$

2. $(x-7)^2=x^2-14x+49$

3. $(9-a)^2=81-18a+a^2$

4. $(2a-3b)^2=4a^2-12ab+9b^2$

5. $(4ax-1)^2=16a^2x^2-8ax+1$

6. $(a^3-b^3)^2=a^6-2a^3b^3+b^6$

7. $(3a^4-5b^2)^2=9a^8-30a^4b^2+25b^4$

8. $(x^2-1)^2=x^4-2x^2+1$

9. $(x^5-3ay^2)^2=x^{10}-6ax^5y^2+9a^2y^4$

10. $(a^7-b^7)^2=a^{14}-2a^7b^7+b^{14}$

11. $(2m-3n)^2=4m^2-12mn+9n^2$

12. $(10x^3-9xy^5)^2=100x^6-180x^4y^5+81x^2y^{10}$

13. $(x^m-y^n)^2=x^{2m}-2x^my^n+y^{2n}$

14. $(a^{x-2}-5)^2=a^{2x-4}-10a^{x-2}+25$

15. $(x^{a+1}-3x^{a-2})^2=x^{2a+2}-6x^{2a-1}+9x^{2a-4}$

EJERCICIO 64

1. $(x+y)(x-y)=x^2-y^2$
2. $(m-n)(m+n)=m^2-n^2$
3. $(a-x)(x+a)=a^2-x^2$
4. $(x^2+a^2)(x^2-a^2)=x^4-a^4$
5. $(2a-1)(1+2a)=4a^2-1$
6. $(n-1)(n+1)=n^2-1$
7. $(1-3ax)(3ax+1)=1-9a^2x^2$
8. $(2m+9)(2m-9)=4m^2-81$
9. $(a^3-b^2)(a^3+b^2)=a^6-b^4$
10. $(y^2-3y)(y^2+3y)=y^4-9y^2$
11. $(1-8xy)(8xy+1)=1-64x^2y^2$
12. $(6x^2-m^2x)(6x^2+m^2x)=36x^4-m^4x^2$
13. $(a^m+b^n)(a^m-b^n)=a^{2m}-b^{2n}$
14. $(3x^a-5y^m)(5y^m+3x^a)=9x^{2a}-25y^{2m}$
15. $(a^{x+1}-2b^{x-1})(2b^{x-1}+a^{x+1})=a^{2x+2}-4b^{2x-2}$

EJERCICIO 65

1. $(x+y+z)(x+y-z)=x^2+2xy+y^2-z^2$
2. $(x-y+z)(x+y-z)=x^2-y^2+2yz-z^2$
3. $(x+y+z)(x-y-z)=x^2-y^2-2yz-z^2$
4. $(m+n+1)(m+n-1)=m^2+2mn+n^2-1$
5. $(m-n-1)(m-n+1)=m^2+n^2-2mn-1$
6. $(x+y-2)(x-y+2)=x^2-y^2+4y-4$
7. $(n^2+2n+1)(n^2-2n-1)=n^4-4n^2-4n-1$
8. $(a^2-2a+3)(a^2+2a+3)=a^4+2a^2+9$
9. $(m^2-m-1)(m^2+m-1)=m^4-3m^2+1$
10. $(2a-b-c)(2a-b+c)=4a^2-4ab+b^2-c^2$
11. $(2x+y-z)(2x-y+z)=4x^2-y^2+2yz-z^2$
12. $(x^2-5x+6)(x^2+5x-6)=x^4-25x^2+60x-36$
13. $(a^2-ab+b^2)(a^2+b^2+ab)=a^4+a^2b^2+b^4$
14. $(x^3-x^2-x)(x^3+x^2+x)=x^6-x^4-2x^3-x^2$

EJERCICIO 66

1. $(a+2)^3=a^3+6a^2+12a+8$
2. $(x-1)^3=x^3-3x^2+3x-1$
3. $(m+3)^3=m^3+9m^2+27m+27$
4. $(n-4)^3=n^3-12n^2+48n-64$
5. $(2x+1)^3=8x^3+12x^2+6x+1$
6. $(1-3y)^3=1-9y+27y^2-27y^3$
7. $(2+y^2)^3=8+12y^2+6y^4+y^6$
8. $(1-2n)^3=1-6n+12n^2-8n^3$
9. $(4n+3)^3=64n^3+144n^2+108n+27$
10. $(a^2-2b)^3=a^6-6a^4b+12a^2b^2-8b^3$
11. $(2x+3y)^3=8x^3+36x^2y+54xy^2+27y^3$
12. $(1-a^2)^3=1-3a^2+3a^4-a^6$

EJERCICIO 67

1. $(a+1)(a+2)=a^2+3a+2$
2. $(x+2)(x+4)=x^2+6x+8$
3. $(x+5)(x-2)=x^2+3x-10$
4. $(m-6)(m-5)=m^2-11m+30$
5. $(x+7)(x-3)=x^2+4x-21$
6. $(x+2)(x-1)=x^2+x-2$
7. $(x-3)(x-1)=x^2-4x+3$
8. $(x-5)(x+4)=x^2-x-20$
9. $(a-11)(a+10)=a^2-a-110$
10. $(n-19)(n+10)=n^2-9n-190$
11. $(a^2+5)(a^2-9)=a^4-4a^2-45$
12. $(x^2-1)(x^2-7)=x^4-8x^2+7$
13. $(n^2-1)(n^2+20)=n^4+19n^2-20$
14. $(n^3+3)(n^3-6)=n^6-3n^3-18$
15. $(x^3+7)(x^3-6)=x^6+x^3-42$
16. $(a^4+8)(a^4-1)=a^8+7a^4-8$
17. $(a^5-2)(a^5+7)=a^{10}+5a^5-14$
18. $(a^6+7)(a^6-9)=a^{12}-2a^6-63$
19. $(ab+5)(ab-6)=a^2b^2-ab-30$
20. $(xy^2-9)(xy^2+12)=x^2y^4+3xy^2-108$
21. $(a^2b^2-1)(a^2b^2+7)=a^4b^4+6a^2b^2-7$
22. $(x^3y^3-6)(x^3y^3+8)=x^6y^6+2x^3y^3-48$
23. $(a^x-3)(a^x+8)=a^{2x}+5a^x-24$
24. $(a^{x+1}-6)(a^{x+1}-5)=a^{2x+2}-11a^{x+1}+30$

EJERCICIO 68

1. $(x+2)^2 = x^2+4x+4$

2. $(x+2)(x+3)= x^2+5x+6$

3. $(x+1)(x-1)= x^2-1$

4. $(x-1)^2 = x^2-2x+1$

5. $(n+3)(n+5)= n^2+8n+15$

6. $(m-3)(m+3)= m^2-9$

7. $(a+b-1)(a+b+1)= a^2+2ab+b^2-1$

8. $(1+b)^3 = 1+3b+3b^2+b^3$

9. $(a^2+4)(a^2-4)= a^4-16$

10. $(3ab-5x^2)^2 = 9a^2b^2-30abx^2+25x^4$

11. $(ab+3)(3-ab) = 9-a^2b^2$

12. $(1-4ax)^2 = 1-8ax+16a^2x^2$

13. $(a^2+8)(a^2-7) = a^4+a^2-56$

14. $(x+y+1)(x-y-1) = x^2-y^2-2y-1$

15. $(1-a)(a+1) = 1-a^2$

16. $(m-8)(m+12) = m^2+4m-96$

17. $(x^2-1)(x^2+3) = x^4+2x^2-3$

18. $(x^3+6)(x^3-8) = x^6-2x^3-48$

19. $(5x^3+6m^4)^2 = 25x^6+60x^3m^4+36m^8$

20. $(x^4-2)(x^4+5)= x^8+3x^4-10$

21. $(1-a+b)(b-a-1)= -1+a^2-2ab+b^2$

22. $(a^x+b^n)(a^x-b^n) = a^{2x}-b^{2n}$

23. $(x^{a+1}-8)(x^{a+1}+9) = x^{2a+2}-x^{a+1}-72$

24. $(a^2b^2+c^2)(a^2b^2-c^2) = a^4b^4-c^4$

25. $(2a+x)^3 = 8a^3+12a^2x+6ax^2+x^3$

26. $(x^2-11)(x^2-2)= x^4-13x^2+22$

27. $(2a^3-5b^4)^2 = 4a^6-20a^3b^4+25b^8$

28. $(a^3+12)(a^3-15)= a^6-3a^3-180$

29. $(m^2-m+n)(n+m+m^2)= m^4+2m^2n+n^2-m^2$

30. $(x^4+7)(x^4-11)= x^8-4x^4-77$

31. $(11-ab)^2 = 121-22ab+a^2b^2$

32. $(x^2y^3-8)(x^2y^3+6) = x^4y^6-2x^2y^3-48$

33. $(a+b)(a-b)(a^2-b^2) = a^4-2a^2b^2+b^4$

34. $(x+1)(x-1)(x^2-2) = x^4-3x^2+2$

35. $(a+3)(a^2+9)(a-3) = a^4-81$

36. $(x+5)(x-5)(x^2+1) = x^4-24x^2-25$

37. $(a+1)(a-1)(a+2)(a-2) = a^4-5a^2+4$

38. $(a+2)(a-3)(a-2)(a+3) = a^4-13a^2+36$

EJERCICIO 69

1. $\dfrac{x^2-1}{x+1}=x-1$

2. $\dfrac{1-x^2}{1-x}=1+x$

3. $\dfrac{x^2-y^2}{x+y}= x-y$

4. $\dfrac{y^2-x^2}{y-x}= y+x$

5. $\dfrac{x^2-4}{x+2}= x-2$

6. $\dfrac{9-x^4}{3-x^2}= 3+x^2$

7. $\dfrac{a^2-4b^2}{a+2b}= a-2b$

8. $\dfrac{25-36x^4}{5-6x^2}=5+6x^2$

9. $\dfrac{4x^2-9m^2n^4}{2x+3mn^2}=2x-3mn^2$

10. $\dfrac{36m^2-49n^2x^4}{6m-7nx^2}=6m+7nx^2$

11. $\dfrac{81a^6-100b^8}{9a^3+10b^4}=9a^3-10b^4$

12. $\dfrac{a^4b^6-4x^8y^{10}}{a^2b^3+2x^4y^5}=a^2b^3+2x^4y^5$

13. $\dfrac{x^{2n}-y^{2n}}{x^n+y^n}=x^n-y^n$

14. $\dfrac{a^{2x+2}-100}{a^{x+1}-10}=a^{x+1}+10$

15. $\dfrac{1-9x^{2m+4}}{1+3x^{m+2}}=1-3x^{m+2}$

16. $\dfrac{(x+y)^2-z^2}{(x+y)-z}=x+y+z$

17. $\dfrac{1-(a+b)^2}{1+(a+b)}=1-a-b$

18. $\dfrac{4-(m+n)^2}{2+(m+n)}=2-m-n$

19. $\dfrac{x^2-(x-y)^2}{x+(x-y)}=y$

20. $\dfrac{(a+x)^2-9}{(a+x)+3}= a+x-3$

EJERCICIO 70

1. $\frac{1+a^3}{1+a}=1-a+a^2$

2. $\frac{1-a^3}{1-a}=1+a+a^2$

3. $\frac{x^3+y^3}{x+y}=x^2-xy+y^2$

4. $\frac{8a^3-1}{2a-1}=4a^2+2a+1$

5. $\frac{8x^3+27y^3}{2x+3y}=4x^2-6xy+9y^2$

6. $\frac{27m^3-125n^3}{3m-5n}=9m^2+15m+25n^2$

7. $\frac{64a^3+343}{4a+7}=16a^2-28a+49$

8. $\frac{216-125y^3}{6-5y}=36+30y+25y^2$

9. $\frac{1+a^3b^3}{1+ab}=1-ab+a^2b^2$

10. $\frac{729-512b^3}{9-8b}=81+72b+64b^2$

11. $\frac{a^3x^3+b^3}{ax+b}=a^2x^2-axb+b^2$

12. $\frac{n^3-m^3x^3}{n-mx}=n^2+nmx+m^2x^2$

13. $\frac{x^6-27y^3}{x^2-3y}=x^4+3x^2y+9y^2$

14. $\frac{8a^9+y^9}{2a^3+y^3}=4a^6-2a^3y^3+y^6$

15. $\frac{1-x^{12}}{1-x^4}=1+x^4+x^8$

16. $\frac{27x^6+1}{3x^2+1}=9x^4-3x^2+1$

17. $\frac{64a^3+b^9}{4a+b^3}=16a^2-4ab^3+b^6$

18. $\frac{a^6-b^6}{a^2-b^2}=a^4+a^2b^2+b^4$

19. $\frac{125-343x^{15}}{5-7x^5}=25+35x^5+49x^{10}$

20. $\frac{n^6+1}{n^2+1}=n^4-n^2+1$

EJERCICIO 71

1. $\frac{x^4-y^4}{x-y}=x^3+x^2y+xy^2+y^3$

2. $\frac{m^5+n^5}{m+n}=m^4-m^3n+m^2n^2-mn^3+n^4$

3. $\frac{a^5-n^5}{a-n}=a^4+a^3n+a^2n^2+an^3+n^4$

4. $\frac{x^6-y^6}{x+y}=x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5$

5. $\frac{a^6-b^6}{a-b}=a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5$

6. $\frac{x^7+y^7}{x+y}=x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6$

7. $\frac{a^7-m^7}{a-m}=a^6+a^5m+a^4m^2+a^3m^3+a^2m^4+am^5+m^6$

8. $\frac{a^8-b^8}{a+b}=a^7-a^6b+a^5b^2-a^4b^3+a^3b^4-a^2b^5+ab^6-b^7$

9. $\frac{x^{10}-y^{10}}{x-y}=x^9+x^8y+x^7y^2+x^6y^3+x^5y^4+x^4y^5+x^3y^6+x^2y^7+xy^8+y^9$

10. $\frac{m^9+n^9}{m+n}=m^8-m^7n+m^6n^2-m^5n^3+m^4n^4-m^3n^5+m^2n^6-mn^7+n^8$

11. $\frac{m^9-n^9}{m-n}=m^8+m^7n+m^6n^2+m^5n^3+m^4n^4+m^3n^5+m^2n^6+mn^7+n^8$

12. $\frac{a^{10}-x^{10}}{a+x}=a^9-a^8x+a^7x^2-a^6x^3+a^5x^4-a^4x^5+a^3x^6-a^2x^7+ax^8-x^9$

13. $\frac{1-n^5}{1-n}=1+n+n^2+n^3+n^4$

14. $\frac{1-a^6}{1-a}=1+a+a^2+a^3+a^4+a^5$

15. $\frac{1+a^7}{1+a}=1-a+a^2-a^3+a^4-a^5+a^6$

16. $\frac{1-m^8}{1+m}=1-m+m^2-m^3+m^4-m^5+m^6-m^7$

17. $\frac{x^4-16}{x-2}=x^3+2x^2+4x+8$

18. $\frac{x^6-64}{x+2}=x^5-2x^4+4x^3-8x^2+16x-32$

19. $\frac{x^7-128}{x-2}=x^6+2x^5+4x^4+8x^3+16x^2+32x+64$

20. $\frac{a^5+243}{a+3}=a^4-3a^3+9a^2-27a+81$

21. $\frac{x^6-729}{x-3}=x^5+3x^4+9x^3+27x^2+81x+243$

22. $\frac{625-x^4}{x+5}=125-25x+5x^2-x^3$

23. $\frac{m^8-256}{m-2}=m^7+2m^6+4m^5+8m^4+16m^3+32m^2+64m+128$

24. $\frac{x^{10}-1}{x-1}=x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1$

25. $\frac{x^5+243y^5}{x+3y}=x^4-3x^3y+9x^2y^2-27xy^3+81y^4$

26. $\frac{16a^4-81b^4}{2a-3b}=8a^3+12a^2b+18ab^2+27b^3$

27. $\frac{64m^6-729n^6}{2m+3n}=32m^5-48m^4n+72m^3n^2-108m^2n^3+162mn^4+243n^5$

28. $\frac{1.024x^{10}-1}{2x-1}=512x^9+256x^8+128x^7+64x^6+32x^5+16x^4+8x^3+4x^2+2x+1$

29. $\frac{512a^9+b^9}{2a+b}=256a^8-128a^7b+64a^6b^2-32a^5b^3+16a^4b^4-8a^3b^5+4a^2b^6-2ab^7+b^8$

30. $\frac{a^6-729}{a-3}=a^5+3a^4+9a^3+27a^2+81a+243$

EJERCICIO 72

1. $\frac{x^6+y^6}{x^2+y^2}=x^4-x^2y^2+y^4$

2. $\frac{a^8-b^8}{a^2+b^2}=a^6-a^4b^2+a^2b^4-b^6$

3. $\frac{m^{10}-n^{10}}{m^2-n^2}=m^8+m^6n^2+m^4n^4+m^2n^6+n^8$

4. $\frac{a^{12}-b^{12}}{a^3+b^3}=a^9-a^6b^3+a^3b^6-b^9$

5. $\frac{a^{12}-x^{12}}{a^3-x^3}=a^9+a^6x^3+a^3x^6+x^9$

6. $\frac{x^{15}+y^{15}}{x^3+y^3}=x^{12}-x^9y^3+x^6y^6-x^3y^9+y^{12}$

7. $\frac{m^{12}+1}{m^4+1}=m^8-m^4+1$

8. $\frac{m^{16}-n^{16}}{m^4-n^4}=m^{12}+m^8n^4+m^4n^8+n^{12}$

9. $\frac{a^{18}-b^{18}}{a^3+b^3}=a^{15}-a^{12}b^3+a^9b^6-a^6b^9+a^3b^{12}-b^{15}$

10. $\frac{x^{20}-y^{20}}{x^5+y^5}=x^{15}-x^{10}y^5+x^5y^{10}-y^{15}$

11. $\frac{m^{21}+n^{21}}{m^3+n^3}=m^{18}-m^{15}n^3+m^{12}n^6-m^9n^9+m^6n^{12}-m^3n^{15}+n^{18}$

12. $\frac{x^{24}-1}{x^6-1}=x^{18}+x^{12}+x^6+1$

13. $\frac{a^{25}+b^{25}}{a^5+b^5}=a^{20}-a^{15}b^5+a^{10}b^{10}-a^5b^{15}+b^{20}$

14 $\frac{a^{30}-m^{30}}{a^6-m^6}=a^{24}+a^{18}m^6+a^{12}m^{12}+a^6m^{18}+m^{24}$

EJERCICIO 73

1. $\frac{x^4-1}{1+x^2}=x^2-1$

2. $\frac{8m^3+n^6}{2m+n^2}=4m^2-2mn^2+n^4$

3. $\frac{1-a^5}{1-a}=1+a+a^2+a^3+a^4$

4. $\frac{x^6-27y^3}{x^2-3y}=x^4+3x^2y+9y^2$

5. $\frac{x^6-49y^6}{x^3+7y^3}=x^3-7y^3$

6. $\frac{a^{14}-b^{14}}{a^2-b^2}=a^{12}+a^{10}b^2+a^8b^4+a^6b^6+a^4b^8+a^2b^{10}+b^{12}$

7. $\dfrac{1+a^3}{1+a}=1-a+a^2$

8. $\dfrac{16x^2y^4-25m^6}{4xy^2+5m^3}=4xy^2-5m^3$

9. $\dfrac{x^{27}+y^{27}}{x^3+y^3}=x^{24}-x^{21}y^3+x^{18}y^6-x^{15}y^9+x^{12}y^{12}-x^9y^{15}+x^6y^{18}-x^3y^{21}+y^{24}$

10. $\dfrac{a^{27}+y^{27}}{a^9+y^9}=a^{18}-a^9y^9+y^{18}$

11. $\dfrac{a^4b^4-64x^6}{a^2b^2+8x^3}=a^2b^2-8x^3$

12. $\dfrac{1-a^2b^4c^8}{1-ab^2c^4}=1+ab^2c^4$

13. $\dfrac{32x^5+243y^5}{2x+3y}=16x^4-24x^3y+36x^2y^2-54xy^3+81y^4$

14. $\dfrac{25-(a+1)^2}{5+(a+1)}=4-a$

15. $\dfrac{1-x^{12}}{1-x^4}=1+x^4+x^8$

16. $\dfrac{64x^6-343y^9}{4x^2-7y^3}=16x^4+28x^2y^3+49y^6$

17. $\dfrac{a^{18}-b^{18}}{a^3+b^3}=a^{15}-a^{12}b^3+a^9b^6-a^6b^9+a^3b^{12}-b^{15}$

18. $\dfrac{(a+x)^2-y^2}{(a+x)-y}=a+x+y$

19. $\dfrac{1+x^{11}}{x+1}=x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1$

20. $\dfrac{x^{40}-y^{40}}{x^8-y^8}=x^{32}+x^{24}y^8+x^{16}y^{16}+x^8y^{24}+y^{32}$

21. $\dfrac{9-36x^{10}}{3+6x^5}=3-6x^5$

22. $\dfrac{x^8-256}{x-2}=x^7+2x^6+4x^5+8x^4+16x^3+32x^2+64x+128$

EJERCICIO 74

1. $x^2-2x+3\div x-1$

$=1^2-2\cdot 1+3$

$=1-2+3=2$

2. $x^3-3x^2+2x-2\div x+1$

$=(-1)^3-3(-1)^2+2(-1)-2$

$=-1-3-2-2=-8$

3. $x^4-x^3+5\div x-2$

$=2^4-2^3+5$

$=16-8+5=13$

4. $a^4-5a^3+2a^2-6\div a+3$

$=(-3)^4-5(-3)^3+2(-3)^2-6$

$=81-5(-27)+2\cdot 9-6$

$=81+135+18-6=228$

5. $m^4+m^3-m^2+5\div m-4$

$=4^4+4^3-4^2+5$

$=256+64-16+5=309$

6. $x^5+3x^4-2x^3+4x^2-2x+2\div x+3$

$=(-3)^5+3(-3)^4-2(-3)^3+4(-3)^2-2(-3)+2$

$=-243+3\cdot 81-2(-27)+4\cdot 9+6+2$

$=-243+243+54+36+8=98$

7. $a^5-2a^3+2a-4\div a-5$

$=5^5-2\cdot 5^3+2\cdot 5-4$

$=3.125-2\cdot 125+10-4$

$=3.125-250+10-4=2.881$

8. $6x^3+x^2+3x+5\div 2x+1$

$=6\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^2+3\left(-\frac{1}{2}\right)+5$

$=6\left(-\frac{1}{8}\right)+\frac{1}{4}-\frac{3}{2}+5=-\frac{3}{4}+\frac{1}{4}-\frac{3}{2}+5$

$=\frac{-3+1-6+20}{4}=\frac{12}{4}=3$

9. $12x^3-21x+90\div 3x-3$

$=12\cdot 1^3-21\cdot 1+90$

$=12-21+90=81$

10. $15x^3-11x^2+10x+18\div 3x+2$

$=15\left(-\frac{2}{3}\right)^3-11\left(-\frac{2}{3}\right)^2+10\left(-\frac{2}{3}\right)+18$

$=15\left(-\frac{8}{27}\right)-11\cdot\frac{4}{9}-\frac{20}{3}+18$

$=-\frac{40}{9}-\frac{44}{9}-\frac{20}{3}+18$

$=\frac{-40-44-60+162}{9}=\frac{18}{9}=2$

11. $5x^4-12x^3+9x^2-22x+21\div 5x-2$

$=5\left(\frac{2}{5}\right)^4-12\left(\frac{2}{5}\right)^3+9\left(\frac{2}{5}\right)^2-22\left(\frac{2}{5}\right)+21$

$=5\cdot\frac{16}{625}-12\cdot\frac{8}{125}+9\cdot\frac{4}{25}-\frac{44}{5}+21$

$=\frac{16}{125}-\frac{96}{125}+\frac{36}{25}-\frac{44}{5}+21$

$=-\frac{80}{125}+\frac{36}{25}-\frac{44}{5}+21$

$=\frac{-80+180-1.100+2.625}{125}=\frac{1.625}{125}=13$

12. $a^6+a^4-8a^2+4a+1\div 2a+3$

$=\left(-\frac{3}{2}\right)^6+\left(-\frac{3}{2}\right)^4-8\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)+1$

$=\frac{729}{64}+\frac{81}{16}-8\cdot\frac{9}{4}-\frac{12}{2}+1$

$=\frac{729}{64}+\frac{81}{16}-18-6+1=\frac{729}{64}+\frac{81}{16}-23$

$=\frac{729+324-1.472}{64}=-\frac{419}{64}$

EJERCICIO 75

1. $x^2-7x+5 \div x-3$

1	−7	5	3
	3	−12	
1	−4	−7	

$=x-4$ Res. -7

2. $a^2-5a+1 \div a+2$

1	−5	1	−2
	−2	14	
1	−7	15	

$=a-7$ Res. 15

3. $x^3-x^2+2x-2 \div x+1$

1	−1	2	−2	−1
	−1	2	−4	
1	−2	4	−6	

$=x^2-2x+4$ Res. -6

4. $x^3-2x^2+x-2 \div x-2$

1	−2	1	−2	2
	2	0	2	
1	0	1	0	

$=x^2+1$ Res. 0

5. $a^3-3a^2-6 \div a+3$

1	−3	0	−6	−3
	−3	18	−54	
1	−6	18	−60	

$=a^2-6a+18$ Res. -60

6. $n^4-5n^3+4n-48 \div n+2$

1	−5	0	4	−48	−2
	−2	14	−28	48	
1	−7	14	−24	0	

$=n^3-7n^2+14n-24$ Res. 0

7. $x^4-3x+5 \div x-1$

1	0	0	−3	5	1
	1	1	1	−2	
1	1	1	−2	3	

$=x^3+x^2+x-2$ Res. 3

8. $x^5+x^4-12x^3-x^2-4x-2 \div x+4$

1	1	−12	−1	−4	−2	−4
	−4	12	0	4	0	
1	−3	0	−1	0	−2	

$=x^4-3x^3-x$ Res. -2

9. $a^5-3a^3+4a-6 \div a-2$

1	0	−3	0	4	−6	2
	2	4	2	4	16	
1	2	1	2	8	10	

$=a^4+2a^3+a^2+2a+8$ Res. 10

10. $x^5-208x^2+2076 \div x-5$

1	0	0	−208	0	2.076	5
	5	25	125	−415	−2.075	
1	5	25	−83	−415	1	

$=x^4+5x^3+25x^2-83x-415$ Res. 1

11. $x^6-3x^5+4x^4-3x^3-x^2+2 \div x+3$

1	−3	4	−3	−1	0	2	−3
	−3	18	−66	207	−618	1.854	
1	−6	22	−69	206	−618	1.856	

$=x^5-6x^4+22x^3-69x^2+206x-618$ Res. 1.856

12. $2x^3-3x^2+7x-5 \div 2x-1$

2	−3	7	−5	$\frac{1}{2}$
	1	−1	3	
2	−2	6	−2	

$=x^2-x+3$ Res. -2

13. $3a^3-4a^2+5a+6 \div 3a+2$

3	−4	5	6	$-\frac{2}{3}$
	−2	4	−6	
3	−6	9	0	

$=a^2-2a+3$ Res. 0

14. $3x^4-4x^3+4x^2-10x+8 \div 3x-1$

3	−4	4	−10	8	$\frac{1}{3}$
	1	−1	1	−3	
3	−3	3	−9	5	

$=x^3-x^2+x-3$ Res. 5

15. $x^6-x^4+\frac{15}{8}x^3+x^2-1 \div 2x+3$

1	0	−1	$\frac{15}{8}$	1	0	−1	$-\frac{3}{2}$
	$-\frac{3}{2}$	$\frac{9}{4}$	$-\frac{15}{8}$	0	$-\frac{3}{2}$	$\frac{9}{4}$	
1	$-\frac{3}{2}$	$\frac{5}{4}$	0	1	$-\frac{3}{2}$	$\frac{5}{4}$	

$=\frac{1}{2}x^5-\frac{3}{4}x^4+\frac{5}{8}x^3+\frac{1}{2}x-\frac{3}{4}$ Res. $\frac{5}{4}$

EJERCICIO 76

1. $x^2-x-6 \div x-3$
Exacta (6 múltiplo de 3)

2. $x^3+4x^2-x-10 \div x+2$
Exacta (10 múltiplo de 2)

3. $2x^4-5x^3+7x^2-9x+3 \div x-1$
Inexacta (1 no anula el polinomio)

4. $x^5+x^4-5x^3-7x+8 \div x+3$
Inexacta (8 no es múltiplo de 3)

5. $4x^3-8x^2-11x-4 \div 2x-1$
Exacta (4 múltiplo de 1)

6. $6x^5+2x^4-3x^3-x^2+3x+3 \div 3x+1$
Inexacta (- 1 no anula el polinomio)

7. $a+1$ **Es factor de** a^3-2a^2+2a+5

$(-1)^3-2(-1)^2+2(-1)+5$

$=-1-2-2+5=0$

No existe residuo, luego a + 1 divide exactamente al polinomio, por lo que se deduce es un factor de este.

8. $x-5$ **divide a** $x^5+6x^4+6x^3-5x^2+2x-10$

$5^5+6\cdot5^4+6\cdot5^3-5\cdot5^2+2\cdot5-10$

$=3125-3750+750+125+10-10=0$

Al sustituir x por 5 en el polinomio, este se anula, entonces x - 5 divide con exactitud a

$x^5+6x^4+6x^3-5x^2+2x-10$

9. $4x-3$ **divide a** $4x^4-7x^3+7x^2-7x+3$

$$4\left(\frac{3}{4}\right)^4-7\left(\frac{3}{4}\right)^3+7\left(\frac{3}{4}\right)^2-7\left(\frac{3}{4}\right)+3$$

$$=\frac{324}{256}-\frac{189}{64}+\frac{63}{16}-\frac{21}{4}+3$$

$$=\frac{324-756+1.008-1.344+768}{256}=\frac{0}{256}=0$$

La variable x del dividendo se reemplaza por 3/4 (4x - 3 = 0 luego x = 3/4) que es el valor de la variable del divisor. Se observa su anulación, por consiguiente, 4x - 3 es un divisor exacto de tal polinomio.

10. 3n + 2 no es factor de $3n^5+2n^4-3n^3-2n^2+6n+7$ **porque 7 no es múltiplo de - 2 / 3, lo cual significa que al reemplazar tal valor en el polinomio resultará un residuo, por ende la división no será exacta y 3n + 2 no se puede concebir como factor de dicho polinomio.**

11. $2a^3-2a^2-4a+16\div a+2$

$$\begin{array}{rrrr} & -4 & 12 & -16 \\ \hline 2 & -6 & 8 & 0 \end{array}$$

Exacta; $coc.\,2a^2-6a+8$

12. $a^4 \quad -a^2+2a+2\div a+1$

$$\begin{array}{rrrrr} & -1 & 1 & 0 & -2 \\ \hline 1 & -1 & 0 & 2 & 0 \end{array}$$

Exacta; $coc.\,a^3-a^2+2$

13. $x^4 \quad +5x-6\div x-1$

$$\begin{array}{rrrrr} & 1 & 1 & 1 & 6 \\ \hline 1 & 1 & 1 & 6 & 0 \end{array}$$

Exacta; $coc.\,x^3+x^2+x+6$

14. $x^6 \quad -39x^4+26x^3-52x^2+29x-30\div x-6$

$$\begin{array}{rrrrrrr} & 6 & 36 & -18 & 48 & -24 & 30 \\ \hline 1 & 6 & -3 & 8 & -4 & 5 & 0 \end{array}$$

Exacta; $coc.\,x^5+6x^4-3x^3+8x^2-4x+5$

15. $a^6-4a^5-a^4+4a^3+a^2-8a+25\div a-4$

$$\begin{array}{rrrrrrr} & 4 & 0 & -4 & 0 & 4 & -16 \\ \hline 1 & 0 & -1 & 0 & 1 & -4 & 9 \end{array}$$

Inexacta; $coc.\,a^5-a^3+a-4$; $Res.\,9$

16. $16x^4-24x^3+37x^2-24x+4\div 4x-1$

$$\begin{array}{rrrrr} & 4 & -5 & 8 & -4 \\ \hline 16 & -20 & 32 & -16 & 0 \end{array}$$

Exacta; $coc.\,4x^3-5x^2+8x-4$

17. $15n^5+25n^4-18n^3-18n^2+17n-11\div 3n+5$

$$\begin{array}{rrrrrr} & -25 & 0 & 30 & -20 & 5 \\ \hline 15 & 0 & -18 & 12 & -3 & -6 \end{array}$$

Inexacta; $coc.\,5n^4-6n^2+4n-1$; $Res.-6$

18. $7x^2-5x\ +k\ \div x-5$

$$\begin{array}{rrr} & 35 & 150 \\ \hline 7 & 30 & 0 \end{array}$$

$Si\,k=-150$ *se anula*

19. $x^3-3x^2+4x+k\ \div x-2$

$$\begin{array}{rrrr} & 2 & -2 & 4 \\ \hline 1 & -1 & 2 & 0 \end{array}$$

$k+4=0$, *luego* $k=-4$

20. $2a^4 \qquad +25a+k\ \div a+3$

$$\begin{array}{rrrrr} & -6 & 18 & -54 & 87 \\ \hline 2 & -6 & 18 & -29 & 0 \end{array}$$

Para $k=-87$ *se cumple que* $k+87=0$

21. $20x^3-7x^2+29x+k\ \div 4x+1$

$$\begin{array}{rrrr} & -5 & 3 & -8 \\ \hline 20 & -12 & 32 & 0 \end{array}$$

$k-8=0$ *entoces* $k=8$

EJERCICIO 77

1. $\frac{x^5+1}{x-1}$ *Inexacta* $Res.\,2$

2. $\frac{a^4+b^4}{a+b}$ *Inexacta* $Res.\,2b^4$

3. $\frac{x^8-1}{x^2+1}$ *Exacta*

4. $\frac{a^{11}+1}{a-1}$ *Inexacta* $Res.\,2$

5. $\frac{a^6+b^6}{a^2+b^2}$ *Inexacta* $Res.\,2b^6$

6. $\frac{x^7-1}{x-1}$ *Exacta*

7. $\frac{x^3-8}{x+2}$ *Inexacta* $Res.-16$

8. $\frac{x^4-16}{x+2}$ *Exacta*

9. $\frac{a^5+32}{a-2}$ *Inexacta* $Res.\,64$

10. $\frac{x^7-128}{x+2}$ *Inexacta* $Res.-256$

11. $\frac{16a^4-81b^4}{2a+3b}$ *Exacta*

12. $\frac{a^3x^6+b^9}{ax^2+b^3}$ *Exacta*

EJERCICIO 78

1. $5x=8x-15$
$5x-8x=-15$
$-3x=-15$
$x=\frac{-15}{-3}$
$x=5$

2. $4x+1=2$
$4x=2-1$
$4x=1$
$x=\frac{1}{4}$

3. $y-5=3y-25$
$y-3y=-25+5$
$-2y=-20$
$y=\frac{-20}{-2}$
$y=10$

4. $5x+6=10x+5$
$5x-10x=5-6$
$-5x=-1$
$x=\frac{-1}{-5}$
$x=\frac{1}{5}$

5. $9y-11=-10+12y$
$9y-12y=-10+11$
$-3y=1$
$y=-\frac{1}{3}$

6. $21-6x=27-8x$
$-6x+8x=27-21$
$2x=6$
$x=\frac{6}{2}$
$x=3$

7. $11x+5x-1=65x-36$
$16x=65x-36+1$
$16x-65x=-35$
$-49x=-35$
$x=\frac{-35}{-49}$
$x=\frac{5}{7}$

8. $8x-4+3x=7x+x+14$
$11x-4=8x+14$
$11x-8x=14+4$
$3x=18$
$x=\frac{18}{3}$
$x=6$

9. $8x+9-12x=4x-13-5x$
$-4x+9=-x-13$
$-4x+x=-13-9$
$-3x=-22$
$x=\frac{-22}{-3}$
$x=\frac{22}{3}$

10. $5y+6y-81=7y+102+65y$
$11y-81=72y+102$
$11y-72y=102+81$
$-61y=183$
$y=\frac{183}{-61}$
$y=-3$

11. $16+7x-5+x=11x-3-x$
$8x+11=10x-3$
$8x-10x=-3-11$
$-2x=-14$
$x=\frac{-14}{-2}$
$x=7$

12. $3x+101-4x-33=108-16x-100$
$-x+68=8-16x$
$-x+16x=8-68$
$15x=-60$
$x=\frac{-60}{15}$
$x=-4$

13. $14-12x+39x-18x=256-60x-657x$
$9x+14=-717x+256$
$9x+717x=256-14$
$726x=242$
$x=\frac{242}{726}$
$x=\frac{1}{3}$

14. $8x-15x-30x-51x=53x+31x-172$
$-88x=84x-172$
$-88x-84x=-172$
$-172x=-172$
$x=\frac{-172}{-172}$
$x=1$

EJERCICIO 79

1. $x-(2x+1)=8-(3x+3)$
$x-2x-1=8-3x-3$
$-x-1=5-3x$
$-x+3x=5+1$
$2x=6$
$x=\frac{6}{2}$
$x=3$

2. $15x-10=6x-(x+2)+(-x+3)$
$15x-10=6x-x-2-x+3$
$15x-10=4x+1$
$15x-4x=10+1$
$x=\frac{11}{11}$
$x=1$

3. $(5-3x)-(-4x+6)=(8x+11)-(3x-6)$
$5-3x+4x-6=8x+11-3x+6$
$x-1=5x+17$
$x-5x=17+1$
$-4x=18$
$x=\frac{18}{-4}$
$x=-\frac{9}{2}$

4. $30x-(-x+6)+(-5x+4)=-(5x+6)+(-8+3x)$

$$30x+x-6-5x+4=-5x-6-8+3x$$
$$26x-2=-2x-14$$
$$26x+2x=-14+2$$
$$28x=-12$$
$$x=-\frac{12}{28}$$
$$x=-\frac{3}{7}$$

5. $15x+(-6x+5)-2-(-x+3)=-(7x+23)-x+(3-2x)$

$$15x-6x+5-2+x-3=-7x-23-x+3-2x$$
$$10x=-10x-20$$
$$10x+10x=-20$$
$$20x=-20$$
$$x=\frac{-20}{20}$$
$$x=-1$$

6. $3x+[-5x-(x+3)]=8x+(-5x-9)$

$$3x+[-5x-x-3]=8x-5x-9$$
$$3x+[-6x-3]=3x-9$$
$$3x-6x-3=3x-9$$
$$-3x-3=3x-9$$
$$-x-1=x-3$$
$$-x-x=-3+1$$
$$-2x=-2\ \Rightarrow\ x=\frac{-2}{-2}=1$$

7. $16x-[3x-(6-9x)]=30x+[-(3x+2)-(x+3)]$

$$16x-[3x-6+9x]=30x+[-3x-2-x-3]$$
$$16x-[12x-6]=30x+[-4x-5]$$
$$16x-12x+6=30x-4x-5$$
$$4x+6=26x-5$$
$$4x-26x=-5-6$$
$$-22x=-11$$
$$x=\frac{-11}{-22}=\frac{1}{2}$$

8. $x-[5+3x-\{5x-(6+x)\}]=-3$

$$x-[5+3x-\{5x-6-x\}]=-3$$
$$x-[5+3x-\{4x-6\}]=-3$$
$$x-[5+3x-4x+6]=-3$$
$$x-[11-x]=-3$$
$$x-11+x=-3$$
$$2x=-3+11$$
$$2x=8\Rightarrow x=\frac{8}{2}=4$$

9. $9x-(5x+1)-\{2+8x-(7x-5)\}+9x=0$

$$9x-5x-1-\{2+8x-7x+5\}+9x=0$$
$$4x-1-\{x+7\}+9x=0$$
$$4x-1-x-7+9x=0$$
$$12x-8=0$$
$$12x=8$$
$$x=\frac{8}{12}$$
$$x=\frac{2}{3}$$

10. $71+[-5x+(-2x+3)]=25-[-(3x+4)-(4x+3)]$

$$71+[-5x-2x+3]=25-[-3x-4-4x-3]$$
$$71+[-7x+3]=25-[-7x-7]$$
$$71-7x+3=25+7x+7$$
$$74-7x=32+7x$$
$$74-32=7x+7x$$
$$42=14x$$
$$\frac{42}{14}=x$$
$$3=x$$

11. $-\{3x+8-[-15+6x-(-3x+2)-(5x+4)]-29\}=-5$

$$-\{3x+8-[-15+6x+3x-2-5x-4]-29\}=-5$$
$$-\{3x-21-[-21+4x]\}=-5$$
$$-\{3x-21+21-4x\}=-5$$
$$-\{-x\}=-5$$
$$x=-5$$

EJERCICIO 80

1. $x+3(x-1)=6-4(2x+3)$

$$x+3x-3=6-8x-12$$
$$4x-3=-8x-6$$
$$4x+8x=-6+3$$
$$12x=-3$$
$$x=\frac{-3}{12}\Rightarrow x=-\frac{1}{4}$$

2. $5(x-1)+16(2x+3)=3(2x-7)-x$

$$5x-5+32x+48=6x-21-x$$
$$37x+43=5x-21$$
$$37x-5x=-21-43$$
$$32x=-64$$
$$x=\frac{-64}{32}\Rightarrow x=-2$$

3. $2(3x+3)-4(5x-3)=x(x-3)-x(x+5)$

$$6x+6-20x+12=x^2-3x-x^2-5x$$
$$-14x+18=-8x$$
$$-14x+8x=-18$$
$$-6x=-18$$
$$x=\frac{-18}{-6}\Rightarrow x=3$$

4. $184-7(2x+5)=301+6(x-1)-6$

$$184-14x-35=301+6x-6-6$$
$$149-14x=289+6x$$
$$149-289=6x+14x$$
$$-140=20x$$
$$\frac{-140}{20}=x$$
$$-7=x$$

5. $7(18-x)-6(3-5x)=-(7x+9)-3(2x+5)-12$

$$126-7x-18+30x=-7x-9-6x-15-12$$
$$108+23x=-13x-36$$
$$23x+13x=-36-108$$
$$36x=-144$$
$$x=\frac{-144}{36}$$
$$x=-4$$

6. $3x(x-3)+5(x+7)-x(x+1)-2(x^2+7)+4=0$

$$3x^2-9x+5x+35-x^2-x-2x^2-14+4=0$$
$$-5x+25=0$$
$$-5x=-25$$
$$x=\frac{-25}{-5}$$
$$x=5$$

7. $-3(2x+7)+(-5x+6)-8(1-2x)-(x-3)=0$

$$-6x-21-5x+6-8+16x-x+3=0$$
$$4x-20=0$$
$$4x=20$$
$$x=\frac{20}{4}$$
$$x=5$$

8. $(3x-4)(4x-3)=(6x-4)(2x-5)$

$$12x^2-9x-16x+12=12x^2-38x+20$$
$$-25x+38x=20-12$$
$$13x=8$$
$$x=\frac{8}{13}$$

9. $(4-5x)(4x-5)=(10x-3)(7-2x)$

$$-20x^2+41x-20=-20x^2+76x-21$$
$$41x-76x=-21+20$$
$$-35x=-1$$
$$x=\frac{-1}{-35}$$
$$x=\frac{1}{35}$$

10. $(x+1)(2x+5)=(2x+3)(x-4)+5$

$$2x^2+7x+5=2x^2-5x-12+5$$
$$7x+5x=-7-5$$
$$12x=-12$$
$$x=\frac{-12}{12}$$
$$x=-1$$

11. $(x-2)^2-(3-x)^2=1$

$$x^2-4x+4-9+6x-x^2=1$$
$$2x-5=1$$
$$2x=6$$
$$x=\frac{6}{2}$$
$$x=3$$

12. $14-(5x-1)(2x+3)=17-(10x+1)(x-6)$

$$14-(10x^2+13x-3)=17-(10x^2-59x-6)$$
$$14-10x^2-13x+3=17-10x^2+59x+6$$
$$-13x-59x=6$$
$$-72x=6$$
$$x=\frac{6}{-72}$$
$$x=-\frac{1}{12}$$

13. $(x-2)^2+x(x-3)=3(x+4)(x-3)-(x+2)(x-1)+2$

$$x^2-4x+4+x^2-3x=3(x^2+x-12)-(x^2+x-2)+2$$
$$2x^2-7x+4=3x^2+3x-36-x^2-x+2+2$$
$$2x^2-7x+4=2x^2+2x-36+4$$
$$-7x-2x=-36$$
$$-9x=-36$$
$$x=\frac{-36}{-9}$$
$$x=4$$

14. $(3x-1)^2-5(x-2)-(2x+3)^2-(5x+2)(x-1)=0$

$$9x^2-6x+1-5x+10-4x^2-12x-9-5x^2+3x+2=0$$
$$-20x+4=0$$
$$-20x=-4$$
$$x=\frac{-4}{-20}$$
$$x=\frac{1}{5}$$

15. $2(x-3)^2-3(x+1)^2+(x-5)(x-3)+4(x^2-5x+1)=4x^2-12$

$$2x^2-12x+18-3x^2-6x-3+x^2-8x+15+4x^2-20x+4=4x^2-12$$
$$4x^2-46x+34=4x^2-12$$
$$-46x=-12-34$$
$$-46x=-46$$
$$x=\frac{-46}{-46}$$
$$x=1$$

16. $5(x-2)^2-5(x+3)^2+(2x-1)(5x+2)-10x^2=0$

$$5x^2-20x+20-5x^2-30x-45+10x^2-x-2-10x^2=0$$
$$-51x-27=0$$
$$-51x=27$$
$$x=\frac{27}{-51}$$
$$x=-\frac{9}{17}$$

17. $x^2-5x+15=x(x-3)-14+5(x-2)+3(13-2x)$

$$x^2-5x+15=x^2-3x-14+5x-10+39-6x$$
$$-5x+15=-4x+15$$
$$-5x+4x=0$$
$$-x=0$$
$$x=0$$

18. $3(5x-6)(3x+2)-6(3x+4)(x-1)-3(9x+1)(x-2)=0$

$$3(15x^2-8x-12)-6(3x^2+x-4)-3(9x^2-17x-2)=0$$
$$45x^2-24x-36-18x^2-6x+24-27x^2+51x+6=0$$
$$21x-6=0$$
$$21x=6$$
$$x=\frac{6}{21}$$
$$x=\frac{2}{7}$$

19. $7(x-4)^2-3(x+5)^2=4(x+1)(x-1)-2$

$$7x^2-56x+112-3x^2-30x-75=4x^2-4-2$$
$$4x^2-86x+37=4x^2-6$$
$$-86x=-6-37$$
$$-86x=-43$$
$$x=\frac{-43}{-86}$$
$$x=\frac{1}{2}$$

20. $5(1-x)^2-6(x^2-3x-7)=x(x-3)-2x(x+5)-2$

$$5-10x+5x^2-6x^2+18x+42=x^2-3x-2x^2-10x-2$$
$$-x^2+8x+47=-x^2-13x-2$$
$$8x+13x=-2-47$$
$$21x=-49$$
$$x=\frac{-49}{21}$$
$$x=-\frac{7}{3}$$

EJERCICIO 81

1. $14x-(3x-2)-[5x+2-(x-1)]=0$

$$14x-3x+2-[5x+2-x+1]=0$$
$$11x+2-[4x+3]=0$$
$$11x+2-4x-3=0$$
$$7x-1=0$$
$$7x=1$$
$$x=\frac{1}{7}$$

2. $(3x-7)^2-5(2x+1)(x-2)=-x^2-[-(3x+1)]$

$$9x^2-42x+49-5(2x^2-3x-2)=-x^2-[-3x-1]$$
$$9x^2+x^2-42x+49-10x^2+15x+10=3x+1$$
$$-27x+59=3x+1$$
$$-27x-3x=1-59$$
$$-30x=-58$$
$$x=\frac{-58}{-30}\Rightarrow x=\frac{29}{15}$$

3. $6x-(2x+1)=-\{-5x+[-(-2x-1)]\}$

$$6x-2x-1=-\{-5x+[2x+1]\}$$
$$4x-1=-\{-5x+2x+1\}$$
$$4x-1=-\{-3x+1\}$$
$$4x-1=3x-1$$
$$4x-3x=-1+1$$
$$x=0$$

4. $2x+3(-x^2-1)=-\{3x^2+2(x-1)-3(x+2)\}$

$$2x-3x^2-3=-\{3x^2+2x-2-3x-6\}$$
$$2x-3x^2-3=-\{3x^2-x-8\}$$
$$2x-3x^2-3=-3x^2+x+8$$
$$2x-x=8+3$$
$$x=11$$

5. $x^2-\{3x+[x(x+1)+4(x^2-1)-4x^2]\}=0$

$$x^2-\{3x+[x^2+x+4x^2-4-4x^2]\}=0$$
$$x^2-\{3x+[x^2+x-4]\}=0$$
$$x^2-\{3x+x^2+x-4\}=0$$
$$x^2-4x-x^2+4=0$$
$$-4x=-4$$
$$x=\frac{-4}{-4}$$
$$x=1$$

6. $3(2x+1)(-x+3)-(2x+5)^2=-[-\{-3(x+5)\}+10x^2]$

$$3(-2x^2+5x+3)-4x^2-20x-25=-[-\{-3x-15\}+10x^2]$$
$$-6x^2+15x+9-4x^2-20x-25=-[3x+15+10x^2]$$
$$-10x^2-5x-16=-3x-15-10x^2$$
$$-5x+3x=16-15$$
$$-2x=1$$
$$x=-\frac{1}{2}$$

7. $(x+1)(x+2)(x-3)=(x-2)(x+1)(x+1)$

$$(x^2+3x+2)(x-3)=(x^2-x-2)(x+1)$$
$$x^3-3x^2+3x^2-9x+2x-6=x^3+x^2-x^2-x-2x-2$$
$$-7x-6=-3x-2$$
$$-7x+3x=-2+6$$
$$-4x=4$$
$$x=\frac{4}{-4}\Rightarrow x=-1$$

8. $(x+2)(x+3)(x-1)=(x+4)(x+4)(x-4)+7$

$$(x^2+5x+6)(x-1)=(x+4)(x^2-16)+7$$
$$x^3-x^2+5x^2-5x+6x-6=x^3-16x+4x^2-64+7$$
$$4x^2+x-6=4x^2-16x-57$$
$$x+16x=-57+6$$
$$17x=-51$$
$$x=\frac{-51}{17}\Rightarrow x=-3$$

9. $(x+1)^3-(x-1)^3=6x(x-3)$

$$x^3+3x^2+3x+1-(x^3-3x^2+3x-1)=6x^2-18x$$
$$x^3+3x^2+3x+1-x^3+3x^2-3x+1=6x^2-18x$$
$$6x^2+2=6x^2-18x$$
$$2=-18x$$
$$\frac{2}{-18}=x\Rightarrow -\frac{1}{9}=x$$

10. $3(x-2)^2(x+5)=3(x+1)^2(x-1)+3$

$$3(x^2-4x+4)(x+5)=3(x^2+2x+1)(x-1)+3$$
$$(3x^2-12x+12)(x+5)=(3x^2+6x+3)(x-1)+3$$
$$3x^3+15x^2-12x^2-60x+12x+60=3x^3-3x^2+6x^2-6x+3x-3+3$$
$$3x^2-48x+60=3x^2-3x$$
$$-48x+3x=-60$$
$$-45x=-60$$
$$x=\frac{-60}{-45}\Rightarrow x=\frac{4}{3}$$

EJERCICIO 82

1. $x\rightarrow N^o\,mayor$

$$x-8\rightarrow N^o\,menor$$
$$x+x-8=106$$
$$2x-8=106$$
$$2x=106+8$$
$$x=\frac{114}{2}$$
$$x=57\rightarrow N^o\,mayor$$

$$x-8\Rightarrow 57-8$$
$$=49\rightarrow N^o\,menor$$

2. $x \rightarrow N^o mayor$

$x-32 \rightarrow N^o menor$

$x+x-32=540$

$2x=540+32$

$2x=572$

$x=\frac{572}{2}$

$x=286 \rightarrow N^o mayor$

$x-32 \Rightarrow 286-32$

$=254 \rightarrow N^o menor$

3. $A+B=1.154\, bs.$

$A-506=B$

$A+A-506=1.154$

$2A=1.154+506$

$2A=1.660$

$A=\frac{1.660}{2}$

$A=830\, bs.$

$A-506=B$

$\Rightarrow 830-506=B$

$324\, bs.=B$

4. $x \rightarrow N^o mayor$

$x-24 \rightarrow N^o menor$

$x+x-24=106$

$2x=106+24$

$x=\frac{130}{2}$

$x=65x \rightarrow N^o mayor$

$x-24 \Rightarrow 65-24$

$=41 \rightarrow N^o menor$

5. $A+B=56\, años$

$A+14=B$

$A+A+14=56$

$2A=56-14$

$A=\frac{42}{2}$

$A=21 años$

$A+14=B$

$21+14=B$

$35\, años=B$

6. $A+B=1.080\, soles$

$A-1.014=B$

$A+A-1.014=1.080$

$2A=1.080+1.014$

$A=\frac{2.094}{2}$

$A=1.047\, soles$

$A-1.014=B$

$\Rightarrow 1.047-1.014=B$

$33\, soles=B$

7. $x \rightarrow N^o mayor$

$x-1 \rightarrow N^o menor$

$x+x-1=103$

$2x=103+1$

$x=\frac{104}{2}$

$x=52 \rightarrow N^o mayor$

$x-1 \Rightarrow 52-1$

$=51 \rightarrow N^o menor$

8. $x \rightarrow N^o menor$

$x+1 \rightarrow N^o medio$

$x+2 \rightarrow N^o mayor$

$x+x+1+x+2=204$

$3x+3=204$

$3x=204-3$

$x=\frac{201}{3}$

$x=67 \rightarrow N^o menor$

$x+1 \Rightarrow 67+1$

$=68 \rightarrow N^o medio$

$x+2 \Rightarrow 67+2$

$=69 \rightarrow N^o mayor$

9. $x \rightarrow N^o 1 \quad x+1 \rightarrow N^o 2$

$x+2 \rightarrow N^o 3 \quad x+3 \rightarrow N^o 4$

$x+x+1+x+2+x+3=74$

$4x+6=74$

$4x=74-6$

$x=\frac{68}{4}$

$x=17 \rightarrow N^o 1$

$x+1 \Rightarrow 17+1$

$=18 \rightarrow N^o 2$

$x+2 \Rightarrow 17+2 \qquad x+3 \Rightarrow 17+3$

$=19 \rightarrow N^o 3 \qquad =20 \rightarrow N^o 4$

10. $2x \rightarrow N^o menor$

$2x+2 \rightarrow N^o mayor$

$2x+2x+2=194$

$4x=194-2$

$x=\frac{192}{4}$

$x=48$

$2x \Rightarrow 2 \cdot 48$

$=96 \rightarrow N^o menor$

$2x+2 \Rightarrow 2 \cdot 48+2$

$=96+2$

$=98 \rightarrow N^o mayor$

11. $x \rightarrow N^o mayor$

$x-1 \rightarrow N^o medio$

$x-2 \rightarrow N^o menor$

$x+x-1+x-2=186$

$3x-3=186$

$3x=189$

$x=\frac{189}{3}$

$x=63 \rightarrow N^o mayor$

$x-1 \Rightarrow 63-1$

$=62 \rightarrow N^o medio$

$x-2 \Rightarrow 63-2$

$=61 \rightarrow N^o menor$

12. $caballo+coche+arreos=325$

$coche+80=caballo$

$coche-25=arreos$

$coche+80+coche+coche-25=325$

$3\, coches+55=325$

$3\, coches=270$

$coche=\frac{270}{3}$

$coche=\$90$

$coche+80=caballo$

$\Rightarrow 90+80=caballo$

$\$170=caballo$

$coche-25=arreos$

$\Rightarrow 90-25=arreos$

$\$65=arreos$

13. $x \to N^o mayor$

$x-32 \to N^o medio$

$x-65 \to N^o menor$

$x+x-32+x-65=200$

$3x-97=200$

$3x=297$

$x=\frac{297}{3}$

$x=99 \to N^o mayor$

$x-32 \Rightarrow 99-32$

$=67 \to N^o medio$

$x-65 \Rightarrow 99-65$

$=34 \to N^o menor$

14. $x \to 1^o\ cesto$

$x-10 \to 2^o cesto$

$x-15 \to 3^o cesto$

$x+x-10+x-15=575$

$3x-25=575$

$3x=600$

$x=\frac{600}{3}$

$x=200 \to 1^o\ cesto$

$x-10 \Rightarrow 200-10$

$=190 \to 2^o\ cesto$

$x-15 \Rightarrow 200-15$

$=185 \to 3^o\ cesto$

15. $x \to mayor$

$x-55 \to medio$

$x-70 \to\ menor$

$x+x-55+x-70=454$

$3x-125=454$

$3x=454+125$

$3x=579$

$x=\frac{579}{3}$

$x=193 \to mayor$

$x-55 \Rightarrow 193-55$

$=138 \to medio$

$x-70 \Rightarrow 193-70$

$=123 \to menor$

16. $x \to 1^o$

$x-20 \to 2^o$

$x-20-40 \to 3^o$

$\Rightarrow x-60 \to 3^o$

$x+x-20+x-60=310$

$3x=310+80$

$3x=390$

$x=\frac{390}{3}$

$x=130 \to 1^o$

$x-20 \Rightarrow 130-20$

$=110 \to 2^o$

$x-60 \Rightarrow 130-60$

$=70 \to 3^o$

17. $x \to mayor$

$x-20 \to menor$

$x-18 \to medio$

$x+x-20+x-18=88$

$3x-38=88$

$3x=88+38$

$3x=126$

$x=\frac{126}{3}$

$x=42 \to mayor$

$x-20 \Rightarrow 42-20$

$=22 \to menor$

$x-18 \Rightarrow 42-18$

$=24 \to medio$

18. $x \to mayor$

$x-36 \to menor$

$x+x-36=642$

$2x-36=642$

$2x=642+36$

$2x=678$

$x=\frac{678}{2}$

$x=339 \to mayor$

$x-36 \Rightarrow 339-36$

$=303 \to menor$

EJERCICIO 83

1. $x \to edad\ Juan$

$3x \to edad Pedro$

$x+3x=40$

$4x=40$

$x=\frac{40}{4}$

$x=10 \to edad\ Juan$

$3x \Rightarrow 3 \cdot 10$

$=30 \to edad Pedro$

2. $x \to Arreos$

$4x \to Caballo$

$x+4x=600$

$5x=600$

$x=\frac{600}{5}$

$x=\$120 \to Arreos$

$4x \Rightarrow 4 \cdot 120$

$=\$\ 480 \to Caballo$

3. $x \to 1^o piso$

$\frac{x}{2} \to 2^o piso$

$x+\frac{x}{2}=48$

$\frac{2x+x}{2}=48$

$3x=48 \cdot 2$

$3x=96$

$x=\frac{96}{3}$

$x=32 Habt. \to 1^o piso$

$\frac{x}{2} \Rightarrow \frac{32}{2}$

$=16 Habt. \to 2^o piso$

4. $A+B+C=300$

$B=2A$

$C=3A$

$\Rightarrow A+2A+3A=300$

$6A=300$

$A=\frac{300}{6}$

$A=50\ colones$

$B=2A \Rightarrow 2 \cdot 50$

$=100\ colones$

$C=3A \Rightarrow 3 \cdot 50$

$=150\ colones$

5. $A+B+C=133$

$A=\frac{B}{2}$

$C=2B$

$\Rightarrow \frac{B}{2}+B+2B=133$

$\frac{B}{2}+3B=133$

$\frac{B+6B}{2}=133$

$7B=133\cdot 2$

$B=\frac{266}{7}$

$B=38\,Sucres$

$A=\frac{B}{2}\Rightarrow \frac{38}{2}$

$=19\,Sucres$

$C=2B\Rightarrow 2\cdot 38$

$=76\,Sucres$

6. $x\rightarrow N^o mayor$

$\frac{x}{6}\rightarrow N^o menor$

$x+\frac{x}{6}=147$

$\frac{6x+x}{6}=147$

$7x=147\cdot 6$

$7x=882$

$x=\frac{882}{7}$

$x=126\rightarrow N^o mayor$

$\frac{x}{6}\Rightarrow \frac{126}{6}$

$=21\rightarrow N^o menor$

7. $A+B+C=140$

$B=\frac{A}{2}\Rightarrow 2B=A$

$B=\frac{C}{4}\Rightarrow 4B=C$

$2B+B+4B=140$

$7B=140$

$B=\frac{140}{7}$

$B=20\,Quetz.$

$A=2B\Rightarrow 2\cdot 20$

$=40\,Quetz.$

$C=4B\Rightarrow 4\cdot 20$

$=80\,Quetz.$

8. $x\rightarrow 1^o parte$

$4x\rightarrow 2^o parte$

$5x\rightarrow 3^o parte$

$x+4x+5x=850$

$10x=850$

$x=\frac{850}{10}$

$x=85\rightarrow 1^o parte$

$4x\Rightarrow 4\cdot 85$

$=340\rightarrow 2^o parte$

$5x\Rightarrow 5\cdot 85$

$=425\rightarrow 3^o parte$

9. $x\rightarrow N^o buscado$

$2x=x+111$

$2x-x=111$

$x=111\rightarrow N^o buscado$

10. $x\rightarrow edad Rosa$

$3x+15\rightarrow edad María$

$x+3x+15=59$

$4x=59-15$

$4x=44$

$x=\frac{44}{4}$

$x=11\rightarrow edad Rosa$

$3x+15\Rightarrow 3\cdot 11+15$

$=33+15$

$=48\rightarrow edad María$

11. $x\rightarrow N^o buscado$

$8x=x+21$

$8x-x=21$

$7x=21$

$x=\frac{21}{7}$

$x=3\rightarrow N^o buscado$

12. $x\rightarrow Mi\ edad$

$3x+7=100$

$3x=100-7$

$3x=93$

$x=\frac{93}{3}$

$x=31\rightarrow Mi\ edad$

13. $x\rightarrow 1^o parte$

$\frac{x}{3}\rightarrow 2^o parte$

$\frac{4x}{3}\rightarrow 3^o parte$

$x+\frac{x}{3}+\frac{4x}{3}=96$

$\frac{3x+x+4x}{3}=96$

$8x=96\cdot 3$

$8x=288$

$x=\frac{288}{8}$

$x=36\rightarrow 1^o parte$

$\frac{x}{3}\Rightarrow \frac{36}{3}=12\rightarrow 2^o parte$

$\frac{4x}{3}\Rightarrow \frac{4\cdot 36}{3}=4\cdot 12=48\rightarrow 3^o parte$

14. $x\rightarrow Edad Enrique$

$2x\rightarrow Edad Pedro$

$3x\rightarrow Edad Juan$

$6x\rightarrow Edad Eugenio$

$x+2x+3x+6x=132$

$12x=132$

$x=\frac{132}{12}$

$x=11\rightarrow Edad Enrique$

$2x\Rightarrow 2\cdot 11=22\rightarrow Edad Pedro$

$3x\Rightarrow 3\cdot 11=33\rightarrow Edad Juan$

$6x\Rightarrow 6\cdot 11=66\rightarrow Edad Eugenio$

EJERCICIO 84

1. $x\rightarrow 1^o parte$

$3x\rightarrow 2^o parte$

$3x-40\rightarrow 3^o parte$

$x+3x+3x-40=254$

$7x=254+40$

$7x=294$

$x=\frac{294}{7}$

$x=42\rightarrow 1^o parte$

$3x\Rightarrow 3\cdot 42=126\rightarrow 2^o parte$

$3x-40\Rightarrow 3\cdot 42-40$

$=126-40=86\rightarrow 3^o parte$

2. $A+B+C=130$

$C=2A$

$B-15=C \Rightarrow B-15=2A$

$B=2A+15$

$A+2A+15+2A=130$

$5A=130-15$

$5A=115$

$A=\frac{115}{5}$

$A=23\,Balb.$

$B=2A+15 \Rightarrow 2\cdot 23+15=46+15=61\,Balb.$

$C=2A \Rightarrow 2\cdot 23=46\,Balb.$

3. $x \rightarrow 1^{o}\,Número$

$\frac{x-8}{2} \rightarrow 2^{o}\,Número$

$x-18 \rightarrow 3^{o}\,Número$

$x+\frac{x-8}{2}+x-18=238$

$2x+\frac{x}{2}-\frac{8}{2}=256$

$\frac{4x+x}{2}=256+4$

$5x=260\cdot 2$

$x=\frac{520}{5}$

$x=104 \rightarrow 1^{o}\,Número$

$\frac{x-8}{2} \Rightarrow \frac{104-8}{2}=\frac{96}{2}=48 \rightarrow 2^{o}\,Número$

$x-18 \Rightarrow 104-18=86 \rightarrow 3^{o}\,Número$

4. $x \rightarrow Costo\ traje$

$\frac{x}{8} \rightarrow Costo\ sombrero$

$x-30 \rightarrow Costo\ bastón$

$x+\frac{x}{8}+x-30=259$

$2x+\frac{x}{8}=259+30$

$\frac{16x+x}{8}=289$

$17x=289\cdot 8$

$x=\frac{2.312}{17}$

$x=\$136 \rightarrow Costo\ traje$

$\frac{x}{8} \Rightarrow \frac{136}{8}=\$17 \rightarrow Costo\ sombrero$

$x-30 \Rightarrow 136-30=\$106 \rightarrow Costo\ bastón$

5. $x \rightarrow 1^{o}\,Número$

$\frac{x-6}{5} \rightarrow 2^{o}\,Número$

$x-6 \rightarrow 3^{o}\,Número$

$x+\frac{x-6}{5}+x-6=72$

$2x+\frac{x}{5}-\frac{6}{5}=78$

$\frac{10x+x}{5}=78+\frac{6}{5}$

$\frac{11x}{5}=\frac{390+6}{5}$

$11x=396$

$x=\frac{396}{11}$

$x=36 \rightarrow 1^{o}\,Número$

$\frac{x-6}{5} \Rightarrow \frac{36-6}{5}$

$=\frac{30}{5}=6 \rightarrow 2^{o}\,Número$

$x-6 \Rightarrow 36-6=30 \rightarrow 3^{o}\,Número$

6. $A+B=99$

$B=3A+19$

$A+3A+19=99$

$4A=99-19$

$4A=80$

$A=\frac{80}{4}$

$A=20\,bs.$

$B=3A+19 \Rightarrow 3\cdot 20+19$

$=60+19=79\,bs.$

7. $x \rightarrow cm\ de\ azúl$

$\frac{x-14}{2} \rightarrow cm\ de\ blanco$

$x+\frac{x-14}{2}=74$

$x+\frac{x}{2}-\frac{14}{2}=74$

$\frac{2x+x}{2}=74+7$

$3x=81\cdot 2$

$x=\frac{162}{3}$

$x=54 \rightarrow cm\ de\ azúl$

$\frac{x-14}{2} \Rightarrow \frac{54-14}{2}$

$=\frac{40}{2}=20 \rightarrow cm\ de\ blanco$

8. $A+B+C=152$

$B=2A-8 \Rightarrow \frac{B+8}{2}=A$

$B-32=C$

$\frac{B+8}{2}+B+B-32=152$

$\frac{B}{2}+\frac{8}{2}+2B=152+32$

$\frac{B+4B}{2}=184-4$

$5B=180\cdot 2$

$B=\frac{360}{5}$

$B=\$72$

$A=\frac{B+8}{2} \Rightarrow \frac{72+8}{2}=\frac{80}{2}=\$40$

$C=B-32 \Rightarrow 72-32=\$40$

9. $x \rightarrow N^{o}\,buscado$

$x-80=220-2x$

$x+2x=220+80$

$3x=300$

$x=\frac{300}{3}$

$x=100 \rightarrow N^{o}\,buscado$

10. $x \rightarrow Tengo\ ahora$

$2x+10=x+60$

$2x-x=60-10$

$x=50\,S/. \rightarrow Tengo\ ahora$

11. $x \rightarrow Parte\ separada$

$x+80 \rightarrow La\ otra\ parte$

$x+x+80=910$

$2x=910-80$

$x=\frac{830}{2}$

$x=415\,cm$

$\Rightarrow 4{,}15\,m \rightarrow Parte\ separada$

$x+80 \Rightarrow 415+80=495\,cm$

$\Rightarrow 4{,}95\,m \rightarrow La\ otra\ parte$

12. $x \to Edad\ padre$

$\frac{x-3}{3} \to Edad\ hijo$

$x + \frac{x-3}{3} = 83$

$x + \frac{x}{3} - \frac{3}{3} = 83$

$\frac{3x+x}{3} = 83+1$

$4x = 84 \cdot 3$

$x = \frac{252}{4}$

$x = 63 \to Edad\ padre$

$\frac{x-3}{3} \Rightarrow \frac{63-3}{3} = \frac{60}{3} = 20 \to Edad\ hijo$

13. $A+B+C = 9.000$

$B+500 = A$

$B-800 = C$

$B+500+B+B-800 = 9.000$

$3B = 9.000+300$

$B = \frac{9.300}{3}$

$B = 3.100\ Votos$

$A = B+500 \Rightarrow 3.100+500 = 3.600\ Votos$

$C = B-800 \Rightarrow 3.100-800 = 2.300\ Votos$

14. $x \to N^o\ buscado$

$8x - 60 = 60 - 7x$

$8x + 7x = 60 + 60$

$15x = 120$

$x = \frac{120}{15}$

$x = 8 \to N^o\ buscado$

15. $x \to Edad\ hombre$

$2x - 17 = 100 - x$

$2x + x = 100 + 17$

$3x = 117$

$x = \frac{117}{3}$

$x = 39 \to Edad\ hombre$

EJERCICIO 85

1. $x \to N^o\ mayor$

$\frac{2x}{3} \to N^o\ menor$

$x + \frac{2x}{3} = 100$

$\frac{3x+2x}{3} = 100$

$5x = 100 \cdot 3$

$x = \frac{300}{5}$

$x = 60 \to N^o\ mayor$

$\frac{2x}{3} \Rightarrow \frac{2 \cdot 60}{3} = 40 \to N^o\ menor$

2. $x \to Edad\ padre$

$\frac{x-15}{2} \to Edad\ hijo$

$x + \frac{x-15}{2} = 60$

$x + \frac{x}{2} - \frac{15}{2} = 60$

$\frac{2x+x}{2} = 60 + \frac{15}{2}$

$\frac{3x}{2} = \frac{120+15}{2}$

$3x = 135$

$x = \frac{135}{3}$

$x = 45 \to Edad\ padre$

$\frac{x-15}{2} \Rightarrow \frac{45-15}{2} = 15 \to Edad\ hijo$

3. $x \to Parte\ mayor$

$x - 232 \to Parte\ menor$

$x + x - 232 = 1.080$

$2x = 1.080 + 232$

$x = \frac{1.312}{2}$

$x = 656 \to Parte\ mayor$

$x - 232 \Rightarrow 656 - 232$

$= 424 \to Parte\ menor$

4. $A + B = 150$

$A - 46 = B$

$A + A - 46 = 150$

$2A = 150 + 46$

$A = \frac{196}{2}$

$A = 98\ Soles$

$B = A - 46 \Rightarrow 98 - 46$

$= 52\ Soles$

5. $x \to Ang.mayor$

$\frac{x+45}{2} \to Ang.\ menor$

$x + \frac{x+45}{2} = 180$

$x + \frac{x}{2} + \frac{45}{2} = 180$

$\frac{2x+x}{2} = 180 - \frac{45}{2}$

$\frac{3x}{2} = \frac{360-45}{2}$

$3x = 315$

$x = \frac{315}{3}$

$x = 105^\circ \to Ang.mayor$

$\frac{x+45}{2} \Rightarrow \frac{105+45}{2} = 75^\circ \to Ang.menor$

6. $x \to N^o\ mayor$

$\frac{x-88}{3} \to N^o\ menor$

$x + \frac{x-88}{3} = 540$

$x + \frac{x}{3} - \frac{88}{3} = 540$

$\frac{3x+x}{3} = 540 + \frac{88}{3}$

$\frac{4x}{3} = \frac{1.620+88}{3}$

$4x = 1.708$

$x = \frac{1.708}{4}$

$x = 427 \to N^o\ mayor$

$\frac{x-88}{3} \Rightarrow \frac{427-88}{3}$

$= \frac{339}{3} = 113 \to N^o\ menor$

7. $x \to N^o\ mayor$

$\frac{x-12}{4} \to N^o\ menor$

$x - \left(\frac{x-12}{4}\right) = 36$

$x - \frac{x}{4} + \frac{12}{4} = 36$

$\frac{4x-x}{4} = 36 - 3$

$3x = 33 \cdot 4$

$3x = 132$

$x = \frac{132}{3}$

$x = 44 \to N^o\ mayor$

$\frac{x-12}{4} \Rightarrow \frac{44-12}{4} = 8 \to N^o\ menor$

8 $x \to Costo\ perro$

$\frac{x}{8} \to Costo\ collar$

$x+\frac{x}{8}=54$

$\frac{8x+x}{8}=54$

$9x=54\cdot 8$

$9x=432$

$x=\frac{432}{9}$

$x=\$48 \to Costo\ perro$

$\frac{x}{8} \Rightarrow \frac{48}{8}=\$6 \to Costo\ collar$

9. $A+B=84$

$A-16=B+20$

$\Rightarrow A-16-20=B$

$A-36=B$

$A+A-36=84$

$2A=84+36$

$A=\frac{120}{2}$

$A=\$60$

$B=A-36 \Rightarrow 60-36=\$24$

10. $x \to N^o\ Señoritas$

$\frac{x-15}{2} \to N^o\ Jovenes$

$x+\frac{x-15}{2}=60$

$x+\frac{x}{2}=60+\frac{15}{2}$

$\frac{2x+x}{2}=\frac{120+15}{2}$

$3x=135$

$x=\frac{135}{3}$

$x=45 \to N^o\ Señoritas$

$\frac{x-15}{2} \Rightarrow \frac{45-15}{2}=15 \to N^o\ jovenes$

11. $x \to Parte\ mayor$

$\frac{x+16}{3} \to Parte\ menor$

$x+\frac{x+16}{3}=160$

$x+\frac{x}{3}=160-\frac{16}{3}$

$\frac{3x+x}{3}=\frac{480-16}{3}$

$4x=464$

$x=\frac{464}{4}$

$x=116 \to Parte\ mayor$

$\frac{x+16}{3} \Rightarrow \frac{116+16}{3}$

$=\frac{132}{3}=44 \to Parte\ menor$

12. $x \to N^o\ mayor$

$\frac{x+150}{3} \to N^o\ menor$

$x+\frac{x+150}{3}=506$

$x+\frac{x}{3}=506-\frac{150}{3}$

$\frac{3x+x}{3}=\frac{1.518-150}{3}$

$4x=1.368$

$x=\frac{1.368}{4}$

$x=342 \to N^o\ mayor$

$\frac{x+150}{3} \Rightarrow \frac{342+150}{3}$

$=\frac{492}{3}=164 \to N^o\ menor$

13. $x \to Estilográfica$

$x-10 \to Lapicero$

$x+x-10=18$

$2x=28$

$x=\frac{28}{2}$

$x=14\ bs. \to Estilográfica$

$x-10 \Rightarrow 14-10=4\ bs. \to Lapicero$

14. $x \to Parte\ roja$

$x+4 \to Parte\ negra$

$x+x+4=84$

$2x=80$

$x=40\ cm \to Parte\ roja$

$x+4 \Rightarrow 40+4=44\ cm \to Parte\ negra$

EJERCICIO 86

1. $2x \to Edad\ actual\ A$

$x \to Edad\ actual\ B$

$2x-10=3(x-10)$

$2x-10=3x-30$

$2x-3x=-30+10$

$-x=-20$

$x=20 \to Edad\ actual\ B$

$2x \Rightarrow 2\cdot 20=40 \to Edad\ actual\ A$

2. $3x \to Edad\ A$

$x \to Edad\ B$

$3x+5=2(x+5)$

$3x+5=2x+10$

$3x-2x=10-5$

$x=5 \to Edad\ B$

$3x \Rightarrow 3\cdot 5=15 \to Edad\ A$

3. $2x \to Tiene\ A$

$x \to Tiene\ B$

$2x-30=x-5$

$2x-x=30-5$

$x=\$25 \to Tiene\ B$

$2x \Rightarrow 2\cdot 25=\$50 \to Tiene\ A$

4. $\frac{x}{2} \to Tiene\ A$

$x \to Tiene\ B$

$\frac{x}{2}+66=2(x-90)$

$\frac{x+132}{2}=2(x-90)$

$x+132=4(x-90)$

$x+132=4x-360$

$-3x=-492$

$x=\frac{-492}{-3}$

$x=164\ colones \to Tiene\ B$

$\frac{x}{2} \Rightarrow \frac{164}{2}=82 \to colones\ Tiene\ A$

5. $x \to N^o\ varones$

$\frac{x}{3} \to N^o\ Srtas$

$\frac{x}{3}+14=x-10$

$\frac{x}{3}-x=-10-14$

$\frac{x-3x}{3}=-24$

$-2x=-72$

$x=36 \to N^o\ varones$

$\frac{x}{3} \Rightarrow \frac{36}{3}=12 \to N^o\ Srtas$

6. $3x \to Edad\ padre$

$x \to Edad\ hijo$

$3x-5=2(x+10)$

$3x-5=2x+20$

$x=25 \to Edad\ hijo$

$3x \Rightarrow 3\cdot 25=75 \to Edad\ padre$

7. $x \rightarrow N^o\ mayor$

$2x-56 \rightarrow N^o\ menor$

$x+2x-56=85$

$3x=141$

$x=\frac{141}{3}$

$x=47 \rightarrow N^o\ mayor$

$2x-56 \Rightarrow 2\cdot 47-56$

$=94-56=38 \rightarrow N^o\ menor$

8. $5x \rightarrow Tiene\ Enrique$

$x \rightarrow Tiene\ su hno.$

$5x-0{,}5=x+0{,}5$

$5x-x=0{,}5+0{,}5$

$4x=1$

$x=\frac{1}{4}$

$x=\$0{,}25 \rightarrow Tiene\ su hno$

$5x \Rightarrow 5\cdot 0{,}25$

$=\$1{,}25 \rightarrow Tiene\ Enrique$

9. $x \rightarrow Bolsa 1$

$x-400 \rightarrow Bolsa 2$

$x+x-400=1.400$

$2x=1.800$

$x=900\ S/. \rightarrow Bolsa 1$

$x-400 \Rightarrow 900-400=500\ S/. \rightarrow Bolsa 2$

10. $4x \rightarrow Días\ Trab. Pedro$

$x \rightarrow Días\ Trab. Enrique$

$4x-15=x+21$

$4x-x=21+15$

$3x=36$

$x=\frac{36}{3}$

$x=12 \rightarrow Días\ Trab. Enrique$

$4x \Rightarrow 4\cdot 12$

$=48 \rightarrow Días\ Trab. Pedro$

11. $2x \rightarrow Edad\ act.\ padre$

$x \rightarrow Edad\ act.\ hijo$

$2x-14=3(x-14)$

$2x-14=3x-42$

$2x-3x=-42+14$

$-x=-28$

$x=28 \rightarrow Edad\ act.\ hijo$

$Edades\ hace\ 14\ años:$

$2x-14 \Rightarrow 2\cdot 28-14$

$=42\ años \rightarrow\ padre$

$x-14 \Rightarrow 28-14$

$=14\ años \rightarrow\ hijo$

12. $3x \rightarrow Edad\ actual\ Juan$

$x \rightarrow Edad\ actual\ hijo$

$3x+22=2(x+22)$

$3x=2x+44-22$

$x=22 \rightarrow Edad\ actual\ hijo$

$3x \Rightarrow 3\cdot 22=66 \rightarrow Edad\ actual\ Juan$

13. $A+B=84 \Rightarrow A=84-B$

$A+80=3(B+4)$

$\Rightarrow 84-B+80=3B+12$

$-B-3B=12-164$

$-4B=-152$

$B=\frac{-152}{-4}$

$B=\$38$

$A=84-B \Rightarrow 84-38=\$46$

EJERCICIO 87

1. $2x \rightarrow N^o\ Sombreros$

$x \rightarrow N^o\ trajes$

$4x+50x=702$

$54x=702$

$x=\frac{702}{54}$

$x=13 \rightarrow N^o\ trajes$

$2x \Rightarrow 2\cdot 13$

$=26 \rightarrow N^o\ Sombreros$

2. $x \rightarrow Cab.$

$x-6 \rightarrow Vacas$

$600x+800(x-6)=40.000$

$600x+800x-4.800=40.000$

$1.400x=44.800$

$x=\frac{44.800}{1.400}$

$x=32 \rightarrow Cab.$

$x-6 \Rightarrow 32-6=26 \rightarrow Vacas$

3. $16-x \rightarrow Prob. reslt.$

$x \rightarrow Prob. no\ reslt.$

$12(16-x)-5x=73$

$192-12x-5x=73$

$-17x=73-192$

$x=\frac{-119}{-17}$

$x=7 \rightarrow Prob. no\ reslt.$

$16-x \Rightarrow 16-7=9 \rightarrow Prob. reslt.$

4. $50-x \rightarrow Días\ trab.$

$x \rightarrow Días\ no\ trab.$

$3(50-x)-2x=90$

$150-3x-2x=90$

$-5x=-60$

$x=\frac{-60}{-5}$

$x=12 \rightarrow Días\ no\ trab.$

$50-x \Rightarrow 50-12=38 \rightarrow Días\ trab.$

5. $35-x \rightarrow Trajes\ de\ 30Q$

$x \rightarrow Trajes\ de\ 25Q$

$30(35-x)+25x=1.015$

$1.050-30x+25x=1.015$

$-5x=1.015-1.050$

$x=\frac{-35}{-5}$

$x=7 \rightarrow Trajes\ de\ 25Q$

$35-x \Rightarrow 35-7=28 \rightarrow Trajes\ de\ 30Q$

6. $x \rightarrow Traje\ Cal.$

$x-7 \rightarrow Traje\ \inf.$

$32x+18(x-7)=1.624$

$32x+18x-126=1.624$

$50x=1.750$

$x=\frac{1.750}{50}$

$x=35\ balb. \rightarrow Traje\ Cal.$

$x-7 \Rightarrow 35-7=28\ balb. \rightarrow Traje\ \inf.$

7. $3x \to N^o\ Lápices$
$x \to N^o\ cuad.$
$0,05 \cdot 3x + 0,06x = 1,47$
$0,15x + 0,06x = 1,47$
$0,21x = 1,47$
$x = \frac{1,47}{0,21}$
$x = 7$
$\Rightarrow 7 \to N^o\ cuad.$
$3x \to 3 \cdot 7 = 21 \to N^o\ Lápices$

8. $3x + 5 \to Sac.Frij.$
$x \to Sac.azúc.$
$6(3x+5) + 5x = 582$
$18x + 30 + 5x = 582$
$23x = 552$
$x = \frac{552}{23}$
$x = 24$
$\Rightarrow 24 \to Sac.azúc.$
$3x + 5 \Rightarrow 3 \cdot 24 + 5$
$= 77 \to Sac.Frij.$

9. $80 - x \to Cedro$
$x \to Caoba$
$0,75(80 - x) + 0,90x = 68,40$
$60 - 0,75x + 0,90x = 68,40$
$0,15x = 8,40$
$x = \frac{8,40}{0,15}$
$x = 56$
$\Rightarrow 56\ pies^3 \to Caoba$
$80 - x \Rightarrow 80 - 56$
$= 24\ pies^3 \to Cedro$

10. $1.050 - x \to P.mayor$
$x \to P.menor$
$3(1.050 - x) - 2x = 1.825$
$3.150 - 3x - 2x = 1.825$
$-5x = 1.825 - 3.150$
$-5x = -1.325$
$x = \frac{-1.325}{-5}$
$x = 265$
$\Rightarrow 265 \to P.menor$
$1.050 - x \Rightarrow 1.050 - 265$
$= 785 \to P.mayor$

EJERCICI0 88

1. $x \to 1^o$
$2x \to 2^o$
$x + 2x - 20 \to 3^o$
$\Rightarrow 3x - 20 \to 3^o$
$x + 2x + 3x - 20 = 196$
$6x = 196 + 20$
$x = \frac{216}{6}$
$x = 36 \to 1^o$
$2x \Rightarrow 2 \cdot 36 = 72 \to 2^o$
$3x - 20 \Rightarrow 3 \cdot 36 - 20$
$= 108 - 20 = 88 \to 3^o$

2. $3x \to Edad\ A$
$x \to Edad\ B$
$3x - 5 = 4(x - 5)$
$3x - 5 = 4x - 20$
$3x - 4x = -20 + 5$
$-x = -15$
$x = 15 \to Edad\ B$
$3x \Rightarrow 3 \cdot 15 = 45 \to Edad\ A$

3. $x \to Par\ zap.$
$2x + 50 \to Traje$
$50(2x + 50) + 35x = 16.000$
$100x + 2.500 + 35x = 16.000$
$135x = 16.000 - 2.500$
$x = \frac{13.500}{135}$
$x = 100\ soles \to Par\ zap.$
$2x + 50 \Rightarrow 2 \cdot 100 + 50 = 250\ soles \to Traje$

4. $x \to Costo\ casa$
$\frac{x}{4} = \frac{x}{6} + 2.000$
$\frac{x}{4} = \frac{x + 12.000}{6}$
$6x = 4x + 48.000$
$2x = 48.000$
$x = \frac{48.000}{2}$
$x = 24.000 bs. \to Costo\ casa$

5. $x \to N^o\ mayor$
$\frac{2x - 156}{3} \to N^o\ menor$
$x + \frac{2x - 156}{3} = 108$
$x + \frac{2x}{3} = 108 + \frac{156}{3}$
$\frac{3x + 2x}{3} = \frac{324 + 156}{3}$
$5x = 480$
$x = \frac{480}{5}$
$x = 96 \to N^o\ mayor$
$\frac{2x - 156}{3} \Rightarrow \frac{2 \cdot 96 - 156}{3}$
$= \frac{36}{3} = 12 \to N^o\ menor$

6. $x \to Ancho$
$461 - 11 = 9x$
$450 = 9x$
$\frac{450}{9} = x$
$50 = x$
$\Rightarrow 50\ pies \to Ancho$

7. $x \to Gasté$
$85 - x = 4x$
$85 = 5x$
$\frac{85}{5} = x$
$17 = x$
$\Rightarrow \$17 \to Gasté$

8. $x \to Edad\ act.B$
$2(x - 12) \to Edad\ A\ hace\ 12\ años$
$2(x - 12) + 24 + 68 = 3(x + 12)$
$2x - 24 + 92 = 3x + 36$
$2x - 3x = 36 - 68$
$-x = -32$
$x = 32 \to Edad\ act.B$
$Edad\ actual\ de\ A:$
$2(x - 12) + 12 \Rightarrow 2(32 - 12) + 12$
$= 2 \cdot 20 + 12 = 52\ años$

9. $x \to mon.\ 10 cts.$
$22 - x \to mon. 5 cts.$
$0,10x + 0,05(22 - x) = 1,85$
$0,10x + 1,10 - 0,05x = 1,85$
$0,05x = 1,85 - 1,10$
$0,05x = 0,75$
$x = \frac{0,75}{0,05}$
$x = 15 \to mon. 10 cts.$
$22 - x \Rightarrow 22 - 15 = 7 \to mon.\ 5 cts.$

10. $x \to N^{o}\,buscado$

$12(x-24)=24(x-27)$

$12x-288=24x-648$

$12x-24x=-648+288$

$-12x=-360$

$x=\dfrac{-360}{-12}$

$x=30 \to N^{o}\,buscado$

11. $x \to c/cab.$

$35x=40(x-10)$

$35x=40x-400$

$-5x=-400$

$x=\dfrac{-400}{-5}$

$x=\$80 \to c/cab.$

12. $x \to N^{o}buscado$

$3x-55=233-x$

$3x+x=233+55$

$4x=288$

$x=\dfrac{288}{4}$

$x=72 \to N^{o}buscado$

13. $x \to N^{o}\,menor$

$x+1 \to N^{o}\,medio$

$x+2 \to N^{o}\,mayor$

$2x+3(x+1)+4(x+2)=740$

$2x+3x+3+4x+8=740$

$9x=740-11$

$x=\dfrac{729}{9}$

$x=81$

$\Rightarrow 81 \to N^{o}\,menor$

$x+1 \Rightarrow 81+1=82 \to N^{o}\,medio$

$x+2 \Rightarrow 81+2=83 \to N^{o}\,mayor$

14. $x \to A\,caballo$

$3x \to En\,auto$

$x-20 \to A\,pie$

$x+3x+x-20=150$

$5x=150+20$

$x=\dfrac{170}{5}$

$x=34Km \to A\,caballo$

$3x \Rightarrow 3\cdot 34=102Km \to En\,auto$

$x-20 \Rightarrow 34-20=14Km \to A\,pie$

15. $x \to Parte\,hijo$

$x+2.000 \to Parte\,hijas$

$3x+2(x+2.000)=16.500$

$3x+2x+4.000=16.500$

$5x=12.500$

$x=\dfrac{12.500}{5}$

$x=2.500$

$\Rightarrow 2.500\ colones \to Parte\,hijo$

$x+2.000 \Rightarrow 2.500+2.000=4.500$

$\Rightarrow 4.500\,colones \to Parte\,hijas$

16. $x \to N^{o}\,mayor$

$x-1 \to N^{o}\,menor$

$x^2-(x-1)^2=31$

$x^2-x^2+2x-1=31$

$2x=32$

$x=16 \to N^{o}\,mayor$

$x-1 \Rightarrow 16-1=15 \to N^{o}\,menor$

17. $3x \to Edad\ A$

$x \to Edad\ B$

$\dfrac{x}{5} \to Edad\ C$

$x-12 \to Edad\ C$

$Luego:$

$\dfrac{x}{5}=x-12$

$x=5(x-12)$

$x=5x-60$

$-4x=-60$

$x=15 \to Edad\ B$

$3x \Rightarrow 3\cdot 15=45 \to Edad\ A$

$x-12 \Rightarrow 15-12=3 \to Edad\ C$

18. $x \to Edad\ act.\,A$

$\dfrac{x+5}{3} \to Edad\ B\,en\,5\,años$

$\dfrac{x+20}{2} \to Edad\ B\,en\,20\,años$

$Luego:$

$\dfrac{x+5}{3}+15=\dfrac{x+20}{2}$

$\dfrac{x+5+45}{3}=\dfrac{x+20}{2}$

$2(x+50)=3(x+20)$

$2x+100=3x+60$

$-x=-40$

$x=40 \to Edad\ act.\,A$

$Edad\ \ actual\,de\,B:$

$\dfrac{x+20}{2}-20 \Rightarrow \dfrac{40+20}{2}-20$

$=30-20=10\,años$

19. $x \to Lunes$

$2x \to Martes$

$4x \to Miercoles$

$8x \to Jueves$

$8x-30 \to Viernes$

$8x-20 \to Sábado$

$x+2x+4x+8x+8x-30+8x-20=911$

$31x-50=911$

$31x=911+50$

$x=\dfrac{961}{31}$

$x=\ \$31 \to Lunes$

$2x \Rightarrow 2\cdot 31=\$62 \to Martes$

$4x \Rightarrow 4\cdot 31=\$124 \to Miercoles$

$8x \Rightarrow 8\cdot 31=\$248 \to Jueves$

$8x-30 \Rightarrow 8\cdot 31-30=\$218 \to Viernes$

$8x-20 \Rightarrow 8\cdot 31-20=\$228 \to Sábado$

20. $x \to N^{o}\,1$

$x-18 \to N^{o}\,2$

$x+x-18=3\cdot 18$

$2x=54+18$

$x=\dfrac{72}{2}$

$x=36 \to N^{o}\,1$

$x-18 \Rightarrow 36-18=18 \to N^{o}\,2$

21. $x \to Tiene\ A$

$3(x-16) \to Tiene\ B$

$x+3(x-16)=36$

$x+3x-48=36$

$4x=84$

$x=\dfrac{84}{4}$

$x=\$21 \to Tiene\ A$

$3(x-16) \Rightarrow 3(21-16)$

$=3\cdot 5=\ \$15 \to Tiene\ B$

22. $3x \to Tiene\ A$

$x \to Tiene\ B$

$\dfrac{x}{2} \to Tiene\,C$

$3x-1-(x-3)=2\left(\dfrac{x}{2}+20\right)$

$3x-1-x+3=x+40$

$2x-x=40-2$

$x=38 \to Tiene\,B$

$3x \Rightarrow 3\cdot 38=\$114 \to Tiene\ A$

$\dfrac{x}{2} \Rightarrow \dfrac{38}{2}=\$19 \to Tiene\,C$

23. $x \to Costo\ tienda$

$\frac{x}{5} - 800 = \frac{x}{7}$

$\frac{x - 4.000}{5} = \frac{x}{7}$

$7x - 28.000 = 5x$

$2x = 28.000$

$x = 14.000$

$\Rightarrow 14.000\,bs. \to Costo\ tienda$

24. $x \to Cab.\,peor$

$2(x+15) \to Cab.mejor$

$x + 2(x+15) = 120$

$x + 2x + 30 = 120$

$3x = 90$

$x = \$30 \to Cab.\,peor$

$2(x+15) = 2(30+15)$

$= 2 \cdot 45 = \$90 \to Cab.mejor$

25. $x \to queda\ A$

$3x \to queda\ B$

$x + 3x = 160$

$4x = 160$

$x = 40$

$Como\,A\ tenía\ 80\,Q.$

$\Rightarrow lo\,que\,perdío\,A$

$= 80 - x \Rightarrow 80 - 40 = 40\,Q.$

26. $2x \to Emp.\,A$

$x \to Emp.\,B$

$2(2x - 400) = x + 400$

$4x - 800 = x + 400$

$3x = 1.200$

$x = \$400 \to Emp.\,B$

$2x \Rightarrow 2 \cdot 400 = \$800 \to Emp.\,A$

27. $4x \to Cab.$

$x \to Vacas$

$4x + 5 = 3(x + 5)$

$4x + 5 = 3x + 15$

$x = 10 \to Vacas$

$4x \Rightarrow 4 \cdot 10 = 40 \to Cab.$

28 $x \to Lunes$

$x + 6 \to Martes$

$x + 12 \to Miercoles$

$x + 18 \to Jueves$

$4x \to Jueves$

$x + 18 = 4x$

$-3x = -18$

$x = \$6 \to Lunes$

$x + 6 \Rightarrow 6 + 6 = \$12 \to Martes$

$x + 12 \Rightarrow 6 + 12 = 18 \to Miercoles$

$4x \Rightarrow 4 \cdot 6 = \$24 \to Jueves$

29. $x \to Tenía\ \ ppio.$

$2x - 50 + 2(2x - 50) - 390 = 0$

$2x - 440 + 4x - 100 = 0$

$6x = 540$

$x = \frac{540}{6}$

$x = 90$

$\Rightarrow 90\,soles \to Tenía\ \ ppio.$

30. $2x \to Largo$

$x \to Ancho$

$(2x - 6)(x + 4) = 2x^2$

$2x^2 + 2x - 24 = 2x^2$

$2x = 24$

$x = 12\,m \to Ancho$

$2x \Rightarrow 2 \cdot 12 = 24m \to Largo$

31. $3x \to Padre\ hace\ 5\,años$

$x \to Hijo\ hace\ 5\,años$

$3x + 10 = 2(x + 10)$

$3x + 10 = 2x + 20$

$x = 10\,años$

$Edad\ actual\ Padre:$

$3x + 5 \Rightarrow 3 \cdot 10 + 5 = 35\,años$

$Edad\ actual\ Hijo:$

$x + 5 \Rightarrow 10 + 5 = 15\,años$

32. $3x \to Edad\ A\,en\,4\,años$

$x \to Edad\ B\,en\,4\,años$

$3x - 6 = 5(x - 6)$

$3x - 6 = 5x - 30$

$-2x = -24$

$x = 12\,años$

$Edad\ Actual\ A:$

$3x - 4 \Rightarrow 3 \cdot 12 - 4 = 32\,años$

$Edad\ Actual\ B:$

$x - 4 \Rightarrow 12 - 4 = 8\,años$

EJERCICIO 89

1. $a^2 + ab = a\,(a + b)$

2. $b + b^2 = b\,(1 + b)$

3. $x^2 + x = x\,(x + 1)$

4. $3a^3 - a^2 = a^2\,(3a - 1)$

5. $x^3 - 4x^4 = x^3\,(1 - 4x)$

6. $5m^2 + 15m^3 = 5m^2\,(1 + 3m)$

7. $ab - bc = b\,(a - c)$

8. $x^2y + x^2z = x^2\,(y + z)$

9. $2a^2x + 6ax^2 = 2ax(a + 3x)$

10. $8m^2 - 12mn = 4m(2m - 3n)$

11. $9a^3x^2 - 18ax^3 = 9ax^2\,(a^2 - 2x)$

12. $15c^3d^2 + 60c^2d^3 = 15c^2d^2\,(c + 4d)$

13. $35m^2n^3 - 70m^3 = 35m^2\,(n^3 - 2m)$

14. $abc + abc^2 = abc(1 + c)$

15. $24a^2xy^2 - 36x^2y^4 = 12xy^2\,(2a^2 - 3xy^2)$

16. $a^3 + a^2 + a = a(a^2 + a + 1)$

17. $4x^2 - 8x + 2 = 2(2x^2 - 4x + 1)$

18. $15y^3 + 20y^2 - 5y = 5y(3y^2 + 4y - 1)$

19. $a^3 - a^2x + ax^2 = a(a^2 - ax + x^2)$

20. $2a^2x + 2ax^2 - 3ax$

$= ax(2a + 2x - 3)$

21. $x^3 + x^5 - x^7 = x^3\,(1 + x^2 - x^4)$

22. $14x^2y^2 - 28x^3 + 56x^4$

$= 14x^2\,(y^2 - 2x + 4x^2)$

23. $34ax^2 + 51a^2y - 68ay^2$

$= 17a\,(2x^2 + 3ay - 4y^2)$

24. $96 - 48mn^2 + 144n^3$

$= 48(2 - mn^2 + 3n^3)$

25. $a^2b^2c^2 - a^2c^2x^2 + a^2c^2y^2$

$= a^2c^2\,(b^2 - x^2 + y^2)$

26. $55m^2n^3x+110m^2n^3x^2-220m^2y^3$
$=55m^2\left(n^3x+2n^3x^2-4y^3\right)$

27. $93a^3x^2y-62a^2x^3y^2-124a^2x$
$=31a^2x\left(3axy-2x^2y^2-4\right)$

28. $x-x^2+x^3-x^4=x\left(1-x+x^2-x^3\right)$

29. $a^6-3a^4+8a^3-4a^2=a^2\left(a^4-3a^2+8a-4\right)$

30. $25x^7-10x^5+15x^3-5x^2$
$=5x^2\left(5x^5-2x^3+3x-1\right)$

31. $x^{15}-x^{12}+2x^9-3x^6=x^6\left(x^9-x^6+2x^3-3\right)$

32. $9a^2-12ab+15a^3b^2-24ab^3$
$=3a\left(3a-4b+5a^2b^2-8b^3\right)$

33. $16x^3y^2-8x^2y-24x^4y^2-40x^2y^3$
$=8x^2y\left(2xy-1-3x^2y-5y^2\right)$

34. $12m^2n+24m^3n^2-36m^4n^3+48m^5n^4$
$=12m^2n\left(1+2mn-3m^2n^2+4m^3n^3\right)$

35. $100a^2b^3c-150ab^2c^2+50ab^3c^3-200abc^2$
$=50abc\left(2ab^2-3bc+b^2c^2-4c\right)$

36. $x^5-x^4+x^3-x^2+x=x\left(x^4-x^3+x^2-x+1\right)$

37. $a^2-2a^3+3a^4-4a^5+6a^6$
$=a^2\left(1-2a+3a^2-4a^3+6a^4\right)$

38. $3a^2b+6ab-5a^3b^2+8a^2bx+4ab^2m$
$=ab\left(3a+6-5a^2b+8ax+4bm\right)$

39. $a^{20}-a^{16}+a^{12}-a^8+a^4-a^2$
$=a^2\left(a^{18}-a^{14}+a^{10}-a^6+a^2-1\right)$

EJERCICIO 90

1. $a(x+1)+b(x+1)=(x+1)(a+b)$
2. $x(a+1)-3(a+1)=(a+1)(x-3)$
3. $2(x-1)+y(x-1)=(x-1)(2+y)$
4. $m(a-b)+(a-b)n=(a-b)(m+n)$
5. $2x(n-1)-3y(n-1)=(n-1)(2x-3y)$
6. $a(n+2)+n+2=(n+2)(a+1)$
7. $x(a+1)-a-1=(a+1)(x-1)$
8. $a^2+1-b\left(a^2+1\right)=\left(a^2+1\right)(1-b)$
9. $3x(x-2)-2y(x-2)=(x-2)(3x-2y)$
10. $1-x+2a(1-x)=(1-x)(1+2a)$
11. $4x(m-n)+n-m=(m-n)(4x-1)$
12. $-m-n+x(m+n)=(m+n)(x-1)$
13. $a^3(a-b+1)-b^2(a-b+1)=(a-b+1)\left(a^3-b^2\right)$
14. $4m\left(a^2+x-1\right)+3n\left(x-1+a^2\right)=\left(a^2+x-1\right)(4m+3n)$
15. $x(2a+b+c)-2a-b-c$
$=x(2a+b+c)-(2a+b+c)=(2a+b+c)(x-1)$
16. $(x+y)(n+1)-3(n+1)=(n+1)(x+y-3)$
17. $(x+1)(x-2)+3y(x-2)=(x-2)(x+1+3y)$
18. $(a+3)(a+1)-4(a+1)=(a+1)(a+3-4)=(a+1)(a-1)$
19. $\left(x^2+2\right)(m-n)+2(m-n)=(m-n)\left(x^2+4\right)$
20. $a(x-1)-(a+2)(x-1)=(x-1)(a-a-2)=-2(x-1)$
21. $5x\left(a^2+1\right)+(x+1)\left(a^2+1\right)=\left(a^2+1\right)(6x+1)$
22. $(a+b)(a-b)-(a-b)(a-b)$
$=(a-b)(a+b-a+b)=2b(a-b)$
23. $(m+n)(a-2)+(m-n)(a-2)=2m(a-2)$
24. $(x+m)(x+1)-(x+1)(x-n)$
$=(x+1)(x+m-x+n)=(x+1)(m+n)$
25. $(x-3)(x-4)+(x-3)(x+4)=(x-3)2x$
26. $(a+b-1)\left(a^2+1\right)-a^2-1=\left(a^2+1\right)(a+b-2)$
27. $(a+b-c)(x-3)-(b-c-a)(x-3)$
$=(x-3)(a+b-c+a-b+c)=(x-3)2a$
28. $3x(x-1)-2y(x-1)+z(x-1)=(x-1)(3x-2y+z)$
29. $a(n+1)-b(n+1)-n-1=(n+1)(a-b-1)$
30. $x(a+2)-a-2+3(a+2)=(a+2)(x+2)$
31. $(1+3a)(x+1)-2a(x+1)+3(x+1)=(x+1)(a+4)$
32. $(3x+2)(x+y-z)-(3x+2)-(x+y-1)(3x+2)$
$=(3x+2)(x+y-z-1-x-y+1)=-z(3x+2)$

EJERCICIO 91

1. $a^2+ab+ax+bx$
$=(a^2+ab)+(ax+bx)$
$=a(a+b)+x(a+b)$
$=(a+b)(a+x)$

2. $am-bm+an-bn$
$=(am-bm)+(an-bn)$
$=m(a-b)+n(a-b)$
$=(a-b)(m+n)$

3. $ax-2bx-2ay+4by$
$=(ax-2bx)-(2ay-4by)$
$=x(a-2b)-2y(a-2b)$
$=(a-2b)(x-2y)$

4. $a^2x^2-3bx^2+a^2y^2-3by^2$
$=(a^2x^2-3bx^2)+(a^2y^2-3by^2)$
$=x^2(a^2-3b)+y^2(a^2-3b)$
$=(a^2-3b)(x^2+y^2)$

5. $3m-2n-2nx^4+3mx^4$
$=(3m+3mx^4)-(2n+2nx^4)$
$=3m(1+x^4)-2n(1+x^4)$
$=(1+x^4)(3m-2n)$

6. $x^2-a^2+x-a^2x$
$=-(a^2+a^2x)+(x^2+x)$
$=-a^2(1+x)+x(x+1)$
$=(x+1)(x-a^2)$

7. $4a^3-1-a^2+4a$
$=(4a^3+4a)-(1+a^2)$
$=4a(a^2+1)-(1+a^2)$
$=(a^2+1)(4a-1)$

8. $x+x^2-xy^2-y^2$
$=(x+x^2)-(xy^2+y^2)$
$=x(1+x)-y^2(x+1)$
$=(x+1)(x-y^2)$

9. $3abx^2-2y^2-2x^2+3aby^2$
$=(3abx^2+3aby^2)-(2y^2+2x^2)$
$=3ab(x^2+y^2)-2(x^2+y^2)$
$=(x^2+y^2)(3ab-2)$

10. $3a-b^2+2b^2x-6ax$
$=(3a-6ax)-(b^2-2b^2x)$
$=3a(1-2x)-b^2(1-2x)$
$=(1-2x)(3a-b^2)$

11. $4a^3x-4a^2b+3bm-3amx$
$=(4a^3x-4a^2b)+(3bm-3amx)$
$=4a^2(ax-b)+3m(b-ax)$
$=4a^2(ax-b)-3m(ax-b)$
$=(ax-b)(4a^2-3m)$

12. $6ax+3a+1+2x$
$=(6ax+3a)+(1+2x)$
$=3a(2x+1)+(2x+1)$
$=(2x+1)(3a+1)$

13. $3x^3-9ax^2-x+3a$
$=(3x^3-x)-(9ax^2-3a)$
$=x(3x^2-1)-3a(3x^2-1)$
$=(3x^2-1)(x-3a)$

14. $2a^2x-5a^2y+15by-6bx$
$=(2a^2x-5a^2y)+(15by-6bx)$
$=a^2(2x-5y)+3b(5y-2x)$
$=a^2(2x-5y)-3b(2x-5y)$
$=(2x-5y)(a^2-3b)$

15. $2x^2y+2xz^2+y^2z^2+xy^3$
$=(2x^2y+xy^3)+(2xz^2+y^2z^2)$
$=xy(2x+y^2)+z^2(2x+y^2)$
$=(2x+y^2)(xy+z^2)$

16. $6m-9n+21nx-14mx$
$=(6m-14mx)-(9n-21nx)$
$=2m(3-7x)-3n(3-7x)$
$=(3-7x)(2m-3n)$

17. $n^2x-5a^2y^2-n^2y^2+5a^2x$
$=(n^2x-n^2y^2)-(5a^2y^2-5a^2x)$
$=n^2(x-y^2)-5a^2(y^2-x)$
$=n^2(x-y^2)+5a^2(x-y^2)$
$=(x-y^2)(n^2+5a^2)$

18. $1+a+3ab+3b$
$=(1+a)+(3ab+3b)$
$=(1+a)+3b(a+1)$
$=(1+a)(3b+1)$

19. $4am^3-12amn-m^2+3n$
$=(4am^3-12amn)-(m^2-3n)$
$=4am(m^2-3n)-(m^2-3n)$
$=(m^2-3n)(4am-1)$

20. $20ax-5bx-2by+8ay$
$=(20ax+8ay)-(5bx+2by)$
$=4a(5x+2y)-b(5x+2y)$
$=(5x+2y)(4a-b)$

21. $3-x^2+2abx^2-6ab$
$=(3-x^2)+(2abx^2-6ab)$
$=(3-x^2)+2ab(x^2-3)$
$=(3-x^2)-2ab(3-x^2)$
$=(3-x^2)(1-2ab)$

22. a^3+a^2+a+1
$=(a^3+a^2)+(a+1)$
$=a^2(a+1)+(a+1)$
$=(a^2+1)(a+1)$

23. $3a^2-7b^2x+3ax-7ab^2$
$=(3a^2+3ax)-(7b^2x+7ab^2)$
$=3a(a+x)-7b^2(x+a)$
$=(a+x)(3a-7b^2)$

24. $2am-2an+2a-m+n-1$
$=(2am-2an+2a)-(m-n+1)$
$=2a(m-n+1)-(m-n+1)$
$=(2a-1)(m-n+1)$

25. $3ax-2by-2bx-6a+3ay+4b$
$=(3ax-6a+3ay)-(2by+2bx-4b)$
$=3a(x-2+y)-2b(y+x-2)$
$=(x+y-2)(3a-2b)$

26. $a^3+a+a^2+1+x^2+a^2x^2$
$=(a^3+a^2+a^2x^2)+(a+1+x^2)$
$=a^2(a+1+x^2)+(a+1+x^2)$
$=(a^2+1)(a+1+x^2)$

27. $3a^3-3a^2b+9ab^2-a^2+ab-3b^2$
$=(3a^3-3a^2b+9ab^2)-(a^2-ab+3b^2)$
$=3a(a^2-ab+3b^2)-(a^2-ab+3b^2)$
$=(3a-1)(a^2-ab+3b^2)$

28. $2x^3-nx^2+2xz^2-nz^2-3ny^2+6xy^2$
$=-(nx^2+nz^2+3ny^2)+(2x^3+2xz^2+6xy^2)$
$=-n(x^2+z^2+3y^2)+2x(x^2+z^2+3y^2)$
$=(2x-n)(x^2+3y^2+z^2)$

29. $3x^3+2axy+2ay^2-3xy^2-2ax^2-3x^2y$
$=(3x^3-3xy^2-3x^2y)+(2axy+2ay^2-2ax^2)$
$=3x(x^2-y^2-xy)+2a(xy+y^2-x^2)$
$=3x(x^2-y^2-xy)-2a(-xy-y^2+x^2)$
$=(3x-2a)(x^2-xy-y^2)$

30 $a^2b^3-n^4+a^2b^3x^2-n^4x^2-3a^2b^3x+3n^4x$
$=(a^2b^3+a^2b^3x^2-3a^2b^3x)-(n^4+n^4x^2-3n^4x)$
$=a^2b^3(1+x^2-3x)-n^4(1+x^2-3x)$
$=(a^2b^3-n^4)(1+x^2-3x)$

EJERCICIO 92

1. $a^2-2ab+b^2=(a-b)^2$

2. $a^2+2ab+b^2=(a+b)^2$

3. $x^2-2x+1=(x-1)^2$

4. $y^4+2y^2+1=(y^2+1)^2$

5. $a^2-10a+25=(a-5)^2$

6. $9-6x+x^2=(3-x)^2$

7. $16+40x^2+25x^4=(4+5x^2)^2$

8. $1-14a+49a^2=(1-7a)^2$

9. $36+12m^2+m^4=(6+m^2)^2$

10. $1-2a^3+a^6=(1-a^3)^2$

11. $a^8+18a^4+81=(a^4+9)^2$

12. $a^6-2a^3b^3+b^6=(a^3-b^3)^2$

13. $4x^2-12xy+9y^2=(2x-3y)^2$

14. $9b^2-30a^2b+25a^4=(3b-5a^2)^2$

15. $1+14x^2y+49x^4y^2=(1+7x^2y)^2$

16. $1-2a^5+a^{10}=(1-a^5)^2$

17. $49m^6-70am^3n^2+25a^2n^4$
$=(7m^3-5an^2)^2$

18. $100x^{10}-60a^4x^5y^6+9a^8y^{12}$
$=(10x^5-3a^4y^6)^2$

19. $121+198x^6+81x^{12}=(11+9x^6)^2$

20. $a^2-24am^2x^2+144m^4x^4$
$=(a-12m^2x^2)^2$

21. $16-104x^2+169x^4=(4-13x^2)^2$

22. $400x^{10}+40x^5+1=(20x^5+1)^2$

23. $\frac{a^2}{4}-ab+b^2=\left(\frac{a}{2}-b\right)^2$

24. $1+\frac{2b}{3}+\frac{b^2}{9}=\left(1+\frac{b}{3}\right)^2$

25. $a^4-a^2b^2+\frac{b^4}{4}=\left(a^2-\frac{b^2}{2}\right)^2$

26. $\frac{1}{25}-\frac{x^2}{3}+\frac{25x^4}{36}=\left(\frac{1}{5}-\frac{5x^2}{6}\right)^2$

27. $16x^6-2x^3y^2+\frac{y^4}{16}=\left(4x^3-\frac{y^2}{4}\right)^2$

28. $\frac{n^2}{9}+2mn+9m^2=\left(\frac{n}{3}+3m\right)^2$

29. $a^2+2a(a+b)+(a+b)^2$
$=(a+a+b)^2=(2a+b)^2$

30. $4-4(1-a)+(1-a)^2$
$=(2-1+a)^2=(1+a)^2$

31. $4m^2-4m(n-m)+(n-m)^2$
$=(2m-n+m)^2=(3m-n)^2$

32. $(m-n)^2+6(m-n)+9$
$=(m-n+3)^2$

33. $(a+x)^2-2(a+x)(x+y)+(x+y)^2$
$=(a+x-x-y)^2=(a-y)^2$

34. $(m+n)^2-2(a-m)(m+n)+(a-m)^2$
$=(m+n-a+m)^2=(2m+n-a)^2$

35. $4(1+a)^2-4(1+a)(b-1)+(b-1)^2$
$=(2+2a-b+1)^2=(2a-b+3)^2$

36. $9(x-y)^2+12(x-y)(x+y)+4(x+y)^2$
$=(3x-3y+2x+2y)^2=(5x-y)^2$

EJERCICIO 93

1. $x^2 - y^2 = (x+y)(x-y)$

2. $a^2 - 1 = (a+1)(a-1)$

3. $a^2 - 4 = (a+2)(a-2)$

4. $9 - b^2 = (3+b)(3-b)$

5. $1 - 4m^2 = (1+2m)(1-2m)$

6. $16 - n^2 = (4+n)(4-n)$

7. $a^2 - 25 = (a+5)(a-5)$

8. $1 - y^2 = (1+y)(1-y)$

9. $4a^2 - 9 = (2a+3)(2a-3)$

10. $25 - 36x^4 = (5+6x^2)(5-6x^2)$

11. $1 - 49a^2b^2 = (1+7ab)(1-7ab)$

12. $4x^2 - 81y^4 = (2x+9y^2)(2x-9y^2)$

13. $a^2b^8 - c^2 = (ab^4+c)(ab^4-c)$

14. $100 - x^2y^6 = (10+xy^3)(10-xy^3)$

15. $a^{10} - 49b^{12} = (a^5+7b^6)(a^5-7b^6)$

16. $25x^2y^4 - 121 = (5xy^2+11)(5xy^2-11)$

17. $100m^2n^4 - 169y^6$
$= (10mn^2+13y^3)(10mn^2-13y^3)$

18. $a^2m^4n^6 - 144 = (am^2n^3+12)(am^2n^3-12)$

19. $196x^2y^4 - 225z^{12}$
$= (14xy^2+15z^6)(14xy^2-15z^6)$

20. $256a^{12} - 289b^4m^{10}$
$= (16a^6+17b^2m^5)(16a^6-17b^2m^5)$

21. $1 - 9a^2b^4c^6d^8 = (1+3ab^2c^3d^4)(1-3ab^2c^3d^4)$

22. $361x^{14} - 1 = (19x^7+1)(19x^7-1)$

23. $\frac{1}{4} - 9a^2 = \left(\frac{1}{2}+3a\right)\left(\frac{1}{2}-3a\right)$

24. $1 - \frac{a^2}{25} = \left(1+\frac{a}{5}\right)\left(1-\frac{a}{5}\right)$

25. $\frac{1}{16} - \frac{4x^2}{49} = \left(\frac{1}{4}+\frac{2x}{7}\right)\left(\frac{1}{4}-\frac{2x}{7}\right)$

26. $\frac{a^2}{36} - \frac{x^6}{25} = \left(\frac{a}{6}+\frac{x^3}{5}\right)\left(\frac{a}{6}-\frac{x^3}{5}\right)$

27. $\frac{x^2}{100} - \frac{y^2z^4}{81} = \left(\frac{x}{10}+\frac{yz^2}{9}\right)\left(\frac{x}{10}-\frac{yz^2}{9}\right)$

28. $\frac{x^6}{49} - \frac{4a^{10}}{121} = \left(\frac{x^3}{7}+\frac{2a^5}{11}\right)\left(\frac{x^3}{7}-\frac{2a^5}{11}\right)$

29. $100m^2n^4 - \frac{1}{16}x^8 = \left(10mn^2+\frac{1}{4}x^4\right)\left(10mn^2-\frac{1}{4}x^4\right)$

30. $a^{2n} - b^{2n} = (a^n+b^n)(a^n-b^n)$

31. $4x^{2n} - \frac{1}{9} = \left(2x^n+\frac{1}{3}\right)\left(2x^n-\frac{1}{3}\right)$

32. $a^{4n} - 225b^4 = (a^{2n}+15b^2)(a^{2n}-15b^2)$

33. $16x^{6m} - \frac{y^{2n}}{49} = \left(4x^{3m}+\frac{y^n}{7}\right)\left(4x^{3m}-\frac{y^n}{7}\right)$

34. $49a^{10n} - \frac{b^{12x}}{81} = \left(7a^{5n}+\frac{b^{6x}}{9}\right)\left(7a^{5n}-\frac{b^{6x}}{9}\right)$

35. $a^{2n}b^{4n} - \frac{1}{25} = \left(a^nb^{2n}+\frac{1}{5}\right)\left(a^nb^{2n}-\frac{1}{5}\right)$

36. $\frac{1}{100} - x^{2n} = \left(\frac{1}{10}+x^n\right)\left(\frac{1}{10}-x^n\right)$

EJERCICIO 94

1. $(x+y)^2 - a^2 = (x+y+a)(x+y-a)$

2. $4 - (a+1)^2 = (2+a+1)(2-a-1) = (3+a)(1-a)$

3. $9 - (m+n)^2 = (3+m+n)(3-m-n)$

4. $(m-n)^2 - 16 = (m-n+4)(m-n-4)$

5. $(x-y)^2 - 4z^2 = (x-y+2z)(x-y-2z)$

6. $(a+2b)^2 - 1 = (a+2b+1)(a+2b-1)$

7. $1 - (x-2y)^2 = (1+x-2y)(1-x+2y)$

8. $(x+2a)^2 - 4x^2$
$= (x+2a+2x)(x+2a-2x) = (3x+2a)(2a-x)$

9. $(a+b)^2 - (c+d)^2 = (a+b+c+d)(a+b-c-d)$

10. $(a-b)^2 - (c-d)^2 = (a-b+c-d)(a-b-c+d)$

11. $(x+1)^2-16x^2$
$=(x+1+4x)(x+1-4x)=(5x+1)(1-3x)$

12. $64m^2-(m-2n)^2$
$=(8m+m-2n)(8m-m+2n)=(9m-2n)(7m+2n)$

13. $(a-2b)^2-(x+y)^2=(a-2b+x+y)(a-2b-x-y)$

14. $(2a-c)^2-(a+c)^2$
$=(2a-c+a+c)(2a-c-a-c)=(3a)(a-2c)$

15. $(x+1)^2-4x^2$
$=(x+1+2x)(x+1-2x)=(3x+1)(1-x)$

16. $36x^2-(a+3x)^2$
$=(6x+a+3x)(6x-a-3x)=(9x+a)(3x-a)$

17. $a^6-(a-1)^2=(a^3+a-1)(a^3-a+1)$

18. $(a-1)^2-(m-2)^2$
$=(a-1+m-2)(a-1-m+2)=(a+m-3)(a-m+1)$

19. $(2x-3)^2-(x-5)^2$
$=(2x-3+x-5)(2x-3-x+5)=(3x-8)(x+2)$

20. $1-(5a+2x)^2=(1+5a+2x)(1-5a-2x)$

21. $(7x+y)^2-81=(7x+y+9)(7x+y-9)$

22. $m^6-(m^2-1)^2=(m^3+m^2-1)(m^3-m^2+1)$

23. $16a^{10}-(2a^2+3)^2=(4a^5+2a^2+3)(4a^5-2a^2-3)$

24. $(x-y)^2-(c+d)^2=(x-y+c+d)(x-y-c-d)$

25. $(2a+b-c)^2-(a+b)^2$
$=(2a+b-c+a+b)(2a+b-c-a-b)$
$=(3a+2b-c)(a-c)$

26. $100-(x-y+z)^2=(10+x-y+z)(10-x+y-z)$

27. $x^2-(y-x)^2=(x+y-x)(x-y+x)=(y)(2x-y)$

28. $(2x+3)^2-(5x-1)^2$
$=(2x+3+5x-1)(2x+3-5x+1)=(7x+2)(4-3x)$

29. $(x-y+z)^2-(y-z+2x)^2$
$=(x-y+z+y-z+2x)(x-y+z-y+z-2x)$
$=(3x)(2z-2y-x)$

30. $(2x+1)^2-(x+4)^2$
$=(2x+1+x+4)(2x+1-x-4)=(3x+5)(x-3)$

31. $(a+2x+1)^2-(x+a-1)^2$
$=(a+2x+1+x+a-1)(a+2x+1-x-a+1)$
$=(2a+3x)(x+2)$

32. $4(x+a)^2-49y^2=(2x+2a+7y)(2x+2a-7y)$

33. $25(x-y)^2-4(x+y)^2$
$=(5x-5y+2x+2y)(5x-5y-2x-2y)$
$=(7x-3y)(3x-7y)$

34. $36(m+n)^2-121(m-n)^2$
$=(6m+6n+11m-11n)(6m+6n-11m+11n)$
$=(17m-5n)(17n-5m)$

EJERCICIO 95

1. $a^2+2ab+b^2-x^2$
$=(a+b)^2-x^2$
$=(a+b+x)(a+b-x)$

2. $x^2-2xy+y^2-m^2$
$=(x-y)^2-m^2$
$=(x-y+m)(x-y-m)$

3. $m^2+2mn+n^2-1$
$=(m+n)^2-1$
$=(m+n+1)(m+n-1)$

4. $a^2-2a+1-b^2$
$=(a-1)^2-b^2$
$=(a-1+b)(a-1-b)$

5. $n^2+6n+9-c^2$
$=(n+3)^2-c^2$
$=(n+3+c)(n+3-c)$

6. $a^2+x^2+2ax-4$
$=(a+x)^2-4$
$=(a+x+2)(a+x-2)$

7. $a^2+4-4a-9b^2$
$=(a-2)^2-9b^2$
$=(a+3b-2)(a-3b-2)$

8. $x^2+4y^2-4xy-1$
$=(x-2y)^2-1$
$=(x-2y+1)(x-2y-1)$

9. $a^2-6ay+9y^2-4x^2$
$=(a-3y)^2-4x^2$
$=(a-3y+2x)(a-3y-2x)$

10. $4x^2+25y^2-36+20xy$
$=(2x+5y)^2-36$
$=(2x+5y+6)(2x+5y-6)$

11. $9x^2-1+16a^2-24ax$
$=(3x-4a)^2-1$
$=(3x-4a+1)(3x-4a-1)$

12. $1+64a^2b^2-x^4-16ab$
$=(8ab-1)^2-x^4$
$=(8ab-1+x^2)(8ab-1-x^2)$

13. $a^2-b^2-2bc-c^2$
$=a^2-\left(b^2+2bc+c^2\right)$
$=a^2-\left(b+c\right)^2$
$=\left(a+b+c\right)\left(a-b-c\right)$

14. $1-a^2+2ax-x^2$
$=1-\left(a^2-2ax+x^2\right)$
$=1-\left(a-x\right)^2$
$=\left(1+a-x\right)\left(1-a+x\right)$

15. $m^2-x^2-2xy-y^2$
$=m^2-\left(x^2+2xy+y^2\right)$
$=m^2-\left(x+y\right)^2$
$=\left(m+x+y\right)\left(m-x-y\right)$

16. c^2-a^2+2a-1
$=c^2-\left(a^2-2a+1\right)$
$=c^2-\left(a-1\right)^2$
$=\left(c+a-1\right)\left(c-a+1\right)$

17. $9-n^2-25-10n$
$=9-\left(n^2+10n+25\right)$
$=9-\left(n+5\right)^2$
$=\left(3+n+5\right)\left(3-n-5\right)$
$=-\left(n+8\right)\left(n+2\right)$

18. $4a^2-x^2+4x-4$
$=4a^2-\left(x^2-4x+4\right)$
$=4a^2-\left(x-2\right)^2$
$=\left(2a+x-2\right)\left(2a-x+2\right)$

19. $1-a^2-9n^2-6an$
$=1-\left(a^2+6an+9n^2\right)$
$=1-\left(a+3n\right)^2$
$=\left(a+3n+1\right)\left(1-a-3n\right)$

20. $25-x^2-16y^2+8xy$
$=25-\left(x^2-8xy+16y^2\right)$
$=25-\left(x-4y\right)^2$
$=\left(5+x-4y\right)\left(5-x+4y\right)$

21. $9x^2-a^2-4m^2+4am$
$=9x^2-\left(a^2-4am+4m^2\right)$
$=9x^2-\left(a-2m\right)^2$
$=\left(3x+a-2m\right)\left(3x-a+2m\right)$

22. $16x^2y^2+12ab-4a^2-9b^2$
$=16x^2y^2-\left(4a^2-12ab+9b^2\right)$
$=16x^2y^2-\left(2a-3b\right)^2$
$=\left(4xy+2a-3b\right)\left(4xy-2a+3b\right)$

23. $-a^2+25m^2-1-2a$
$=25m^2-\left(a^2+2a+1\right)$
$=25m^2-\left(a+1\right)^2$
$=\left(5m+a+1\right)\left(5m-a-1\right)$

24. $49x^4-25x^2-9y^2+30xy$
$=49x^4-\left(25x^2-30xy+9y^2\right)$
$=49x^4-\left(5x-3y\right)^2$
$=\left(7x^2+5x-3y\right)\left(7x^2-5x+3y\right)$

25. $a^2-2ab+b^2-c^2-2cd-d^2$
$=\left(a-b\right)^2-\left(c+d\right)^2$
$=\left(a-b+c+d\right)\left(a-b-c-d\right)$

26. $x^2+2xy+y^2-m^2+2mn-n^2$
$=\left(x+y\right)^2-\left(m-n\right)^2$
$=\left(x+y+m-n\right)\left(x+y-m+n\right)$

27. $a^2+4b^2+4ab-x^2-2ax-a^2$
$=\left(a^2+4ab+4b^2\right)-\left(x^2+2ax+a^2\right)$
$=\left(a+2b\right)^2-\left(x+a\right)^2$
$=\left(a+2b+x+a\right)\left(a+2b-x-a\right)$
$=\left(2a+2b+x\right)\left(2b-x\right)$

28. $x^2+4a^2-4ax-y^2-9b^2+6by$
$=\left(x^2-4ax+4a^2\right)-\left(y^2-6by+9b^2\right)$
$=\left(x-2a\right)^2-\left(y-3b\right)^2$
$=\left(x-2a+y-3b\right)\left(x-2a-y+3b\right)$

29. $m^2-x^2+9n^2+6mn-4ax-4a^2$
$=\left(m^2+6mn+9n^2\right)-\left(x^2+4ax+4a^2\right)$
$=\left(m+3n\right)^2-\left(x+2a\right)^2$
$=\left(m+3n+x+2a\right)\left(m+3n-x-2a\right)$

30. $9x^2+4y^2-a^2-12xy-25b^2-10ab$
$=\left(9x^2-12xy+4y^2\right)-\left(a^2+10ab+25b^2\right)$
$=\left(3x-2y\right)^2-\left(a+5b\right)^2$
$=\left(3x-2y+a+5b\right)\left(3x-2y-a-5b\right)$

31. $2am-x^2-9+a^2+m^2-6x$
$=\left(a^2+2am+m^2\right)-\left(x^2+6x+9\right)$
$=\left(a+m\right)^2-\left(x+3\right)^2$
$=\left(a+m+x+3\right)\left(a+m-x-3\right)$

32. $x^2-9a^4+6a^2b+1+2x-b^2$
$=\left(x^2+2x+1\right)-\left(b^2-6a^2b+9a^4\right)$
$=\left(x+1\right)^2-\left(b-3a^2\right)^2$
$=\left(x+1+b-3a^2\right)\left(x+1-b+3a^2\right)$

33. $16a^2-1-10m+9x^2-24ax-25m^2$
$=\left(16a^2-24ax+9x^2\right)-\left(25m^2+10m+1\right)$
$=\left(4a-3x\right)^2-\left(5m+1\right)^2$
$=\left(4a-3x+5m+1\right)\left(4a-3x-5m-1\right)$

34. $9m^2-a^2+2acd-c^2d^2+100-60m$
$=-\left(a^2-2acd+c^2d^2\right)+\left(9m^2-60m+100\right)$
$=\left(3m-10\right)^2-\left(a-cd\right)^2$
$=\left(3m-10+a-cd\right)\left(3m-10-a+cd\right)$

35. $4a^2-9x^2+49b^2-30xy-25y^2-28ab$
$=\left(4a^2-28ab+49b^2\right)-\left(9x^2+30xy+25y^2\right)$
$=\left(2a-7b\right)^2-\left(3x+5y\right)^2$
$=\left(2a-7b+3x+5y\right)\left(2a-7b-3x-5y\right)$

36. $225a^2-169b^2+1+30a+26bc-c^2$
$=\left(225a^2+30a+1\right)-\left(169b^2-26bc+c^2\right)$
$=\left(15a+1\right)^2-\left(13b-c\right)$
$=\left(15a+1+13b-c\right)\left(15a+1-13b+c\right)$

37. $x^2-y^2+4+4x-1-2y$
$=\left(x^2+4x+4\right)-\left(y^2+2y+1\right)$
$=\left(x+2\right)^2-\left(y+1\right)^2$
$=\left(x+2+y+1\right)\left(x+2-y-1\right)$
$=\left(x+y+3\right)\left(x-y+1\right)$

38. $a^2-16-x^2+36+12a-8x$
$=\left(a^2+12a+36\right)-\left(x^2+8x+16\right)$
$=\left(a+6\right)^2-\left(x+4\right)^2$
$=\left(a+6+x+4\right)\left(a+6-x-4\right)$
$=\left(a+x+10\right)\left(a-x+2\right)$

EJERCICIO 96

1. a^4+a^2+1

$\quad +a^2 \qquad -a^2$

$\left(a^4+2a^2+1\right)-a^2$

$=\left(a^2+1\right)^2-a^2$

$=\left(a^2+a+1\right)\left(a^2-a+1\right)$

2. $m^4+m^2n^2+n^4$

$\quad +m^2n^2 \qquad -m^2n^2$

$m^4+2m^2n^2+n^4-m^2n^2$

$=\left(m^2+n^2\right)^2-m^2n^2$

$=\left(m^2+n^2+mn\right)\left(m^2+n^2-mn\right)$

3. x^8+3x^4+4

$\quad +\ x^4 \qquad -x^4$

$x^8+4x^4+4-x^4$

$=\left(x^4+2\right)^2-x^4$

$=\left(x^4+x^2+2\right)\left(x^4-x^2+2\right)$

4. a^4+2a^2+9

$\quad +4a^2 \qquad -4a^2$

$a^4+6a^2+9-4a^2$

$=\left(a^2+3\right)^2-4a^2$

$=\left(a^2+2a+3\right)\left(a^2-2a+3\right)$

5. $a^4-3a^2b^2+b^4$

$\quad +\ a^2b^2 \qquad -\ a^2b^2$

$a^4-2a^2b^2+b^4-a^2b^2$

$=\left(a^2-b^2\right)^2-a^2b^2$

$=\left(a^2+ab-b^2\right)\left(a^2-ab-b^2\right)$

6. x^4-6x^2+1

$\quad +4x^2 \qquad -4x^2$

$x^4-2x^2+1-4x^2$

$=\left(x^2-1\right)^2-4x^2$

$=\left(x^2+2x-1\right)\left(x^2-2x-1\right)$

7. $4a^4+3a^2b^2+9b^4$

$\quad +9a^2b^2 \qquad -9a^2b^2$

$4a^4+12a^2b^2+9b^4-9a^2b^2$

$=\left(2a^2+3b^2\right)^2-9a^2b^2$

$=\left(2a^2+3ab+3b^2\right)\left(2a^2-3ab+3b^2\right)$

8. $4x^4-29x^2+25$

$\quad +\ 9x^2 \qquad -9x^2$

$4x^4-20x^2+25-9x^2$

$=\left(2x^2-5\right)^2-9x^2$

$=\left(2x^2+3x-5\right)\left(2x^2-3x-5\right)$

9. $x^8+4x^4y^4+16y^8$

$\quad +4x^4y^4 \qquad -4x^4y^4$

$x^8+8x^4y^4+16y^8-4x^4y^4$

$=\left(x^4+4y^4\right)^2-4x^4y^4$

$=\left(x^4+2x^2y^2+4y^4\right)\left(x^4-2x^2y^2+4y^4\right)$

10. $16m^4-25m^2n^2+9n^4$

$\quad +\ m^2n^2 \qquad -m^2n^2$

$16m^4-24m^2n^2+9n^4-m^2n^2$

$=\left(4m^2-3n^2\right)^2-m^2n^2$

$=\left(4m^2+mn-3n^2\right)\left(4m^2-mn-3n^2\right)$

11. $25a^4+54a^2b^2+49b^4$

$\quad +16a^2b^2 \qquad -16a^2b^2$

$25a^4+70a^2b^2+49b^4-16a^2b^2$

$=\left(5a^2+7b^2\right)^2-16a^2b^2$

$=\left(5a^2+4ab+7b^2\right)\left(5a^2-4ab+7b^2\right)$

12. $36x^4-109x^2y^2+49y^4$

$\quad +\ 25x^2y^2 \qquad -25x^2y^2$

$36x^4-\ 84x^2y^2+49y^4-\ 25x^2y^2$

$=\left(6x^2-7y^2\right)^2-25x^2y^2$

$=\left(6x^2+5xy-7y^2\right)\left(6x^2-5xy-7y^2\right)$

13. $81m^8+2m^4+1$

$\quad +16m^4 \qquad -16m^4$

$81m^8+18m^4+1-16m^4$

$=\left(9m^4+1\right)^2-16m^4$

$=\left(9m^4+4m^2+1\right)\left(9m^4-4m^2+1\right)$

14. c^4-45c^2+100

$\quad +25c^2 \qquad -25c^2$

$c^4-20c^2+100-25c^2$

$=\left(c^2-10\right)^2-25c^2$

$=\left(c^2+5c-10\right)\left(c^2-5c-10\right)$

15. $4a^8-53a^4b^4+49b^8$

$\quad +25a^4b^4 \qquad -25a^4b^4$

$4a^8-28a^4b^4+49b^8-25a^4b^4$

$=\left(2a^4-7b^4\right)^2-25a^4b^4$

$=\left(2a^4+5a^2b^2-7b^4\right)\left(2a^4-5a^2b^2-7b^4\right)$

16. $49+76n^2+64n^4$

$\quad +36n^2 \qquad -36n^2$

$49+112n^2+64n^4-36n^2$

$=\left(7+8n^2\right)^2-36n^2$

$=\left(8n^2+6n+7\right)\left(8n^2-6n+7\right)$

17. $25x^4-139x^2y^2+81y^4$

$\quad +\ 49x^2y^2 \qquad -49x^2y^2$

$25x^4-\ 90x^2y^2+81y^4-49x^2y^2$

$=\left(5x^2-9y^2\right)^2-49x^2y^2$

$=\left(5x^2+7xy-9y^2\right)\left(5x^2-7xy-9y^2\right)$

18. $49x^8+76x^4y^4+100y^8$

$\quad +64x^4y^4 \qquad -64x^4y^4$

$49x^8+140x^4y^4+100y^8-64x^4y^4$

$=\left(7x^4+10y^4\right)^2-64x^4y^4$

$=\left(7x^4+8x^2y^2+10y^4\right)\left(7x^4-8x^2y^2+10y^4\right)$

19. $4-108x^2+121x^4$

$\quad +\ 64x^2 \qquad -64x^2$

$4-44x^2+121x^4\ -64x^2$

$=\left(2-11x^2\right)^2-64x^2$

$=\left(2+8x-11x^2\right)\left(2-8x-11x^2\right)$

20. $121x^4-133x^2y^4+36y^8$

$\quad +\ x^2y^4 \qquad -x^2y^4$

$121x^4-132x^2y^4+36y^8-x^2y^4$

$=\left(11x^2-6y^4\right)^2-x^2y^4$

$=\left(11x^2+xy^2-6y^4\right)\left(11x^2-xy^2-6y^4\right)$

21. $144+23n^6+9n^{12}$

$\quad +49n^6 \qquad -49n^6$

$144+72n^6+9n^{12}-49n^6$

$=\left(12+3n^6\right)^2-49n^6$

$=\left(12+7n^3+3n^6\right)\left(12-7n^3+3n^6\right)$

22. $16-9c^4+c^8$

$\quad + \ c^4 \qquad -c^4$

$16-8c^4+c^8-c^4$

$=\left(4-c^4\right)^2-c^4$

$=\left(4+c^2-c^4\right)\left(4-c^2-c^4\right)$

23. $64a^4-169a^2b^4+81b^8$

$\quad + \ 25a^2b^4 \qquad -25a^2b^4$

$64a^4-144a^2b^4+81b^8-25a^2b^4$

$=\left(8a^2-9b^4\right)^2-25a^2b^4$

$=\left(8a^2+5ab-9b^4\right)\left(8a^2-5ab-9b^4\right)$

24. $225+ \ 5m^2 \ +m^4$

$\quad +25m^2 \qquad -25m^2$

$225+30m^2+m^4-25m^2$

$=\left(15+m^2\right)^2-25m^2$

$=\left(m^2+5m+15\right)\left(m^2-5m+15\right)$

25. $1-126a^2b^4+169a^4b^8$

$\quad +100a^2b^4 \qquad -100a^2b^4$

$1-26a^2b^4 \ +169a^4b^8-100a^2b^4$

$=\left(1-13a^2b^4\right)^2-100a^2b^4$

$=\left(1+10ab^2-13a^2b^4\right)\left(1-10ab^2-13a^2b^4\right)$

26. $x^4y^4+21x^2y^2+121$

$\quad + \ x^2y^2 \qquad -x^2y^2$

$x^4y^4+22x^2y^2+121-x^2y^2$

$=\left(x^2y^2+11\right)^2-x^2y^2$

$=\left(x^2y^2+xy+11\right)\left(x^2y^2-xy+11\right)$

27. $49c^8+ \ 75c^4m^2n^2+196m^4n^4$

$\quad + \ 121c^4m^2n^2 \qquad -121c^4m^2n^2$

$49c^8+196c^4m^2n^2+196m^4n^4-121c^4m^2n^2$

$=\left(7c^4+14m^2n^2\right)^2-121c^4m^2n^2$

$=\left(7c^4+11c^2mn+14m^2n^2\right)\left(7c^4-11c^2mn+14m^2n^2\right)$

28. $81a^4b^8-292a^2b^4x^8+256x^{16}$

$\quad + \ 4a^2b^4x^8 \qquad -4a^2b^4x^8$

$81a^4b^8-288a^2b^4x^8+256x^{16}-4a^2b^4x^8$

$=\left(9a^2b^4-16x^8\right)^2-4a^2b^4x^8$

$=\left(9a^2b^4+2ab^2x^4-16x^8\right)\left(9a^2b^4-2ab^2x^4-16x^8\right)$

EJERCICIO 97

1. $x^4 \qquad +64y^4$

$\quad +16x^2y^2 \qquad -16x^2y^2$

$x^4+16x^2y^2+64y^4-16x^2y^2$

$=\left(x^2+8y^2\right)^2-16x^2y^2$

$=\left(x^2+4xy+8y^2\right)\left(x^2-4xy+8y^2\right)$

2. $4x^8 \qquad +y^8$

$\quad +4x^4y^4 \qquad -4x^4y^4$

$4x^8+4x^4y^4+y^8- \ 4x^4y^4$

$=\left(2x^4+y^4\right)^2- \ 4x^4y^4$

$=\left(2x^4+2x^2y^2+y^4\right)\left(2x^4-2x^2y^2+y^4\right)$

3. $a^4 \qquad +324b^4$

$\quad +36a^2b^2 \qquad -36a^2b^2$

$a^4+ \ 36a^2b^2+324b^4-36a^2b^2$

$=\left(a^2+18b^2\right)^2-36a^2b^2$

$=\left(a^2+6ab+18b^2\right)\left(a^2-6ab+18b^2\right)$

4. $4m^4 \qquad +81n^4$

$\quad +36m^2n^2 \qquad -36m^2n^2$

$4m^4+36m^2n^2+81n^4-36m^2n^2$

$=\left(2m^2+9n^2\right)^2-36m^2n^2$

$=\left(2m^2+6mn+9n^2\right)\left(2m^2-6mn+9n^2\right)$

5. $4 \qquad +625x^8$

$\quad +100x^4 \qquad -100x^4$

$4+100x^4+625x^8-100x^4$

$=\left(2+25x^4\right)^2-100x^4$

$=\left(25x^4+10x^2+2\right)\left(25x^4-10x^2+2\right)$

6. $64 \qquad +a^{12}$

$\quad +16a^6 \qquad -16a^6$

$64+16a^6+a^{12}-16a^6$

$=\left(8+a^6\right)^2-16a^6$

$=\left(a^6+4a^3+8\right)\left(a^6-4a^3+8\right)$

7.
$$\begin{array}{l} 1 \qquad\quad +4n^4 \\ \underline{+\ 4n^2 \qquad -\ 4n^2} \\ 1+4n^2+4n^4-4n^2 \\ =\left(1+2n^2\right)^2-4n^2 \\ =\left(2n^2+2n+1\right)\left(2n^2-2n+1\right) \end{array}$$

8.
$$\begin{array}{l} 64x^8 \qquad\qquad +y^8 \\ \underline{+16x^4y^4 \qquad -16x^4y^4} \\ 64x^8+16x^4y^4+y^8-16x^4y^4 \\ =\left(8x^4+y^4\right)^2-16x^4y^4 \\ =\left(8x^4+4x^2y^2+y^4\right)\left(8x^4-4x^2y^2+y^4\right) \end{array}$$

9.
$$\begin{array}{l} 81a^4 \qquad\qquad +64b^4 \\ \underline{+144a^2b^2 \qquad -144a^2b^2} \\ 81a^4+144a^2b^2+64b^4-144a^2b^2 \\ =\left(9a^2+8b^2\right)^2-144a^2b^2 \\ =\left(9a^2+12ab+8b^2\right)\left(9a^2-12ab+8b^2\right) \end{array}$$

EJERCICIO 98

1. $x^2+7x+10=(x+5)(x+2)$

2. $x^2-5x+6=(x-3)(x-2)$

3. $x^2+3x-10=(x+5)(x-2)$

4. $x^2+x-2=(x+2)(x-1)$

5. $a^2+4a+3=(a+3)(a+1)$

6. $m^2+5m-14=(m+7)(m-2)$

7. $y^2-9y+20=(y-5)(y-4)$

8. $x^2-x-6=(x-3)(x+2)$

9. $x^2-9x+8=(x-8)(x-1)$

10. $c^2+5c-24=(c+8)(c-3)$

11. $x^2-3x+2=(x-2)(x-1)$

12. $a^2+7a+6=(a+6)(a+1)$

13. $y^2-4y+3=(y-3)(y-1)$

14. $n^2-8n+12=(n-6)(n-2)$

15. $x^2+10x+21=(x+7)(x+3)$

16. $a^2+7a-18=(a+9)(a-2)$

17. $m^2-12m+11=(m-11)(m-1)$

18. $x^2-7x-30=(x-10)(x+3)$

19. $n^2+6n-16=(n+8)(n-2)$

20. $a^2-21a+20=(a-20)(a-1)$

21. $y^2+y-30=(y+6)(y-5)$

22. $a^2-11a+28=(a-7)(a-4)$

23. $n^2-6n-40=(n-10)(n+4)$

24. $x^2-5x-36=(x-9)(x+4)$

25. $a^2-2a-35=(a-7)(a+5)$

26. $x^2+14x+13=(x+13)(x+1)$

27. $a^2-14a+33=(a-11)(a-3)$

28. $m^2+13m-30=(m+15)(m-2)$

29. $c^2-13c-14=(c-14)(c+1)$

30. $x^2+15x+56=(x+8)(x+7)$

31. $x^2-15x+54=(x-9)(x-6)$

32. $a^2+7a-60=(a+12)(a-5)$

33. $x^2-17x-60$
$$\begin{array}{r|ll} 60 & 2 & \\ 30 & 2 & 2\cdot2\cdot5=20 \\ 15 & 3 & 3\cdot1=3 \\ 5 & 5 & \Rightarrow 20-3=17 \\ 1 & & =(x-20)(x+3) \end{array}$$

34. $x^2+8x-180$
$$\begin{array}{r|ll} 180 & 2 & \\ 90 & 2 & 2\cdot3\cdot3=18 \\ 45 & 3 & 2\cdot5=10 \\ 15 & 3 & \Rightarrow 18-10=8 \\ 5 & 5 & =(x+18)(x-10) \\ 1 & & \end{array}$$

35. $m^2-20m-300$
$$\begin{array}{r|ll} 300 & 2 & \\ 150 & 2 & 2\cdot3\cdot5=30 \\ 75 & 3 & 2\cdot5=10 \\ 25 & 5 & \Rightarrow 30-10=20 \\ 5 & 5 & =(m-30)(m+10) \\ 1 & & \end{array}$$

36. $x^2+x-132$
$$\begin{array}{r|ll} 132 & 2 & \\ 66 & 2 & 2\cdot2\cdot3=12 \\ 33 & 3 & 11\cdot1=11 \\ 11 & 11 & \Rightarrow 12-11=1 \\ 1 & & =(x+12)(x-11) \end{array}$$

37. $m^2-2m-168$
$$\begin{array}{r|ll} 168 & 2 & \\ 84 & 2 & 2\cdot2\cdot3=12 \\ 42 & 2 & 2\cdot7=14 \\ 21 & 3 & \Rightarrow 14-12=2 \\ 7 & 7 & =(m-14)(m+12) \\ 1 & & \end{array}$$

38. $c^2+24c+135$
$$\begin{array}{r|ll} 135 & 3 & \\ 45 & 3 & 3\cdot3=9 \\ 15 & 3 & 3\cdot5=15 \\ 5 & 5 & \Rightarrow 15+9=24 \\ 1 & & =(c+15)(c+9) \end{array}$$

39. $m^2-41m+400$
$$\begin{array}{r|ll} 400 & 2 & \\ 200 & 2 & 2^4=16 \\ 100 & 2 & 5^2=25 \\ 50 & 2 & \Rightarrow 25+16=41 \\ 25 & 5 & =(m-25)(m-16) \\ 5 & 5 & \\ 1 & & \end{array}$$

40. $a^2+a-380$
$$\begin{array}{r|ll} 380 & 2 & \\ 190 & 2 & 2^2\cdot5=20 \\ 95 & 5 & 19\cdot1=19 \\ 19 & 19 & \Rightarrow 20-19=1 \\ 1 & & =(a+20)(a-19) \end{array}$$

41. $x^2+12x-364$
$$\begin{array}{r|ll} 364 & 2 & \\ 182 & 2 & 2\cdot13=26 \\ 91 & 7 & 2\cdot7=14 \\ 13 & 13 & \Rightarrow 26-14=12 \\ 1 & & =(x+26)(x-14) \end{array}$$

42. $a^2+42a+432$

$$\begin{array}{r|ll} 432 & 2 & \\ 216 & 2 & 2^3\cdot 3=24 \\ 108 & 2 & 2\cdot 3^2=18 \\ 54 & 2 & \Rightarrow 24+18=42 \\ 27 & 3 & =(a+24)(a+18) \\ 9 & 3 & \\ 3 & 3 & \\ 1 & & \end{array}$$

43. $m^2-30m-675$

$$\begin{array}{r|ll} 675 & 3 & \\ 225 & 5 & 3\cdot 5\cdot 3=45 \\ 45 & 3 & 3\cdot 5=15 \\ 15 & 3 & \Rightarrow 45-15=30 \\ 5 & 5 & =(m-45)(m+15) \\ 1 & & \end{array}$$

44. $y^2+50y+336$

$$\begin{array}{r|ll} 336 & 3 & \\ 112 & 2 & 3\cdot 7\cdot 2=42 \\ 56 & 2 & 2^3=8 \\ 28 & 2 & \Rightarrow 42+8=50 \\ 14 & 2 & =(y+42)(y+8) \\ 7 & 7 & \\ 1 & & \end{array}$$

45. $x^2-2x-528$

$$\begin{array}{r|ll} 528 & 2 & \\ 264 & 2 & 2^3\cdot 3=24 \\ 132 & 2 & 2\cdot 11=22 \\ 66 & 2 & \Rightarrow 24-22=2 \\ 33 & 3 & =(x-24)(x+22) \\ 11 & 11 & \\ 1 & & \end{array}$$

46. $n^2+43n+432$

$$\begin{array}{r|ll} 432 & 2 & \\ 216 & 2 & 2^4=16 \\ 108 & 2 & 3^3=27 \\ 54 & 2 & \Rightarrow 16+27=43 \\ 27 & 3 & =(n+16)(n+27) \\ 9 & 3 & \\ 3 & 3 & \\ 1 & & \end{array}$$

47. $c^2-4c-320$

$$\begin{array}{r|ll} 320 & 2 & \\ 160 & 2 & 2^4=16 \\ 80 & 2 & 2^2\cdot 5=20 \\ 40 & 2 & \Rightarrow 20-16=4 \\ 20 & 2 & =(c-20)(c+16) \\ 10 & 2 & \\ 5 & 5 & \\ 1 & & \end{array}$$

48. $m^2-8m-1.008$

$$\begin{array}{r|ll} 1.008 & 2 & \\ 504 & 2 & 2^2\cdot 7=28 \\ 252 & 2 & 2^2\cdot 3^2=36 \\ 126 & 2 & \Rightarrow 36-28=8 \\ 63 & 3 & =(m-36)(m+28) \\ 21 & 3 & \\ 7 & 7 & \\ 1 & & \end{array}$$

EJERCICIO 99

1. $x^4+5x^2+4=(x^2+4)(x^2+1)$

2. $x^6-6x^3-7=(x^3-7)(x^3+1)$

3. $x^8-2x^4-80=(x^4-10)(x^4+8)$

4. $x^2y^2+xy-12=(xy+4)(xy-3)$

5. $(4x)^2-2(4x)-15=(4x-5)(4x+3)$

6. $(5x)^2+13(5x)+42=(5x+7)(5x+6)$

7. $x^2+2ax-15a^2=(x+5a)(x-3a)$

8. $a^2-4ab-21b^2=(a-7b)(a+3b)$

9. $(x-y)^2+2(x-y)-24=(x-y+6)(x-y-4)$

10. $-x^2+4x+5$

$=-(x^2-4x-5)=-(x-5)(x+1)=(5-x)(x+1)$

11. $x^{10}+x^5-20=(x^5+5)(x^5-4)$

12. $m^2+mn-56n^2=(m+8n)(m-7n)$

13. $x^4+7ax^2-60a^2$

$=(x^2+12a)(x^2-5a)$

14. $(2x)^2-4(2x)+3$

$=(2x-3)(2x-1)$

15. $(m-n)^2+5(m-n)-24$

$=(m-n+8)(m-n-3)$

16. x^8+x^4-240

$$\begin{array}{r|ll} 240 & 2 & \\ 120 & 2 & 2^4=16 \\ 60 & 2 & 3\cdot 5=15 \\ 30 & 2 & \Rightarrow 16-15=1 \\ 15 & 3 & =(x^4+16)(x^4-15) \\ 5 & 5 & \\ 1 & & \end{array}$$

17. $-y^2+2y+15$

$=-(y^2-2y-15)$

$=-(y-5)(y+3)=(5-y)(y+3)$

18. $a^4b^4-2a^2b^2-99$

$$\begin{array}{r|ll} 99 & 3 & 3\cdot 3=9 \\ 33 & 3 & 11\cdot 1=11 \\ 11 & 11 & \Rightarrow 11-9=2 \\ 1 & & =(a^2b^2-11)(a^2b^2+9) \end{array}$$

19. $c^2+11cd+28d^2$

$=(c+7d)(c+4d)$

20. $25x^2-5(5x)-84$

$$\begin{array}{r|ll} 84 & 2 & \\ 42 & 2 & 2^2\cdot 3=12 \\ 21 & 3 & 7\cdot 1=7 \\ 7 & 7 & \Rightarrow 12-7=5 \\ 1 & & =(5x-12)(5x+7) \end{array}$$

21. $a^2-21ab+98b^2$

$$\begin{array}{r|ll} 98 & 2 & 2\cdot 7=14 \\ 49 & 7 & 7\cdot 1=7 \\ 7 & 7 & \Rightarrow 14+7=21 \\ 1 & & =(a-14b)(a-7b) \end{array}$$

22. $x^4y^4+x^2y^2-132$

$$\begin{array}{r|lll} 132 & 2 & & \\ 66 & 2 & & 2\cdot2\cdot3=12 \\ 33 & 3 & & 11\cdot1=11 \\ 11 & 11 & \Rightarrow 12-11=1 & \\ 1 & & & =\left(x^2y^2+12\right)\left(x^2y^2-11\right) \end{array}$$

23. $-x^4+2x^2+48$

$=-\left(x^4-2x^2-48\right)$

$=-\left(x^2-8\right)\left(x^2+6\right)=\left(8-x^2\right)\left(x^2+6\right)$

24. $(c+d)^2-18(c+d)+65$

$=(c+d-13)(c+d-5)$

25. $a^2+2axy-440x^2y^2$

$$\begin{array}{r|lll} 440 & 2 & & \\ 220 & 2 & & 2^2\cdot5=20 \\ 110 & 2 & & 11\cdot2=22 \\ 55 & 5 & \Rightarrow 22-20=2 & \\ 11 & 11 & & =(a+22xy)(a-20xy) \\ 1 & & & \end{array}$$

26. $m^6n^6-21m^3n^3+104$

$$\begin{array}{r|lll} 104 & 2 & & \\ 52 & 2 & & 2^3=8 \\ 26 & 2 & & 13\cdot1=13 \\ 13 & 13 & \Rightarrow 13+8=21 & \\ 1 & & & =\left(m^3n^3-13\right)\left(m^3n^3-8\right) \end{array}$$

27. $-n^2+5n+14$

$=-\left(n^2-5n-14\right)$

$=-(n-7)(n+2)=(7-n)(n+2)$

28. x^6+x^3-930

$$\begin{array}{r|lll} 930 & 2 & & \\ 465 & 5 & & 2\cdot5\cdot3=30 \\ 93 & 3 & & 31\cdot1=31 \\ 31 & 31 & \Rightarrow 31-30=1 & \\ 1 & & & =\left(x^3+31\right)\left(x^3-30\right) \end{array}$$

29. $\left(4x^2\right)^2-8\left(4x^2\right)-105$

$$\begin{array}{r|lll} 105 & 5 & & \\ 21 & 3 & & 5\cdot3=15 \\ 7 & 7 & & 7\cdot1=7 \\ 1 & & \Rightarrow 15-7=8 & \end{array}$$

$=\left(4x^2-15\right)\left(4x^2+7\right)$

30. $x^4+5abx^2-36a^2b^2$

$=\left(x^2+9ab\right)\left(x^2-4ab\right)$

31. $a^4-a^2b^2-156b^4$

$$\begin{array}{r|lll} 156 & 2 & & \\ 78 & 2 & & 2^2\cdot3=12 \\ 39 & 3 & & 13\cdot1=13 \\ 13 & 13 & \Rightarrow 13-12=1 & \\ 1 & & & =\left(a^2-13b^2\right)\left(a^2+12b^2\right) \end{array}$$

32. $-x^2+4ax+21a^2$

$=-\left(x^2-4ax-21a^2\right)$

$=-(x-7a)(x+3a)$

$=(7a-x)(x+3a)$

33. $x^8y^8-15x^4y^4-100a^2$

$=\left(x^4y^4-20a\right)\left(x^4y^4+5a\right)$

34. $(a-1)^2+3(a-1)-108$

$$\begin{array}{r|lll} 108 & 2 & & \\ 54 & 2 & & 2^2\cdot3=12 \\ 27 & 3 & & 3^2=9 \\ 9 & 3 & \Rightarrow 12-9=3 & \\ 3 & 3 & =(a-1+12)(a-1-9) & \\ 1 & & =(a+11)(a-10) & \end{array}$$

35. $m^2+abcm-56a^2b^2c^2$

$=(m+8abc)(m-7abc)$

36. $\left(7x^2\right)^2+24\left(7x^2\right)+128$

$$\begin{array}{r|lll} 128 & 4 & & \\ 32 & 4 & & 4^2=16 \\ 8 & 4 & & 4\cdot2=8 \\ 2 & 2 & \Rightarrow 16+8=24 & \\ 1 & & & =\left(7x^2+16\right)\left(7x^2+8\right) \end{array}$$

EJERCICIO 100

1. $2x^2+3x-2$

$=4x^2+3(2x)-4$

$=\dfrac{(2x+4)(2x-1)}{2}$

$=(x+2)(2x-1)$

2. $3x^2-5x-2$

$=9x^2-5(3x)-6$

$=\dfrac{(3x-6)(3x+1)}{3}$

$=(x-2)(3x+1)$

3. $6x^2+7x+2$

$=36x^2+7(6x)+12$

$=\dfrac{(6x+4)(6x+3)}{2\cdot3}$

$=(3x+2)(2x+1)$

4. $5x^2+13x-6$

$=25x^2+13(5x)-30$

$=\dfrac{(5x+15)(5x-2)}{5}$

$=(x+3)(5x-2)$

5. $6x^2-5x-6$

$=36x^2-5(6x)-36$

$=\dfrac{(6x-9)(6x+4)}{3\cdot2}$

$=(2x-3)(3x+2)$

6. $12x^2-x-6$

$=144x^2-1(12x)-72$

$=\dfrac{(12x-9)(12x+8)}{3\cdot4}$

$=(4x-3)(3x+2)$

7. $4a^2+15a+9$

$=16a^2+15(4a)+36$

$=\dfrac{(4a+12)(4a+3)}{4}$

$=(a+3)(4a+3)$

8. $10a^2+11a+3$

$=100a^2+11(10a)+30$

$=\dfrac{(10a+6)(10a+5)}{2\cdot5}$

$=(5a+3)(2a+1)$

9. $12m^2-13m-35$

$=144m^2-13(12m)-420$

$$\begin{array}{r|lll} 420 & 4 & & 4\cdot7=28 \\ 105 & 5 & & 5\cdot3=15 \\ 21 & 3 & \Rightarrow 28-15=13 & \\ 7 & 7 & =\dfrac{(12m-28)(12m+15)}{4\cdot3} & \\ 1 & & =(3m-7)(4m+5) & \end{array}$$

10. $20y^2+y-1$

$=400y^2+1(20y)-20$

$=\dfrac{(20y+5)(20y-4)}{5\cdot4}$

$=(4y+1)(5y-1)$

11. $8a^2-14a-15$

$=64a^2-14(8a)-120$

$$\begin{array}{r|lll} 120 & 2 & & \\ 60 & 2 & & 2\cdot2\cdot5=20 \\ 30 & 2 & & 2\cdot3=6 \\ 15 & 3 & \Rightarrow 20-6=14 & \\ 5 & 5 & =\dfrac{(8a-20)(8a+6)}{4\cdot2} & \\ 1 & & =(2a-5)(4a+3) & \end{array}$$

12. $7x^2-44x-35$

$=49x^2-44(7x)-245$

$$\begin{array}{r|ll} 245 & 5 & \\ 49 & 7 & 7\cdot 7=49 \\ 7 & 7 & 5\cdot 1=5 \\ 1 & & \Rightarrow 49-5=44 \end{array}$$

$$=\frac{(7x-49)(7x+5)}{7}$$

$=(x-7)(7x+5)$

13. $15m^2+16m-15$

$=225m^2+16(15m)-225$

$$\begin{array}{r|ll} 225 & 5 & \\ 45 & 5 & 5\cdot 5=25 \\ 9 & 3 & 3\cdot 3=9 \\ 3 & 3 & \Rightarrow 25-9=16 \\ 1 & & \end{array}$$

$$=\frac{(15m+25)(15m-9)}{5\cdot 3}$$

$=(3m+5)(5m-3)$

14. $2a^2+5a+2$

$=4a^2+5(2a)+4$

$$=\frac{(2a+4)(2a+1)}{2}$$

$=(a+2)(2a+1)$

15. $12x^2-7x-12$

$=144x^2-7(12x)-144$

$$\begin{array}{r|ll} 144 & 2 & \\ 72 & 2 & 2^4=16 \\ 36 & 2 & 3^2=9 \\ 18 & 2 & \Rightarrow 16-9=7 \\ 9 & 3 & \\ 3 & 3 & \\ 1 & & \end{array}$$

$$=\frac{(12x-16)(12x+9)}{4\cdot 3}$$

$=(3x-4)(4x+3)$

16. $9a^2+10a+1$

$=81a^2+10(9a)+9$

$$=\frac{(9a+9)(9a+1)}{9}$$

$=(a+1)(9a+1)$

17. $20n^2-9n-20$

$=400n^2-9(20)-400$

$$\begin{array}{r|ll} 400 & 2 & \\ 200 & 2 & 2^4=16 \\ 100 & 2 & 5^2=25 \\ 50 & 2 & \Rightarrow 25-16=9 \\ 25 & 5 & \\ 5 & 5 & \\ 1 & & \end{array}$$

$$=\frac{(20n-25)(20n+16)}{5\cdot 4}$$

$=(4n-5)(5n+4)$

18. $21x^2+11x-2$

$=441x^2+11(21x)-42$

$$=\frac{(21x+14)(21x-3)}{7\cdot 3}$$

$=(3x+2)(7x-1)$

19. $15m^2+m-6$

$=225m^2+15m-90$

$$=\frac{(15m+10)(15m-9)}{5\cdot 3}$$

$=(3m+2)(5m-3)$

20. $15a^2-8a-12$

$=225a^2-8(15a)-180$

$$\begin{array}{r|ll} 180 & 2 & \\ 90 & 2 & 2\cdot 3^2=18 \\ 45 & 5 & 5\cdot 2=10 \\ 9 & 3 & \Rightarrow 18-10=8 \\ 3 & 3 & \\ 1 & & \end{array}$$

$$=\frac{(15a-18)(15a+10)}{3\cdot 5}$$

$=(5a-6)(3a+2)$

21. $9x^2+37x+4$

$=81x^2+37(9x)+36$

$$=\frac{(9x+36)(9x+1)}{9}$$

$=(x+4)(9x+1)$

22. $20n^2+44n-15$

$=400n^2+44(20n)-300$

$$\begin{array}{r|ll} 300 & 2 & \\ 150 & 2 & 2\cdot 5^2=50 \\ 75 & 5 & 2\cdot 3=6 \\ 15 & 5 & \Rightarrow 50-6=44 \\ 3 & 3 & \\ 1 & & \end{array}$$

$$=\frac{(20n+50)(20n-6)}{10\cdot 2}$$

$=(2n+5)(10n-3)$

23. $14m^2-31m-10$

$=196m^2-31(14m)-140$

$$\begin{array}{r|ll} 140 & 2 & \\ 70 & 2 & 5\cdot 7=35 \\ 35 & 5 & 2\cdot 2=4 \\ 7 & 7 & \Rightarrow 35-4=31 \\ 1 & & \end{array}$$

$$=\frac{(14m-35)(14m+4)}{7\cdot 2}$$

$=(2m-5)(7m+2)$

24. $2x^2+29x+90$

$=4x^2+29(2x)+180$

$$\begin{array}{r|ll} 180 & 2 & \\ 90 & 2 & 2^2\cdot 5=20 \\ 45 & 3 & 3^2=9 \\ 15 & 3 & \Rightarrow 20+9=29 \\ 5 & 5 & \\ 1 & & \end{array}$$

$$=\frac{(2x+20)(2x+9)}{2}$$

$=(2x+10)(2x+9)$

25. $20a^2-7a-40$

$=400a^2-7(20a)-800$

$$\begin{array}{r|ll} 800 & 4 & \\ 200 & 4 & 4^2\cdot 2=32 \\ 50 & 5 & 5^2=25 \\ 10 & 5 & \Rightarrow 32-25=7 \\ 2 & 2 & \\ 1 & & \end{array}$$

$$=\frac{(20a-32)(20a+25)}{4\cdot 5}$$

$=(5a-8)(4a+5)$

26. $4n^2+n-33$

$=16n^2+4n-132$

$$\begin{array}{r|ll} 132 & 2 & \\ 66 & 2 & 2^2\cdot 3=12 \\ 33 & 3 & 11\cdot 1=11 \\ 11 & 11 & \Rightarrow 12-11=1 \\ 1 & & \end{array}$$

$$=\frac{(4n+12)(4n-11)}{4}$$

$=(n+3)(4n-11)$

27. $30x^2+13x-10$

$=900x^2+13(30x)-300$

$$\begin{array}{r|ll} 300 & 2 & \\ 150 & 2 & 5^2=25 \\ 75 & 5 & 2^2\cdot 3=12 \\ 15 & 5 & \Rightarrow 25-12=13 \\ 3 & 3 & \\ 1 & & \end{array}$$

$$=\frac{(30x+25)(30x-12)}{5\cdot 6}$$

$=(6x+5)(5x-2)$

EJERCICIO 101

1. $6x^4+5x^2-6$

$=\left(6x^2\right)^2+5\left(6x^2\right)-36$

$=\dfrac{\left(6x^2+9\right)\left(6x^2-4\right)}{3\cdot 2}$

$=\left(2x^2+3\right)\left(3x^2-2\right)$

2. $5x^6+4x^3-12$

$=\left(5x^3\right)^2+4\left(5x^3\right)-60$

$=\dfrac{\left(5x^3+10\right)\left(5x^3-6\right)}{5}$

$=\left(x^3+2\right)\left(5x^3-6\right)$

3. $10x^8+29x^4+10$

$=\left(10x^4\right)^2+29\left(10x^4\right)+100$

$=\dfrac{\left(10x^4+25\right)\left(10x^4+4\right)}{5\cdot 2}$

$=\left(2x^4+5\right)\left(5x^4+2\right)$

4. $6a^2x^2+5ax-21$

$(6ax)^2+5(6ax)-126$

126	2	
63	3	$2\cdot 7=14$
21	7	$3\cdot 3=9$
3	3	$\Rightarrow 14-9=5$
1		$=\dfrac{\left(6ax+14\right)\left(6ax-9\right)}{2\cdot 3}$

$=\left(3ax+7\right)\left(2ax-3\right)$

5. $20x^2y^2+9xy-20$

$=\left(20xy\right)^2+9\left(20xy\right)-400$

400	2	
200	2	$5^2=25$
100	2	$2^4=16$
50	2	$\Rightarrow 25-16=9$
25	5	$=\dfrac{\left(20xy+25\right)\left(20xy-16\right)}{5\cdot 4}$
5	5	$=\left(4xy+5\right)\left(5xy-4\right)$
1		

6. $15x^2-ax-2a^2$

$=(15x)^2-a(15x)-30a^2$

$=\dfrac{\left(15x-6a\right)\left(15x+5a\right)}{3\cdot 5}$

$=\left(5x-2a\right)\left(3x+a\right)$

7. $-10x^2-7x+12$

$=-\left(10x^2+7x-12\right)$

$=-\left[(10x)^2+7(10x)-120\right]$

120	2	
60	2	$5\cdot 3=15$
30	2	$2^3=8$
15	5	$\Rightarrow 15-8=7$
3	3	$=\dfrac{-\left[\left(10x+15\right)\left(10x-8\right)\right]}{5\cdot 2}$
1		$=-\left(2x+3\right)\left(5x-4\right)$

$=\left(2x+3\right)\left(4-5x\right)$

8. $21x^2-29xy-72y^2$

$=441x^2-29\left(21xy\right)-1.512y^2$

1.512	2	
756	2	$2^3\cdot 7=56$
378	2	$3^3=27$
189	3	$\Rightarrow 56-27=29$
63	3	$=\dfrac{\left(21x-56y\right)\left(21x+27y\right)}{7\cdot 3}$
21	3	$=\left(3x-8y\right)\left(7x+9y\right)$
7	7	
1		

9. $6m^2-13am-15a^2$

$=36m^2-13(6am)-90a^2$

$=\dfrac{\left(6m-18a\right)\left(6m+5a\right)}{6}$

$=\left(m-3a\right)\left(6m+5a\right)$

10. $14x^4-45x^2-14$

$=\left(14x^2\right)^2-45\left(14x^2\right)-196$

196	2	
98	2	$7\cdot 7=49$
49	7	$2\cdot 2=4$
7	7	$\Rightarrow 49-4=45$
1		$=\dfrac{\left(14x^2-49\right)\left(14x^2+4\right)}{7\cdot 2}$

$=\left(2x^2-7\right)\left(7x^2+2\right)$

11. $30a^2-13ab-3b^2$

$=900a^2-13(30ab)-90b^2$

$=\dfrac{\left(30a-18b\right)\left(30a+5b\right)}{6\cdot 5}$

$=\left(5a-3b\right)\left(6a+b\right)$

12. $7x^6-33x^3-10$

$=\left(7x^3\right)^2-33\left(7x^3\right)-70$

$=\dfrac{\left(7x^3-35\right)\left(7x^3+2\right)}{7}$

$=\left(x^3-5\right)\left(7x^3+2\right)$

13. $-3a^2+13a+30$

$=-\left(3a^2-13a-30\right)$

$=-\left(9a^2-13(3a)-90\right)$

$=-\dfrac{\left(3a-18\right)\left(3a+5\right)}{3}$

$=-\left(a-6\right)\left(3a+5\right)$

$=\left(6-a\right)\left(3a+5\right)$

14. $-6x^8+7x^4+5$

$=-\left(6x^8-7x^4-5\right)$

$=-\left(\left(6x^4\right)^2-7\left(6x^4\right)-30\right)$

$=-\dfrac{\left(6x^4-10\right)\left(6x^4+3\right)}{2\cdot 3}$

$=-\left(3x^4-5\right)\left(2x^4+1\right)$

$=\left(5-3x^4\right)\left(2x^4+1\right)$

15. $6a^2-ax-15x^2$

$=36a^2-6ax-90x^2$

$=\dfrac{\left(6a-10x\right)\left(6a+9x\right)}{2\cdot 3}$

$=\left(3a-5x\right)\left(2a+3x\right)$

16. $4x^2+7mnx-15m^2n^2$

$=(4x)^2+7mn(4x)-60m^2n^2$

$=\dfrac{\left(4x+12mn\right)\left(4x-5mn\right)}{4}$

$=\left(x+3mn\right)\left(4x-5mn\right)$

17. $18a^2+17ay-15y^2$

$=(18a)^2+17y(18a)-270y^2$

270	2	
135	5	$3^3=27$
27	3	$5\cdot 2=10$
9	3	$\Rightarrow 27-10=17$
3	3	$=\dfrac{\left(18a+27y\right)\left(18a-10y\right)}{9\cdot 2}$
1		$=\left(2a+3y\right)\left(9a-5y\right)$

18. $-8x^4+2x^2+15$

$=-\left(8x^4-2x^2-15\right)$

$=-\left(\left(8x^2\right)^2-2\left(8x^2\right)-120\right)$

$=-\dfrac{\left(8x^2-12\right)\left(8x^2+10\right)}{4\cdot 2}$

$=-\left(2x^2-3\right)\left(4x^2+5\right)$

$=\left(3-2x^2\right)\left(4x^2+5\right)$

19. $-25x^8+5x^4+6$

$=-\left(25x^8-5x^4-6\right)$

$=-\left(\left(25x^4\right)^2-5\left(25x^4\right)-150\right)$

$=-\dfrac{\left(25x^4-15\right)\left(25x^4+10\right)}{5\cdot 5}$

$=-\left(5x^4-3\right)\left(5x^4+2\right)$

$=\left(3-5x^4\right)\left(5x^4+2\right)$

20. $30x^{10}-91x^5-30$

$=\left(30x^5\right)^2-91\left(30x^5\right)-900$

$=\dfrac{\left(30x^5-100\right)\left(30x^5+9\right)}{10\cdot 3}$

$=\left(3x^5-10\right)\left(10x^5+3\right)$

21. $30m^2+17am-21a^2$

$=(30m)^2+17a(30m)-630a^2$

630	2	
315	5	$5\cdot 7=35$
63	3	$2\cdot 3^2=18$
21	3	$\Rightarrow 35-18=17$
7	7	$=\dfrac{(30m+35a)(30m-18a)}{5\cdot 6}$
1		$=(6m+7a)(5m-3a)$

22. $-15a^2+16a-4$

$=-\left(15a^2-16a+4\right)$

$=-\left((15a)^2-16(15a)+60\right)$

$=-\dfrac{(15a-10)(15a-6)}{5\cdot 3}$

$=-(3a-2)(5a-2)$

$=(3a-2)(2-5a)$

23. $-6y^2+11xy-4x^2$

$=-\left(6y^2-11xy+4x^2\right)$

$=-\left((6y)^2-11x(6y)+24x^2\right)$

$=-\dfrac{(6y-8x)(6y-3x)}{2\cdot 3}$

$=-(3y-4x)(2y-x)$

$=(4x-3y)(2y-x)$

24. $-20a^2+27ab-9b^2$

$=-\left(20a^2-27ab+9b^2\right)$

$=-\left((20a)^2-27b(20a)+180b^2\right)$

180	2	
90	2	$2^2\cdot 3=12$
45	5	$5\cdot 3=15$
9	3	$\Rightarrow 15+12=27$
3	3	$=-\dfrac{(20a-15b)(20a-12b)}{5\cdot 4}$
1		$=-(4a-3b)(5a-3b)$

$=(4a-3b)(3b-5a)$

EJERCICIO 102

1. $a^3+3a^2+3a+1=(a+1)^3$

2. $27-27x+9x^2-x^3=(3-x)^3$

3. $m^3+3m^2n+3mn^2+n^3=(m+n)^3$

4. $1-3a+3a^2-a^3=(1-a)^3$

5. $8+12a^2+6a^4+a^6=\left(2+a^2\right)^3$

6. $125x^3+75x^2+15x+1=(5x+1)^3$

7. $8a^3-36a^2b+54ab^2-27b^3=(2a-3b)^3$

8. $27m^3+108m^2n+144mn^2+64n^3=(3m+4n)^3$

9. $x^3-3x^2+3x+1=$ *No es cubo perfecto*

10. $1+12a^2b-6ab-8a^3b^3=$ *No es cubo perfecto*

11. $125a^3+150a^2b+60ab^2+8b^3=(5a+2b)^3$

12. $8+36x+54x^2+27x^3=(2+3x)^3$

13. $8-12a^2-64a^4-a^6=$ *No es cubo perfecto*

14. $a^6+3a^4b^3+3a^2b^6+b^9=\left(a^2+b^3\right)^3$

15. $x^9-9x^6y^4+27x^3y^8-27y^{12}=\left(x^3-3y^4\right)^3$

16. $64x^3+240x^2y+300xy^2+125y^3=(4x+5y)^3$

17. $216-756a^2+882a^4-343a^6=\left(6-7a^2\right)^3$

18. $125x^{12}+600x^8y^5+960x^4y^{10}+512y^{15}=\left(5x^4+8y^5\right)^3$

19. $a^{18}+3a^{12}+3a^6+1=\left(a^6+1\right)^3$

20. $m^3-3am^2n+3a^2mn^2-a^3n^3=(m-an)^3$

21. $1+18a^2b^3+108a^4b^6+216a^6b^9=\left(1+6a^2b^3\right)^3$

22. $64x^9-240x^6y^4+300x^3y^8-125y^{12}=\left(4x^3-5y^4\right)^3$

EJERCICIO 103

1. $1+a^3=(1+a)(1-a+a^2)$

2. $1-a^3=(1-a)(1+a+a^2)$

3. $x^3+y^3=(x+y)(x^2-xy+y^2)$

4. $m^3-n^3=(m-n)(m^2+mn+n^2)$

5. $a^3-1=(a-1)(a^2+a+1)$

6. $y^3+1=(y+1)(y^2-y+1)$

7. $y^3-1=(y-1)(y^2+y+1)$

8. $8x^3-1=(2x-1)(4x^2+2x+1)$

9. $1-8x^3=(1-2x)(1+2x+4x^2)$

10. $x^3-27=(x-3)(x^2+3x+9)$

11. $a^3+27=(a+3)(a^2-3a+9)$

12. $8x^3+y^3=(2x+y)(4x^2-2xy+y^2)$

13. $27a^3-b^3=(3a-b)(9a^2+3ab+b^2)$

14. $64+a^6=(4+a^2)(16-4a^2+a^4)$

15. $a^3-125=(a-5)(a^2+5a+25)$

16. $1-216m^3=(1-6m)(1+6m+36m^2)$

17. $8a^3+27b^6=(2a+3b^2)(4a^2-6ab^2+9b^4)$

18. $x^6-b^9=(x^2-b^3)(x^4+x^2b^3+b^6)$

19. $8x^3-27y^3=(2x-3y)(4x^2+6xy+9y^2)$

20. $1+343n^3=(1+7n)(1-7n+49n^2)$

21. $64a^3-729=(4a-9)(16a^2+36a+81)$

22. $a^3b^3-x^6=(ab-x^2)(a^2b^2+abx^2+x^4)$

23. $512+27a^9=(8+3a^3)(64-24a^3+9a^6)$

24. $x^6-8y^{12}=(x^2-2y^4)(x^4+2x^2y^4+4y^8)$

25. $1+729x^6=(1+9x^2)(1-9x^2+81x^4)$

26. $27m^3+64n^9=(3m+4n^3)(9m^2-12mn^3+16n^6)$

27. $343x^3+512y^6=(7x+8y^2)(49x^2-56xy^2+64y^4)$

28. $x^3y^6-216y^9=(xy^2-6y^3)(x^2y^4+6xy^5+36y^6)$

29. $a^3b^3x^3+1=(abx+1)(a^2b^2x^2-abx+1)$

30. $x^9+y^9=(x^3+y^3)(x^6-x^3y^3+y^6)$

31. $1.000x^3-1=(10x-1)(100x^2+10x+1)$

32. $a^6+125b^{12}=(a^2+5b^4)(a^4-5a^2b^4+25b^8)$

33. $x^{12}+y^{12}=(x^4+y^4)(x^8-x^4y^4+y^8)$

34. $1-27a^3b^3=(1-3ab)(1+3ab+9a^2b^2)$

35. $8x^6+729=(2x^2+9)(4x^4-18x^2+81)$

36. $a^3+8b^{12}=(a+2b^4)(a^2-2ab^4+4b^8)$

37. $8x^9-125y^3z^6=(2x^3-5yz^2)(4x^6+10x^3yz^2+25y^2z^4)$

38. $27m^6+343n^9=(3m^2+7n^3)(9m^4-21m^2n^3+49n^6)$

39. $216-x^{12}=(6-x^4)(36+6x^4+x^8)$

EJERCICIO 104

1. $1+(x+y)^3=(1+x+y)\left[1-(x+y)+(x+y)^2\right]$

$=(1+x+y)(1-x-y+x^2+2xy+y^2)$

2. $1-(a+b)^3=\left[1-(a+b)\right]\left[1+(a+b)+(a+b)^2\right]$

$=(1-a-b)(1+a+b+a^2+2ab+b^2)$

3. $27+(m-n)^3=(3+m-n)\left[9-3(m-n)+(m-n)^2\right]$

$=(3+m-n)(9-3m+3n+m^2-2mn+n^2)$

4. $(x-y)^3-8=(x-y-2)\left[(x-y)^2+2(x-y)+4\right]$

$=(x-y-2)(x^2-2xy+y^2+2x-2y+4)$

5. $(x+2y)^3+1=(x+2y+1)\left[(x+2y)^2-(x+2y)+1\right]$

$=(x+2y+1)(x^2+4xy+4y^2-x-2y+1)$

6. $1-(2a-b)^3=\left[1-(2a-b)\right]\left[1+(2a-b)+(2a-b)^2\right]$

$=(1-2a+b)(1+2a-b+4a^2-4ab+b^2)$

7. $a^3+(a+1)^3=(a+a+1)\left[a^2-a(a+1)+(a+1)^2\right]$

$=(2a+1)(a^2-a^2-a+a^2+2a+1)$

$=(2a+1)(a^2+a+1)$

8. $8a^3-(a-1)^3=\left[2a-(a-1)\right]\left[4a^2+2a(a-1)+(a-1)^2\right]$

$=(2a-a+1)(4a^2+2a^2-2a+a^2-2a+1)$

$=(a+1)(7a^2-4a+1)$

9. $27x^3-(x-y)^3$
$=[3x-(x-y)][9x^2+3x(x-y)+(x-y)^2]$
$=(3x-x+y)(9x^2+3x^2-3xy+x^2-2xy+y^2)$
$=(2x+y)(13x^2-5xy+y^2)$

10. $(2a-b)^3-27$
$=(2a-b-3)[(2a-b)^2+3(2a-b)+9]$
$=(2a-b-3)(4a^2-4ab+b^2+6a-3b+9)$

11. $x^6-(x+2)^3$
$=[x^2-(x+2)][x^4+x^2(x+2)+(x+2)^2]$
$=(x^2-x-2)(x^4+x^3+2x^2+x^2+4x+4)$
$=(x^2-x-2)(x^4+x^3+3x^2+4x+4)$

12. $(a+1)^3+(a-3)^3$
$=(a+1+a-3)[(a+1)^2-(a-3)(a+1)+(a-3)^2]$
$=(2a-2)(a^2+2a+1-a^2+2a+3+a^2-6a+9)$
$=(2a-2)(a^2-2a+13)$

13. $(x-1)^3-(x+2)^3$
$=[x-1-(x+2)][(x-1)^2+(x-1)(x+2)+(x+2)^2]$
$=(x-1-x-2)(x^2-2x+1+x^2+x-2+x^2+4x+4)$
$=-3(3x^2+3x+3)=-9(x^2+x+1)$

14. $(x-y)^3-(x+y)^3$
$=[(x-y)-(x+y)][(x-y)^2+(x-y)(x+y)+(x+y)^2]$
$=(x-y-x-y)(x^2-2xy+y^2+x^2-y^2+x^2+2xy+y^2)$
$=-2y(3x^2+y^2)$

15. $(m-2)^3+(m-3)^3$
$=[(m-2)+(m-3)][(m-2)^2-(m-2)(m-3)+(m-3)^2]$
$=(m-2+m-3)(m^2-4m+4-m^2+5m-6+m^2-6m+9)$
$=(2m-5)(m^2-5m+7)$

16. $(2x-y)^3+(3x+y)^3$
$=[(2x-y)+(3x+y)][(2x-y)^2-(2x-y)(3x+y)+(3x+y)^2]$
$=(2x-y+3x+y)(4x^2-4xy+y^2-6x^2+xy+y^2+9x^2+6xy+y^2)$
$=(5x)(7x^2+3xy+3y^2)$

17. $8(a+b)^3+(a-b)^3$
$=[2(a+b)+(a-b)][4(a+b)^2-2(a+b)(a-b)+(a-b)^2]$
$=(2a+2b+a-b)(4a^2+8ab+4b^2-2a^2+2b^2+a^2-2ab+b^2)$
$=(3a+b)(3a^2+6ab+7b^2)$

18. $64(m+n)^3-125$
$=[4(m+n)-5][16(m+n)^2+4(m+n)(5)+25]$
$=(4m+4n-5)(16m^2+32mn+16n^2+20m+20n+25)$

EJERCICIO 105

1. $a^5+1=(a+1)(a^4-a^3+a^2-a+1)$
2. $a^5-1=(a-1)(a^4+a^3+a^2+a+1)$
3. $1-x^5=(1-x)(1+x+x^2+x^3+x^4)$
4. a^7+b^7
$=(a+b)(a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6)$
5. m^7-n^7
$=(m-n)(m^6+m^5n+m^4n^2+m^3n^3+m^2n^4+mn^5+n^6)$
6. $a^5+243=(a+3)(a^4-3a^3+9a^2-27a+81)$
7. $32-m^5=(2-m)(16+8m+4m^2+2m^3+m^4)$
8. $1+243x^5=(1+3x)(1-3x+9x^2-27x^3+81x^4)$
9. x^7+128
$=(x+2)(x^6-2x^5+4x^4-8x^3+16x^2-32x+64)$
10. $243-32b^5=(3-2b)(81+54b+36b^2+24b^3+16b^4)$
11. $a^5+b^5c^5=(a+bc)(a^4-a^3bc+a^2b^2c^2-ab^3c^3+b^4c^4)$
12. $m^7-a^7x^7$
$=(m-ax)(m^6+m^5ax+m^4a^2x^2+m^3a^3x^3+m^2a^4x^4+ma^5x^5+a^6x^6)$
13. $1+x^7=(1+x)(1-x+x^2-x^3+x^4-x^5+x^6)$
14. x^7-y^7
$=(x-y)(x^6+x^5y+x^4y^2+x^3y^3+x^2y^4+xy^5+y^6)$
15. $a^7+2.187$
$=(a+3)(a^6-3a^5+9a^4-27a^3+81a^2-243a+729)$
16. $1-128a^7=(1-2a)(1+2a+4a^2+8a^3+16a^4+32a^5+64a^6)$
17. $x^{10}+32y^5=(x^2+2y)(x^8-2x^6y+4x^4y^2-8x^2y^3+16y^4)$
18. $1+128x^4$
$=(1+2x^2)(1-2x^2+4x^4-8x^6+16x^8-32x^{10}+64x^{12})$

EJERCICIO 106

1. $5a^2+a=a(5a+1)$

2. $m^2+2mx+x^2=(m+x)^2$

3. $a^2+a-ab-b=(a^2+a)-(ab-b)$

$=a(a+1)-b(a+1)$

$=(a+1)(a-b)$

4. $x^2-36=x^2-6^2=(x+6)(x-6)$

5. $9x^2-6xy+y^2=(3x-y)^2$

6. $x^2-3x-4=(x-1)(x+4)$

7. $6x^2-x-2$

$=36x^2-6x-12$

$=\dfrac{(6x-4)(6x+3)}{3\cdot 2}$

$=(3x-2)(2x+1)$

8. $1+x^3=(1+x)(1-x+x^2)$

9. $27a^3-1=(3a-1)(9a^2+3a+1)$

10. $x^5+m^5=(x+m)(x^4-x^3m+x^2m^2-xm^3+m^4)$

11. $a^3-3a^2b+5ab^2=a(a^2-3ab+5b^2)$

12. $2xy-6y+xz-3z$

$=2y(x-3)+z(x-3)=(x-3)(2y+z)$

13. $1-4b+4b^2=(1-2b)^2$

14. $4x^4+3x^2y^2+y^4$

$\quad +x^2y^2 \qquad -x^2y^2$

$4x^4+4x^2y^2+y^4-x^2y^2$

$=(2x^2+y^2)^2-x^2y^2$

$=(2x^2+xy+y^2)(2x^2-xy+y^2)$

15. $x^8-6x^4y^4+y^8$

$\quad +2x^4y^4 \qquad -4x^4y^4$

$x^8-2x^4y^4+y^8-4x^4y^4$

$=(x^4-y^4)^2-4x^4y^4$

$=(x^4+2x^2y^2-y^4)(x^4-2x^2y^2-y^4)$

16. $a^2-a-30=(a-6)(a+5)$

17. $15m^2+11m-14$

$=225m^2+11(15m)-210$

$=\dfrac{(15m+21)(15m-10)}{3\cdot 5}$

$=(5m+7)(3m-2)$

18. $a^6+1=(a^2+1)(a^4-a^2+1)$

19. $8m^3-27y^6=(2m-3y^2)(4m^2+6my^2+9y^4)$

20. $16a^2-24ab+9b^2=(4a-3b)^2$

21. $1+a^7=(1+a)(1-a+a^2-a^3+a^4-a^5+a^6)$

22. $8a^3-12a^2+6a-1=(2a-1)^3$

23. $1-m^2=(1+m)(1-m)$

24. $x^4+4x^2-21=(x^2+7)(x^2-3)$

25. $125a^6+1=(5a^2+1)(25a^4-5a^2+1)$

26. $a^2+2ab+b^2-m^2$

$=(a+b)^2-m^2=(a+b+m)(a+b-m)$

27. $8a^2b+16a^3b-24a^2b^2=8a^2b(1+2a-3b)$

28. x^5-x^4+x-1

$=(x^5-x^4)+(x-1)=x^4(x-1)+(x-1)=(x^4+1)(x-1)$

29. $6x^2+19x-20$

$=36x^2+19(6x)-120$

120	2	
60	2	$2^3\cdot 3=24$
30	2	$5\cdot 1=5$
15	3	$\Rightarrow\ 24-5=19$
5	5	$=\dfrac{(6x+24)(6x-5)}{6}$
1		$=(x+4)(6x-5)$

30. $25x^4-81y^2=(5x^2+9y)(5x^2-9y)$

31. $1-m^3=(1-m)(1+m+m^2)$

32. $x^2-a^2+2xy+y^2+2ab-b^2$

$=(x^2+2xy+y^2)-(a^2-2ab+b^2)$

$=(x+y)^2-(a-b)^2$

$=(x+y+a-b)(x+y-a+b)$

33. $21m^5n-7m^4n^2+7m^3n^3-7m^2n=7m^2n(3m^3-m^2n+mn^2-1)$

34. $a(x+1)-b(x+1)+c(x+1)=(a-b+c)(x+1)$

35. $4+4(x-y)+(x-y)^2=(2+x-y)^2$

36. $1-a^2b^4=(1+ab^2)(1-ab^2)$

37. $b^2+12ab+36a^2=(b+6a)^2$

38. $x^6+4x^3-77=(x^3+11)(x^3-7)$

39. $15x^4-17x^2-4$

$=\left(15x^2\right)^2-17\left(15x^2\right)-60$

$=\dfrac{\left(15x^2-20\right)\left(15x^2+3\right)}{5\cdot 3}$

$=\left(3x^2-4\right)\left(5x^2+1\right)$

40. $1+\left(a-3b\right)^3$

$=\left(1+a-3b\right)\left[1-\left(a-3b\right)+\left(a-3b\right)^2\right]$

$=\left(1+a-3b\right)\left(1-a+3b+a^2-6ab+9b^2\right)$

41. $x^4+x^2\quad +25$

$\quad +9x^2\qquad -9x^2$

$x^4+10x^2+25-9x^2$

$=\left(x^2+5\right)^2-\ 9x^2$

$=\left(x^2+3x+5\right)\left(x^2-3x+5\right)$

42. a^8-28a^4+36

$\quad +16a^4\qquad -16a^4$

$a^8-12a^4+36-16a^4$

$=\left(a^4-6\right)^2-16a^4$

$=\left(a^4+4a^2-6\right)\left(a^4-4a^2-6\right)$

43. $343+8a^3=\left(7+2a\right)\left(49-14a+4a^2\right)$

44. $12a^2bx-15a^2by=3a^2b\left(4x-5y\right)$

45. $x^2+2xy-15y^2=\left(x+5y\right)\left(x-3y\right)$

46. $6am-4an-2n+3m$

$=\left(6am+3m\right)-\left(4an+2n\right)$

$=3m\left(2a+1\right)-2n\left(2a+1\right)$

$=\left(3m-2n\right)\left(2a+1\right)$

47. $81a^6-4b^2c^8=\left(9a^3+2bc^4\right)\left(9a^3-2bc^4\right)$

48. $16-\left(2a+b\right)^2=\left(4+2a+b\right)\left(4-2a-b\right)$

49. $20-x-x^2$

$=-\left(x^2+x-20\right)$

$=-\left(x+5\right)\left(x-4\right)$

$=\left(x+5\right)\left(4-x\right)$

50. $n^2+n-42=\left(n+7\right)\left(n-6\right)$

51. $a^2-d^{\,2}+n^2-c^2-2an-2cd$

$=\left(a^2-2an+n^2\right)-\left(c^2+2cd+d^{\,2}\right)$

$=\left(a-n\right)^2-\left(c+d\right)^2$

$=\left(a-n+c+d\right)\left(a-n-c-d\right)$

52. $1+216x^9=\left(1+6x^3\right)\left(1-6x^3+36x^6\right)$

53 $x^3-64=\left(x-4\right)\left(x^2+4x+16\right)$

54. $x^3-64x^4=x^3\left(1-64x\right)$

55. $18ax^5y^3-36x^4y^3-54x^2y^8=18x^2y^3\left(ax^3-2x^2-3y^5\right)$

56. $49a^2b^2-14ab+1=\left(7ab-1\right)$

57. $\left(x+1\right)^2-81=\left(x+1+9\right)\left(x+1-9\right)=\left(x+10\right)\left(x-8\right)$

58. $a^2-\left(b+c\right)^2=\left(a+b+c\right)\left(a-b-c\right)$

59. $\left(m+n\right)^2-6\left(m+n\right)+9=\left(m+n-3\right)^2$

60. $7x^2+31x-20$

$=(7x)^2+31(7x)-140$

140	2	
70	2	$7\cdot 5=35$
35	5	$2^2=4$
7	7	$\Rightarrow 35-4=31$
1		$=\dfrac{(7x+35)(7x-4)}{7}$

$=\left(x+5\right)\left(7x-4\right)$

61. $9a^3+63a-45a^2$

$=9a\left(a^2+7-5a\right)$

62. $ax+a-x-1$

$=\left(ax+a\right)-\left(x+1\right)$

$=a\left(x+1\right)-\left(x+1\right)$

$=\left(a-1\right)\left(x+1\right)$

63. $81x^4+25y^2-90x^2y$

$\Rightarrow 81x^4-90x^2y+25y^2$

$=\left(9x^2-5y\right)^2$

64. $1-27b^2+b^4$

$\quad +25b^2\qquad -25b^2$

$1-2b^2+b^4-25b^2$

$=\left(1-b^2\right)^2-\ 25b^2$

$=\left(1+5b-b^2\right)\left(1-5b\ -b^2\right)$

65. $m^4+m^2n^2+n^4$

$\quad +m^2n^2\qquad -m^2n^2$

$m^4+2m^2n^2+n^4-m^2n^2$

$=\left(m^2+n^2\right)^2-m^2n^2$

$=\left(m^2+mn+n^2\right)\left(m^2-mn+n^2\right)$

66. $c^4-4d^{\,4}=\left(c^2+2d^{\,2}\right)\left(c^2-2d^{\,2}\right)$

67. $15x^4-15x^3+20x^2=5x^2\left(3x^2-3x+4\right)$

68. a^2-x^2-a-x

$=\left(a^2-x^2\right)-\left(a+x\right)=\left(a+x\right)\left(a-x\right)-\left(a+x\right)=\left(a+x\right)\left(a-x-1\right)$

69. x^4-8x^2-240

240	4	
60	4	$4\cdot 5=20$
15	3	$4\cdot 3=12$
5	5	$\Rightarrow 20-12=8$
1		$=\left(x^2-20\right)\left(x^2+12\right)$

70. $6m^4+7m^2-20$

$=\left(6m^2\right)^2+7\left(6m^2\right)-120$

120	4	
30	3	$4\cdot 2=8$
10	5	$5\cdot 3=15$
2	2	$\Rightarrow 15-8=7$
1		$=\dfrac{\left(6m^2+15\right)\left(6m^2-8\right)}{3\cdot 2}$

$=\left(2m^2+5\right)\left(3m^2-4\right)$

71. $9n^2+4a^2-12an \Rightarrow 9n^2-12an+4a^2$

$=\left(3n-2a\right)^2$

72. $2x^2+2=2\left(x^2+1\right)$

73. $7a\left(x+y-1\right)-3b\left(x+y-1\right)=\left(7a-3b\right)\left(x+y-1\right)$

74. $x^2+3x-18=\left(x+6\right)\left(x-3\right)$

75. $\left(a+m\right)^2-\left(b+n\right)^2=\left(a+m+b+n\right)\left(a+m-b-n\right)$

76 $x^3+6x^2y+12xy^2+8y^3=\left(x+2y\right)^3$

77. $8a^2-22a-21$

$=(8a)^2-22(8a)-168$

168	4	
42	2	$3\cdot 2=6$
21	3	$4\cdot 7=28$
7	7	$\Rightarrow 28-6=22$
1		$=\dfrac{\left(8a-28\right)\left(8a+6\right)}{4\cdot 2}$

$=\left(2a-7\right)\left(4a+3\right)$

78. $1+18ab+81a^2b^2=\left(1+9ab\right)^2$

79. $4a^6-1=\left(2a^3+1\right)\left(2a^3-1\right)$

80. x^6-4x^3-480

480	4	
120	3	$5\cdot 4=20$
40	4	$4\cdot 3\cdot 2=24$
10	5	$\Rightarrow 24-20=4$
2	2	$=\left(x^3-24\right)\left(x^3+20\right)$
1		

81. $ax-bx+b-a-by+ay$

$=\left(ax+ay-a\right)-\left(bx+by-b\right)$

$=a\left(x+y-1\right)-b\left(x+y-1\right)$

$=\left(a-b\right)\left(x+y-1\right)$

82 $6am-3m-2a+1$

$=\left(6am-3m\right)-\left(2a-1\right)$

$=3m\left(2a-1\right)-\left(2a-1\right)=\left(3m-1\right)\left(2a-1\right)$

83. $15+14x-8x^2$

$=-\left(8x^2-14x-15\right)$

$=-\left[(8x)^2-14(8x)-120\right]$

120	4	
30	3	$4\cdot 5=20$
10	5	$3\cdot 2=6$
2	2	$\Rightarrow 20-6=14$
1		$=-\dfrac{\left(8x-20\right)\left(8x+6\right)}{4\cdot 2}$

$=-\left(2x-5\right)\left(4x+3\right)$

$=\left(5-2x\right)\left(4x+3\right)$

84. $a^{10}-a^8+a^6+a^4=a^4\left(a^6-a^4+a^2+1\right)$

85. $2x\left(a-1\right)-a+1=2x\left(a-1\right)-\left(a-1\right)=\left(a-1\right)\left(2x-1\right)$

86. $\left(m+n\right)\left(m-n\right)+3n\left(m-n\right)=\left(m-n\right)\left(m+n+3n\right)=\left(m-n\right)\left(m+4n\right)$

87. $a^2-b^3+2b^3x^2-2a^2x^2$

$=-\left(b^3-2b^3x^2\right)+\left(a^2-2a^2x^2\right)$

$=-b^3\left(1-2x^2\right)+a^2\left(1-2x^2\right)=\left(1-2x^2\right)\left(a^2-b^3\right)$

88. $2am-3b-c-cm-3bm+2a$

$=\left(2am-cm-3bm\right)-\left(3b+c-2a\right)$

$=m\left(2a-c-3b\right)-\left(3b+c-2a\right)$

$=m\left(2a-3b-c\right)+\left(2a-3b-c\right)$

$=\left(2a-3b-c\right)\left(m+1\right)$

89. $x^2-\dfrac{2}{3}x+\dfrac{1}{9}=\left(x-\dfrac{1}{3}\right)^2$

90. $4a^{2n}-b^{4n}$

$=\left(2a^n+b^{2n}\right)\left(2a^n-b^{2n}\right)$

91. $81x^2-\left(a+x\right)^2=\left(9x+a+x\right)\left(9x-a-x\right)=\left(10x+a\right)\left(8x-a\right)$

92. $a^2+9-6a-16x^2$

$=\left(a^2-6a+9\right)-16x^2=\left(a-3\right)^2-16x^2=\left(a-3+4x\right)\left(a-4x-3\right)$

93. $9a^2-x^2-4+4x$

$=9a^2-\left(x^2-4x+4\right)=9a^2-\left(x-2\right)^2=\left(3a+x-2\right)\left(3a-x+2\right)$

94. $9x^2-y^2+3x-y$

$=\left(9x^2-y^2\right)+\left(3x-y\right)=\left(3x+y\right)\left(3x-y\right)+\left(3x-y\right)=\left(3x-y\right)\left(3x+y+1\right)$

95. $x^2-x-72=\left(x-9\right)\left(x+8\right)$

96. $36a^4-120a^2b^2+49b^4$

$\quad +36a^2b^2 \qquad -36a^2b^2$

$36a^4-84a^2b^2+49b^4-36a^2b^2$

$=\left(6a^2-7b^2\right)^2-36a^2b^2$

$=\left(6a^2+6ab-7b^2\right)\left(6a^2-6ab-7b^2\right)$

97. $a^2-m^2-9n^2-6mn+4ab+4b^2$

$=\left(a^2+4ab+4b^2\right)-\left(m^2+6mn+9n^2\right)$

$=\left(a+2b\right)^2-\left(m+3n\right)^2$

$=\left(a+2b+m+3n\right)\left(a+2b-m-3n\right)$

98. $1-\dfrac{4}{9}a^8=\left(1+\dfrac{2}{3}a^4\right)\left(1-\dfrac{2}{3}a^4\right)$

99. $81a^8 \qquad +64b^{12}$

$\quad +144a^4b^6 \qquad -144a^4b^6$

$81a^8+144a^4b^6+64b^{12}-144a^4b^6$

$=\left(9a^4+8b^6\right)^2-144a^4b^6$

$=\left(9a^4-12a^2b^3+8b^6\right)\left(9a^4+12a^2b^3+8b^6\right)$

100. $49x^2-77x+30$

$=(49x)^2-77(49x)+1.470$

1.470	7	
210	7	
30	5	
6	6	$7\cdot 5=35$
1		$7\cdot 6=42$

$\Rightarrow 42+35=77$

$=\dfrac{\left(49x-35\right)\left(49x-42\right)}{7\cdot 7}$

$=\left(7x-5\right)\left(7x-6\right)$

101. $x^2-2abx-35a^2b^2$

$\quad +36a^2b^2-36a^2b^2$

$x^2-2abx+a^2b^2-36a^2b^2$

$=\left(x-ab\right)^2-36a^2b^2$

$=\left(x-ab+6ab\right)\left(x-ab-6ab\right)$

$=\left(x+5ab\right)\left(x-7ab\right)$

102. $125x^3-225x^2+135x-27=(5x-3)^3$

103. $(a-2)^2-(a+3)^2$

$=(a-2+a+3)(a-2-a-3)=(2a+1)(-5)$

104. $4a^2m+12a^2n-5bm-15bn$

$=(4a^2m+12a^2n)-(5bm+15bn)$

$=4a^2(m+3n)-5b(m+3n)=(m+3n)(4a^2-5b)$

105. $1+6x^3+9x^6=(1+3x^3)^2$

106. $a^4+3a^2b-40b^2=(a^2+8b)(a^2-5b)$

107. $m^3+8a^3x^3=(m+2ax)(m^2-2axm+4a^2x^2)$

108. $1-9x^2+24xy-16y^2$

$=-[(9x^2-24xy+16y^2)-1]$

$=-[(3x-4y)^2-1]$

$=-[(3x-4y+1)(3x-4y-1)]$

$=(3x-4y+1)(1+4y-3x)$

109. $1+11x+24x^2$

$=(24x)^2+11(24x)+24$

$=\dfrac{(24x+8)(24x+3)}{8\cdot 3}$

$=(3x+1)(8x+1)$

110. $9x^2y^3-27x^3y^3-9x^5y^3=9x^2y^3(1-3x-x^3)$

111. $(a^2+b^2-c^2)^2-9x^2y^2$

$=(a^2+b^2-c^2+3xy)(a^2+b^2-c^2-3xy)$

112. $8(a+1)^3-1$

$=[2(a+1)-1][4(a+1)^2+2(a+1)+1]$

$=(2a+2-1)(4a^2+8a+4+2a+2+1)$

$=(2a+1)(4a^2+10a+7)$

113. $100x^4y^6-121m^4$

$=(10x^2y^3+11m^2)(10x^2y^3-11m^2)$

114. $(a^2+1)^2+5(a^2+1)-24$

$=(a^2+1+8)(a^2+1-3)=(a^2+9)(a^2-2)$

115. $1+1.000x^6=(1+10x^2)(1-10x^2+100x^4)$

116. $49a^2-x^2-9y^2+6xy$

$=-[(x^2-6xy+9y^2)-49a^2]$

$=-[(x-3y)^2-49a^2]$

$=-(7a+x-3y)(-7a+x-3y)$

$=(7a+x-3y)(7a-x+3y)$

117. $x^4-y^2+4x^2+4-4yz-4z^2$

$=(x^4+4x^2+4)-(y^2+4yz+4z^2)$

$=(x^2+2)^2-(y+2z)^2$

$=(x^2+2+y+2z)(x^2+2-y-2z)$

118. $a^3-64=(a-4)(a^2+4a+16)$

119. $a^5+x^5=(a+x)(a^4-a^3x+a^2x^2-ax^3+x^4)$

120. $a^6-3a^3b-54b^2=(a^3-9b)(a^3+6b)$

121. $165+4x-x^2$

$=-(x^2-4x-165)$

165	5	
33	3	$5\cdot 3=15$
11	11	$11\cdot 1=11$
1		$\Rightarrow 15-11=4$

$=-(x-15)(x+11)$

$=(15-x)(x+11)$

122. a^4+a^2+1

$\quad +a^2 \quad -a^2$

$a^4+2a^2+1-a^2$

$=(a^2+1)^2-a^2$

$=(a^2+a+1)(a^2-a+1)$

123. $\dfrac{x^2}{4}-\dfrac{y^6}{81}=\left(\dfrac{x}{2}+\dfrac{y^3}{9}\right)\left(\dfrac{x}{2}-\dfrac{y^3}{9}\right)$

124. $16x^2+\dfrac{8xy}{5}+\dfrac{y^2}{25}=\left(4x+\dfrac{y}{5}\right)^2$

125. $a^4b^4+4a^2b^2-96$

$\quad +100-100$

$a^4b^4+4a^2b^2+\quad 4\ -100$

$=(a^2b^2+2)^2-100$

$=(a^2b^2+2+10)(a^2b^2+2-10)$

$=(a^2b^2+12)(a^2b^2-8)$

126. $8a^2x+7y+21by-7ay-8a^3x+24a^2bx$

$=\left(7y+21by-7ay\right)+\left(8a^2x-8a^3x+24a^2bx\right)$

$=7y\left(1+3b-a\right)+8a^2x\left(1-a+3b\right)$

$=\left(1+3b-a\right)\left(7y+8a^2x\right)$

127. x^4+11x^2-390

390	3	
130	2	$13\cdot2=26$
65	5	$5\cdot3=15$
13	13	$\Rightarrow 26-15=11$
1		$=\left(x^2+26\right)\left(x^2-15\right)$

128. $7+33m-10m^2$

$=-\left(10m^2-33m-7\right)$

$=-\left[(10m)^2-33(10m)-70\right]$

$=-\dfrac{\left(10m-35\right)\left(10m+2\right)}{5\cdot2}$

$=-\left(2m-7\right)\left(5m+1\right)$

$=\left(7-2m\right)\left(5m+1\right)$

129. $4\left(a+b\right)^2-9\left(c+d\right)^2$

$=\left[2\left(a+b\right)+3\left(c+d\right)\right]\left[2\left(a+b\right)-3\left(c+d\right)\right]$

$=\left(2a+2b+3c+3d\right)\left(2a+2b-3c-3d\right)$

130. $729-125x^3y^{12}=\left(9-5xy^4\right)\left(81+45xy^4+25x^2y^8\right)$

131. $\left(x+y\right)^2+x+y=\left(x+y\right)^2+\left(x+y\right)=\left(x+y\right)\left(x+y+1\right)$

132. $4-\left(a^2+b^2\right)+2ab$

$4-a^2-b^2+2ab$

$+b^2\qquad -b^2$

$4-a^2\qquad +2ab-b^2$

$=4-\left(a^2-2ab+b^2\right)$

$=4-\left(a-b\right)^2$

$=\left(2+a-b\right)\left(2-a+b\right)$

133. x^3-y^3+x-y

$=\left(x^3-y^3\right)+\left(x-y\right)$

$=\left(x-y\right)\left(x^2+xy+y^2\right)+\left(x-y\right)=\left(x-y\right)\left(x^2+xy+y^2+1\right)$

134. $a^2-b^2+a^3-b^3$

$=\left(a^2-b^2\right)+\left(a^3-b^3\right)$

$=\left(a+b\right)\left(a-b\right)+\left(a-b\right)\left(a^2+ab+b^2\right)$

$=\left(a-b\right)\left(a+b+a^2+ab+b^2\right)$

EJERCICIO 107

1. $3ax^2-3a$

$=3a\left(x^2-1\right)=3a\left(x+1\right)\left(x-1\right)$

2. $3x^2-3x-6$

$=3\left(x^2-x-2\right)=3\left(x-2\right)\left(x+1\right)$

3. $2a^2x-4abx+2b^2x$

$=2x\left(a^2-2ab+b^2\right)$

$=2x\left(a-b\right)^2=2x\left(a-b\right)\left(a-b\right)$

4. $2a^3-2$

$=2\left(a^3-1\right)=2\left(a-1\right)\left(a^2+a+1\right)$

5. a^3-3a^2-28a

$=a\left(a^2-3a-28\right)=a\left(a-7\right)\left(a+4\right)$

6. x^3-4x+x^2-4

$=\left(x^3+x^2\right)-\left(4x+4\right)$

$=x^2\left(x+1\right)-4\left(x+1\right)$

$=\left(x^2-4\right)\left(x+1\right)=\left(x+2\right)\left(x-2\right)\left(x+1\right)$

7. $3ax^3+3ay^3$

$=3a\left(x^3+y^3\right)$

$=3a\left(x+y\right)\left(x^2-xy+y^2\right)$

8. $4ab^2-4abn+an^2$

$=a\left(4b^2-4bn+n^2\right)=a\left(2b-n\right)^2$

9. x^4-3x^2-4

$=\left(x^2-4\right)\left(x^2+1\right)$

$=\left(x+2\right)\left(x-2\right)\left(x^2+1\right)$

10. a^3-a^2-a+1

$=\left(a^3-a^2\right)-\left(a-1\right)$

$=a^2\left(a-1\right)-\left(a-1\right)$

$=\left(a-1\right)\left(a^2-1\right)$

$=\left(a-1\right)\left(a+1\right)\left(a-1\right)$

11. $2ax^2-4ax+2a$

$=2a\left(x^2-2x+1\right)=2a\left(x-1\right)^2$

12. x^3-x+x^2y-y

$=\left(x^3-x\right)+\left(x^2y-y\right)$

$=x\left(x^2-1\right)+y\left(x^2-1\right)$

$=\left(x^2-1\right)\left(x+y\right)=\left(x+1\right)\left(x-1\right)\left(x+y\right)$

13. $2a^3+6a^2-8a$

$=2a\left(a^2+3a-4\right)=2a\left(a+4\right)\left(a-1\right)$

14. $16x^3-48x^2y+36xy^2$

$=4x\left(4x^2-12xy+9y^2\right)=4x\left(2x-3y\right)^2$

15. $3x^3-x^2y-3xy^2+y^3$

$=\left(3x^3-x^2y\right)-\left(3xy^2-y^3\right)$

$=x^2\left(3x-y\right)-y^2\left(3x-y\right)$

$=\left(3x-y\right)\left(x^2-y^2\right)=\left(3x-y\right)\left(x+y\right)\left(x-y\right)$

16. $5a^4+5a$

$=5a\left(a^3+1\right)=5a\left(a+1\right)\left(a^2-a+1\right)$

17. $6ax^2-ax-2a$

$=a\left(6x^2-x-2\right)$

$=a\left(36x^2-6x-12\right)$

$=\dfrac{a\left(6x-4\right)\left(6x+3\right)}{2\cdot 3}$

$=a\left(3x-2\right)\left(2x+1\right)$

18. n^4-81

$=\left(n^2-9\right)\left(n^2+9\right)=\left(n+3\right)\left(n-3\right)\left(n^2+9\right)$

19. $8ax^2-2a$

$=2a\left(4x^2-1\right)=2a\left(2x-1\right)\left(2x+1\right)$

20. ax^3+10ax^2+25ax

$=ax\left(x^2+10x+25\right)=ax\left(x+5\right)\left(x+5\right)$

21. x^3-6x^2-7x

$=x\left(x^2-6x-7\right)$

$=x\left(x^2-6x-7+16-16\right)$

$=x\left(x^2-6x+9-16\right)$

$=x\left[\left(x-3\right)^2-4^2\right]$

$=x\left(x-3+4\right)\left(x-3-4\right)$

$=x\left(x+1\right)\left(x-7\right)$

22. $m^3+3m^2-16m-48$

$=\left(m^3+3m^2\right)-\left(16m+48\right)$

$=m^2\left(m+3\right)-16\left(m+3\right)$

$=\left(m+3\right)\left(m^2-16\right)$

$=\left(m+3\right)\left(m-4\right)\left(m+4\right)$

23. $x^3-6x^2y+12xy^2-8y^3$

$=\left(x-2y\right)^3=\left(x-2y\right)\left(x-2y\right)\left(x-2y\right)$

24. $\left(a+b\right)\left(a^2-b^2\right)-\left(a^2-b^2\right)$

$=\left(a^2-b^2\right)\left(a+b-1\right)$

$=\left(a+b\right)\left(a-b\right)\left(a+b-1\right)$

25. $32a^5x-48a^3bx+18ab^2x$

$=2ax\left(16a^4-24a^2b+9b^2\right)$

$=2ax\left(4a^2-3b\right)^2$

$=2ax\left(4a^2-3b\right)\left(4a^2-3b\right)$

26. $x^4-x^3+x^2-x$

$=x\left(x^3-x^2+x-1\right)$

$=x\left[x^2\left(x-1\right)+\left(x-1\right)\right]=x\left[\left(x-1\right)\left(x^2+1\right)\right]$

27. $4x^2+32x-36$

$=16x^2+32\left(4x\right)-144$

$$\begin{array}{r|ll}
144 & 2 & \\
72 & 2 & 2^2=4 \\
36 & 4 & 4\cdot 3\cdot 3=36 \\
9 & 3 & \Rightarrow 36-4=32 \\
3 & 3 & =\dfrac{\left(4x+36\right)\left(4x-4\right)}{4} \\
1 & & =\left(x+9\right)\left(4x-4\right) \\
 & & =4\left(x+9\right)\left(x-1\right)
\end{array}$$

28. $a^4-\left(a+2\right)^2$

$=\left(a^2+a+2\right)\left(a^2-a-2\right)$

$=\left(a^2+a+2\right)\left(a-2\right)\left(a+1\right)$

29. x^6-25x^3-54

$=\left(x^3-27\right)\left(x^3+2\right)$

$=\left(x-3\right)\left(x^2+3x+9\right)\left(x^3+2\right)$

30. a^6+a

$=a\left(a^5+1\right)$

$=a\left(a+1\right)\left(a^4-a^3+a^2-a+1\right)$

31. $a^3b+2a^2bx+abx^2-aby^2$

$=ab\left(a^2+2ax+x^2-y^2\right)$

$=ab\left[\left(a+x\right)^2-y^2\right]$

$=ab\left(a+x+y\right)\left(a+x-y\right)$

32. $3abm^2-3ab$

$=3ab\left(m^2-1\right)$

$=3ab\left(m+1\right)\left(m-1\right)$

33. $81x^4y+3xy^4$

$=3xy\left(27x^3+y^3\right)$

$=3xy\left(3x+y\right)\left(9x^2-3xy+y^2\right)$

34. a^4-a^3+a-1

$=\left(a^4-a^3\right)+\left(a-1\right)$

$=a^3\left(a-1\right)+\left(a-1\right)$

$=\left(a-1\right)\left(a^3+1\right)$

$=\left(a-1\right)\left(a+1\right)\left(a^2-a+1\right)$

35. $x-3x^2-18x^3$

$=x\left(1-3x-18x^2\right)$

$=-x\left(18x^2+3x-1\right)$

$=-x\left[\left(18x\right)^2+3\left(18x\right)-18\right]$

$=\dfrac{-x\left(18x+6\right)\left(18x-3\right)}{6\cdot 3}$

$=-x\left(3x+1\right)\left(6x-1\right)$

$=x\left(3x+1\right)\left(1-6x\right)$

36. $6ax-2bx+6ab-2b^2$

$=\left(6ax-2bx\right)+\left(6ab-2b^2\right)$

$=2x\left(3a-b\right)+2b\left(3a-b\right)$

$=\left(3a-b\right)\left(2x+2b\right)$

$=2\left(x+b\right)\left(3a-b\right)$

37. am^3-7am^2+12am

$=am\left(m^2-7m+12\right)$

$=am\left(m-4\right)\left(m-3\right)$

38. $4a^2x^3-4a^2$

$=4a^2\left(x^3-1\right)$

$=4a^2\left(x-1\right)\left(x^2+x+1\right)$

39. $28x^3y-7xy^3$

$=7xy\left(4x^2-y^2\right)$

$=7xy\left(2x-y\right)\left(2x+y\right)$

40. $3abx^2-3abx-18ab$

$=3ab\left(x^2-x-6\right)$

$=3ab\left(x-3\right)\left(x+2\right)$

41. x^4-8x^2-128

$$\begin{array}{r|ll}
128 & 4 & \\
32 & 4 & 4\cdot 4=16 \\
8 & 4 & 4\cdot 2=8 \\
2 & 2 & \Rightarrow 16-8=8 \\
1 & & =\left(x^2-16\right)\left(x^2+8\right) \\
 & & =\left(x+4\right)\left(x-4\right)\left(x^2+8\right)
\end{array}$$

42. $18x^2y+60xy^2+50y^3$

$=2y\left(9x^2+30xy+25y^2\right)$

$=2y\left(3x+5y\right)\left(3x+5y\right)$

43. $\left(x^2-2xy\right)\left(a+1\right)+y^2\left(a+1\right)$
$=\left(a+1\right)\left(x^2-2xy+y^2\right)$
$=\left(a+1\right)\left(x-y\right)\left(x-y\right)$

44. $x^3+2x^2y-3xy^2$
$=x\left(x^2+2xy-3y^2\right)$
$=x\left(x+3y\right)\left(x-y\right)$

45. $a^2x-4b^2x+2a^2y-8b^2y$
$=\left(a^2x-4b^2x\right)+\left(2a^2y-8b^2y\right)$
$=x\left(a^2-4b^2\right)+2y\left(a^2-4b^2\right)$
$=\left(a^2-4b^2\right)\left(x+2y\right)$
$=\left(a-2b\right)\left(a+2b\right)\left(x+2y\right)$

46. $45a^2x^4-20a^2$
$=5a^2\left(9x^4-4\right)$
$=5a^2\left(3x^2+2\right)\left(3x^2-2\right)$

47. $a^4-\left(a-12\right)^2$
$=\left(a^2+a-12\right)\left(a^2-a+12\right)$
$=\left(a+4\right)\left(a-3\right)\left(a^2-a+12\right)$

48. bx^2-b-x^2+1
$=\left(bx^2-x^2\right)-\left(b-1\right)$
$=x^2\left(b-1\right)-\left(b-1\right)$
$=\left(b-1\right)\left(x^2-1\right)$
$=\left(b-1\right)\left(x+1\right)\left(x-1\right)$

49. $2x^4+6x^3-56x^2$
$=2x^2\left(x^2+3x-28\right)$
$=2x^2\left(x+7\right)\left(x-4\right)$

50. $30a^2-55a-50$
$=(30a)^2-55(30a)-1.500$

$$\begin{array}{r|l}1.500 & 2\\ 750 & 2\\ 375 & 5\\ 75 & 5\\ 15 & 5\\ 3 & 3\\ 1 & \end{array}$$

$5\cdot 2^2=20$
$5^2\cdot 3=75$
$\Rightarrow 75-20=55$

$=\dfrac{\left(30a-75\right)\left(30a+20\right)}{15\cdot 2}$
$=\left(2a-5\right)\left(15a+10\right)$
$=5\left(2a-5\right)\left(3a+2\right)$

51. $9\left(x-y\right)^3-\left(x-y\right)$
$=\left(x-y\right)\left[9\left(x-y\right)^2-1\right]$
$=\left(x-y\right)\left[3\left(x-y\right)+1\right]\left[3\left(x-y\right)-1\right]$
$=\left(x-y\right)\left(3x-3y+1\right)\left(3x-3y-1\right)$

52. $6a^2x-9a^3-ax^2$
$=a\left(6ax-9a^2-x^2\right)$
$=-a\left(x^2-6ax+9a^2\right)$
$=-a\left(x-3a\right)\left(x-3a\right)$
$=a\left(3a-x\right)\left(x-3a\right)$

53. $64a-125a^4$
$=a\left(64-125a^3\right)$
$=a\left(4-5a\right)\left(16+20a+25a^2\right)$

54. $70x^4+26x^3-24x^2$
$=2x^2\left(35x^2+13x-12\right)$
$=2x^2\left[(35x)^2+13(35x)-420\right]$

$$\begin{array}{r|l}420 & 2\\ 210 & 2\\ 105 & 5\\ 21 & 3\\ 7 & 7\\ 1 & \end{array}$$

$2^2\cdot 7=28$
$3\cdot 5=15$
$\Rightarrow 28-15=13$

$=\dfrac{\left(35x+28\right)\left(35x-15\right)}{7\cdot 5}$
$=2x^2\left(5x+4\right)\left(7x-3\right)$

55. $a^7+6a^6-55a^3$
$=a^3\left(a^4+6a^3-55\right)$
$=a^3\left(a^2+11\right)\left(a^2-5\right)$

56. $16a^5b-56a^3b^3+49ab^5$
$=ab\left(16a^4-56a^2b^2+49b^4\right)$
$=ab\left(4a^2-7b^2\right)^2$

57. $7x^6+32a^2x^4-15a^4x^2$
$=x^2\left(7x^4+32a^2x^2-15a^4\right)$
$=x^2\left[\left(7x^2\right)^2+32a^2\left(7x^2\right)-105a^4\right]$

$$\begin{array}{r|l}105 & 5\\ 21 & 3\\ 7 & 7\\ 1 & \end{array}$$

$5\cdot 7=35$
$3\cdot 1=3$
$\Rightarrow 35-3=32$

$=\dfrac{x^2\left(7x^2+35a^2\right)\left(7x^2-3a^2\right)}{7}$
$=x^2\left(x^2+5a^2\right)\left(7x^2-3a^2\right)$

58. $x^{2m+2}-x^2y^{2n}$
$=\left(x^{m+1}+xy^n\right)\left(x^{m+1}-xy^n\right)$
$=x\left(x^m+y^n\right)x\left(x^m-y^n\right)$
$=x^2\left(x^m+y^n\right)\left(x^m-y^n\right)$

59. $2x^4+5x^3-54x-135$
$=\left(2x^4-54x\right)+\left(5x^3-135\right)$
$=2x\left(x^3-27\right)+5\left(x^3-27\right)$
$=\left(x^3-27\right)\left(2x+5\right)$
$=\left(x-3\right)\left(x^2+3x+9\right)\left(2x+5\right)$

60. $ax^3+ax^2y+axy^2-2ax^2-2axy-2ay^2$
$=a\left(x^3+x^2y+xy^2-2x^2-2xy-2y^2\right)$
$=a\left[\left(x^3+x^2y+xy^2\right)-\left(2x^2+2xy+2y^2\right)\right]$
$=a\left[x\left(x^2+xy+y^2\right)-2\left(x^2+xy+y^2\right)\right]$
$=a\left(x-2\right)\left(x^2+xy+y^2\right)$

61. $\left(x+y\right)^4-1$
$=\left[\left(x+y\right)^2+1\right]\left[\left(x+y\right)^2-1\right]$
$=\left(x^2+2xy+y^2+1\right)\left(x+y+1\right)\left(x+y-1\right)$

62. $3a^5+3a^3+3a$
$=3a\left(a^4+a^2+1\right)$
$=3a\left(a^4+a^2+1+a^2-a^2\right)$
$=3a\left[\left(a^4+2a^2+1\right)-a^2\right]$
$=3a\left[\left(a^2+1\right)^2-a^2\right]$
$=3a\left(a^2+a+1\right)\left(a^2-a+1\right)$

EJERCICIO 108

1. $1-a^8$

$=(1+a^4)(1-a^4)$

$=(1+a^4)(1+a^2)(1-a^2)$

$=(1+a^4)(1+a^2)(1+a)(1-a)$

2. a^6-1

$=(a^3+1)(a^3-1)$

$=(a+1)(a^2-a+1)(a-1)(a^2+a+1)$

3. x^4-41x^2+400

400	4	
100	5	$4^2=16$
20	5	$5^2=25$
4	4	$\Rightarrow 25+16=41$
1		

$=(x^2-25)(x^2-16)$

$=(x+5)(x-5)(x+4)(x-4)$

4. $a^4-2a^2b^2+b^4$

$=(a^2-b^2)^2$

$=[(a+b)(a-b)]^2=(a+b)^2(a-b)^2$

5. x^5+x^3-2x

$=x(x^4+x^2-2)$

$=x(x^2+2)(x^2-1)$

$=x(x^2+2)(x+1)(x-1)$

6. $2x^4+6x^3-2x-6$

$=(2x^4+6x^3)-(2x+6)$

$=2x^3(x+3)-2(x+3)$

$=(2x^3-2)(x+3)$

$=2(x^3-1)(x+3)$

$=2(x-1)(x^2+x+1)(x+3)$

7. $3x^4-243$

$=3(x^4-81)$

$=3(x^2+9)(x^2-9)$

$=3(x^2+9)(x+3)(x-3)$

8. $16x^4-8x^2y^2+y^4$

$=(4x^2-y^2)^2$

$=[(2x+y)(2x-y)]^2$

$=(2x+y)(2x-y)(2x+y)(2x-y)$

9. $9x^4+9x^3y-x^2-xy$

$=(9x^4+9x^3y)-(x^2+xy)$

$=9x^3(x+y)-x(x+y)$

$=(9x^3-x)(x+y)$

$=x(9x^2-1)(x+y)$

$=x(3x+1)(3x-1)(x+y)$

10. $12ax^4+33ax^2-9a$

$=3a(4x^4+11x^2-3)$

$=3a[(4x^2)^2+11(4x^2)-12]$

$=\dfrac{3a(4x^2+12)(4x^2-1)}{4}$

$=3a(x^2+3)(4x^2-1)$

$=3a(x^2+3)(2x+1)(2x-1)$

11. x^8-y^8

$=(x^4+y^4)(x^4-y^4)$

$=(x^4+y^4)(x^2+y^2)(x^2-y^2)$

$=(x^4+y^4)(x^2+y^2)(x+y)(x-y)$

12. x^6-7x^3-8

$=(x^3-8)(x^3+1)$

$=(x-2)(x^2+2x+4)(x^3+1)$

$=(x-2)(x^2+2x+4)(x+1)(x^2-x+1)$

13. $64-x^6$

$=(8+x^3)(8-x^3)$

$=(2+x)(4-2x+x^2)(2-x)(4+2x+x^2)$

14. $a^5-a^3b^2-a^2b^3+b^5$

$=(a^5-a^3b^2)-(a^2b^3-b^5)$

$=a^3(a^2-b^2)-b^3(a^2-b^2)$

$=(a^2-b^2)(a^3-b^3)$

$=(a+b)(a-b)(a-b)(a^2+ab+b^2)$

15. $8x^4+6x^2-2$

$=2(4x^4+3x^2-1)$

$=2[(4x^2)^2+3(4x^2)-4]$

$=\dfrac{2(4x^2+4)(4x^2-1)}{4}$

$=2(x^2+1)(4x^2-1)=2(x^2+1)(2x+1)(2x-1)$

16 a^4-25a^2+144

144	4	
36	4	$4^2=16$
9	3	$3^2=9$
3	3	$\Rightarrow 16+9=25$
1		

$=(a^2-16)(a^2-9)$

$=(a+4)(a-4)(a+3)(a-3)$

17. $a^2x^3-a^2y^3+2ax^3-2ay^3$

$=(a^2x^3-a^2y^3)+(2ax^3-2ay^3)$

$=a^2(x^3-y^3)+2a(x^3-y^3)$

$=(a^2+2a)(x^3-y^3)$

$=a(a+2)(x-y)(x^2+xy+y^2)$

18. $a^4+2a^3-a^2-2a$

$=(a^4+2a^3)-(a^2+2a)$

$=a^3(a+2)-a(a+2)$

$=(a^3-a)(a+2)$

$=a(a^2-1)(a+2)$

$=a(a+1)(a-1)(a+2)$

19. $1-2a^3+a^6$

$=(a^3-1)^2$

$=[(a-1)(a^2+a+1)]^2$

$=(a-1)^2(a^2+a+1)^2$

20. m^6-729

$=(m^2-9)(m^4+9m^2+81)$

$=(m+3)(m-3)(m^4+9m^2+81+9m^2-9m^2)$

$=(m+3)(m-3)[(m^4+18m^2+81)-9m^2]$

$=(m+3)(m-3)[(m^2+9)^2-9m^2]$

$=(m+3)(m-3)(m^2+3m+9)(m^2-3m+9)$

21. x^5-x

$=x(x^4-1)$

$=x(x^2+1)(x^2-1)$

$=x(x^2+1)(x+1)(x-1)$

22. $x^5 - x^3y^2 + x^2y^3 - y^5$

$= \left(x^5 - x^3y^2\right) + \left(x^2y^3 - y^5\right)$

$= x^3\left(x^2 - y^2\right) + y^3\left(x^2 - y^2\right)$

$= \left(x^3 + y^3\right)\left(x^2 - y^2\right)$

$= \left(x + y\right)\left(x^2 - xy + y^2\right)\left(x + y\right)\left(x - y\right)$

23. $a^4b - a^3b^2 - a^2b^3 + ab^4$

$= \left(a^4b - a^3b^2\right) - \left(a^2b^3 - ab^4\right)$

$= a^3b\left(a - b\right) - ab^3\left(a - b\right)$

$= \left(a^3b - ab^3\right)\left(a - b\right)$

$= ab\left(a^2 - b^2\right)\left(a - b\right)$

$= ab\left(a + b\right)\left(a - b\right)\left(a - b\right)$

24. $5a^4 - 3.125$

$= 5\left(a^4 - 625\right)$

$= 5\left(a^2 + 25\right)\left(a^2 - 25\right)$

$= 5\left(a^2 + 25\right)\left(a + 5\right)\left(a - 5\right)$

25. $\left(a^2 + 2a\right)^2 - 2\left(a^2 + 2a\right) - 3$

$= \left(a^2 + 2a\right)^2 - 2\left(a^2 + 2a\right) - 3 + 4 - 4$

$= \left\{\left[\left(a^2 + 2a\right)^2 - 2\left(a^2 + 2a\right) + 1\right] - 4\right\}$

$= \left(a^2 + 2a - 1\right)^2 - 4$

$= \left(a^2 + 2a - 1 + 2\right)\left(a^2 + 2a - 1 - 2\right)$

$= \left(a^2 + 2a + 1\right)\left(a^2 + 2a - 3\right)$

$= \left(a + 1\right)^2\left(a^2 + 2a - 3 + 4 - 4\right)$

$= \left(a + 1\right)^2\left[\left(a^2 + 2a + 1\right) - 4\right]$

$= \left(a + 1\right)^2\left[\left(a + 1\right)^2 - 4\right]$

$= \left(a + 1\right)^2\left(a + 1 + 2\right)\left(a + 1 - 2\right)$

$= \left(a + 1\right)^2\left(a + 3\right)\left(a - 1\right)$

26. $a^2x^3 + 2ax^3 - 8a^2 - 16a$

$= \left(a^2x^3 + 2ax^3\right) - \left(8a^2 + 16a\right)$

$= ax^3\left(a + 2\right) - 8a\left(a + 2\right)$

$= \left(ax^3 - 8a\right)\left(a + 2\right)$

$= a\left(x^3 - 8\right)\left(a + 2\right)$

$= a\left(x - 2\right)\left(x^2 + 2x + 4\right)\left(a + 2\right)$

27. $1 - a^6b^6$

$= \left(1 - a^3b^3\right)\left(1 + a^3b^3\right)$

$= \left(1 - ab\right)\left(1 + ab + a^2b^2\right)\left(1 + a^3b^3\right)$

$= \left(1 - ab\right)\left(1 + ab + a^2b^2\right)\left(1 + ab\right)\left(1 - ab + a^2b^2\right)$

28. $5ax^3 + 10ax^2 - 5ax - 10a$

$= \left(5ax^3 + 10ax^2\right) - \left(5ax + 10a\right)$

$= 5ax^2\left(x + 2\right) - 5a\left(x + 2\right)$

$= \left(5ax^2 - 5a\right)\left(x + 2\right)$

$= 5a\left(x^2 - 1\right)\left(x + 2\right)$

$= 5a\left(x + 1\right)\left(x - 1\right)\left(x + 2\right)$

29. $a^2x^2 + b^2y^2 - b^2x^2 - a^2y^2$

$= \left(a^2x^2 - b^2x^2\right) + \left(b^2y^2 - a^2y^2\right)$

$= x^2\left(a^2 - b^2\right) + y^2\left(b^2 - a^2\right)$

$= x^2\left(a^2 - b^2\right) - y^2\left(a^2 - b^2\right)$

$= \left(a^2 - b^2\right)\left(x^2 - y^2\right)$

$= \left(a + b\right)\left(a - b\right)\left(x + y\right)\left(x - y\right)$

30. $x^8 + x^4 - 2$

$= \left(x^4 + 2\right)\left(x^4 - 1\right)$

$= \left(x^4 + 2\right)\left(x^2 + 1\right)\left(x^2 - 1\right)$

$= \left(x^4 + 2\right)\left(x^2 + 1\right)\left(x + 1\right)\left(x - 1\right)$

31. $a^4 + a^3 - 9a^2 - 9a$

$= \left(a^4 + a^3\right) - \left(9a^2 + 9a\right)$

$= a^3\left(a + 1\right) - 9a\left(a + 1\right)$

$= \left(a^3 - 9a\right)\left(a + 1\right)$

$= a\left(a^2 - 9\right)\left(a + 1\right)$

$= a\left(a + 3\right)\left(a - 3\right)\left(a + 1\right)$

32. $a^2x^2 + a^2x - 6a^2 - x^2 - x + 6$

$= \left(a^2x^2 + a^2x - 6a^2\right) - \left(x^2 + x - 6\right)$

$= a^2\left(x^2 + x - 6\right) - \left(x^2 + x - 6\right)$

$= \left(a^2 - 1\right)\left(x^2 + x - 6\right)$

$= \left(a + 1\right)\left(a - 1\right)\left(x + 3\right)\left(x - 2\right)$

33. $16m^4 - 25m^2 + 9$

$= \left(16m^2\right)^2 - 25\left(16m^2\right) + 144$

$$\begin{array}{r|ll} 144 & 4 & \\ 36 & 4 & 4^2 = 16 \\ 9 & 3 & 3^2 = 9 \\ 3 & 3 & \Rightarrow 16 + 9 = 25 \\ 1 & & \end{array}$$

$= \dfrac{\left(16m^2 - 16\right)\left(16m^2 - 9\right)}{16}$

$= \left(m^2 - 1\right)\left(16m^2 - 9\right)$

$= \left(m + 1\right)\left(m - 1\right)\left(4m + 3\right)\left(4m - 3\right)$

34. $3abx^2 - 12ab + 3bx^2 - 12b$

$= \left(3abx^2 + 3bx^2\right) - \left(12ab + 12b\right)$

$= 3bx^2\left(a + 1\right) - 12b\left(a + 1\right)$

$= \left(3bx^2 - 12b\right)\left(a + 1\right)$

$= 3b\left(x^2 - 4\right)\left(a + 1\right)$

$= 3b\left(x + 2\right)\left(x - 2\right)\left(a + 1\right)$

35. $3a^2m + 9am - 30m + 3a^2 + 9a - 30$

$= \left(3a^2m + 9am - 30m\right) + \left(3a^2 + 9a - 30\right)$

$= 3m\left(a^2 + 3a - 10\right) + 3\left(a^2 + 3a - 10\right)$

$= \left(3m + 3\right)\left(a^2 + 3a - 10\right)$

$= 3\left(m + 1\right)\left(a + 5\right)\left(a - 2\right)$

36. $a^3x^2 - 5a^3x + 6a^3 + x^2 - 5x + 6$

$= \left(a^3x^2 - 5a^3x + 6a^3\right) + \left(x^2 - 5x + 6\right)$

$= a^3\left(x^2 - 5x + 6\right) + \left(x^2 - 5x + 6\right)$

$= \left(a^3 + 1\right)\left(x^2 - 5x + 6\right)$

$= \left(a + 1\right)\left(a^2 - a + 1\right)\left(x - 3\right)\left(x - 2\right)$

37. $x^2\left(x^2 - y^2\right) - \left(2x - 1\right)\left(x^2 - y^2\right)$

$= \left(x^2 - y^2\right)\left[x^2 - \left(2x - 1\right)\right]$

$= \left(x + y\right)\left(x - y\right)\left(x^2 - 2x + 1\right)$

$= \left(x + y\right)\left(x - y\right)\left(x - 1\right)^2$

38. $a\left(x^3 + 1\right) + 3ax\left(x + 1\right)$

$= a\left(x + 1\right)\left(x^2 - x + 1\right) + 3ax\left(x + 1\right)$

$= \left(x + 1\right)\left[a\left(x^2 - x + 1\right) + 3ax\right]$

$= \left(x + 1\right)\left(ax^2 - ax + a + 3ax\right)$

$= \left(x + 1\right)\left(ax^2 + 2ax + a\right)$

$= a\left(x + 1\right)\left(x^2 + 2x + 1\right)$

$= a\left(x + 1\right)\left(x + 1\right)^2 = a\left(x + 1\right)^3$

EJERCICIO 109

1. $x^9 - xy^8$

$= x\left(x^8 - y^8\right)$

$= x\left(x^4 - y^4\right)\left(x^4 + y^4\right)$

$= x\left(x^2 + y^2\right)\left(x^2 - y^2\right)\left(x^4 + y^4\right)$

$= x\left(x^2 + y^2\right)\left(x + y\right)\left(x - y\right)\left(x^4 + y^4\right)$

2. $x^5 - 40x^3 + 144x$

$= x\left(x^4 - 40x^2 + 144\right)$

$= x\left(x^2 - 36\right)\left(x^2 - 4\right)$

$= x\left(x - 6\right)\left(x + 6\right)\left(x + 2\right)\left(x - 2\right)$

3. $a^6 + a^3b^3 - a^4 - ab^3$

$= \left(a^6 + a^3b^3\right) - \left(a^4 + ab^3\right)$

$= a^3\left(a^3 + b^3\right) - a\left(a^3 + b^3\right)$

$= \left(a^3 - a\right)\left(a^3 + b^3\right)$

$= a\left(a^2 - 1\right)\left(a + b\right)\left(a^2 - ab + b^2\right)$

$= a\left(a + 1\right)\left(a - 1\right)\left(a + b\right)\left(a^2 - ab + b^2\right)$

4. $4x^4 - 8x^2 + 4$

$= 4\left(x^4 - 2x^2 + 1\right)$

$= 4\left(x^2 - 1\right)\left(x^2 - 1\right)$

$= 4\left(x + 1\right)^2\left(x - 1\right)^2$

5. $a^7 - ab^6$

$= a\left(a^6 - b^6\right)$

$= a\left(a^3 + b^3\right)\left(a^3 - b^3\right)$

$= a\left(a + b\right)\left(a^2 - ab + b^2\right)\left(a - b\right)\left(a^2 + ab + b^2\right)$

6. $2a^4 - 2a^3 - 4a^2 - 2a^2b^2 + 2ab^2 + 4b^2$

$= \left(2a^4 - 2a^3 - 4a^2\right) - \left(2a^2b^2 - 2ab^2 - 4b^2\right)$

$= 2a^2\left(a^2 - a - 2\right) - 2b^2\left(a^2 - a - 2\right)$

$= 2\left(a^2 - a - 2\right)\left(a^2 - b^2\right)$

$= 2\left(a - 2\right)\left(a + 1\right)\left(a + b\right)\left(a - b\right)$

7. $x^6 + 5x^5 - 81x^2 - 405x$

$= \left(x^6 + 5x^5\right) - \left(81x^2 + 405x\right)$

$= x^5\left(x + 5\right) - 81x\left(x + 5\right)$

$= \left(x^5 - 81x\right)\left(x + 5\right)$

$= x\left(x^4 - 81\right)\left(x + 5\right)$

$= x\left(x^2 - 9\right)\left(x^2 + 9\right)\left(x + 5\right)$

$= x\left(x + 3\right)\left(x - 3\right)\left(x^2 + 9\right)\left(x + 5\right)$

8. $3 - 3a^6$

$= 3\left(1 - a^6\right)$

$= 3\left(1 - a^3\right)\left(1 + a^3\right)$

$= 3\left(1 - a\right)\left(1 + a + a^2\right)\left(1 + a\right)\left(1 - a + a^2\right)$

9. $4ax^2\left(a^2 - 2ax + x^2\right) - a^3 + 2a^2x - ax^2$

$= 4ax^2\left(a - x\right)^2 - a\left(a^2 - 2ax + x^2\right)$

$= 4ax^2\left(a - x\right)^2 - a\left(a - x\right)^2$

$= \left(4ax^2 - a\right)\left(a - x\right)^2$

$= a\left(4x^2 - 1\right)\left(a - x\right)^2$

$= a\left(2x + 1\right)\left(2x - 1\right)\left(a - x\right)^2$

10. $x^7 + x^4 - 81x^3 - 81$

$= \left(x^7 + x^4\right) - \left(81x^3 + 81\right)$

$= x^4\left(x^3 + 1\right) - 81\left(x^3 + 1\right)$

$= \left(x^4 - 81\right)\left(x^3 + 1\right)$

$= \left(x^2 + 9\right)\left(x^2 - 9\right)\left(x + 1\right)\left(x^2 - x + 1\right)$

$= \left(x^2 + 9\right)\left(x + 3\right)\left(x - 3\right)\left(x + 1\right)\left(x^2 - x + 1\right)$

11. $x^{17} - x$

$= x\left(x^{16} - 1\right)$

$= x\left(x^8 + 1\right)\left(x^8 - 1\right)$

$= x\left(x^8 + 1\right)\left(x^4 + 1\right)\left(x^4 - 1\right)$

$= x\left(x^8 + 1\right)\left(x^4 + 1\right)\left(x^2 + 1\right)\left(x^2 - 1\right)$

$= x\left(x^8 + 1\right)\left(x^4 + 1\right)\left(x^2 + 1\right)\left(x + 1\right)\left(x - 1\right)$

12. $3x^6 - 75x^4 - 48x^2 + 1.200$

$= \left(3x^6 - 75x^4\right) - \left(48x^2 - 1.200\right)$

$= 3x^4\left(x^2 - 25\right) - 48\left(x^2 - 25\right)$

$= \left(3x^4 - 48\right)\left(x^2 - 25\right)$

$= 3\left(x^4 - 16\right)\left(x + 5\right)\left(x - 5\right)$

$= 3\left(x^2 + 4\right)\left(x^2 - 4\right)\left(x + 5\right)\left(x - 5\right)$

$= 3\left(x^2 + 4\right)\left(x + 2\right)\left(x - 2\right)\left(x + 5\right)\left(x - 5\right)$

13. $a^6x^2 - x^2 + a^6x - x$

$= \left(a^6x^2 - x^2\right) + \left(a^6x - x\right)$

$= x^2\left(a^6 - 1\right) + x\left(a^6 - 1\right)$

$= \left(x^2 + x\right)\left(a^6 - 1\right)$

$= x\left(x + 1\right)\left(a^3 - 1\right)\left(a^3 + 1\right)$

$= x\left(x + 1\right)\left(a - 1\right)\left(a^2 + a + 1\right)\left(a + 1\right)\left(a^2 - a + 1\right)$

14. $\left(a^2 - ax\right)\left(x^4 - 82x^2 + 81\right)$

$= a\left(a - x\right)\left(x^2 - 81\right)\left(x^2 - 1\right)$

$= a\left(a - x\right)\left(x + 9\right)\left(x - 9\right)\left(x + 1\right)\left(x - 1\right)$

EJERCICIO 110

1. $x^3 + x^2 - x - 1$

$$\begin{array}{rrrr|l} 1 & 1 & -1 & -1 & +1 \\ & 1 & 2 & 1 & \\ \hline 1 & 2 & 1 & 0 & \end{array}$$

$= \left(x^2 + 2x + 1\right)\left(x - 1\right)$

$= \left(x + 1\right)^2\left(x - 1\right)$

2. $x^3 - 4x^2 + x + 6$

$$\begin{array}{rrrr|l} 1 & -4 & 1 & 6 & -1 \\ & -1 & 5 & -6 & \\ \hline 1 & -5 & 6 & 0 & \end{array}$$

$= \left(x^2 - 5x + 6\right)\left(x + 1\right)$

$= \left(x - 3\right)\left(x - 2\right)\left(x + 1\right)$

3. $a^3 - 3a^2 - 4a + 12$

$$\begin{array}{rrrr|l} 1 & -3 & -4 & 12 & +2 \\ & 2 & -2 & -12 & \\ \hline 1 & -1 & -6 & 0 & \end{array}$$

$= \left(a^2 - a - 6\right)\left(a - 2\right)$

$= \left(a - 3\right)\left(a + 2\right)\left(a - 2\right)$

4. $m^3 - 12m + 16$

$$\begin{array}{rrrr|l} 1 & 0 & -12 & 16 & +2 \\ & 2 & 4 & -16 & \\ \hline 1 & 2 & -8 & 0 & \end{array}$$

$= \left(m^2 + 2m - 8\right)\left(m - 2\right)$

$= \left(m + 4\right)\left(m - 2\right)\left(m - 2\right)$

$= \left(m + 4\right)\left(m - 2\right)^2$

5. $2x^3-x^2-18x+9$

$$\begin{array}{rrrr|r} 2 & -1 & -18 & 9 & +3 \\ & 6 & 15 & -9 & \\ \hline 2 & 5 & -3 & 0 & \end{array}$$

$=\left(2x^2+5x-3\right)\left(x-3\right)$

$$\begin{array}{rrr|r} 2 & 5 & -3 & -3 \\ & -6 & 3 & \\ \hline 2 & -1 & 0 & \end{array}$$

$=\left(2x-1\right)\left(x-3\right)\left(x+3\right)$

6. $a^3+a^2-13a-28$

$$\begin{array}{rrrr|r} 1 & 1 & -13 & -28 & +4 \\ & 4 & 20 & 28 & \\ \hline 1 & 5 & 7 & 0 & \end{array}$$

$=\left(a^2+5a+7\right)\left(a-4\right)$

7. x^3+2x^2+x+2

$$\begin{array}{rrrr|r} 1 & 2 & 1 & 2 & -2 \\ & -2 & 0 & -2 & \\ \hline 1 & 0 & 1 & 0 & \end{array}$$

$=\left(x^2+1\right)\left(x+2\right)$

8. n^3-7n+6

$$\begin{array}{rrrr|r} 1 & 0 & -7 & 6 & +1 \\ & 1 & 1 & -6 & \\ \hline 1 & 1 & -6 & 0 & \end{array}$$

$=\left(n^2+n-6\right)\left(n-1\right)$

$=\left(n+3\right)\left(n-2\right)\left(n-1\right)$

9. x^3-6x^2+32

$$\begin{array}{rrrr|r} 1 & -6 & 0 & 32 & -2 \\ & -2 & 16 & -32 & \\ \hline 1 & -8 & 16 & 0 & \end{array}$$

$=\left(x^2-8x+16\right)\left(x+2\right)$

$=\left(x-4\right)^2\left(x+2\right)$

10. $6x^3+23x^2+9x-18$

$$\begin{array}{rrrr|r} 6 & 23 & 9 & -18 & -3 \\ & -18 & -15 & 18 & \\ \hline 6 & 5 & -6 & 0 & \end{array}$$

$=\left(6x^2+5x-6\right)\left(x+3\right)$

$=\left[(6x)^2+5(6x)-36\right]\left(x+3\right)$

$=\dfrac{\left(6x+9\right)\left(6x-4\right)\left(x+3\right)}{3\cdot 2}$

$=\left(2x+3\right)\left(3x-2\right)\left(x+3\right)$

11. $x^4-4x^3+3x^2+4x-4$

$$\begin{array}{rrrrr|r} 1 & -4 & 3 & 4 & -4 & +1 \\ & 1 & -3 & 0 & 4 & \\ \hline 1 & -3 & 0 & 4 & 0 & \end{array}$$

$=\left(x^3-3x^2+4\right)\left(x-1\right)$

$$\begin{array}{rrrr|r} 1 & -3 & 0 & 4 & -1 \\ & -1 & 4 & -4 & \\ \hline 1 & -4 & 4 & 0 & \end{array}$$

$=\left(x^2-4x+4\right)\left(x-1\right)\left(x+1\right)$

$=\left(x-2\right)^2\left(x-1\right)\left(x+1\right)$

12. $x^4-2x^3-13x^2+14x+24$

$$\begin{array}{rrrrr|r} 1 & -2 & -13 & 14 & 24 & +2 \\ & 2 & 0 & -26 & -24 & \\ \hline 1 & 0 & -13 & -12 & 0 & \end{array}$$

$=\left(x^3-13x-12\right)\left(x-2\right)$

$$\begin{array}{rrrr|r} 1 & 0 & -13 & -12 & -1 \\ & -1 & 1 & 12 & \\ \hline 1 & -1 & -12 & 0 & \end{array}$$

$=\left(x^2-x-12\right)\left(x-2\right)\left(x+1\right)$

$=\left(x-4\right)\left(x+3\right)\left(x-2\right)\left(x+1\right)$

13. $a^4-15a^2-10a+24$

$$\begin{array}{rrrrr|r} 1 & 0 & -15 & -10 & 24 & +1 \\ & 1 & 1 & -14 & -24 & \\ \hline 1 & 1 & -14 & -24 & 0 & \end{array}$$

$=\left(a^3+a^2-14a-24\right)\left(a-1\right)$

$$\begin{array}{rrrr|r} 1 & 1 & -14 & -24 & -2 \\ & -2 & 2 & 24 & \\ \hline 1 & -1 & -12 & 0 & \end{array}$$

$=\left(a^2-a-12\right)\left(a-1\right)\left(a+2\right)$

$=\left(a-4\right)\left(a+3\right)\left(a-1\right)\left(a+2\right)$

14. $n^4-27n^2-14n+120$

$$\begin{array}{rrrrr|r} 1 & 0 & -27 & -14 & 120 & +2 \\ & 2 & 4 & -46 & -120 & \\ \hline 1 & 2 & -23 & -60 & 0 & \end{array}$$

$=\left(n^3+2n^2-23n-60\right)\left(n-2\right)$

$$\begin{array}{rrrr|r} 1 & 2 & -23 & -60 & -3 \\ & -3 & 3 & 60 & \\ \hline 1 & -1 & -20 & 0 & \end{array}$$

$=\left(n^2-n-20\right)\left(n-2\right)\left(n+3\right)$

$=\left(n-5\right)\left(n+4\right)\left(n-2\right)\left(n+3\right)$

15. $x^4+6x^3+3x+140$

$$\begin{array}{rrrrr|r} 1 & 6 & 0 & 3 & 140 & -4 \\ & -4 & -8 & 32 & -140 & \\ \hline 1 & 2 & -8 & 35 & 0 & \end{array}$$

$=\left(x^3+2x^2-8x+35\right)\left(x+4\right)$

$$\begin{array}{rrrr|r} 1 & 2 & -8 & 35 & -5 \\ & -5 & 15 & -35 & \\ \hline 1 & -3 & 7 & 0 & \end{array}$$

$=\left(x^2-3x+7\right)\left(x+4\right)\left(x+5\right)$

16. $8a^4-18a^3-75a^2+46a+120$

$$\begin{array}{rrrrr|r} 8 & -18 & -75 & 46 & 120 & -2 \\ & -16 & 68 & 14 & -120 & \\ \hline 8 & -34 & -7 & 60 & 0 & \end{array}$$

$=\left(8a^3-34a^2-7a+60\right)\left(a+2\right)$

$$\begin{array}{rrrr|r} 8 & -34 & -7 & 60 & +4 \\ & 32 & -8 & -60 & \\ \hline 8 & -2 & -15 & 0 & \end{array}$$

$=\left(8a^2-2a-15\right)\left(a+2\right)\left(a-4\right)$

$=\left[(8a)^2-2(8a)-120\right]\left(a+2\right)\left(a-4\right)$

$$\begin{array}{r|ll} 120 & 4 & \\ 30 & 3 & 4\cdot 3=12 \\ 10 & 2 & 5\cdot 2=10 \\ 5 & 5 & \Rightarrow 12-10=2 \\ 1 & & \end{array}$$

$=\dfrac{\left(8a-12\right)\left(8a+10\right)\left(a+2\right)\left(a-4\right)}{4\cdot 2}$

$=\left(2a-3\right)\left(4a+5\right)\left(a+2\right)\left(a-4\right)$

17. x^4-22x^2-75

$$\begin{array}{rrrrr|r} 1 & 0 & -22 & 0 & -75 & +5 \\ & 5 & 25 & 15 & 75 & \\ \hline 1 & 5 & 3 & 15 & 0 & \end{array}$$

$=\left(x^3+5x^2+3x+15\right)\left(x-5\right)$

$$\begin{array}{rrrr|r} 1 & 5 & 3 & 15 & -5 \\ & -5 & 0 & -15 & \\ \hline 1 & 0 & 3 & 0 & \end{array}$$

$=\left(x^2+3\right)\left(x-5\right)\left(x+5\right)$

18. $15x^4+94x^3-5x^2-164x+60$

$15 \quad 94 \quad -5 \quad -164 \quad 60 \;\mid\; +1$
$\quad 15 \quad 109 \quad 104 \quad -60$
$\overline{15 \quad 109 \quad 104 \quad -60 \quad 0}$

$=\left(15x^3+109x^2+104x-60\right)\left(x-1\right)$

$15 \quad 109 \quad 104 \quad -60 \;\mid\; -6$
$\quad -90 \quad -114 \quad 60$
$\overline{15 \quad 19 \quad -10 \quad 0}$

$=\left(15x^2+19x-10\right)\left(x-1\right)\left(x+6\right)$

$=\left[(15x)^2+19(15x)-150\right]\left(x-1\right)\left(x+6\right)$

$=\dfrac{\left(15x+25\right)\left(15x-6\right)\left(x-1\right)\left(x+6\right)}{5\cdot 3}$

$=\left(3x+5\right)\left(5x-2\right)\left(x-1\right)\left(x+6\right)$

19. $x^5-21x^3+16x^2+108x-144$

$1 \quad 0 \quad -21 \quad 16 \quad 108 \quad -144 \;\mid\; +2$
$\quad 2 \quad 4 \quad -34 \quad -36 \quad 144$
$\overline{1 \quad 2 \quad -17 \quad -18 \quad 72 \quad 0}$

$=\left(x^4+2x^3-17x^2-18x+72\right)\left(x-2\right)$

$1 \quad 2 \quad -17 \quad -18 \quad 72 \;\mid\; -3$
$\quad -3 \quad 3 \quad 42 \quad -72$
$\overline{1 \quad -1 \quad -14 \quad 24 \quad 0}$

$=\left(x^3-x^2-14x+24\right)\left(x-2\right)\left(x+3\right)$

$1 \quad -1 \quad -14 \quad 24 \;\mid\; +2$
$\quad 2 \quad 2 \quad -24$
$\overline{1 \quad 1 \quad -12 \quad 0}$

$=\left(x^2+x-12\right)\left(x-2\right)\left(x+3\right)\left(x-2\right)$

$=\left(x+4\right)\left(x-3\right)\left(x-2\right)\left(x+3\right)\left(x-2\right)$

$=\left(x+4\right)\left(x-3\right)\left(x+3\right)\left(x-2\right)^2$

20. $a^5-23a^3-6a^2+112a+96$

$1 \quad 0 \quad -23 \quad -6 \quad 112 \quad 96 \;\mid\; -1$
$\quad -1 \quad 1 \quad 22 \quad -16 \quad -96$
$\overline{1 \quad -1 \quad -22 \quad 16 \quad 96 \quad 0}$

$=\left(a^4-a^3-22a^2+16a+96\right)\left(a+1\right)$

$1 \quad -1 \quad -22 \quad 16 \quad 96 \;\mid\; -2$
$\quad -2 \quad 6 \quad 32 \quad -96$
$\overline{1 \quad -3 \quad -16 \quad 48 \quad 0}$

$=\left(a^3-3a^2-16a+48\right)\left(a+1\right)\left(a+2\right)$

$1 \quad -3 \quad -16 \quad 48 \;\mid\; +3$
$\quad 3 \quad 0 \quad -48$
$\overline{1 \quad 0 \quad -16 \quad 0}$

$=\left(a^2-16\right)\left(a+1\right)\left(a+2\right)\left(a-3\right)$

$=\left(a+4\right)\left(a-4\right)\left(a+1\right)\left(a+2\right)\left(a-3\right)$

21. $4x^5+3x^4-108x^3-25x^2+522x+360$

$4 \quad 3 \quad -108 \quad -25 \quad 522 \quad 360 \;\mid\; -2$
$\quad -8 \quad 10 \quad 196 \quad -342 \quad -360$
$\overline{4 \quad -5 \quad -98 \quad 171 \quad 180 \quad 0}$

$=\left(4x^4-5x^3-98x^2+171x+180\right)\left(x+2\right)$

$4 \quad -5 \quad -98 \quad 171 \quad 180 \;\mid\; +3$
$\quad 12 \quad 21 \quad -231 \quad -180$
$\overline{4 \quad 7 \quad -77 \quad -60 \quad 0}$

$=\left(4x^3+7x^2-77x-60\right)\left(x+2\right)\left(x-3\right)$

$4 \quad 7 \quad -77 \quad -60 \;\mid\; +4$
$\quad 16 \quad 92 \quad 60$
$\overline{4 \quad 23 \quad 15 \quad 0}$

$=\left(4x^2+23x+15\right)\left(x+2\right)\left(x-3\right)\left(x-4\right)$

$=\left[(4x)^2+23(4x)+60\right]\left(x+2\right)\left(x-3\right)\left(x-4\right)$

$=\left[\left(4x+20\right)\left(4x+3\right)\left(x+2\right)\left(x-3\right)\left(x-4\right)\right]\div 4$

$=\left(x+5\right)\left(4x+3\right)\left(x+2\right)\left(x-3\right)\left(x-4\right)$

22. $n^5-30n^3-25n^2-36n-180$

$1 \quad 0 \quad -30 \quad -25 \quad -36 \quad -180 \;\mid\; -2$
$\quad -2 \quad 4 \quad 52 \quad -54 \quad 180$
$\overline{1 \quad -2 \quad -26 \quad 27 \quad -90 \quad 0}$

$=\left(n^4-2n^3-26n^2+27n-90\right)\left(n+2\right)$

$1 \quad -2 \quad -26 \quad 27 \quad -90 \;\mid\; -5$
$\quad -5 \quad 35 \quad -45 \quad 90$
$\overline{1 \quad -7 \quad 9 \quad -18 \quad 0}$

$=\left(n^3-7n^2+9n-18\right)\left(n+2\right)\left(n+5\right)$

$1 \quad -7 \quad 9 \quad -18 \;\mid\; +6$
$\quad 6 \quad -6 \quad 18$
$\overline{1 \quad -1 \quad 3 \quad 0}$

$=\left(n^2-n+3\right)\left(n+2\right)\left(n+5\right)\left(n-6\right)$

23. $6x^5-13x^4-81x^3+112x^2+180x-144$

$6 \quad -13 \quad -81 \quad 112 \quad 180 \quad -144 \;\mid\; +2$
$\quad 12 \quad -2 \quad -166 \quad -108 \quad 144$
$\overline{6 \quad -1 \quad -83 \quad -54 \quad 72 \quad 0}$

$=\left(6x^4-x^3-83x^2-54x+72\right)\left(x-2\right)$

$6 \quad -1 \quad -83 \quad -54 \quad 72 \;\mid\; -3$
$\quad -18 \quad 57 \quad 78 \quad -72$
$\overline{6 \quad -19 \quad -26 \quad 24 \quad 0}$

$=\left(6x^3-19x^2-26x+24\right)\left(x-2\right)\left(x+3\right)$

$6 \quad -19 \quad -26 \quad 24 \;\mid\; +4$
$\quad 24 \quad 20 \quad -24$
$\overline{6 \quad 5 \quad -6 \quad 0}$

$=\left(6x^2+5x-6\right)\left(x-2\right)\left(x+3\right)\left(x-4\right)$

$=\left[(6x)^2+5(6x)-36\right]\left(x-2\right)\left(x+3\right)\left(x-4\right)$

$=\dfrac{\left(6x+9\right)\left(6x-4\right)\left(x-2\right)\left(x+3\right)\left(x-4\right)}{3\cdot 2}$

$=\left(2x+3\right)\left(3x-2\right)\left(x-2\right)\left(x+3\right)\left(x-4\right)$

24. $x^5-25x^3+x^2-25$

$1 \quad 0 \quad -25 \quad 1 \quad 0 \quad -25 \;\mid\; -1$
$\quad -1 \quad 1 \quad 24 \quad -25 \quad 25$
$\overline{1 \quad -1 \quad -24 \quad 25 \quad -25 \quad 0}$

$=\left(x^4-x^3-24x^2+25x-25\right)\left(x+1\right)$

$1 \quad -1 \quad -24 \quad 25 \quad -25 \;\mid\; +5$
$\quad 5 \quad 20 \quad -20 \quad 25$
$\overline{1 \quad 4 \quad -4 \quad 5 \quad 0}$

$=\left(x^3+4x^2-4x+5\right)\left(x+1\right)\left(x-5\right)$

$1 \quad 4 \quad -4 \quad 5 \;\mid\; -5$
$\quad -5 \quad 5 \quad -5$
$\overline{1 \quad -1 \quad 1 \quad 0}$

$=\left(x^2-x+1\right)\left(x+1\right)\left(x-5\right)\left(x+5\right)$

25. $2a^5-8a^4+3a-12$

$2 \quad -8 \quad 0 \quad 0 \quad 3 \quad -12 \;\mid\; +4$
$\quad 8 \quad 0 \quad 0 \quad 0 \quad 12$
$\overline{2 \quad 0 \quad 0 \quad 0 \quad 3 \quad 0}$

$=\left(2a^4+3\right)\left(a-4\right)$

26. $x^5+2x^4-15x^3-3x^2-6x+45$

$1 \quad 2 \quad -15 \quad -3 \quad -6 \quad 45 \;\mid\; +3$
$\quad 3 \quad 15 \quad 0 \quad -9 \quad -45$
$\overline{1 \quad 5 \quad 0 \quad -3 \quad -15 \quad 0}$

$=\left(x^4+5x^3-3x-15\right)\left(x-3\right)$

$1 \quad 5 \quad 0 \quad -3 \quad -15 \;\mid\; -5$
$\quad -5 \quad 0 \quad 0 \quad 15$
$\overline{1 \quad 0 \quad 0 \quad -3 \quad 0}$

$=\left(x^3-3\right)\left(x-3\right)\left(x+5\right)$

27. $x^6+6x^5+4x^4-42x^3-113x^2-108x-36$

$1 \quad 6 \quad 4 \quad -42 \quad -113 \quad -108 \quad -36 \;\mid\; -1$
$\quad -1 \quad -5 \quad 1 \quad 41 \quad 72 \quad 36$
$\overline{1 \quad 5 \quad -1 \quad -41 \quad -72 \quad -36 \quad 0}$

$=\left(x^5+5x^4-x^3-41x^2-72x-36\right)\left(x+1\right)$

$1 \quad 5 \quad -1 \quad -41 \quad -72 \quad -36 \;\mid\; -1$
$\quad -1 \quad -4 \quad 5 \quad 36 \quad 36$
$\overline{1 \quad 4 \quad -5 \quad -36 \quad -36 \quad 0}$

$=\left(x^4+4x^3-5x^2-36x-36\right)\left(x+1\right)\left(x+1\right)$

$1 \quad 4 \quad -5 \quad -36 \quad -36 \;\mid\; -2$
$\quad -2 \quad -4 \quad 18 \quad 36$
$\overline{1 \quad 2 \quad -9 \quad -18 \quad 0}$

$=\left(x^3+2x^2-9x-18\right)\left(x+1\right)^2\left(x+2\right)$

$1 \quad 2 \quad -9 \quad -18 \;\mid\; -2$
$\quad -2 \quad 0 \quad 18$
$\overline{1 \quad 0 \quad -9 \quad 0}$

$=\left(x^2-9\right)\left(x+1\right)^2\left(x+2\right)^2$

$=\left(x+3\right)\left(x-3\right)\left(x+1\right)^2\left(x+2\right)^2$

28. $a^6-32a^4+18a^3+247a^2-162a-360$

$$\begin{array}{rrrrrrr|r} 1 & 0 & -32 & 18 & 247 & -162 & -360 & -1 \\ & -1 & 1 & 31 & -49 & -198 & 360 & \\ \hline 1 & -1 & -31 & 49 & 198 & -360 & 0 & \end{array}$$

$=\left(a^5-a^4-31a^3+49a^2+198a-360\right)\left(a+1\right)$

$$\begin{array}{rrrrrr|r} 1 & -1 & -31 & 49 & 198 & -360 & +2 \\ & 2 & 2 & -58 & -18 & 360 & \\ \hline 1 & 1 & -29 & -9 & 180 & 0 & \end{array}$$

$=\left(a^4+a^3-29a^2-9a+180\right)\left(a+1\right)\left(a-2\right)$

$$\begin{array}{rrrrr|r} 1 & 1 & -29 & -9 & 180 & +3 \\ & 3 & 12 & -51 & -180 & \\ \hline 1 & 4 & -17 & -60 & 0 & \end{array}$$

$=\left(a^3+4a^2-17a-60\right)\left(a+1\right)\left(a-2\right)\left(a-3\right)$

$$\begin{array}{rrrr|r} 1 & 4 & -17 & -60 & -3 \\ & -3 & -3 & 60 & \\ \hline 1 & 1 & -20 & 0 & \end{array}$$

$=\left(a^2+a-20\right)\left(a+1\right)\left(a-2\right)\left(a-3\right)\left(a+3\right)$

$=\left(a+5\right)\left(a-4\right)\left(a+1\right)\left(a-2\right)\left(a-3\right)\left(a+3\right)$

30. $2x^6-10x^5-34x^4+146x^3+224x^2-424x-480$

$$\begin{array}{rrrrrrr|r} 2 & -10 & -34 & 146 & 224 & -424 & -480 & -1 \\ & -2 & 12 & 22 & -168 & -56 & 480 & \\ \hline 2 & -12 & -22 & 168 & 56 & -480 & 0 & \end{array}$$

$=\left(2x^5-12x^4-22x^3+168x^2+56x-480\right)\left(x+1\right)$

$$\begin{array}{rrrrrr|r} 2 & -12 & -22 & 168 & 56 & -480 & -2 \\ & -4 & 32 & -20 & -296 & 480 & \\ \hline 2 & -16 & 10 & 148 & -240 & 0 & \end{array}$$

$=\left(2x^4-16x^3+10x^2+148x-240\right)\left(x+1\right)\left(x+2\right)$

$$\begin{array}{rrrrr|r} 2 & -16 & 10 & 148 & -240 & +2 \\ & 4 & -24 & -28 & 240 & \\ \hline 2 & -12 & -14 & 120 & 0 & \end{array}$$

$=\left(2x^3-12x^2-14x+120\right)\left(x+1\right)\left(x+2\right)\left(x-2\right)$

$$\begin{array}{rrrr|r} 2 & -12 & -14 & 120 & +4 \\ & 8 & -16 & -120 & \\ \hline 2 & -4 & -30 & 0 & \end{array}$$

$=\left(2x^2-4x-30\right)\left(x+1\right)\left(x+2\right)\left(x-2\right)\left(x-4\right)$

$=\left[(2x)^2-4(2x)-60\right]\left(x+1\right)\left(x+2\right)\left(x-2\right)\left(x-4\right)$

$=\dfrac{\left(2x-10\right)\left(2x+6\right)\left(x+1\right)\left(x+2\right)\left(x-2\right)\left(x-4\right)}{2}$

$=\left(x-5\right)\left(2x+6\right)\left(x+1\right)\left(x+2\right)\left(x-2\right)\left(x-4\right)$

$=2\left(x-5\right)\left(x+3\right)\left(x+1\right)\left(x+2\right)\left(x-2\right)\left(x-4\right)$

29. $x^6-41x^4+184x^2-144$

$$\begin{array}{rrrrrrr|r} 1 & 0 & -41 & 0 & 184 & 0 & -144 & +1 \\ & 1 & 1 & -40 & -40 & 144 & 144 & \\ \hline 1 & 1 & -40 & -40 & 144 & 144 & 0 & \end{array}$$

$=\left(x^5+x^4-40x^3-40x^2+144x+144\right)\left(x-1\right)$

$$\begin{array}{rrrrrr|r} 1 & 1 & -40 & -40 & 144 & 144 & -1 \\ & -1 & 0 & 40 & 0 & -144 & \\ \hline 1 & 0 & -40 & 0 & 144 & 0 & \end{array}$$

$=\left(x^4-40x^2+144\right)\left(x-1\right)\left(x+1\right)$

$=\left(x^2-36\right)\left(x^2-4\right)\left(x-1\right)\left(x+1\right)$

$=\left(x+6\right)\left(x-6\right)\left(x+2\right)\left(x-2\right)\left(x-1\right)\left(x+1\right)$

31. $a^6-8a^5+6a^4+103a^3-344a^2+396a-144$

$$\begin{array}{rrrrrrr|r} 1 & -8 & 6 & 103 & -344 & 396 & -144 & +2 \\ & 2 & -12 & -12 & 182 & -324 & 144 & \\ \hline 1 & -6 & -6 & 91 & -162 & 72 & 0 & \end{array}$$

$=\left(a^5-6a^4-6a^3+91a^2-162a+72\right)\left(a-2\right)$

$$\begin{array}{rrrrrr|r} 1 & -6 & -6 & 91 & -162 & 72 & +2 \\ & 2 & -8 & -28 & 126 & -72 & \\ \hline 1 & -4 & -14 & 63 & -36 & 0 & \end{array}$$

$=\left(a^4-4a^3-14a^2+63a-36\right)\left(a-2\right)\left(a-2\right)$

$$\begin{array}{rrrrr|r} 1 & -4 & -14 & 63 & -36 & +3 \\ & 3 & -3 & -51 & 36 & \\ \hline 1 & -1 & -17 & 12 & 0 & \end{array}$$

$=\left(a^3-a^2-17a+12\right)\left(a-2\right)^2\left(a-3\right)$

$$\begin{array}{rrrr|r} 1 & -1 & -17 & 12 & -4 \\ & -4 & 20 & -12 & \\ \hline 1 & -5 & 3 & 0 & \end{array}$$

$=\left(a^2-5a+3\right)\left(a-2\right)^2\left(a-3\right)\left(a+4\right)$

32. $x^7-20x^5-2x^4+64x^3+40x^2-128$

$$\begin{array}{rrrrrrrr|r} 1 & 0 & -20 & -2 & 64 & 40 & 0 & -128 & +2 \\ & 2 & 4 & -32 & -68 & -8 & 64 & 128 & \\ \hline 1 & 2 & -16 & -34 & -4 & 32 & 64 & 0 & \end{array}$$

$=\left(x^6+2x^5-16x^4-34x^3-4x^2+32x+64\right)\left(x-2\right)$

$$\begin{array}{rrrrrrr|r} 1 & 2 & -16 & -34 & -4 & 32 & +64 & -2 \\ & -2 & 0 & 32 & 4 & 0 & -64 & \\ \hline 1 & 0 & -16 & -2 & 0 & 32 & 0 & \end{array}$$

$=\left(x^5-16x^3-2x^2+32\right)\left(x-2\right)\left(x+2\right)$

$$\begin{array}{rrrrrr|r} 1 & 0 & -16 & -2 & 0 & 32 & +4 \\ & 4 & 16 & 0 & -8 & -32 & \\ \hline 1 & 4 & 0 & -2 & -8 & 0 & \end{array}$$

$=\left(x^4+4x^3-2x-8\right)\left(x-2\right)\left(x+2\right)\left(x-4\right)$

$$\begin{array}{rrrrr|r} 1 & 4 & 0 & -2 & -8 & -4 \\ & -4 & 0 & 0 & 8 & \\ \hline 1 & 0 & 0 & -2 & 0 & \end{array}$$

$=\left(x^3-2\right)\left(x-2\right)\left(x+2\right)\left(x-4\right)\left(x+4\right)$

EJERCICIO 111

1. a^2x, ax^2 *mcd* ax

2. ab^2c, a^2bc *mcd* abc

3. $2x^2y, x^2y^3$ *mcd* x^2y

4. $6a^2b^3, 15a^3b^4$

6	3	15	3
2	2	5	5
1		1	

$3\cdot 2=6 \quad 3\cdot 5=15$

mcd $3 \Rightarrow 3a^2b^3$

5. $8am^3n, 20x^2m^2$

8	2	20	2
4	2	10	2
2	2	5	5
1		1	

$2^3=8 \quad 2^2\cdot 5=20$

mcd $2^2=4$

$\Rightarrow 4m^2$

6. $18mn^2, 27a^2m^3n^4$

18	3	27	3
6	3	9	3
2	2	3	3
1		1	

$3^2\cdot 2=18 \quad 3^3=27$

mcd $3^2=9$

$\Rightarrow 9mn^2$

7. $15a^2b^3c, 24ab^2x, 36b^4x^2$

15	5	24	3	36	3
3	3	8	4	12	4
1		2	2	3	3
		1		1	

$5\cdot 3=15 \quad 3\cdot 4\cdot 2=24 \quad 3^2\cdot 4=36$

mcd $3 \Rightarrow 3b^2$

8. $12x^2yz^3, 18xy^2z, 24x^3yz^2$

12	3	18	3	24	3
4	2	6	3	8	4
2	2	2	2	2	2
1		1		1	

$3\cdot 2^2=12 \quad 3^2\cdot 2=18 \quad 3\cdot 4\cdot 2=24$

mcd $3\cdot 2 \Rightarrow 6xyz$

9. $28a^2b^3c^4, 35a^3b^4c^5, 42a^4b^5c^6$

28	7	35	7	42	7
4	4	5	5	6	6
1		1		1	

$7\cdot 4=28 \quad 7\cdot 5=35 \quad 7\cdot 6=42$

mcd $7 \Rightarrow 7a^2b^3c^4$

10. $72x^3y^4z^4, 96x^2y^2z^3, 120x^4y^5z^7$

72	8	96	8	120	8
9	3	12	3	15	3
3	3	4	4	5	5
1		1		1	

$8\cdot 3^2=72 \quad 8\cdot 3\cdot 4=96 \quad 8\cdot 3\cdot 5=120$

mcd $8\cdot 3 \Rightarrow 24x^2y^2z^3$

11. $42am^2n, 56m^3n^2x, 70m^4n^2y$

42	14	56	14	70	14
3	3	4	4	5	5
1		1		1	

$14\cdot 3=42 \quad 14\cdot 4=56 \quad 14\cdot 5=70$

mcd $14 \Rightarrow 14m^2n$

12. $75a^4b^3c^2, 150a^5b^7x^2, 225a^3b^6y^2$

75	15	150	15	225	15
5	5	10	5	15	5
1		2	2	3	3
		1		1	

$15\cdot 5=75 \quad 15\cdot 5\cdot 2=150 \quad 15\cdot 5\cdot 3=225$

mcd $15\cdot 5 \Rightarrow 75a^3b^3$

13. $4a^2b, 8a^3b^2, 2a^2bc, 10ab^3c^2$

4	2	8	2	2	2	10	2
2	2	4	4	1		5	5
1		1				1	

$2^2=4 \quad 2\cdot 4=8 \quad 2=2 \quad 2\cdot 5=10$

mcd $2 \Rightarrow 2ab$

14. $38a^2x^6y^4, 76mx^4y^7, 95x^5y^6$

38	19	76	19	95	19
2	2	4	4	5	5
1		1		1	

$19\cdot 2=38 \quad 19\cdot 4=76 \quad 19\cdot 5=95$

mcd $19 \Rightarrow 19x^4y^4$

EJERCICIO 112

1. $2a^2+2ab=2a(a+b)$

$4a^2-4ab=2^2a(a-b)$

mcd $2a$

2. $6x^3y-6x^2y=2\cdot 3x^2y(x-1)$

$9x^3y^2+18x^2y^2=3^2x^2y^2(x+2)$

mcd $3x^2y$

3. $12a^2b^3=4\cdot 3a^2b^3$

$4a^3b^2-8a^2b^3=4a^2b^2(a-2b)$

mcd $4a^2b^2$

4. $ab+b=b(a+1)$

$a^2+a=a(a+1)$

mcd $a+1$

5. $x^2-x=x(x-1)$

$x^3-x^2=x^2(x-1)$

mcd $x(x-1)$

6. $30ax^2-15x^3=5\cdot 3x^2(2a-x)$

$10axy^2-20x^2y^2=5\cdot 2xy^2(a-2x)$

mcd $5x$

7. $18a^2x^3y^4=6\cdot 3a^2x^3y^4$

$6a^2x^2y^4-18a^2xy^4=6a^2xy^4(x-3)$

mcd $6a^2xy^4$

8. $5a^2-15a=5a(a-3)$

$a^3-3a^2=a^2(a-3)$

mcd $a(a-3)$

9. $3x^3+15x^2=3x^2(x+5)$

$ax^2+5ax=ax(x+5)$

mcd $x(x+5)$

10. $a^2-b^2=(a+b)(a-b)$

$a^2-2ab+b^2=(a-b)^2$

mcd $(a-b)$

11. $m^3+n^3=(m+n)(m^2-mn+n^2)$

$3am+3an=3a(m+n)$

mcd $m+n$

12. $x^2-4=(x+2)(x-2)$

$x^3-8=(x-2)(x^2+2x+4)$

mcd $x-2$

13. $2ax^2+4ax=2ax(x+2)$

$x^3-x^2-6x=x(x^2-x-6)=x(x-3)(x+2)$

$mcd \quad x(x+2)$

14. $9x^2-1=(3x+1)(3x-1)$

$9x^2-6x+1=(3x-1)^2$

$mcd \quad 3x-1$

15. $4a^2+4ab+b^2=(2a+b)^2$

$2a^2-2ab+ab-b^2=(2a^2-2ab)+(ab-b^2)$

$=2a(a-b)+b(a-b)$

$=(2a+b)(a-b)$

$mcd \quad 2a+b$

16. $3x^2+3x-60=3(x^2+x-20)$

$=3(x+5)(x-4)$

$6x^2-18x-24=6(x^2-3x-4)$

$=3\cdot 2(x-4)(x+1)$

$mcd \quad 3(x-4)$

17. $8x^3+y^3=(2x+y)(4x^2-2xy+y^2)$

$4ax^2-ay^2=a(4x^2-y^2)$

$=a(2x+y)(2x-y)$

$mcd \quad 2x+y$

18. $2a^3-12a^2b+18ab^2=2a(a^2-6ab+9b^2)$

$=2a(a-3b)^2$

$a^3x-9ab^2x=ax(a^2-9b^2)$

$=ax(a+3b)(a-3b)$

$mcd \quad a(a-3b)$

19. $ac+ad-2bc-2bd=(ac+ad)-(2bc+2bd)$

$=a(c+d)-2b(c+d)$

$=(a-2b)(c+d)$

$2c^2+4cd+2d^2=2(c^2+2cd+d^2)$

$=2(c+d)^2$

$mcd \quad c+d$

20. $3a^2m^2+6a^2m-45a^2=3a^2(m^2+2m-15)$

$=3a^2(m+5)(m-3)$

$6am^2x+24amx-30ax=6ax(m^2-4m-5)$

$=3\cdot 2ax(m+5)(m-1)$

$mcd \quad 3a(m+5)$

21. $4x^4-y^2=(2x^2+y)(2x^2-y)$

$(2x^2+y)^2=(2x^2+y)(2x^2+y)$

$mcd \quad 2x^2+y$

22. $3x^5-3x=3x(x^4-1)=3x(x^2+1)(x^2-1)$

$=3x(x^2+1)(x+1)(x-1)$

$9x^3-9x=9x(x^2-1)=3^2x(x+1)(x-1)$

$mcd \quad 3x(x+1)(x-1)$

23. $a^2+ab=a(a+b)$

$ab+b^2=b(a+b)$

$a^3+a^2b=a^2(a+b)$

$mcd \quad a+b$

24. $2x^3-2x^2=2x^2(x-1)$

$3x^2-3x=3x(x-1)$

$4x^3-4x^2=4x^2(x-1)$

$mcd \quad x(x-1)$

25. $x^4-9x^2=x^2(x^2-9) \quad =x^2(x+3)(x-3)$

$x^4-5x^3+6x^2=x^2(x^2-5x+6)=x^2(x-3)(x-2)$

$x^4-6x^3+9x^2=x^2(x^2-6x+9)=x^2(x-3)^2$

$mcd \quad x^2(x-3)$

26. $a^3b+2a^2b^2+ab^3=ab(a^2+2ab+b^2)=ab(a+b)^2$

$a^4b-a^2b^3=a^2b(a^2-b^2) \quad =a^2b(a+b)(a-b)$

$mcd \quad ab(a+b)$

27. $2x^2+2x-4=2(x^2+x-2) \quad =2(x+2)(x-1)$

$2x^2-8x+6=2(x^2-4x+3)=2(x-3)(x-1)$

$2x^3-2=2(x^2-1) \quad =2(x+1)(x-1)$

$mcd \quad 2(x-1)$

28. $ax^3-2ax^2-8ax=ax(x^2-2x-8) \quad =ax(x-4)(x+2)$

$ax^2-ax-6a=a(x^2-x-6) \quad =a(x-3)(x+2)$

$a^2x^3-3a^2x^2-10a^2x=a^2x(x^2-3x-10)=a^2x(x-5)(x+2)$

$mcd \quad a(x+2)$

29. $2an^4-16an^2+32a=2a(n^4-8n^2+16)$

$=2a(n^2-4)(n^2-4)=2a(n+2)^2(n-2)^2$

$2an^3-8an=2an(n^2-4) \quad =2an(n+2)(n-2)$

$2a^2n^3+16a^2=2a^2(n^3+8) \quad =2a^2(n+2)(n^2-2n+4)$

$mcd \quad 2a(n+2)$

30. $4a^2+8a-12=4\left(a^2+2a-3\right)=2^2\left(a+3\right)\left(a-1\right)$

$2a^2-6a+4=2\left(a^2-3a+2\right)=2\left(a-2\right)\left(a-1\right)$

$6a^2+18a-24=6\left(a^2+3a-4\right)=3\cdot 2\left(a+4\right)\left(a-1\right)$

$mcd \quad 2\left(a-1\right)$

31. $4a^2-b^2=\left(2a+b\right)\left(2a-b\right)$

$8a^3+b^3=\left(2a+b\right)\left(4a^2-2ab+b^2\right)$

$4a^2+4ab+b^2=\left(2a+b\right)^2$

$mcd \quad 2a+b$

32. $x^2-2x-8=\left(x-4\right)\left(x+2\right)$

$x^2-x-12=\left(x-4\right)\left(x+3\right)$

$x^3-9x^2+20x=x\left(x^2-9x+20\right)=x\left(x-5\right)\left(x-4\right)$

$mcd \quad x-4$

33. $a^2+a=a\left(a+1\right)$

$a^3-6a^2-7a=a\left(a^2-6a-7\right)=a\left(a-7\right)\left(a+1\right)$

$a^6+a=a\left(a^5+1\right) \quad =a\left(a+1\right)\left(a^4-a^3+a^2-a+1\right)$

$mcd \quad a\left(a+1\right)$

34. $x^3+27=\left(x+3\right)\left(x^2-3x+9\right)$

$2x^2-6x+18=2\left(x^2-3x+9\right)$

$x^4-3x^3+9x^2=x^2\left(x^2-3x+9\right)$

$mcd \quad x^2-3x+9$

35. $x^2+ax-6a^2=\left(x+3a\right)\left(x-2a\right)$

$x^2+2ax-3a^2=\left(x+3a\right)\left(x-a\right)$

$x^2+6ax+9a^2=\left(x+3a\right)^2$

$mcd \quad x+3a$

36. $54x^3+250=2\left(27x^3+125\right) \quad =2\left(3x+5\right)\left(9x^2-15x+25\right)$

$18ax^2-50a=2a\left(9x^2-25\right) \quad =2a\left(3x+5\right)\left(3x-5\right)$

$18x^2+60x+50=2\left(9x^2+30x+25\right)=2\left(3x+5\right)^2$

$mcd \quad 2\left(3x+5\right)$

37. $\left(x^2-1\right)^2=\left[\left(x+1\right)\left(x-1\right)\right]^2$

$x^2-4x-5=\left(x-5\right)\left(x+1\right)$

$x^4-1=\left(x^2+1\right)\left(x^2-1\right)=\left(x^2+1\right)\left(x+1\right)\left(x-1\right)$

$mcd \quad x+1$

38. $4ax^2-28ax=4ax\left(x-7\right)$

$a^2x^3-8a^2x^2+7a^2x=a^2x\left(x^2-8x+7\right) \quad =a^2x\left(x-7\right)\left(x-1\right)$

$ax^4-15ax^3+56ax^2=ax^2\left(x^2-15x+56\right)=ax^2\left(x-8\right)\left(x-7\right)$

$mcd \quad ax\left(x-7\right)$

39. $3a^2-6a=3a\left(a-2\right)$

$a^3-4a=a\left(a^2-4\right)=a\left(a+2\right)\left(a-2\right)$

$a^2b-2ab=ab\left(a-2\right)$

$a^2-a-2=\left(a-2\right)\left(a+1\right)$

$mcd \quad a-2$

40. $3x^2-x=x\left(3x-1\right)$

$27x^3-1=\left(3x-1\right)\left(9x^2+3x+1\right)$

$9x^2-6x+1=\left(3x-1\right)^2$

$3ax-a+6x-2=\left(3ax-a\right)+\left(6x-2\right)$

$=a\left(3x-1\right)+2\left(3x-1\right)$

$=\left(a+2\right)\left(3x-1\right)$

$mcd \quad 3x-1$

41. $a^4-1=\left(a^2+1\right)\left(a^2-1\right)=\left(a^2+1\right)\left(a+1\right)\left(a-1\right)$

$a^3+a^2+a+1=\left(a^3+a^2\right)+\left(a+1\right)$

$=a^2\left(a+1\right)+\left(a+1\right)=\left(a^2+1\right)\left(a+1\right)$

$a^3x+a^2x+ax+x=\left(a^3x+a^2x\right)+\left(ax+x\right)$

$=a^2x\left(a+1\right)+x\left(a+1\right)$

$=\left(a^2x+x\right)\left(a+1\right)=x\left(a^2+1\right)\left(a+1\right)$

$a^5+a^3+a^2+1=\left(a^5+a^3\right)+\left(a^2+1\right)$

$=a^3\left(a^2+1\right)+\left(a^2+1\right)$

$=\left(a^3+1\right)\left(a^2+1\right)=\left(a+1\right)\left(a^2-a+1\right)\left(a^2+1\right)$

$mcd \quad \left(a^2+1\right)\left(a+1\right)$

42. $2m^2+4mn+2n^2=2\left(m^2+2mn+n^2\right)$

$=2\left(m+n\right)^2$

$m^3+m^2n+mn^2+n^3=m^2\left(m+n\right)+n^2\left(m+n\right)$

$=\left(m^2+n^2\right)\left(m+n\right)$

$m^3+n^3=\left(m+n\right)\left(m^2-mn+n^2\right)$

$m^3-mn^2=m\left(m^2-n^2\right)$

$=m\left(m+n\right)\left(m-n\right)$

$mcd \quad m+n$

43. $a^3-3a^2+3a-1=\left(a^3-1\right)-3a\left(a-1\right)$

$=\left(a-1\right)\left(a^2+a+1\right)-3a\left(a-1\right)$

$=\left(a-1\right)\left(a^2+a+1-3a\right)$

$=\left(a-1\right)\left(a^2-2a+1\right)=\left(a-1\right)\left(a-1\right)^2$

$a^2-2a+1=\left(a-1\right)^2$

$a^3-a=a\left(a^2-1\right)=a\left(a+1\right)\left(a-1\right)$

$a^2-4a+3=\left(a-3\right)\left(a-1\right)$

mcd $a-1$

44. $16a^3x+54x=2x\left(8a^3+27\right)$

$=2x\left(2a+3\right)\left(4a^2-6a+9\right)$

$12a^2x^2-42ax^2-90x^2=6x^2\left(2a^2-7a-15\right)$

$=6x^2\left[(2a)^2-7\left(2a\right)-30\right]$

$=\left[6x^2\left(2a-10\right)\left(2a+3\right)\right]\div 2$

$=3\cdot 2x^2\left(a-5\right)\left(2a+3\right)$

$32a^3x+24a^2x-36ax=4ax\left(8a^2+6a-9\right)$

$=4ax\left[(8a)^2+6(8a)-72\right]$

$=\left[4ax\left(8a+12\right)\left(8a-6\right)\right]\div 4$

$=2^2ax\left(2a+3\right)\left(8a-6\right)$

$=2^3ax\left(2a+3\right)\left(4a-3\right)$

$32a^4x-144a^2x+162x=2x\left(16a^4-72a^2+81\right)$

$=2x\left(4a^2-9\right)^2$

$=2x\left[\left(2a+3\right)\left(2a-3\right)\right]^2$

mcd $2x\left(2a+3\right)$

45. $\left(xy+y^2\right)^2=x^2y^2+2xy^3+y^4$

$=y^2\left(x^2+2xy+y^2\right)=y^2\left(x+y\right)^2$

$x^2y-2xy^2-3y^3=y\left(x^2-2xy-3y^2\right)=y\left(x-3y\right)\left(x+y\right)$

$ax^3y+ay^4=ay\left(x^3+y^3\right) \quad =ay\left(x+y\right)\left(x^2-xy+y^2\right)$

$x^2y-y^3=y\left(x^2-y^2\right) \quad =y\left(x+y\right)\left(x-y\right)$

mcd $y\left(x+y\right)$

46. $2a^2-am+4a-2m=\left(2a^2-am\right)+\left(4a-2m\right)$

$=a\left(2a-m\right)+2\left(2a-m\right)$

$=\left(a+2\right)\left(2a-m\right)$

$2am^2-m^3=m^2\left(2a-m\right)$

$6a^2+5am-4m^2=\left[(6a)^2+5m(6a)-24m^2\right]$

$=\left[\left(6a+8m\right)\left(6a-3m\right)\right]\div 2\cdot 3$

$=\left(3a+4m\right)\left(2a-m\right)$

$16a^2+72am-40m^2=8\left(2a^2+9am-5m^2\right)$

$=8\left[(2a)^2+9m(2a)-10m\right]$

$=8\left[\left(2a+10m\right)\left(2a-m\right)\right]\div 2$

$=8\left(a+5m\right)\left(2a-m\right)$

mcd $2a-m$

47. $12ax-6ay+24bx-12by=\left(12ax-6ay\right)+\left(24bx-12by\right)$

$=6a\left(2x-y\right)+12b\left(2x-y\right)$

$=\left(6a+12b\right)\left(2x-y\right)$

$=3\cdot 2\left(a+2b\right)\left(2x-y\right)$

$3a^3+24b^3=3\left(a^3+8b^3\right)$

$=3\left(a+2b\right)\left(a^2-2ab+4b^2\right)$

$9a^2+9ab-18b^2=9\left(a^2+ab-2b^2\right)$

$=3^2\left(a+2b\right)\left(a-b\right)$

$12a^2+24ab=12a\left(a+2b\right)=4\cdot 3a\left(a+2b\right)$

mcd $3\left(a+2b\right)$

48. $5a^2+5ax+5ay+5xy=5a\left(a+x\right)+5y\left(a+x\right)$

$=\left(5a+5y\right)\left(a+x\right)=5\left(a+y\right)\left(a+x\right)$

$15a^3-15ax^2+15a^2y-15x^2y=15a^2\left(a+y\right)-15x^2\left(a+y\right)$

$=\left(15a^2-15x^2\right)\left(a+y\right)$

$=15\left(a^2-x^2\right)\left(a+y\right)$

$=5\cdot 3\left(a+x\right)\left(a-x\right)\left(a+y\right)$

$20a^3-20ay^2+20a^2x-20xy^2=20a^2\left(a+x\right)-20y^2\left(a+x\right)$

$=\left(20a^2-20y^2\right)\left(a+x\right)$

$=20\left(a^2-y^2\right)\left(a+x\right)$

$=5\cdot 4\left(a+y\right)\left(a-y\right)\left(a+x\right)$

mcd $5\left(a+y\right)\left(a+x\right)$

EJERCICIO 113

1.

$$12x^2 + 8x + 1 \quad \mid \; 2x^2 - 5x - 3$$
$$-12x^2 + 30x + 18 \qquad 6$$
$$38x + 19 \quad (\div 2)$$
$$2x^2 - 5x - 3 \quad \mid \; 2x + 1$$
$$-2x^2 - x \qquad x$$
$$-6x - 3 \quad (\div -3)$$
$$2x + 1 \quad \mid \; 2x + 1$$
$$-2x - 1 \qquad 1 \qquad mcd \;\; 2x + 1$$

2. $3(2a^3 - a^2 - 6a) \;\; ; \;\; (6a^2 - 2a - 20) \div 2$

$$6a^3 - 3a^2 - 18a \quad \mid \; 3a^2 - a - 10$$
$$-6a^3 + 2a^2 + 20a \qquad 2a$$
$$-a^2 + 2a \quad (\div -a)$$
$$3a^2 - a - 10 \quad \mid \; a - 2$$
$$-3a^2 + 6a \qquad 3a$$
$$5a - 10 \quad (\div 2)$$
$$a - 2 \quad \mid \; a - 2$$
$$-a + 2 \qquad 1 \qquad mcd \;\; a - 2$$

3. $3(5a^3 - 6a^2x + ax^2) \;\; ; \;\; 3a^3 - 4a^2x + ax^2$

$$= 15a^3 - 18a^2x + 3ax^2 \qquad = a(3a^2 - 4ax + x^2)$$
$$= a(15a^2 - 18ax + 3x^2)$$
$$15a^2 - 18ax + 3x^2 \quad \mid \; 3a^2 - 4ax + x^2$$
$$-15a^2 + 20ax - 5x^2 \qquad 5$$
$$2ax - 2x^2 \quad (\div 2x)$$
$$3a^2 - 4ax + x^2 \quad \mid \; a - x$$
$$-3a^2 + 3ax \qquad 3a$$
$$-ax + x^2 \quad (\div -x)$$
$$a - x \quad \mid \; a - x$$
$$-a + x \qquad 1 \qquad mcd \;\; a(a - x)$$

4. $(2x^3 + 4x^2 - 4x + 6) \div 2$

$$x^3 + 2x^2 - 2x + 3 \quad \mid \; x^3 + x^2 - x + 2$$
$$-x^3 - x^2 + x - 2 \qquad 1$$
$$x^2 - x + 1$$
$$x^3 + x^2 - x + 2 \quad \mid \; x^2 - x + 1$$
$$-x^3 + x^2 - x \qquad x$$
$$2x^2 - 2x + 2 \quad (\div 2)$$
$$x^2 - x + 1 \quad \mid \; x^2 - x + 1$$
$$-x^2 + x - 1 \qquad 1 \qquad mcd \;\; x^2 - x + 1$$

5. $8a^4 - 6a^3x + 7a^2x^2 - 3ax^3 \;\; ; \;\; 2a^3 + 3a^2x - 2ax^2$

$$= a(8a^3 - 6a^2x + 7ax^2 - 3x^3) \qquad = a(2a^2 + 3ax - 2x^2)$$
$$8a^3 - 6a^2x + 7ax^2 - 3x^3 \quad \mid \; 2a^2 + 3ax - 2x^2 \; (3x)$$
$$-8a^3 - 12a^2x + 8ax^2 \qquad 4a$$
$$-18a^2x + 15ax^2 - 3x^3 \;\; \div \; -3$$
$$6a^2x + 9ax^2 - 6x^3 \quad \mid \; 6a^2x - 5ax^2 + x^3$$
$$-6a^2x + 5ax^2 - x^3 \qquad 1$$
$$14ax^2 - 7x^3 \;\; \div 7x^2$$
$$6a^2x - 5ax^2 + x^3 \quad \mid \; 2a - x$$
$$-6a^2x + 3ax^2 \qquad 3ax$$
$$-2ax^2 + x^3 \;\; \div \; -x^2$$
$$2a - x \quad \mid \; 2a - x$$
$$-2a + x \qquad 1 \qquad mcd \;\; a(2a - x)$$

6. $(12ax^4 - 3ax^3 + 26ax^2 - 5ax + 10a) \div a$

$$12x^4 - 3x^3 + 26x^2 - 5x + 10 \quad \mid \; 3x^4 + 3x^3 - 4x^2 + 5x - 15 \; (5)$$
$$-12x^4 - 12x^3 + 16x^2 - 20x + 60 \qquad 4$$
$$-15x^3 + 42x^2 - 25x + 70 \quad (-1)$$
$$15x^4 + 15x^3 - 20x^2 + 25x - 75 \quad \mid \; 15x^3 - 42x^2 + 25x - 70 \; (19)$$
$$-15x^4 + 42x^3 - 25x^2 + 70x \qquad x$$
$$57x^3 - 45x^2 + 95x - 75 \quad (5)$$
$$285x^3 - 798x^2 + 475x - 1.330 \quad \mid \; 285x^3 - 225x^2 + 475x - 375$$
$$-285x^3 + 225x^2 - 475x + 375 \qquad 1$$
$$-573x^2 - 955 \quad (\div -191)$$
$$285x^3 - 225x^2 + 475x - 375 \quad \mid \; 3x^2 + 5$$
$$-285x^3 - 475x \qquad 95x$$
$$-225x^2 - 375 \quad (\div -75)$$
$$3x^2 + 5 \quad \mid \; 3x^2 + 5$$
$$-3x^2 - 5 \qquad 1 \qquad mcd \;\; 3x^2 + 5$$

7.

$$6x^4 - 4x^3y - 3x^2y^2 + 5xy^3 - 2y^4 \quad \mid \; 3x^3 - 2x^2y + 9xy^2 - 6y^3 \; (7)$$
$$-6x^4 + 4x^3y - 18x^2y^2 + 12xy^3 \qquad 2x$$
$$-21x^2y^2 + 17xy^3 - 2y^4 \quad (\div -y^2)$$
$$21x^3 - 14x^2y + 63xy^2 - 42y^3 \quad \mid \; 21x^2 - 17xy + 2y^2$$
$$-21x^3 + 17x^2y - 2xy^2 \qquad x$$
$$3x^2y + 61xy^2 - 42y^3 \quad (\div y)$$
$$21x^2 - 17xy + 2y^2 \quad \mid \; 3x^2 + 61xy - 42y^2$$
$$-21x^2 + 427xy + 294y^2 \qquad 7$$
$$-444xy + 296y^2 \quad (\div -148y)$$
$$3x^2 + 61xy - 42y^2 \quad \mid \; 3x - 2y$$
$$-3x^2 + 2xy \qquad x$$
$$63xy - 42y^2 \quad (\div 21y)$$
$$3x - 2y \quad \mid \; 3x - 2y$$
$$-3x + 2y \qquad 1 \qquad mcd \;\; 3x - 2y$$

8. $(ax^4+3ax^3-2ax^2+6ax-8a)\div a$;

$(x^4+4x^3-x^2-4x)\div x$

$x^4+3x^3-2x^2+6x-8 \quad | \; \underline{x^3+4x^2-x-4}$

$\underline{-x^4-4x^3+x^2+4x} \qquad x$

$-x^3-x^2+10x-8 \quad (-1)$

$x^3+4x^2-x-4 \quad | \; \underline{x^3+x^2-10x+8}$

$\underline{-x^3-x^2+10x-8} \qquad 1$

$3x^2+9x-12 \quad (\div 3)$

$x^3+x^2-10x+8 \quad | \; \underline{x^2+3x-4}$

$\underline{-x^3-3x^2+4x} \qquad x$

$-2x^2-6x+8 \quad (\div -2)$

$x^2+3x-4 \quad | \; \underline{x^2+3x-4}$

$\underline{-x^2-3x+4} \qquad 1 \qquad mcd \;\; x^2+3x-4$

9. $3(2m^4-4m^3-m^2+6m-3)$;

$(3m^5-6m^4+8m^3-10m^2+5m)\div m$

$6m^4-12m^3-3m^2+18m-9 \quad | \; \underline{3m^4-6m^3+8m^2-10m+5}$

$\underline{-6m^4+12m^3-16m^2+20m-10} \qquad 2$

$-19m^2+38m-19 \quad (\div -19)$

$3m^4-6m^3+8m^2-10m+5 \quad | \; \underline{m^2-2m+1}$

$\underline{-3m^4+6m^3-3m^2} \qquad 3m^2$

$5m^2-10m+5 \quad (\div 5)$

$m^2-2m+1 \quad | \; \underline{m^2-2m+1}$

$\underline{-m^2+2m-1} \qquad 1 \qquad mcd \;\; m^2-2m+1$

10. *Factor común* a

$7(3a^5-6a^4+16a^3-2a^2+5a)$; $3(7a^5-14a^4+33a^3+4a^2-10a)$

$=21a^5-42a^4+112a^3-14a^2+35a$; $=21a^5-42a^4+99a^3+12a^2-30a$

$21a^4-42a^3+112a^2-14a+35 \quad | \; \underline{21a^4-42a^3+99a^2+12a-30}$

$\underline{-21a^4+42a^3-99a^2-12a+30} \qquad 1$

$13a^2-26a+65 \quad (\div 13)$

$21a^4-42a^3+99a^2+12a-30 \quad | \; \underline{a^2-2a+5}$

$\underline{-21a^4+42a^3-105a^2} \qquad 21a^2$

$-6a^2+12a-30 \quad (\div -6)$

$a^2-2a+5 \quad | \; \underline{a^2-2a+5}$

$\underline{-a^2+2a-5} \qquad 1 \qquad mcd \;\; a(a^2-2a+5)$

11. $(45ax^3+75ax^2-18ax-30a)\div 3a$;

$(24ax^3+40ax^2-30ax-50a)\div 2a$

$4(15x^3+25x^2-6x-10)$; $5(12x^3+20x^2-15x-25)$

$60x^3+100x^2-24x-40 \quad | \; \underline{60x^3+100x^2-75x-125} \quad (\div 5)$

$\underline{-60x^3-100x^2+75x+125} \qquad 1$

$51x+85 \quad (\div 17)$

$12x^3+20x^2-15x-25 \quad | \; \underline{3x+5}$

$\underline{-12x^3-20x^2} \qquad 4x^2$

$-15x-25 \quad (\div -5)$

$3x+5 \quad | \; \underline{3x+5}$

$\underline{-3x-5} \qquad 1 \qquad mcd \;\; a(3x+5)$

12. *Factor común* 2:

$2x^3+2ax^2+2a^2x+2a^3$; $10x^3+4ax^2+10a^2x+4a^3$

$5x^3+2ax^2+5a^2x+2a^3 \quad | \; \underline{x^3+ax^2+a^2x+a^3}$

$\underline{-5x^3-5ax^2-5a^2x-5a^3} \qquad 5$

$-3ax^2-3a^3 \quad (\div -3a)$

$x^3+ax^2+a^2x+a^3 \quad | \; \underline{x^2+a^2}$

$\underline{-x^3-a^2x} \qquad x$

$ax^2+a^3 \quad (\div a)$

$x^2+a^2 \quad | \; \underline{x^2+a^2}$

$\underline{-x^2-a^2} \qquad 1 \qquad mcd \;\; 2(x^2+a^2)$

13. *Factor común* 3 :

$9x^3+15ax^2+3a^2x-3a^3$;

$12x^3+21ax^2+6a^2x-3a^3 \ (3)$

$12x^3+21ax^2+6a^2x-3a^3 \ \lfloor \ 3x^3+5ax^2+a^2x-a^3$

$-12x^3-20ax^2-4a^2x+4a^3 \quad 4$

$ax^2+2a^2x+a^3 \quad (\div a)$

$3x^3+5ax^2+a^2x-a^3 \ \lfloor \ x^2+2ax+a^2$

$-3x^3-6ax^2-3a^2x \quad 3x$

$-ax^2-2a^2x-a^3 \quad (\div -a)$

$x^2+2ax+a^2 \ \lfloor \ x^2+2ax+a^2$

$-x^2-2ax-a^2 \quad 1 \qquad mcd \ \ 3\left(x^2+2ax+a^2\right)$

14. *Factor común* $2ab$:

$8a^4b+4a^3b^2+4ab^4$; $12a^4b-18a^3b^2+12a^2b^3-6ab^4$

$3\left(4a^3+2a^2b+2b^3\right) \quad 6a^3-9a^2b+6ab^2-3b^3$

$12a^3+6a^2b+6b^3 \ \lfloor \ 6a^3-9a^2b+6ab^2-3b^3$

$-12a^3+18a^2b-12ab^2+6b^3 \quad 2$

$24a^2b-12ab^2+12b^3 \quad (\div 12b)$

$6a^3-9a^2b+6ab^2-3b^3 \ \lfloor \ 2a^2-ab+b^2$

$-6a^3+3a^2b-3ab^2 \quad 3a$

$-6a^2b+3ab^2-3b^3 \quad (\div -3b)$

$2a^2-ab+b^2 \ \lfloor \ 2a^2-ab+b^2$

$-2a^2+ab-b^2 \quad 1 \qquad mcd \ \ 2ab\left(2a^2-ab+b^2\right)$

15. *Factor común* $3a^2n^2$:

$9a^5n^2-33a^4n^3+27a^3n^4-6a^2n^5$;

$9a^5n^2+12a^4n^3-21a^3n^4+6a^2n^5$

$3a^3-11a^2n+9an^2-2n^3 \ \lfloor \ 3a^3+4a^2n-7an^2+2n^3 \ (5)$

$-3a^3-4a^2n+7an^2-2n^3 \quad 1$

$-15a^2n+16an^2-4n^3 \quad (\div -n)$

$15a^3+20a^2n-35an^2+10n^3 \ \lfloor \ 15a^2-16an+4n^2 \ (12)$

$-15a^3+16a^2n-4an^2 \quad a$

$36a^2n-39an^2+10n^3 \quad (\div n)$

$180a^2-192an+48n^2 \ \lfloor \ 36a^2-39an+10n^2$

$-180a^2+195an-50n^2 \quad 5$

$3an-2n^2 \quad (\div n)$

$36a^2-39an+10n^2 \ \lfloor \ 3a-2n$

$-36a^2+24an \quad 12a$

$-15an+10n^2 \quad (\div -5n)$

$3a-2n \ \lfloor \ 3a-2n$

$-3a+2n \quad 1 \qquad mcd \ \ 3a^2n^2\left(3a-2n\right)$

16. $a^7-a^6+a^4+1 \ \lfloor \ a^5-2a^4+a^3+a-1 \ (a)$

$-a^7+2a^6-a^5-a^3+a^2 \quad a^2$

$a^6-a^5+a^4-a^3+a^2+1$

$a^6-2a^5+a^4+a^2-a \ \lfloor \ a^6-a^5+a^4-a^3+a^2+1$

$-a^6+a^5-a^4+a^3-a^2-1 \quad 1$

$-a^5+a^3-a-1 \quad (-1)$

$a^6-a^5+a^4-a^3+a^2+1 \ \lfloor \ a^5-a^3+a+1$

$-a^6+a^4-a^2-a \quad a$

$-a^5+2a^4-a^3-a+1 \quad (-1)$

$a^5-a^3+a+1 \ \lfloor \ a^5-2a^4+a^3+a-1$

$-a^5+2a^4-a^3-a+1 \quad 1$

$2a^4-2a^3+2 \quad (\div 2)$

$a^5-2a^4+a^3+a-1 \ \lfloor \ a^4-a^3+1$

$-a^5+a^4-a \quad a$

$-a^4+a^3-1 \quad (-1)$

$a^4-a^3+1 \ \lfloor \ a^4-a^3+1$

$-a^4+a^3-1 \quad 1 \qquad mcd \ \ a^4-a^3+1$

17. *Factor común* $2a$:

$6ax^4-4ax^3+6ax^2-10ax+4a$;

$36ax^4-24ax^3-18ax^2+48ax-24a$

$18x^4-12x^3-9x^2+24x-12 \ \lfloor \ 3x^4-2x^3+3x^2-5x+2 \ (9)$

$-18x^4+12x^3-18x^2+30x-12 \quad 6$

$-27x^2+54x-24 \quad (-1)$

$27x^4-18x^3+27x^2-45x+18 \ \lfloor \ 27x^2-54x+24 \quad (4x)$

$-27x^4+54x^3-24x^2 \quad x^2$

$36x^3+3x^2-45x+18 \quad (\div 3)$

$108x^3-216x^2+96x \ \lfloor \ 12x^3+x^2-15x+6 \ (25)$

$-108x^3-9x^2+135x-54 \quad 9$

$-225x^2+231x-54 \quad (\div -3)$

$300x^3+25x^2-375x+150 \ \lfloor \ 75x^2-77x+18 \ (111)$

$-300x^3+308x^2-72x \quad 4x$

$333x^2-447x+150 \quad (\div 3)$

$8.325x^2-8.547x+1.998 \ \lfloor \ 111x^2-149x+50 \ (2.628)$

$-8.325x^2+11.175x-3.750 \quad 75$

$2.628x-1.752$

$291.708x^2-391.572x+131.400 \ \lfloor \ 2.628x-1.752 \ (\div 876)$

$-291.708x^2+194.472x \quad 111x$

$-197.100x+131.400 \quad (\div -65.700)$

$3x-2 \ \lfloor \ 3x-2$

$-3x+2 \quad 1 \qquad mcd \ \ 2a\left(3x-2\right)$

EJERCICIO 114

1.

$2x^3-5x^2-6x+9 \;|\; 2x^2-5x-3$
$-2x^3+5x^2+3x \qquad x$
$-3x+9 \quad (\div -3)$

$2x^2-5x-3 \;|\; x-3$
$-2x^2+6x \qquad 2x$
$x-3$

$x^3-2x^2-5x+6 \;|\; x-3 \quad (x)$
$-x^3+3x^2 \qquad x^2$
x^2-5x+6

$x^2-3x \;|\; x^2-5x+6$
$+5x-6 \qquad 1$
$2x-6 \quad (\div 2)$

$x^2-5x+6 \;|\; x-3$
$-x^2+2x \qquad x$
$-3x+6 \quad (\div -3)$

$x-3 \;|\; x-3$
$-x+3 \qquad 1 \qquad mcd \quad x-3$

2.

$8x^3+6x^2y-3xy^2-y^3 \;|\; 2x^3-x^2y-2xy^2+y^3$
$-8x^3+4x^2y+8xy^2-4y^3 \qquad 4$
$10x^2y+5xy^2-5y^3 \quad (\div 5y)$

$2x^3-x^2y-2xy^2+y^3 \;|\; 2x^2+xy-y^2$
$-2x^3-x^2y+xy^2 \qquad x$
$-2x^2y-xy^2+y^3 \quad (\div -y)$

$2x^2+xy-y^2 \;|\; 2x^2+xy-y^2$
$-2x^2-xy+y^2 \qquad 1$
0

$6x^2-xy-y^2 \;|\; 2x^2+xy-y^2$
$-6x^2-3xy+3y^2 \qquad 3$
$-4xy+2y^2 \quad (\div -2y)$

$2x^2+xy-y^2 \;|\; 2x-y$
$-2x^2+xy \qquad x$
$2xy-y^2 \quad (\div y)$

$2x-y \;|\; 2x-y$
$-2x+y \qquad 1 \qquad mcd \quad 2x-y$

3.

$(x^4+x^3-x^2-x)\div x$;
$(2x^3+2x^2-2x-2)\div 2$

$x^3+x^2-x-1 \;|\; x^3+x^2-x-1$
$-x^3-x^2+x+1 \qquad 1$
0

$5x^3-5x^2+2x-2 \;|\; x^3+x^2-x-1 \quad (10)$
$-5x^3-5x^2+5x+5 \qquad 5$
$-10x^2+7x+3 \quad (-1)$

$10x^3+10x^2-10x-10 \;|\; 10x^2-7x-3 \quad (17)$
$-10x^3+7x^2+3x \qquad x$
$17x^2-7x-10$

$170x^2-119x-51 \;|\; 17x^2-7x-10$
$-170x^2+70x+100 \qquad 10$
$-49x+49 \quad (\div -49)$

$17x^2-7x-10 \;|\; x-1$
$-17x^2+17x \qquad 17x$
$10x-10 \quad (\div 10)$

$x-1 \;|\; x-1$
$-x+1 \qquad 1 \qquad mcd \quad x-1$

4.

$3a^4+9a^3x+4a^2x^2-3ax^3+2x^4 \;|\; a^4+3a^3x+a^2x^2-3ax^3-2x^4$
$-3a^4-9a^3x-3a^2x^2+9ax^3+6x^4 \qquad 3$
$a^2x^2+6ax^3+8x^4 \quad (\div x^2)$

$a^4+3a^3x+a^2x^2-3ax^3-2x^4 \;|\; a^2+6ax+8x^2 \quad (3a)$
$-a^4-6a^3x-8a^2x^2 \qquad a^2$
$-3a^3x-7a^2x^2-3ax^3-2x^4 \quad (\div -x)$

$3a^3+18a^2x+24ax^2 \;|\; 3a^3+7a^2x+3ax^2+2x^3 \quad (11)$
$-3a^3-7a^2x-3ax^2-2x^3 \qquad 1$
$11a^2x+21ax^2-2x^3 \quad (\div x)$

$33a^3+77a^2x+33ax^2+22x^3 \;|\; 11a^2+21ax-2x^2 \quad (14)$
$-33a^3-63a^2x+6ax^2 \qquad 3a$
$14a^2x+39ax^2+22x^3 \quad (\div x)$

$154a^2+294ax-28x^2 \;|\; 14a^2+39ax+22x^2$
$-154a^2-429ax-242x^2 \qquad 11$
$-135ax-270x^2 \quad (\div -135x)$

$14a^2+39ax+22x^2 \;|\; a+2x$
$-14a^2-28ax \qquad 14a$
$11ax+22x^2 \quad (\div 11x)$

$a+2x \;|\; a+2x$
$-a-2x \qquad 1$
0

$4a^3+8a^2x-ax^2-2x^3 \;|\; a+2x$
$-4a^3-8a^2x \qquad 4a^2$
$-ax^2-2x^3 \quad (\div -x^2)$

$a+2x \;|\; a+2x$
$-a-2x \qquad 1 \qquad mcd \quad a+2x$

5. *Factor común x*

$2(2x^5+2x^4-2x^2-2x)$

$4x^4+4x^3-4x-4 \;|\; 4x^3-4x^2+3x-3 \quad (2)$
$-4x^4+4x^3-3x^2+3x \qquad x$
$8x^3-3x^2-x-4$

$8x^3-8x^2+6x-6 \;|\; 8x^3-3x^2-x-4 \quad (5)$
$-8x^3+3x^2+x+4 \qquad 1$
$-5x^2+7x-2 \quad (-1)$

$40x^3-15x^2-5x-20 \;|\; 5x^2-7x+2 \quad (41)$
$-40x^3+56x^2-16x \qquad 8x$
$41x^2-21x-20$

$205x^2-287x+82 \;|\; 41x^2-21x-20$
$-205x^2+105x+100 \qquad 5$
$-182x+182 \quad (\div -182)$

$41x^2-21x-20 \;|\; x-1$
$-41x^2+41x \qquad 41x$
$20x-20 \quad (\div 20)$

$x-1 \;|\; x-1$
$-x+1 \qquad 1$
0

$3x^6-4x^4-3x^3+4x \;|\; x-1 \quad (3x^4)$
$-3x^6+3x^5 \qquad 3x^5$
$3x^5-4x^4-3x^3+4x$

Continua.

Continuación.

5.

$$3x^5-3x^4 \quad \big| \; 3x^5-4x^4-3x^3+4x \;(\div x)$$
$$-3x^5+4x^4+3x^3-4x \quad 1$$
$$x^4+3x^3-4x \quad (\div x)$$

$$3x^4-4x^3-3x^2 \quad +4 \quad \big| \; x^3+3x^2-4 \;(13)$$
$$-3x^4-9x^3 \quad +12x \quad 3x$$
$$-13x^3-3x^2+12x+4 \quad (\div -1)$$

$$13x^3+39x^2 \quad -52 \quad \big| \; 13x^3+3x^2-12x-4 \;(3)$$
$$-13x^3-3x^2+12x+4 \quad 1$$
$$36x^2+12x-48 \quad (\div 12)$$

$$39x^3+9x^2-36x-12 \; \big| \; 3x^2+x-4$$
$$-39x^3-13x^2+52x \quad 13x$$
$$-\;4x^2+16x-12 \quad (\div -4)$$

$$3x^2+\;x\;-4 \; \big| \; x^2-4x+3$$
$$-3x^2+12x-9 \quad 3$$
$$13x-13 \quad (\div 13)$$

$$x^2-4x+3 \; \big| \; x-1$$
$$-x^2+x \quad x$$
$$-3x+3 \quad (\div -3)$$

$$x-1 \; \big| \; x-1$$
$$-x+1 \quad 1 \qquad mcd \quad x(x-1)$$

EJERCICIO 115

1. a^2 , ab^2 $mcm\ a^2b^2$

2. x^2y , xy^2 $mcm\ x^2y^2$

3. ab^2c , a^2bc $mcm\ a^2b^2c$

4. a^2x^3 , a^3bx^2 $mcm\ a^3bx^3$

5. $6m^2n$, $4m^3$ $mcm\ 12m^3n$

6. $9ax^3y$, $15x^2y^5$ $mcm\ 45ax^3y^5$

7. a^3 , ab^2 , a^2b $mcm\ a^3b^2$

8. x^2y , xy^2 , xy^3z $mcm\ x^2y^3z$

9. $2ab^2$, $4a^2b$, $8a^3$ $mcm\ 8a^3b^2$

10. $3x^2y^3z$, $4x^3y^3z^2$, $6x^4$ $mcm\ 12x^4y^3z^2$

11. $6mn^2$, $9m^2n^3$, $12m^3n$ $mcm\ 36m^3n^3$

12. $3a^2$, $4b^2$, $8x^2$ $mcm\ 24a^2b^2x^2$

13. $5x^2$, $10xy$, $15xy^2$ $mcm\ 30x^2y^2$

14. ax^3y^2 , a^3xy , $a^2x^2y^3$ $mcm\ a^3x^3y^3$

15. $4ab$, $6a^2$, $3b^2$ $mcm\ 12a^2b^2$

16. $3x^3$, $6x^2$, $9x^4y^2$ $mcm\ 18x^4y^2$

17. $9a^2bx$, $12ab^2x^2$, $18a^3b^3x$ $mcm\ 36a^3b^3x^2$

18. $10m^2$, $15mn^2$, $20n^3$ $mcm\ 60m^2n^3$

19. $18a^3$, $24b^2$, $36ab^3$ $mcm\ 72a^3b^3$

20. $20m^2n^3$, $24m^3n$, $30mn^2$

20	24	30	6	
20	4	5	5	$6\cdot5\cdot4=120$
4	4	1	4	$\Rightarrow mcm\ 120m^3n^3$
1	1			

21. ab^2 , bc^2 , a^2c^3 , b^3c^3 $mcm\ a^2b^3c^3$

22. $2x^2y$, $8xy^3$, $4a^2x^3$, $12a^3$ $mcm\ 24a^3x^3y^3$

23. $6a^2$, $9x$, $12ay^2$, $18x^3y$ $mcm\ 36a^2x^3y^2$

24. $15mn^2$, $10m^2$, $20n^3$, $25mn^4$

15	10	20	25	5	
3	2	4	5	5	$5^2\cdot3\cdot2^2=300$
3	2	4	1	3	$\Rightarrow mcm\ 300m^2n^4$
1	2	4		2	
	1	2		2	
		1			

25. $24a^2x^3$, $36a^2y^4$, $40x^2y^5$, $60a^3y^6$

24	36	40	60	2	
12	18	20	30	2	
6	9	10	15	3	$2^3\cdot3^2\cdot5=360$
2	3	10	5	5	$\Rightarrow mcm\ 360a^3x^3y^6$
2	3	2	1	3	
2	1	2		2	
1		1			

26. $3a^3$, $8ab$, $10b^2$, $12a^2b^3$, $16a^2b^2$

3	8	10	12	16	2	
3	4	5	6	8	2	
3	2	5	3	4	2	$2^4\cdot3\cdot5=240$
3	1	5	3	2	2	$\Rightarrow mcm\ 240a^3b^3$
3		5	3	1	3	
1		5	1		5	
		1				

EJERCICIO 116

1. $2a = 2a$

$4x - 8 = 2^2(x-2)$

$mcm\ 4a(x-2)$

2. $ab - b^2 = b(a-b)$

$3b^2 = 3b^2$

$mcm\ 3b^2(a-b)$

3. $x^2y + xy^2 = xy(x+y)$

$x^2y = x^2y$

$mcm\ x^2y(x+y)$

4. $4 + 8a = 2^2(1+2a)$

$8 = 2^3$

$mcm\ 2^3(1+2a) = 8(1+2a)$

5. $6a^2b = 2\cdot 3a^2b$

$3a^2b^2 + 6ab^3 = 3ab^2(a+2b)$

$mcm\ 6a^2b^2(a+2b)$

6. $14x^2 = 7\cdot 2x^2$

$6x^2 + 4xy = 2x(3x+2y)$

$mcm\ 14x^2(3x+2y)$

7. $9m = 3^2m$

$6mn^2 - 12mn = 2\cdot 3mn(n-2)$

$mcm\ 2\cdot 3^2mn(n-2)$

$\Rightarrow 18mn(n-2)$

8. $15 = 5\cdot 3$

$3x + 6 = 3(x+2)$

$mcm\ 15(x+2)$

9. $10 = 5\cdot 2$

$5 - 15b = 5(1-3b)$

$mcm\ 10(1-3b)$

10. $4ax - 12ay = 2^2a(x-3y)$

$36a^2 = 2^2\cdot 3^2a^2$

$mcm\ 2^2\cdot 3^2a^2(x-3y)$

$\Rightarrow 36a^2(x-3y)$

11. $12xy^2 = 2^2\cdot 3xy^2$

$2ax^2y^3 + 5x^2y^3 = x^2y^3(2a+5)$

$mcm\ 12x^2y^3(2a+5)$

12. $mn^3 - mn^2 = mn^2(n-1)$

$mn = mn$

$m^2 = m^2$

$mcm\ m^2n^2(n-1)$

13. $3a^2 - 6ab = 3a(a-2b)$

$2a^2 = 2a^2$

$6ab = 2\cdot 3ab$

$mcm\ 6a^2b(a-2b)$

14. $5x^5 - 5x^4 = 5x^4(x-1)$

$xy^2 = xy^2$

$x^2y^3 = x^2y^3$

$mcm\ 5x^4y^3(x-1)$

15. $27a^4b + 81a^3b^2 = 3^3a^3b(a+3b)$

$9a^2 = 3^2a^2$

$18b^3 = 2\cdot 3^2b^3$

$mcm\ 2\cdot 3^3a^3b^3(a+3b)$

$\Rightarrow 54a^3b^3(a+3b)$

16. $9x^3y + 9xy^3 = 3^2xy(x^2+y^2)$

$10 = 5\cdot 2$

$6x^2 = 2\cdot 3x^2$

$mcm\ 5\cdot 2\cdot 3^2x^2y(x^2+y^2)$

$\Rightarrow 90x^2y(x^2+y^2)$

17. $x^2y - xy = xy(x-1)$

$x^3 + x^2 = x^2(x+1)$

$4x = 2^2x$

$mcm\ 2^2x^2y(x-1)(x+1)$

$\Rightarrow 4x^2y(x^2-1)$

18. $6m^2 + 18m = 2\cdot 3m(m+3)$

$8m - 24 = 2^3(m-3)$

$24 = 2^3\cdot 3$

$mcm\ 2^3\cdot 3m(m+3)(m-3)$

$\Rightarrow 24m(m^2-9)$

19. $3ax + 3a = 3a(x+1)$

$2a^2b^2 = 2a^2b^2$

$6x - 18 = 2\cdot 3(x-3)$

$mcm\ 6a^2b^2(x+1)(x-3)$

20. $x^2 + 4x + 4 = (x+2)^2$

$x^3 + x^2 - 2x = x(x^2+x-2)$

$= x(x+2)(x-1)$

$x^2 = x^2$

$mcm\ x^2(x+2)^2(x-1)$

21. $9a^2x - 18a^2y = 3^2a^2(x-2y)$

$x^2 - 4xy + 4y^2 = (x-2y)^2$

$6ab = 2\cdot 3ab$

$mcm\ 3^2\cdot 2a^2b(x-2y)^2$

$\Rightarrow 18a^2b(x-2y)^2$

22. $9x^4 - 36x^2 = 3^2x^2(x^2-4)$

$= 3^2x^2(x+2)(x-2)$

$3x^3 - 3x^2 - 18x = 3x(x^2-x-6)$

$= 3x(x-3)(x+2)$

$6x^3 = 2\cdot 3x^3$

$mcm\ 2\cdot 3^2x^3(x+2)(x-2)(x-3)$

$\Rightarrow 18x^3(x^2-4)(x-3)$

23. $4x^3 - 12x^2y + 9xy^2 = x(4x^2 - 12xy + 9y^2)$

$= x(2x-3y)^2$

$2x^4 - 3x^3y = x^3(2x-3y)$

$a^2x^2 = a^2x^2$

$mcm\ a^2x^3(2x-3y)^2$

24. $9x^2 - 45x = 3^2x(x-5)$

$12x^2y^2 = 3\cdot 2^2x^2y^2$

$8x^3 = 2^3x^3$

$mcm\ 3^2\cdot 2^3x^3y^2(x-5)$

$\Rightarrow 72x^3y^2(x-5)$

25. $n^2x^2 + n^2y^2 = n^2(x^2+y^2)$

$nx^2 + 2nxy + ny^2 = n(x^2+2xy+y^2)$

$= n(x+y)^2$

$2n = 2n$

$an^3 = an^3$

$mcm\ 2an^3(x+y)^2(x^2+y^2)$

26. $4x^3 + 24x^2 + 36x = 2^2x(x^2+6x+9)$

$= 2^2x(x+3)^2$

$2x^3 - 8x^2 + 8x = 2x(x^2-4x+4)$

$= 2x(x-2)^2$

$x^3 + x^2 - 6x = x(x^2+x-6)$

$= x(x+3)(x-2)$

$8x^2 = 2^3x^2$

$mcm\ 8x^2(x+3)^2(x-2)^2$

27. $6x^3+6x^2=2\cdot 3x^2(x+1)$

$2x^2-2x+2=2(x^2-x+1)$

$x^3+1=(x+1)(x^2-x+1)$

$3x^3=3x^3$

$mcm\ \ 2\cdot 3x^3(x+1)(x^2-x+1)$

$\Rightarrow 6x^3(x+1)(x^2-x+1)$

28. $ax-a+bx-b=a(x-1)+b(x-1)=(a+b)(x-1)$

$a^2+2ab+b^2=(a+b)^2$

$4xy^2=2^2xy^2$

$3x^3-3x^2=3x^2(x-1)$

$mcm\ \ 2^2\cdot 3x^2y^2(a+b)^2(x-1)\Rightarrow 12x^2y^2(a+b)^2(x-1)$

29. $5ab^3-5b^4=5b^3(a-b)$

$12a^2-24ab+12b^2=2^2\cdot 3(a^2-2ab+b^2)$

$=2^2\cdot 3(a-b)^2$

$6a^2b=2\cdot 3a^2b$

$4b=2^2b$

$2a=2a$

$mcm=5\cdot 2^2\cdot 3a^2b^3(a-b)^2\ \Rightarrow\ 60a^2b^3(a-b)^2$

30. $14x+14=7\cdot 2(x+1)$

$x^2+2x+1=(x+1)^2$

$7x^2+7=7(x^2+1)$

$28x=7\cdot 2^2x$

$x^2+1=(x^2+1)$

$mcm\ \ 28x(x+1)^2(x^2+1)$

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1. $6x-6=2\cdot 3(x-1)$

$3x+3=3(x+1)$

$mcm\ 2\cdot 3(x-1)(x+1)$

$\Rightarrow 6(x^2-1)$

2. $10x^2-40=5\cdot 2(x^2-4)$

$=5\cdot 2(x+2)(x-2)$

$5x+10=5(x+2)$

$mcm\ 5\cdot 2(x+2)(x-2)$

$\Rightarrow 10(x^2-4)$

3. $x^2-4y^2=(x+2y)(x-2y)$

$x^3+2x^2y=x^2(x+2y)$

$mcm\ \ x^2(x+2y)(x-2y)$

$\Rightarrow x^2(x^2-4y^2)$

4. $x^2-6x+9=(x-3)^2$

$3a^2x-9a^2=3a^2(x-3)$

$mcm\ \ 3a^2(x-3)^2$

5. $4a^2-12ab+9b^2=(2a-3b)^2$

$4a^2-9b^2=(2a+3b)(2a-3b)$

$mcm\ \ (2a+3b)(2a-3b)^2$

6. $a^3+2a^2b+ab^2=a(a^2+2ab+b^2)$

$=a(a+b)^2$

$a^3+a^2b=a^2(a+b)$

$mcm\ \ a^2(a+b)^2$

7. $2bx^2+6bx-8b=2b(x^2+3x-4)$

$=2b(x+4)(x-1)$

$3ax+12a=3a(x+4)$

$mcm\ \ 2b\cdot 3a(x+4)(x-1)$

$\Rightarrow 6ab(x+4)(x-1)$

8. $x^2+2x-15=(x+5)(x-3)$

$x^3-25x=x(x^2-25)$

$=x(x+5)(x-5)$

$mcm\ \ x(x+5)(x-5)(x-3)$

$\Rightarrow x(x^2-25)(x-3)$

9. $(x-1)^2=(x-1)^2$

$x^2-1=(x+1)(x-1)$

$mcm\ (x+1)(x-1)^2$

10. $(x+1)^2=(x+1)^2$

$x^2+1=x^2+1$

$mcm\ (x^2+1)(x+1)^2$

11. $x^3+y^3=(x+y)(x^2-xy+y^2)$

$(x+y)^3=(x+y)^3$

$mcm\ \ (x+y)^3(x^2-xy+y^2)$

12. $x^3-y^3=(x-y)(x^2+xy+y^2)$

$(x-y)^3=(x-y)^3$

$mcm\ \ (x-y)^3(x^2+xy+y^2)$

13. $4x^2-7x-2=(4x)^2-7(4x)-8$

$=[(4x-8)(4x+1)]\div 4$

$=(x-2)(4x+1)$

$x^2+3x-10=(x+5)(x-2)$

$mcm\ \ (x-2)(4x+1)(x+5)$

14. $a^2+a-30=(a+6)(a-5)$

$a^2+3a-18=(a+6)(a-3)$

$mcm\ (a+6)(a-3)(a-5)$

15. $x^4+2x^3-15x^2=x^2(x^2+2x-15)$

$=x^2(x+5)(x-3)$

$x^3-9x+5x^2-45=x(x^2-9)+5(x^2-9)$

$=(x+5)(x^2-9)$

$=(x+5)(x+3)(x-3)$

$mcm\ x^2(x+5)(x+3)(x-3)$

$\Rightarrow\ x^2(x+5)(x^2-9)$

16. $x^6-4x^3-32=(x^3-8)(x^3+4)$

$=(x-2)(x^2+2x+4)(x^3+4)$

$ax^4+2ax^3+4ax^2=ax^2(x^2+2x+4)$

$mcm\ ax^2(x-2)(x^2+2x+4)(x^3+4)$

17. $12(x^2-y^2)=2^2\cdot 3(x+y)(x-y)$

$8(x-y)^2=2^3(x-y)^2$

$mcm\ 2^3\cdot 3(x+y)(x-y)^2\Rightarrow 24(x+y)(x-y)^2$

18. $5(x+y)^2=5(x+y)^2$

$10(x^2+y^2)=5\cdot 2(x^2+y^2)$

$mcm\ 10(x^2+y^2)(x+y)^2$

19. $6a(m+n)^3=2\cdot 3a(m+n)^3$

$4a^2b(m^3+n^3)=2^2a^2b(m+n)(m^2-mn+n^2)$

$mcm\ 2^2\cdot 3a^2b(m+n)^3(m^2-mn+n^2)$

$\Rightarrow 12a^2b(m+n)^3(m^2-mn+n^2)$

20. $ax(m-n)^3=ax(m-n)^3$

$x^3(m^3-n^3)=x^3(m-n)(m^2+mn+n^2)$

$mcm\quad ax^3(m-n)^3(m^2+mn+n^2)$

21. $2a^2+2a=2a(a+1)$

$3a^2-3a=3a(a-1)$

$a^4-a^2=a^2(a^2-1)=a^2(a+1)(a-1)$

$mcm\quad 2\cdot 3a^2(a+1)(a-1)\Rightarrow 6a^2(a^2-1)$

22. $x^2+2x=x(x+2)$

$x^3-2x^2=x^2(x-2)$

$x^2-4=(x+2)(x-2)$

$mcm\ x^2(x-2)(x+2)\Rightarrow x^2(x^2-4)$

23. $x^2+x-2=(x+2)(x-1)$

$x^2-4x+3=(x-3)(x-1)$

$x^2-x-6=(x-3)(x+2)$

$mcm(x+2)(x-1)(x-3)$

24. $4+12a+9a^2=(2+3a)^2$

$3a^2+14a+8=(3a)^2+14(3a)+24$

$=[(3a+12)(3a+2)]\div 3\ =(a+4)(3a+2)$

$6a^2+13a+6=(6a)^2+13(6a)+36$

$=[(6a+9)(6a+4)]\div 3\cdot 2\ =(2a+3)(3a+2)$

$mcm\ (a+4)(2a+3)(3a+2)^2$

25. $10x^2+10=5\cdot 2(x^2+1)$

$15x+15=5\cdot 3(x+1)$

$5x^2-5=5(x^2-1)=5(x+1)(x-1)$

$mcm\ 5\cdot 2\cdot 3(x^2+1)(x+1)(x-1)\Rightarrow 30(x^2+1)(x^2-1)$

26. $ax-2bx+ay-2by=x(a-2b)+y(a-2b)$

$=(x+y)(a-2b)$

$x^2+xy=x(x+y)$

$x^2-xy=x(x-y)$

$mcm\ x(x+y)(x-y)(a-2b)\Rightarrow x(x^2-y^2)(a-2b)$

27. $4a^2b+4ab^2=2^2ab(a+b)$

$6a-6b=2\cdot 3(a-b)$

$15a^2-15b^2=5\cdot 3(a^2-b^2)=5\cdot 3(a+b)(a-b)$

$mcm\ \ 2^2\cdot 3\cdot 5ab(a+b)(a-b)\ \Rightarrow 60ab(a^2-b^2)$

28. $x^2-25=(x+5)(x-5)$

$x^3-125=(x-5)(x^2+5x+25)$

$2x+10=2(x+5)$

$mcm\ \ 2(x+5)(x-5)(x^2+5x+25)$

29. $a^3b-6a^2b^2+9ab^3=ab(a^2-6ab+9b^2)$

$=ab(a-3b)^2$

$a^2-2ab-3b^2=(a-3b)(a+b)$

$ab^2+b^3=b^2(a+b)$

$mcm\quad ab^2(a-3b)^2(a+b)$

30. $2m^2+2mn=2m(m+n)$

$4mn-4n^2=2^2n(m-n)$

$6m^3n-6mn^3=2\cdot 3mn(m^2-n^2)=2\cdot 3mn(m+n)(m-n)$

$mcm\quad 2^2\cdot 3mn(m+n)(m-n)\ \Rightarrow 12mn(m^2-n^2)$

31. $20(x^2-y^2)=2^2\cdot 5(x+y)(x-y)$

$15(x-y)^2=5\cdot 3(x-y)^2$

$12(x+y)^2=2^2\cdot 3(x+y)^2$

$mcm\ \ 2^2\cdot 5\cdot 3(x+y)^2(x-y)^2\ \Rightarrow 60(x+y)^2(x-y)^2$

32. $ax^2+5ax-14a=a(x^2+5x-14)\ =a(x+7)(x-2)$

$x^3+14x^2+49x=x(x^2+14x+49)=x(x+7)^2$

$x^4+7x^3-18x^2=x^2(x^2+7x-18)=x^2(x+9)(x-2)$

$mcm\ \ ax^2\ (x+7)^2(x-2)(x+9)$

33. $2x^3-12x^2+18x=2x\left(x^2-6x+9\right)=2x\left(x-3\right)^2$

$3x^4-27x^2=3x^2\left(x^2-9\right) \quad =3x^2\left(x+3\right)\left(x-3\right)$

$5x^3+30x^2+45x=5x\left(x^2+6x+9\right)=5x\left(x+3\right)^2$

$mcm\ 2\cdot 3\cdot 5x^2\left(x+3\right)^2\left(x-3\right)^2 \Rightarrow 30x^2\left(x+3\right)^2\left(x-3\right)^2$

34. $3-3a^2=3\left(1-a^2\right)=3\left(1+a\right)\left(1-a\right)$

$6+6a=2\cdot 3\left(1+a\right)$

$12+12a^2=2^2\cdot 3\left(1+a^2\right)$

$9-9a=3^2\left(1-a\right)$

$mcm\ 2^2\cdot 3^2\left(1+a^2\right)\left(1+a\right)\left(1-a\right)$

$\Rightarrow 36\left(1+a^2\right)\left(1-a^2\right)=36\left(1-a^4\right)$

35. $2\left(3n-2\right)^2=2\left(3n-2\right)^2$

$135n^3-40=5\left(27n^3-8\right)=5\left(3n-2\right)\left(9n^2+6n+4\right)$

$12n-8=2^2\left(3n-2\right)$

$mcm\ 5\cdot 2^2\left(3n-2\right)^2\left(9n^2+6n+4\right)$

$\Rightarrow 20\left(3n-2\right)^2\left(9n^2+6n+4\right)$

36. $12mn+8m-3n-2=4m\left(3n+2\right)-\left(3n+2\right)$

$=\left(4m-1\right)\left(3n+2\right)$

$48m^2n-3n+32m^2-2=16m^2\left(3n+2\right)-\left(3n+2\right)$

$=\left(16m^2-1\right)\left(3n+2\right)$

$=\left(4m+1\right)\left(4m-1\right)\left(3n+2\right)$

$6n^2-5n-6=(6n)^2-5(6n)-36$

$=\left[\left(6n-9\right)\left(6n+4\right)\right]\div 3\cdot 2$

$=\left(2n-3\right)\left(3n+2\right)$

$mcm\quad \left(4m+1\right)\left(4m-1\right)\left(3n+2\right)\left(2n-3\right)$

$\Rightarrow \left(16m^2-1\right)\left(3n+2\right)\left(2n-3\right)$

37. $15a^2x^5+16a^2x^4-15a^2x^3=a^2x^3\left(15x^2+16x-15\right)$

$=a^2x^3\left[(15x)^2+16(15x)-225\right]$

$=a^2x^3\left[\left(15x+25\right)\left(15x-9\right)\div 5\cdot 3\right]$

$=a^2x^3\left(3x+5\right)\left(5x-3\right)$

$18x^3+60x^2+50x=2x\left(9x^2+30x+25\right)$

$=2x\left[(9x)^2+30(9x)+225\right]$

$=2x\left[\left(9x+15\right)\left(9x+15\right)\div 3\cdot 3\right]$

$=2x\left(3x+5\right)^2$

$12ax^3+20ax^2=2^2ax^2\left(3x+5\right)$

$mcm\ =2^2a^2x^3\left(3x+5\right)^2\left(5x-3\right)$

$=4a^2x^3\left(3x+5\right)^2\left(5x-3\right)$

38. $16+8x^2+x^4=\left(4+x^2\right)^2$

$16-8x^2+x^4=\left(4-x^2\right)^2 \quad =\left(2-x\right)^2\left(2+x\right)^2$

$16-x^4=\left(4+x^2\right)\left(4-x^2\right)=\left(4+x^2\right)\left(2+x\right)\left(2-x\right)$

$mcm\quad \left(4+x^2\right)^2\left(2+x\right)^2\left(2-x\right)^2$

39. $1+a^2=1+a^2$

$\left(1+a\right)^2=\left(1+a\right)^2$

$1+a^3=\left(1+a\right)\left(1-a+a^2\right)$

$mcm\ \left(1+a\right)^2\left(1-a+a^2\right)\left(1+a^2\right)$

40. $8n^2-10n-3=(8n)^2-10(8n)-24$

$=\left[\left(8n-12\right)\left(8n+2\right)\right]\div 4\cdot 2=\left(2n-3\right)\left(4n+1\right)$

$20n^2+13n+2=(20n)^2+13(20n)+40$

$=\left[\left(20n+8\right)\left(20n+5\right)\right]\div 4\cdot 5=\left(5n+2\right)\left(4n+1\right)$

$10n^2-11n-6=(10n)^2-11(10n)-60$

$=\left[\left(10n-15\right)\left(10n+4\right)\right]\div 5\cdot 2=\left(2n-3\right)\left(5n+2\right)$

$mcm\quad \left(2n-3\right)\left(5n+2\right)\left(4n+1\right)$

41. $6a^2+ab-2b^2=36a^2+6ab-12b^2$

$=\left[\left(6a+4b\right)\left(6a-3b\right)\right]\div 2\cdot 3$

$=\left(3a+2b\right)\left(2a-b\right)$

$15a^2+22ab+8b^2=(15a)^2+22b(15a)+120b^2$

$=\left[\left(15a+12b\right)\left(15a+10b\right)\right]\div 3\cdot 5$

$=\left(5a+4b\right)\left(3a+2b\right)$

$10a^2+3ab-4b^2=(10a)^2+3b(10a)-40b^2$

$=\left[\left(10a+8b\right)\left(10a-5b\right)\right]\div 2\cdot 5$

$=\left(5a+4b\right)\left(2a-b\right)$

$mcm\quad \left(3a+2b\right)\left(2a-b\right)\left(5a+4b\right)$

42. $12x^2+5xy-2y^2=(12x)^2+5y(12x)-24y^2$

$=\left[\left(12x+8y\right)\left(12x-3y\right)\right]\div 4\cdot 3$

$=\left(3x+2y\right)\left(4x-y\right)$

$15x^2+13xy+2y^2=(15x)^2+13y(15x)+30y^2$

$=\left[\left(15x+10y\right)\left(15x+3y\right)\right]\div 5\cdot 3$

$=\left(3x+2y\right)\left(5x+y\right)$

$20x^2-xy-y^2=(20x)^2-y(20x)-20y^2$

$=\left[\left(20x-5y\right)\left(20x+4y\right)\right]\div 5\cdot 4$

$=\left(4x-y\right)\left(5x+y\right)$

$mcm\quad \left(4x-y\right)\left(5x+y\right)\left(3x+2y\right)$

43. $6b^2x^2+6b^2x^3=2\cdot 3b^2x^2(1+x)$

$3a^2x-3a^2x^2=3a^2x(1-x)$

$1-x^4=(1+x^2)(1-x^2)=(1+x^2)(1+x)(1-x)$

$mcm\ 2\cdot 3a^2b^2x^2\ (1+x^2)(1+x)(1-x)$

$\Rightarrow 6a^2b^2x^2\ (1+x^2)(1-x^2)$

44. $x^4+8x-4x^3-32=x^3(x-4)+8(x-4)$

$=(x^3+8)(x-4)$

$=(x+2)(x^2-2x+4)(x-4)$

$a^2x^4-2a^2x^3-8a^2x^2=a^2x^2(x^2-2x-8)$

$=a^2x^2(x-4)(x+2)$

$2x^4-4x^3+8x^2=2x^2(x^2-2x+4)$

$mcm\ \ 2a^2x^2(x+2)(x^2-2x+4)(x-4)$

45. $x^4-10x^2+9=(x^4-10x^2+9+4x^2-4x^2)$

$=(x^4-6x^2+9)-4x^2$

$=(x^2-3)^2-4x^2$

$=(x^2-2x-3)(x^2+2x-3)$

$=(x-3)(x+1)(x+3)(x-1)$

$x^2+4x+3=(x^2+4x+3+1-1)$

$=(x^2+4x+4)-1$

$=(x+2)^2-1$

$=(x+2+1)(x+2-1)=(x+3)(x+1)$

$x^2-4x+3=(x^2-4x+3+1-1)$

$=(x^2-4x+4)-1$

$=(x-2)^2-1$

$=(x-2+1)(x-2-1)=(x-1)(x-3)$

$x^3-9x+x^2-9=x^2(x+1)-9(x+1)$

$=(x^2-9)(x+1)$

$=(x+3)(x-3)(x+1)$

$mcm\ \ (x+3)(x-3)(x+1)(x-1)\ \ \Rightarrow (x^2-9)(x^2-1)$

46. $1-a^3=(1-a)(1+a+a^2)$

$1-a=(1-a)$

$1-a^2=(1+a)(1-a)$

$1-2a+a^2=(1-a)^2$

$mcm\ (1-a)^2(1+a+a^2)(1+a)$

47. $a(ab-b^2)^2=a[b(a-b)]^2\ \ =ab^2(a-b)^2$

$b(a^2+ab)^2=b[a(a+b)]^2\ \ =a^2b(a+b)^2$

$a^4b^2-a^2b^4=a^2b^2(a^2-b^2)=a^2b^2(a+b)(a-b)$

$a^2b-ab^2=ab(a-b)$

$mcm\ \ a^2b^2(a+b)^2(a-b)^2$

48. $m^3-27n^3=(m-3n)(m^2+3mn+9n^2)$

$m^2-9n^2=(m-3n)(m+3n)$

$m^2-6mn+9n^2=(m-3n)^2$

$m^2+3mn+9n^2=(m^2+3mn+9n^2)$

$mcm\ \ (m-3n)^2(m^2+3mn+9n^2)(m+3n)$

EJERCICIO 118

1. $\dfrac{a^2}{ab}=\dfrac{a}{b}$

2. $\dfrac{2a}{8a^2b}=\dfrac{1}{4ab}$

3. $\dfrac{x^2y^2}{x^3y^3}=\dfrac{1}{xy}$

4. $\dfrac{ax^3}{4x^5y}=\dfrac{a}{4x^2y}$

5. $\dfrac{6m^2n^3}{3m}=2mn^3$

6. $\dfrac{9x^2y^3}{24a^2x^3y^4}=\dfrac{3}{8a^2xy}$

7. $\dfrac{8m^4n^3x^2}{24mn^2x^2}=\dfrac{m^3n}{3}$

8. $\dfrac{12x^3y^4z^5}{32xy^2z}=\dfrac{3x^2y^2z^4}{8}$

9. $\dfrac{12a^2b^3}{60a^3b^5x^6}=\dfrac{1}{5ab^2x^6}$

10. $\dfrac{21mn^3x^6}{28m^4n^2x^2}=\dfrac{3nx^4}{4m^3}$

11. $\dfrac{42a^2c^3n}{26a^4c^5m}=\dfrac{21n}{13a^2c^2m}$

12. $\dfrac{7x^3y^4z^6}{34x^7y^8z^{10}}=\dfrac{1}{2x^4y^4z^4}$

13. $\dfrac{30x^6y^2}{45a^3x^4z^3}=\dfrac{2x^2y^2}{3a^3z^3}$

14. $\dfrac{a^5b^7}{3a^8b^9c}=\dfrac{1}{3a^3b^2c}$

15. $\dfrac{21a^8b^{10}c^{12}}{63a^4bc^2}=\dfrac{a^4b^9c^{10}}{3}$

16. $\dfrac{54x^9y^{11}z^{13}}{63x^{10}y^{12}z^{15}}=\dfrac{6}{7xyz^2}$

17. $\dfrac{15a^{12}b^{15}c^{20}}{75a^{11}b^{16}c^{22}}=\dfrac{a}{5bc^2}$

18. $\dfrac{75a^7m^5}{100a^3m^{12}n^3}=\dfrac{3a^4}{4m^7n^3}$

EJERCICIO 119

1. $\frac{3ab}{2a^2x+2a^3}=\frac{3ab}{2a^2(x+a)}=\frac{3b}{2a(x+a)}$

2. $\frac{xy}{3x^2y-3xy^2}=\frac{xy}{3xy(x-y)}=\frac{1}{3(x-y)}$

3. $\frac{2ax+4bx}{3ay+6by}=\frac{2x(a+2b)}{3y(a+2b)}=\frac{2x}{3y}$

4. $\frac{x^2-2x-3}{x-3}=\frac{(x-3)(x+1)}{x-3}=x+1$

5. $\frac{10a^2b^3c}{80(a^3-a^2b)}=\frac{a^2b^3c}{8a^2(a-b)}=\frac{b^3c}{8(a-b)}$

6. $\frac{x^2-4}{5ax+10a}=\frac{(x+2)(x-2)}{5a(x+2)}=\frac{x-2}{5a}$

7. $\frac{3x^2-4x-15}{x^2-5x+6}$

$=\frac{(3x)^2-4(3x)-45}{(x-3)(x-2)}$

$=\frac{[(3x-9)(3x+5)]\div 3}{(x-3)(x-2)}=\frac{(x-3)(3x+5)}{(x-3)(x-2)}=\frac{3x+5}{x-2}$

8. $\frac{15a^2bn-45a^2bm}{10a^2b^2n-30a^2b^2m}=\frac{15a^2b(n-3m)}{10a^2b^2(n-3m)}=\frac{3}{2b}$

9. $\frac{x^2-y^2}{x^2+2xy+y^2}=\frac{(x+y)(x-y)}{(x+y)^2}=\frac{x-y}{x+y}$

10. $\frac{3x^2y+15xy}{x^2-25}=\frac{3xy(x+5)}{(x+5)(x-5)}=\frac{3xy}{x-5}$

11. $\frac{a^2-4ab+4b^2}{a^3-8b^3}=\frac{(a-2b)^2}{(a-2b)(a^2+2ab+4b^2)}=\frac{a-2b}{a^2+2ab+4b^2}$

12. $\frac{x^3+4x^2-21x}{x^3-9x}=\frac{x(x^2+4x-21)}{x(x^2-9)}=\frac{(x+7)(x-3)}{(x+3)(x-3)}=\frac{x+7}{x+3}$

13. $\frac{6x^2+5x-6}{15x^2-7x-2}$

$=\frac{(6x)^2+5(6x)-36}{(15x)^2-7(15x)-30}$

$=\frac{[(6x+9)(6x-4)]\div 3\cdot 2}{[(15x-10)(15x+3)]\div 5\cdot 3}=\frac{(2x+3)(3x-2)}{(3x-2)(5x+1)}=\frac{2x+3}{5x+1}$

14. $\frac{a^3+1}{a^4-a^3+a-1}=\frac{a^3+1}{a^3(a-1)+(a-1)}=\frac{a^3+1}{(a^3+1)(a-1)}=\frac{1}{a-1}$

15. $\frac{2ax+ay-4bx-2by}{ax-4a-2bx+8b}$

$=\frac{2x(a-2b)+y(a-2b)}{x(a-2b)-4(a-2b)}=\frac{(2x+y)(a-2b)}{(x-4)(a-2b)}=\frac{2x+y}{x-4}$

16. $\frac{a^2-ab-6b^2}{a^3x-6a^2bx+9ab^2x}$

$=\frac{(a-3b)(a+2b)}{ax(a^2-6b+9b^2)}=\frac{(a-3b)(a+2b)}{ax(a-3b)^2}=\frac{a+2b}{ax(a-3b)}$

17. $\frac{m^2+n^2}{m^4-n^4}=\frac{m^2+n^2}{(m^2+n^2)(m^2-n^2)}=\frac{1}{m^2-n^2}$

18. $\frac{x^3+y^3}{(x+y)^3}=\frac{(x+y)(x^2-xy+y^2)}{(x+y)^3}=\frac{x^2-xy+y^2}{(x+y)^2}$

19. $\frac{(m-n)^2}{m^2-n^2}=\frac{(m-n)^2}{(m+n)(m-n)}=\frac{m-n}{m+n}$

20. $\frac{(a-x)^3}{a^3-x^3}=\frac{(a-x)^3}{(a-x)(a^2+ax+x^2)}=\frac{(a-x)^2}{a^2+ax+x^2}$

21. $\frac{a^2-a-20}{a^2-7a+10}=\frac{(a-5)(a+4)}{(a-5)(a-2)}=\frac{a+4}{a-2}$

22. $\frac{(1-a^2)^2}{a^2+2a+1}=\frac{[(a+1)(1-a)]^2}{(a+1)^2}=(1-a)^2$

23. $\frac{a^4b^2-a^2b^4}{a^4-b^4}=\frac{a^2b^2(a^2-b^2)}{(a^2+b^2)(a^2-b^2)}=\frac{a^2b^2}{a^2+b^2}$

24. $\frac{x^2-y^2}{x^3-y^3}=\frac{(x+y)(x-y)}{(x-y)(x^2+xy+y^2)}=\frac{x+y}{x^2+xy+y^2}$

25. $\frac{24a^3b+8a^2b^2}{36a^4+24a^3b+4a^2b^2}$

$=\frac{8a^2b(3a+b)}{4a^2(9a^2+6ab+b^2)}=\frac{2b(3a+b)}{(3a+b)^2}=\frac{2b}{3a+b}$

26. $\frac{n^3-n}{n^2-5n-6}$

$=\frac{n(n^2-1)}{(n-6)(n+1)}=\frac{n(n+1)(n-1)}{(n-6)(n+1)}=\frac{n(n-1)}{n-6}$

27. $\frac{8n^3+1}{8n^3-4n^2+2n}$

$=\frac{(2n+1)(4n^2-2n+1)}{2n(4n^2-2n+1)}=\frac{2n+1}{2n}$

28. $\dfrac{a^2-(b-c)^2}{(a+b)^2-c^2}$

$=\dfrac{(a+b-c)(a-b+c)}{(a+b-c)(a+b+c)}=\dfrac{a-b+c}{a+b+c}$

29. $\dfrac{(a+b)^2-(c-d)^2}{(a+c)^2-(b-d)^2}$

$=\dfrac{(a+b+c-d)(a+b-c+d)}{(a+b+c-d)(a-b+c+d)}$

$=\dfrac{a+b-c+d}{a-b+c+d}$

30. $\dfrac{3x^3+9x^2}{x^2+6x+9}=\dfrac{3x^2(x+3)}{(x+3)^2}=\dfrac{3x^2}{x+3}$

31. $\dfrac{10a^2(a^3+b^3)}{6a^4-6a^3b+6a^2b^2}$

$=\dfrac{10a^2(a+b)(a^2-ab+b^2)}{6a^2(a^2-ab+b^2)}=\dfrac{5(a+b)}{3}$

32. $\dfrac{a(4a^2-8ab)}{x(3a^2-6ab)}=\dfrac{4a^2(a-2b)}{3ax(a-2b)}=\dfrac{4a}{3x}$

33. $\dfrac{x^3-6x^2}{x^2-12x+36}=\dfrac{x^2(x-6)}{(x-6)^2}=\dfrac{x^2}{x-6}$

34. $\dfrac{(x-4y)^2}{x^5-64x^2y^3}$

$=\dfrac{(x-4y)^2}{x^2(x^3-64y^3)}$

$=\dfrac{(x-4y)^2}{x^2(x-4y)(x^2+4xy+16y^2)}$

$=\dfrac{x-4y}{x^2(x^2+4xy+16y^2)}$

35. $\dfrac{x^3-3xy^2}{x^4-6x^2y^2+9y^4}=\dfrac{x(x^2-3y^2)}{(x^2-3y^2)^2}=\dfrac{x}{x^2-3y^2}$

36. $\dfrac{m^3n+3m^2n+9mn}{m^3-27}$

$=\dfrac{mn(m^2+3m+9)}{(m-3)(m^2+3m+9)}=\dfrac{mn}{m-3}$

37. $\dfrac{x^4-8x^2+15}{x^4-9}=\dfrac{(x^2-5)(x^2-3)}{(x^2+3)(x^2-3)}=\dfrac{x^2-5}{x^2+3}$

38. $\dfrac{a^4+6a^2-7}{a^4+8a^2-9}=\dfrac{(a^2+7)(a^2-1)}{(a^2+9)(a^2-1)}=\dfrac{a^2+7}{a^2+9}$

39. $\dfrac{3x^2+19x+20}{6x^2+17x+12}$

$=\dfrac{(3x)^2+19(3x)+60}{(6x)^2+17(6x)+72}$

$=\dfrac{[(3x+15)(3x+4)]\div 3}{[(6x+8)(6x+9)]\div 2\cdot 3}$

$=\dfrac{(x+5)(3x+4)}{(3x+4)(2x+3)}=\dfrac{x+5}{2x+3}$

40. $\dfrac{4a^4-15a^2-4}{a^2-8a-20}$

$=\dfrac{(4a^2)^2-15(4a^2)-16}{(a-10)(a+2)}$

$=\dfrac{[(4a^2-16)(4a^2+1)]\div 4}{(a-10)(a+2)}$

$=\dfrac{(a^2-4)(4a^2+1)}{(a-10)(a+2)}$

$=\dfrac{(a+2)(a-2)(4a^2+1)}{(a-10)(a+2)}$

$=\dfrac{(a-2)(4a^2+1)}{a-10}$

41. $\dfrac{125a+a^4}{2a^3+20a^2+50a}$

$=\dfrac{a(125+a^3)}{2a(a^2+10a+25)}$

$=\dfrac{(5+a)(25-5a+a^2)}{2(a+5)^2}$

$=\dfrac{a^2-5a+25}{2(a+5)}$

42. $\dfrac{a^2n^2-36a^2}{an^2+an-30a}$

$=\dfrac{a^2(n^2-36)}{a(n^2+n-30)}$

$=\dfrac{a(n+6)(n-6)}{(n+6)(n-5)}$

$=\dfrac{a(n-6)}{n-5}$

43. $\dfrac{3m^2+5mn-8n^2}{m^3-n^3}$

$=\dfrac{(3m)^2+5n(3m)-24n^2}{(m-n)(m^2+mn+n^2)}$

$=\dfrac{[(3m+8n)(3m-3n)]\div 3}{(m-n)(m^2+mn+n^2)}$

$=\dfrac{(3m+8n)(m-n)}{(m-n)(m^2+mn+n^2)}$

$=\dfrac{3m+8n}{m^2+mn+n^2}$

44. $\dfrac{15a^3b-18a^2b}{20a^2b^2-24ab^2}$

$=\dfrac{3a^2b(5a-6)}{4ab^2(5a-6)}=\dfrac{3a}{4b}$

45. $\dfrac{9x^2-24x+16}{9x^4-16x^2}$

$=\dfrac{(3x-4)^2}{x^2(9x^2-16)}$

$=\dfrac{(3x-4)^2}{x^2(3x+4)(3x-4)}=\dfrac{3x-4}{x^2(3x+4)}$

46. $\dfrac{16a^2x-25x}{12a^3-7a^2-10a}$

$=\dfrac{x(16a^2-25)}{a(12a^2-7a-10)}$

$=\dfrac{x(4a+5)(4a-5)}{a[(12a)^2-7(12a)-120]}$

$=\dfrac{x(4a+5)(4a-5)}{a[(12a-15)(12a+8)]\div 3\cdot 4}$

$=\dfrac{x(4a+5)(4a-5)}{a(4a-5)(3a+2)}=\dfrac{x(4a+5)}{a(3a+2)}$

47. $\dfrac{8x^4-xy^3}{4x^4-4x^3y+x^2y^2}$

$=\dfrac{x(8x^3-y^3)}{x^2(4x^2-4xy+y^2)}$

$=\dfrac{(2x-y)(4x^2+2xy+y^2)}{x(2x-y)^2}$

$=\dfrac{4x^2+2xy+y^2}{x(2x-y)}$

48. $$\frac{3an-4a-6bn+8b}{6n^2-5n-4}$$
$$=\frac{3n\left(a-2b\right)-4\left(a-2b\right)}{(6n)^2-5(6n)-24}$$
$$=\frac{\left(3n-4\right)\left(a-2b\right)}{\left[\left(6n-8\right)\left(6n+3\right)\right]\div 2\cdot 3}$$
$$=\frac{\left(3n-4\right)\left(a-2b\right)}{\left(3n-4\right)\left(2n+1\right)}=\frac{a-2b}{2n+1}$$

49. $$\frac{x^4-49x^2}{x^3+2x^2-63x}$$
$$=\frac{\left(x^2+7x\right)\left(x^2-7x\right)}{x\left(x^2+2x-63\right)}$$
$$=\frac{x^2\left(x+7\right)\left(x-7\right)}{x\left(x+9\right)\left(x-7\right)}=\frac{x\left(x+7\right)}{x+9}$$

50. $$\frac{x^4+x-x^3y-y}{x^3-x-x^2y+y}$$
$$=\frac{x^3\left(x-y\right)+\left(x-y\right)}{x^2\left(x-y\right)-\left(x-y\right)}$$
$$=\frac{\left(x^3+1\right)\left(x-y\right)}{\left(x^2-1\right)\left(x-y\right)}$$
$$=\frac{\left(x+1\right)\left(x^2-x+1\right)}{\left(x+1\right)\left(x-1\right)}$$
$$=\frac{x^2-x+1}{x-1}$$

51. $$\frac{2x^3+6x^2-x-3}{x^3+3x^2+x+3}$$
$$=\frac{2x^2\left(x+3\right)-\left(x+3\right)}{x^2\left(x+3\right)+\left(x+3\right)}$$
$$=\frac{\left(2x^2-1\right)\left(x+3\right)}{\left(x^2+1\right)\left(x+3\right)}=\frac{2x^2-1}{x^2+1}$$

52. $$\frac{a^3m-4am+a^3n-4an}{a^4-4a^3-12a^2}$$
$$=\frac{am\left(a^2-4\right)+an\left(a^2-4\right)}{a^2\left(a^2-4a-12\right)}$$
$$=\frac{a\left(m+n\right)\left(a^2-4\right)}{a^2\left(a-6\right)\left(a+2\right)}$$
$$=\frac{\left(m+n\right)\left(a+2\right)\left(a-2\right)}{a\left(a-6\right)\left(a+2\right)}$$
$$=\frac{\left(m+n\right)\left(a-2\right)}{a\left(a-6\right)}$$

53. $$\frac{4a^2-\left(x-3\right)^2}{\left(2a+x\right)^2-9}$$
$$=\frac{\left(2a+x-3\right)\left(2a-x+3\right)}{\left(2a+x+3\right)\left(2a+x-3\right)}$$
$$=\frac{2a-x+3}{2a+x+3}$$

54. $$\frac{m-am+n-an}{1-3a+3a^2-a^3}$$
$$=\frac{m\left(1-a\right)+n\left(1-a\right)}{\left(1-a^3\right)-3a\left(1-a\right)}$$
$$=\frac{\left(m+n\right)\left(1-a\right)}{\left(1-a\right)\left(1+a+a^2\right)-3a\left(1-a\right)}$$
$$=\frac{\left(m+n\right)\left(1-a\right)}{\left(1-a\right)\left(1-2a+a^2\right)}$$
$$=\frac{\mathrm{m}+n}{a^2-2a+1}=\frac{m+n}{\left(a-1\right)^2}$$

55. $$\frac{6x^2+3}{42x^5-9x^3-15x}$$
$$=\frac{3\left(2x^2+1\right)}{3x\left(14x^4-3x^2-5\right)}$$
$$=\frac{2x^2+1}{x\left[\left(14x^2\right)^2-3\left(14x^2\right)-70\right]}$$
$$=\frac{2x^2+1}{x\left[\left(14x^2-10\right)\left(14x^2+7\right)\right]\div 2\cdot 7}$$
$$=\frac{2x^2+1}{x\left(7x^2-5\right)\left(2x^2+1\right)}=\frac{1}{x\left(7x^2-5\right)}$$

56. $$\frac{a^2-a^3-1+a}{a^2+1-a^3-a}$$
$$=\frac{a^2\left(1-a\right)-\left(1-a\right)}{a^2\left(1-a\right)+\left(1-a\right)}$$
$$=\frac{\left(a^2-1\right)\left(1-a\right)}{\left(a^2+1\right)\left(1-a\right)}=\frac{a^2-1}{a^2+1}$$

57. $$\frac{8x^3+12x^2y+6xy^2+y^3}{6x^2+xy-y^2}$$
$$=\frac{6xy\left(2x+y\right)+\left(8x^3+y^3\right)}{(6x)^2+y(6x)-6y^2}$$
$$=\frac{6xy\left(2x+y\right)+\left(2x+y\right)\left(4x^2-2xy+y^2\right)}{\left[\left(6x+3y\right)\left(6x-2y\right)\right]\div 3\cdot 2}$$
$$=\frac{\left(2x+y\right)\left(4x^2+4xy+y^2\right)}{\left(2x+y\right)\left(3x-y\right)}=\frac{\left(2x+y\right)^2}{3x-y}$$

58. $$\frac{8n^3-125}{25-20n+4n^2}$$
$$=\frac{\left(2n-5\right)\left(4n^2+10n+25\right)}{\left(2n-5\right)^2}$$
$$=\frac{4n^2+10n+25}{2n-5}$$

59. $$\frac{6-x-x^2}{15+2x-x^2}$$
$$=\frac{-\left(x^2+x-6\right)}{-\left(x^2-2x-15\right)}$$
$$=\frac{\left(x+3\right)\left(x-2\right)}{\left(x-5\right)\left(x+3\right)}=\frac{x-2}{x-5}$$

60. $$\frac{3+2x-8x^2}{4+5x-6x^2}$$
$$=\frac{-\left(8x^2-2x-3\right)}{-\left(6x^2-5x-4\right)}$$
$$=\frac{(8x)^2-2(8x)-24}{(6x)^2-5(6x)-24}$$
$$=\frac{\left[\left(8x-6\right)\left(8x+4\right)\right]\div 2\cdot 4}{\left[\left(6x-8\right)\left(6x+3\right)\right]\div 2\cdot 3}$$
$$=\frac{\left(4x-3\right)\left(2x+1\right)}{\left(3x-4\right)\left(2x+1\right)}=\frac{4x-3}{3x-4}$$

61. $$\frac{m^2n^2+3mn-10}{4-4mn+m^2n^2}$$
$$=\frac{\left(mn+5\right)\left(mn-2\right)}{\left(mn-2\right)^2}=\frac{mn+5}{mn-2}$$

62. $$\frac{x^3+x^2y-4b^2x-4b^2y}{4b^2-4bx+x^2}$$
$$=\frac{x^2\left(x+y\right)-4b^2\left(x+y\right)}{\left(2b-x\right)^2}$$
$$=\frac{\left(x^2-4b^2\right)\left(x+y\right)}{\left(x-2b\right)^2}$$
$$=\frac{\left(x+2b\right)\left(x-2b\right)\left(x+y\right)}{\left(x-2b\right)^2}$$
$$=\frac{\left(x+2b\right)\left(x+y\right)}{x-2b}$$

63. $$\frac{x^6+x^3-2}{x^4-x^3y-x+y}$$

$$=\frac{(x^3+2)(x^3-1)}{x^3(x-y)-(x-y)}$$

$$=\frac{(x^3+2)(x^3-1)}{(x^3-1)(x-y)}=\frac{x^3+2}{x-y}$$

64. $$\frac{(x^2-x-2)(x^2-9)}{(x^2-2x-3)(x^2+x-6)}$$

$$=\frac{(x-2)(x+1)(x+3)(x-3)}{(x-3)(x+1)(x+3)(x-2)}=1$$

65. $$\frac{(a^2-4a+4)(4a^2-4a+1)}{(a^2+a-6)(2a^2-5a+2)}$$

$$=\frac{(a-2)^2(2a-1)^2}{(a+3)(a-2)[(2a)^2-5(2a)+4]}$$

$$=\frac{(a-2)(2a-1)^2}{(a+3)[(2a-4)(2a-1)]\div 2}$$

$$=\frac{(a-2)(2a-1)^2}{(a+3)(a-2)(2a-1)}=\frac{2a-1}{a+3}$$

66. $$\frac{(x^3-3x)(x^3-1)}{(x^4+x^3+x^2)(x^2-1)}$$

$$=\frac{x(x^2-3)(x-1)(x^2+x+1)}{x^2(x^2+x+1)(x+1)(x-1)}$$

$$=\frac{x^2-3}{x(x+1)}$$

67. $$\frac{(4n^2+4n-3)(n^2+7n-30)}{(2n^2-7n+3)(4n^2+12n+9)}$$

$$=\frac{[(4n)^2+4(4n)-12](n+10)(n-3)}{[(2n)^2-7(2n)+6](2n+3)^2}$$

$$=\frac{(n+10)(n-3)[(4n+6)(4n-2)]\div 2\cdot 2}{(2n+3)^2[(2n-6)(2n-1)]\div 2\cdot}$$

$$=\frac{(n+10)(n-3)(2n+3)(2n-1)}{(2n+3)^2(n-3)(2n-1)}=\frac{n+10}{2n+3}$$

68. $$\frac{(x^6-y^6)(x+y)}{(x^3-y^3)(x^3+x^2y+xy^2+y^3)}$$

$$=\frac{(x^3+y^3)(x^3-y^3)(x+y)}{(x^3-y^3)[x^2(x+y)+y^2(x+y)]}$$

$$=\frac{(x^3+y^3)(x+y)}{(x^2+y^2)(x+y)}=\frac{x^3+y^3}{x^2+y^2}$$

69. x^3+3x^2-4

$$\begin{array}{rrrr|r} 1&3&0&-4&-2\\ &-2&-2&4&\\ \hline 1&1&-2&0& \end{array}$$

$=(x+2)(x^2+x-2)$

$=(x+2)(x+2)(x-1)$

$=(x+2)^2(x-1)$

$x^3+x^2-8x-12$

$$\begin{array}{rrrr|r} 1&1&-8&-12&-2\\ &-2&2&12&\\ \hline 1&-1&-6&0& \end{array}$$

$=(x+2)(x^2-x-6)$

$=(x+2)(x-3)(x+2)$

$=(x+2)^2(x-3)$

$$\Rightarrow \frac{(x+2)^2(x-1)}{(x+2)^2(x-3)}=\frac{x-1}{x-3}$$

70. $x^3-x^2-8x+12$

$$\begin{array}{rrrr|r} 1&-1&-8&12&2\\ &2&2&-12&\\ \hline 1&1&-6&0& \end{array}$$

$=(x-2)(x^2+x-6)$

$=(x-2)(x+3)(x-2)$

$=(x-2)^2(x+3)$

$x^4-2x^3-7x^2+20x-12$

$$\begin{array}{rrrrr|r} 1&-2&-7&20&-12&2\\ &2&0&-14&12&\\ \hline 1&0&-7&6&0& \end{array}$$

$(x-2)(x^3-7x+6)$

$$\begin{array}{rrrr|r} 1&0&-7&6&2\\ &2&4&-6&\\ \hline 1&2&-3&0& \end{array}$$

$=(x-2)(x-2)(x^2+2x-3)$

$=(x-2)^2(x+3)(x-1)$

$$\Rightarrow \frac{(x-2)^2(x+3)}{(x-2)^2(x+3)(x-1)}$$

$$=\frac{1}{x-1}$$

71. x^4-7x^2-2x+8

$$\begin{array}{rrrrr|r} 1&0&-7&-2&8&-2\\ &-2&4&6&-8&\\ \hline 1&-2&-3&4&0& \end{array}$$

$(x+2)(x^3-2x^2-3x+4)$

$$\begin{array}{rrrr|r} 1&-2&-3&4&1\\ &1&-1&-4&\\ \hline 1&-1&-4&0& \end{array}$$

$(x+2)(x-1)(x^2-x-4)$

$x^4-2x^3-9x^2+10x+24$

$$\begin{array}{rrrrr|r} 1&-2&-9&10&24&-2\\ &-2&8&2&-24&\\ \hline 1&-4&-1&12&0& \end{array}$$

$(x+2)(x^3-4x^2-x+12)$

$$\begin{array}{rrrr|r} 1&-4&-1&12&3\\ &3&-3&-12&\\ \hline 1&-1&-4&0& \end{array}$$

$(x+2)(x-3)(x^2-x-4)$

$$\Rightarrow \frac{(x+2)(x-1)(x^2-x-4)}{(x+2)(x-3)(x^2-x-4)}=\frac{x-1}{x-3}$$

72. $a^5-a^3-a^2+1$

$=a^2(a^3-1)-(a^3-1)$

$=(a^2-1)(a^3-1)$

$=(a-1)^2(a+1)(a^2+a+1)$

$a^5-2a^4-6a^3+8a^2+5a-6$

$$\begin{array}{rrrrrr|r} 1&-2&-6&8&5&-6&1\\ &1&-1&-7&1&6&\\ \hline 1&-1&-7&1&6&0& \end{array}$$

$(a-1)(a^4-a^3-7a^2+a+6)$

$$\begin{array}{rrrrr|r} 1&-1&-7&1&6&1\\ &1&0&-7&-6&\\ \hline 1&0&-7&-6&0& \end{array}$$

$(a-1)^2(a^3-7a-6)$

$$\begin{array}{rrrr|r} 1&0&-7&-6&-1\\ &-1&1&6&\\ \hline 1&-1&-6&0& \end{array}$$

$(a-1)^2(a+1)(a^2-a-6)$

$=(a-1)^2(a+1)(a-3)(a+2)$

$$\Rightarrow \frac{(a-1)^2(a+1)(a^2+a+1)}{(a-1)^2(a+1)(a-3)(a+2)}$$

$$=\frac{a^2+a+1}{(a-3)(a+2)}$$

EJERCICIO 120

1. $\frac{4-4x}{6x-6}=\frac{4(1-x)}{6(x-1)}=-\frac{4(x-1)}{6(x-1)}=-\frac{2}{3}$

2. $\frac{a^2-b^2}{b^2-a^2}=-\frac{(a^2-b^2)}{(a^2-b^2)}=-1$

3. $\frac{m^2-n^2}{(n-m)^2}=\frac{(m+n)(m-n)}{(n-m)(n-m)}=\frac{(m+n)(m-n)}{(m-n)(m-n)}=\frac{m+n}{m-n}$

4. $\frac{x^2-x-12}{16-x^2}=\frac{(x-4)(x+3)}{(4-x)(4+x)}=-\frac{(x-4)(x+3)}{(x-4)(4+x)}=-\frac{x+3}{4+x}$

5. $\frac{3y-6x}{2mx-my-2nx+ny}$

$=\frac{3(y-2x)}{2x(m-n)-y(m-n)}$

$=\frac{3(y-2x)}{(2x-y)(m-n)}=-\frac{3(2x-y)}{(2x-y)(m-n)}=-\frac{3}{m-n}$

6. $\frac{2x^2-9x-5}{10+3x-x^2}$

$=-\frac{\left[(2x)^2-9(2x)-10\right]}{x^2-3x-10}$

$=-\frac{(2x-10)(2x+1)}{(x-5)(x+2)}=-\frac{(x-5)(2x+1)}{(x-5)(x+2)}=-\frac{2x+1}{x+2}$

7. $\frac{8-a^3}{a^2+2a-8}$

$=\frac{(2-a)(4+2a+a^2)}{(a+4)(a-2)}=-\frac{(a-2)(4+2a+a^2)}{(a+4)(a-2)}=-\frac{a^2+2a+4}{a+4}$

8. $\frac{a^2+a-2}{n-an-m+am}$

$=\frac{(a+2)(a-1)}{n(1-a)-m(1-a)}=\frac{(a+2)(a-1)}{(n-m)(1-a)}=\frac{(a+2)(a-1)}{(m-n)(a-1)}=\frac{a+2}{m-n}$

9. $\frac{4x^2-4xy+y^2}{5y-10x}=\frac{(2x-y)^2}{5(y-2x)}=-\frac{(2x-y)^2}{5(2x-y)}=-\frac{2x-y}{5}$

10. $\frac{3mx-nx-3my+ny}{ny^2-nx^2-3my^2+3mx^2}$

$=\frac{3m(x-y)-n(x-y)}{-3m(y^2-x^2)+n(y^2-x^2)}$

$=\frac{(3m-n)(x-y)}{(n-3m)(y^2-x^2)}=\frac{(3m-n)(x-y)}{(3m-n)(x^2-y^2)}=\frac{x-y}{(x+y)(x-y)}=\frac{1}{x+y}$

11. $\frac{9-6x+x^2}{x^2-7x+12}=\frac{(x-3)^2}{(x-4)(x-3)}=\frac{x-3}{x-4}$

12. $\frac{a^2-b^2}{b^3-a^3}$

$=\frac{(a+b)(a-b)}{(b-a)(b^2+ab+a^2)}$

$=-\frac{(a+b)(a-b)}{(a-b)(b^2+ab+a^2)}=-\frac{a+b}{b^2+ab+a^2}$

13. $\frac{3ax-3bx-6a+6b}{2b-2a-bx+ax}$

$=\frac{3x(a-b)-6(a-b)}{2(b-a)-x(b-a)}$

$=\frac{3(x-2)(a-b)}{(2-x)(b-a)}=\frac{3(x-2)(a-b)}{(x-2)(a-b)}=3$

14. $\frac{a^2-x^2}{x^2-ax-3x+3a}$

$=\frac{(a+x)(a-x)}{x(x-a)-3(x-a)}$

$=\frac{(a+x)(a-x)}{(x-3)(x-a)}=\frac{(a+x)(a-x)}{(3-x)(a-x)}=\frac{a+x}{3-x}$

15. $\frac{3bx-6x}{8-b^3}$

$=\frac{3x(b-2)}{(2-b)(4+2b+b^2)}$

$=-\frac{3x(b-2)}{(b-2)(4+2b+b^2)}=-\frac{3x}{4+2b+b^2}$

16. $\frac{(1-a)^3}{a-1}=-\frac{(1-a)^3}{(1-a)}=-(1-a)^2$

17. $\frac{2x^3-2x^2y-2xy^2}{3y^3+3xy^2-3x^2y}$

$=\frac{2x(x^2-xy-y^2)}{3y(y^2+xy-x^2)}=-\frac{2x(x^2-xy-y^2)}{3y(x^2-xy-y^2)}=-\frac{2x}{3y}$

18. $\frac{(a-b)^3}{(b-a)^2}=\frac{(a-b)^3}{(a-b)^2}=a-b$

19. $\dfrac{2x^2-22x+60}{75-3x^2}$

$=\dfrac{\left[(2x)^2-22(2x)+120\right]}{3\left(25-x^2\right)}$

$=\dfrac{(2x-12)(2x-10)}{3(5-x)(5+x)}$

$=\dfrac{(2x-12)(x-5)}{3(5-x)(5+x)}=\dfrac{2(6-x)(5-x)}{3(5-x)(5+x)}=\dfrac{2(6-x)}{3(x+5)}$

20. $\dfrac{6an^2-3b^2n^2}{b^4-4ab^2+4a^2}$

$=\dfrac{3n^2\left(2a-b^2\right)}{\left(b^2-2a\right)^2}=\dfrac{3n^2\left(2a-b^2\right)}{\left(2a-b^2\right)\left(2a-b^2\right)}=\dfrac{3n^2}{2a-b^2}$

21. $\dfrac{(x-y)^2-z^2}{(y+z)-x^2}$

$=\dfrac{(x-y-z)(x-y+z)}{(y+z-x)(y+z+x)}$

$=\dfrac{(z+y-x)(y-x-z)}{(z+y-x)(y+z+x)}=\dfrac{y-x-z}{x+y+z}$

22. $\dfrac{3a^2-3ab}{bd-ad-bc+ac}$

$=\dfrac{3a(a-b)}{d(b-a)-c(b-a)}$

$=\dfrac{3a(a-b)}{(d-c)(b-a)}=\dfrac{3a(a-b)}{(c-d)(a-b)}=\dfrac{3a}{c-d}$

23. $\dfrac{(x-5)^3}{125-x^3}$

$=\dfrac{(x-5)^3}{(5-x)\left(25+5x+x^2\right)}$

$=-\dfrac{(x-5)^3}{(x-5)\left(x^2+5x+25\right)}=-\dfrac{(x-5)^2}{x^2+5x+25}$

24. $\dfrac{13x-6-6x^2}{6x^2-13x+6}=-\dfrac{\left(6x^2-13x+6\right)}{6x^2-13x+6}=-1$

25. $\dfrac{2x^3-2xy^2+x^2-y^2}{2xy^2+y^2-2x^3-x^2}$

$=\dfrac{2x\left(x^2-y^2\right)+\left(x^2-y^2\right)}{y^2(2x+1)-x^2(2x+1)}$

$=\dfrac{(2x+1)\left(x^2-y^2\right)}{(2x+1)\left(y^2-x^2\right)}=-\dfrac{\left(x^2-y^2\right)}{\left(x^2-y^2\right)}=-1$

26. $\dfrac{30x^2y-45xy^2-20x^3}{8x^3+27y^3}$

$=\dfrac{5x\left(6xy-9y^2-4x^2\right)}{(2x+3y)\left(4x^2-6xy+9y^2\right)}$

$=-\dfrac{5x\left(4x^2-6xy+9y^2\right)}{(2x+3y)\left(4x^2-6xy+9y^2\right)}=-\dfrac{5x}{2x+3y}$

27. $\dfrac{n+1-n^3-n^2}{n^3-n-2n^2+2}$

$=\dfrac{(n+1)-n^2(n+1)}{n\left(n^2-1\right)-2\left(n^2-1\right)}$

$=\dfrac{\left(1-n^2\right)(n+1)}{(n-2)\left(n^2-1\right)}=\dfrac{\left(1-n^2\right)(n+1)}{(2-n)\left(1-n^2\right)}=\dfrac{n+1}{2-n}$

28. $\dfrac{(x-2)^2\left(x^2+x-12\right)}{(2-x)(3-x)^2}$

$=\dfrac{(2-x)^2(x+4)(x-3)}{(2-x)(3-x)^2}$

$=\dfrac{(x-2)(x+4)(3-x)}{(3-x)^2}=\dfrac{(x-2)(x+4)}{3-x}$

29. $\dfrac{5x^3-15x^2y}{90x^3y^2-10x^5}$

$=\dfrac{5x^2(x-3y)}{10x^3\left(9y^2-x^2\right)}$

$=\dfrac{(x-3y)}{2x(3y-x)(3y+x)}$

$=-\dfrac{x-3y}{2x(x-3y)(3y+x)}=\dfrac{-1}{2x(3y+x)}$

30. $\dfrac{\left(x^2-1\right)\left(x^2-8x+16\right)}{\left(x^2-4x\right)\left(1-x^2\right)}$

$=\dfrac{(x+1)(x-1)(x-4)(x-4)}{x(x-4)(1+x)(1-x)}=\dfrac{(1-x)(4-x)}{x(1-x)}=\dfrac{4-x}{x}$

EJERCICIO 121

1. $\dfrac{a^4-a^3x+a^2x^2-ax^3}{a^4-a^3x-2a^2x^2+2ax^3}$ *Factor común a*

$$a^3-a^2x+ax^2-x^3 \;\Big|\; a^3-a^2x-2ax^2+2x^3$$
$$-a^3+a^2x+2ax^2-2x^3 \qquad 1$$
$$3ax^2-3x^3 \div 3x^2$$

$$a^3-a^2x-2ax^2+2x^3 \;\Big|\; a-x$$
$$-a^3+a^2x \qquad a^2-2x^2$$
$$-2ax^2+2x^3 \qquad mcd=a(a-x)$$
$$+2ax^2-2x^3 \qquad =a^2-ax$$

$$a^4-a^3x+a^2x^2-ax^3 \;\Big|\; a^2-ax$$
$$-a^4+a^3x \qquad a^2+x^2$$
$$a^2x^2-ax^3$$
$$-a^2x^2+ax^3$$

$$a^4-a^3x-2a^2x^2+2ax^3 \;\Big|\; a^2-ax$$
$$-a^4+a^3x \qquad a^2-2x^2$$
$$-2a^2x^2+2ax^3$$
$$+2a^2x^2-2ax^3$$

$$=\frac{a^2+x^2}{a^2-2x^2} \quad \textit{Rta.}$$

2. $\dfrac{x^4+3x^3+4x^2-3x-5}{x^4+3x^3+6x^2+3x+5}$

$$x^4+3x^3+4x^2-3x-5 \;\Big|\; x^4+3x^3+6x^2+3x+5$$
$$-x^4-3x^3-6x^2-3x-5 \qquad 1$$
$$-2x^2-6x-10 \quad \div -2$$

$$x^4+3x^3+6x^2+3x+5 \;\Big|\; x^2+3x+5$$
$$-x^4-3x^3-5x^2 \qquad x^2$$
$$x^2+3x+5$$

$$x^2+3x+5 \;\Big|\; x^2+3x+5$$
$$-x^2-3x-5 \qquad 1 \qquad mcd \quad x^2+3x+5$$

$$\Rightarrow x^4+3x^3+4x^2-3x-5 \;\Big|\; x^2+3x+5$$
$$-x^4-3x^3-5x^2 \qquad x^2-1$$
$$-x^2-3x-5$$
$$x^2+3x+5$$

$$x^4+3x^3+6x^2+3x+5 \;\Big|\; x^2+3x+5$$
$$-x^4-3x^3-5x^2 \qquad x^2+1$$
$$x^2+3x+5$$
$$-x^2-3x-5$$

$$=\frac{x^2-1}{x^2+1} \quad \textit{Rta.}$$

3. $\dfrac{2ax^4-ax^3-ax^2-2ax+2a\ (3)}{3ax^4-4ax^3+ax^2+3ax-3a\ (2)}$ *Factor común a*

$$6x^4-3x^3-3x^2-6x+6 \;\Big|\; 6x^4-8x^3+2x^2+6x-6 \quad (5)$$
$$-6x^4+8x^3-2x^2-6x+6 \qquad 1$$
$$5x^3-5x^2-12x+12 \quad (6)$$

$$30x^4-40x^3+10x^2+30x-30 \;\Big|\; 30x^3-30x^2-72x+72 \ (\div 6)$$
$$-30x^4+30x^3+72x^2-72x \qquad x$$
$$-10x^3+82x^2-42x-30 \quad (\div -2)$$

$$5x^3-5x^2-12x+12 \;\Big|\; 5x^3-41x^2+21x+15 \quad (12)$$
$$-5x^3+41x^2-21x-15 \qquad 1$$
$$36x^2-33x-3 \quad (\div 3)$$

$$60x^3-492x^2+252x+180 \;\Big|\; 12x^2-11x-1 \quad (437)$$
$$-60x^3+55x^2+5x \qquad 5x$$
$$-437x^2+257x+180 \quad (-12)$$

$$5.244x^2-4.807x-437 \;\Big|\; 5.244x^2-3.084x-2.160 (\div 12)$$
$$-5.244x^2+3.084x+2.160 \qquad 1$$
$$-1723x+1723 \quad (\div -1.723)$$

$$437x^2-257x-180 \;\Big|\; x-1$$
$$-437x^2+437x \qquad 437x$$
$$180x-180 \quad (\div 180)$$

$$x-1 \;\Big|\; x-1 \qquad mcd=a(x-1)$$
$$-x+1 \qquad 1 \qquad =ax-a$$

$$\Rightarrow 2ax^4-ax^3-ax^2-2ax+2a \;\Big|\; ax-a$$
$$-2ax^4+2ax^3 \qquad 2x^3+x^2-2$$
$$ax^3-ax^2$$
$$-ax^3+ax^2$$
$$-2ax+2a$$
$$+2ax-2a$$

$$3ax^4-4ax^3+ax^2+3ax-3a \;\Big|\; ax-a$$
$$-3ax^4+3ax^3 \qquad 3x^3-x^2+3$$
$$-ax^3+ax^2$$
$$+ax^3-ax^2$$
$$3ax-3a$$
$$-3ax+3a$$

$$=\frac{2x^3+x^2-2}{3x^3-x^2+3} \quad \textit{Rta.}$$

4. $\dfrac{6x^3-13x^2+18x-8 \quad (10)}{10x^3-9x^2+11x+12 \quad (6)}$

$$\begin{array}{l|l} 60x^3-130x^2+180x-80 & 60x^3-54x^2+66x+72 \quad (\div 6) \\ \hline -60x^3+54x^2-66x-72 & 1 \end{array}$$

$$-76x^2+114x-152 \quad (\div -38)$$

$$\begin{array}{l|l} 10x^3-9x^2+11x+12 & 2x^2-3x+4 \\ \hline -10x^3+15x^2-20x & 5x \end{array}$$

$$6x^2-9x+12 \quad (\div 3)$$

$$\begin{array}{l|l} 2x^2-3x+4 & 2x^2-3x+4 \\ \hline -2x^2+3x-4 & 1 \end{array} \qquad mcd = 2x^2-3x+4$$

$$\begin{array}{l|l} 6x^3-13x^2+18x-8 & 2x^2-3x+4 \\ \hline -6x^3+9x^2-12x & 3x-2 \end{array}$$

$$-4x^2+6x-8$$
$$+4x^2-6x+8$$

$$\begin{array}{l|l} 10x^3-9x^2+11x+12 & 2x^2-3x+4 \\ \hline -10x^3+15x^2-20x & 5x+3 \end{array}$$

$$6x^2-9x+12$$
$$-6x^2+9x-12$$

$$=\frac{3x-2}{5x+3} \quad Rta.$$

5. $\dfrac{x^4-2x^3y+2x^2y^2-xy^3 \quad (2)}{2x^4-5x^3y+4x^2y^2-xy^3}$ *Factor común x*

$$\begin{array}{l|l} 2x^3-4x^2y+4xy^2-2y^3 & 2x^3-5x^2y+4xy^2-y^3 \\ \hline -2x^3+5x^2y-4xy^2+y^3 & 1 \end{array}$$

$$x^2y - y^3 \quad (\div y)$$

$$\begin{array}{l|l} 2x^3-5x^2y+4xy^2-y^3 & x^2-y^2 \quad (5) \\ \hline -2x^3+2xy^2 & 2x \end{array}$$

$$-5x^2y+6xy^2-y^3 \quad (\div -y)$$

$$\begin{array}{l|l} 5x^2-5y^2 & 5x^2-6xy+y^2 \\ \hline -5x^2+6xy-y^2 & 1 \end{array}$$

$$6xy-6y^2 \quad (\div 6y)$$

$$\begin{array}{l|l} 5x^2-6xy+y^2 & x-y \\ \hline -5x^2+5xy & 5x \end{array}$$

$$-xy+y^2 \quad (\div -y)$$

$$\begin{array}{l|l} x-y & x-y \\ \hline -x+y & 1 \end{array} \qquad mcd\ x(x-y)=x^2-xy$$

Continúa

Continuación

5.
$$\begin{array}{l|l} x^4-2x^3y+2x^2y^2-xy^3 & x^2-xy \\ \hline -x^4+x^3y & x^2-xy+y^2 \end{array}$$

$$-x^3y+2x^2y^2-xy^3$$
$$+x^3y-x^2y^2$$
$$x^2y^2-xy^3$$
$$-x^2y^2+xy^3$$

$$\begin{array}{l|l} 2x^4-5x^3y+4x^2y^2-xy^3 & x^2-xy \\ \hline -2x^4+2x^3y & 2x^2-3xy+y^2 \end{array}$$

$$-3x^3y+4x^2y^2$$
$$+3x^3y-3x^2y^2$$
$$x^2y^2-xy^3$$
$$-x^2y^2+xy^3$$

$$=\frac{x^2-xy+y^2}{2x^2-3xy+y^2} \quad Rta.$$

6. $\dfrac{2a^5-a^4+2a^3+2a^2+3 \quad (3)}{3a^5-a^4+3a^3+4a^2+5 \quad (2)}$

$$\begin{array}{l|l} 6a^5-3a^4+6a^3+6a^2+9 & 6a^5-2a^4+6a^3+8a^2+10 \quad (\div 2) \\ \hline -6a^5+2a^4-6a^3-8a^2-10 & 1 \end{array}$$

$$-a^4-2a^2-1 \quad (\div -1)$$

$$\begin{array}{l|l} 3a^5-a^4+3a^3+4a^2+5 & a^4+2a^2+1 \\ \hline -3a^5-6a^3-3a & 3a \end{array}$$

$$-a^4-3a^3+4a^2-3a+5 \quad (-1)$$

$$\begin{array}{l|l} a^4+2a^2+1 & a^4+3a^3-4a^2+3a-5 \\ \hline -a^4-3a^3+4a^2-3a+5 & 1 \end{array}$$

$$-3a^3+6a^2-3a+6 \quad (\div -3)$$

$$\begin{array}{l|l} a^4+3a^3-4a^2+3a-5 & a^3-2a^2+a-2 \\ \hline -a^4+2a^3-a^2+2a & a \end{array}$$

$$5a^3-5a^2+5a-5 \quad (\div 5)$$

$$\begin{array}{l|l} a^3-2a^2+a-2 & a^3-a^2+a-1 \\ \hline -a^3+a^2-a+1 & 1 \end{array}$$

$$-a^2-1 \quad (\div -1)$$

$$\begin{array}{l|l} a^3-a^2+a-1 & a^2+1 \\ \hline -a^3-a & a \end{array}$$

$$-a^2-1 \quad (\div -1)$$

$$\begin{array}{l|l} a^2+1 & a^2+1 \\ \hline -a^2-1 & 1 \end{array} \qquad mcd = a^2+1$$

Continúa

Continuación

6.

$$\begin{array}{l|l} \Rightarrow 2a^5-a^4+2a^3+2a^2+3 & a^2+1 \\ \hline -2a^5 \qquad -2a^3 & 2a^3-a^2+3 \\ \hline -a^4 \qquad +2a^2+3 & \\ +a^4 \qquad +a^2 & \\ \hline 3a^2+3 & \\ -3a^2-3 & \\ \hline \end{array}$$

$$\begin{array}{l|l} 3a^5-a^4+3a^3+4a^2+5 & a^2+1 \\ \hline -3a^5 \qquad -3a^3 & 3a^3-a^2+5 \\ \hline -a^4 \qquad +4a^2+5 & \\ +a^4 \qquad +a^2 & \\ \hline 5a^2+5 & \\ -5a^2-5 & \\ \hline \end{array}$$

$$=\frac{2a^3-a^2+3}{3a^3-a^2+5} \quad Rta.$$

7.

$$\begin{array}{l|l} 1-x-x^3+x^4 & 1-2x-x^2-2x^3+x^4 \\ \hline -1+2x+x^2+2x^3-x^4 & 1 \\ \hline \end{array}$$

$$x+x^2+x^3 \quad (\div x)$$

$$\begin{array}{l|l} 1-2x-x^2-2x^3+x^4 & 1+x+x^2 \quad (3) \\ \hline -1-x-x^2 & 1 \\ \hline \end{array}$$

$$-3x-2x^2-2x^3+x^4 \quad (\div -x)$$

$$\begin{array}{l|l} 3+3x+3x^2 & 3+2x+2x^2-x^3 \\ \hline -3-2x-2x^2+x^3 & 1 \\ \hline \end{array}$$

$$x+x^2+x^3 \quad (\div x)$$

$$\begin{array}{l|l} 3+2x+2x^2-x^3 & 1+x+x^2 \\ \hline -3-3x-3x^2 & 3 \\ \hline \end{array}$$

$$-x-x^2-x^3 \quad (\div -x)$$

$$\begin{array}{l|l} 1+x+x^2 & 1+x+x^2 \\ \hline -1-x-x^2 & 1 \\ \hline \end{array} \qquad mcd=1+x+x^2$$

$$\begin{array}{l|l} \Rightarrow 1-x \qquad -x^3+x^4 & 1+x+x^2 \\ \hline -1-x-x^2 & 1-2x+x^2 \\ \hline -2x-x^2-x^3+x^4 & \\ +2x+2x^2+2x^3 & \\ \hline x^2+x^3+x^4 & \\ -x^2-x^3-x^4 & \\ \hline \end{array}$$

Continúa

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7.

$$\begin{array}{l|l} 1-2x-x^2-2x^3+x^4 & 1+x+x^2 \\ \hline -1-x-x^2 & 1-3x+x^2 \\ \hline -3x-2x^2-2x^3+x^4 & \\ +3x+3x^2+3x^3 & \\ \hline x^2+x^3+x^4 & \\ -x^2-x^3-x^4 & \\ \hline \end{array}$$

$$=\frac{1-2x+x^2}{1-3x+x^2} \quad Rta.$$

8. $2m^3+2m^2n-mn^2-n^3$

$$=m\left(2m^2-n^2\right)+n\left(2m^2-n^2\right)=\left(m+n\right)\left(2m^2-n^2\right)$$

$$3m^3+3m^2n+mn+n^2$$

$$=m\left(3m^2+n\right)+n\left(3m^2+n\right) \quad =\left(m+n\right)\left(3m^2+n\right)$$

$$mcd=m+n$$

$$\begin{array}{l|l} 2m^3+2m^2n-mn^2-n^3 & m+n \\ \hline -2m^3-2m^2n & 2m^2-n^2 \\ \hline -mn^2-n^3 & \\ +mn^2+n^3 & \\ \hline \end{array}$$

$$\begin{array}{l|l} 3m^3+3m^2n+mn+n^2 & m+n \\ \hline -3m^3-3m^2n & 3m^2+n \\ \hline mn+n^2 & \\ -mn-n^2 & \\ \hline \end{array}$$

$$=\frac{2m^2-n^2}{3m^2+n} \quad Rta.$$

9.

$$\frac{6a^5+3a^4-4a^3-2a^2+10a+5}{3a^6+7a^4-a^2+15} \quad (2)$$

$$\begin{array}{l|l} 6a^6 \quad +14a^4 \quad -2a^2 \quad +30 & 6a^5+3a^4-4a^3-2a^2+10a+5 \\ \hline -6a^6-3a^5+4a^4+2a^3-10a^2-5a & a \\ \hline \end{array}$$

$$-3a^5+18a^4+2a^3-12a^2-5a+30 \quad (\div -1)$$

$$\begin{array}{l|l} 6a^5+3a^4-4a^3-2a^2+10a+5 & 3a^5-18a^4-2a^3+12a^2+5a-30 \\ \hline -6a^5+36a^4+4a^3-24a^2-10a+60 & 2 \\ \hline \end{array}$$

$$39a^4 \quad -26a^2 \quad +65 \quad (\div 13)$$

$$\begin{array}{l|l} 3a^5-18a^4-2a^3+12a^2+5a-30 & 3a^4-2a^2+5 \\ \hline -3a^5 \quad +2a^3 \quad -5a & a \\ \hline \end{array}$$

$$-18a^4 \quad +12a^2 \quad -30 \quad (\div -6)$$

$$\begin{array}{l|l} 3a^4-2a^2+5 & 3a^4-2a^2+5 \\ \hline -3a^4+2a^2-5 & 1 \\ \hline \end{array} \qquad mcd=3a^4-2a^2+5$$

Continúa

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9.

$$
\begin{array}{l|l}
6a^5+3a^4-4a^3-2a^2+10a+5 & 3a^4-2a^2+5 \\ \hline
-6a^5\quad +4a^3\quad -10a & 2a+1 \\
3a^4\quad -2a^2\quad +5 & \\
-3a^4\quad +2a^2\quad -5 &
\end{array}
\qquad
\begin{array}{l|l}
3a^6+7a^4-a^2+15 & 3a^4-2a^2+5 \\ \hline
-3a^6+2a^4-5a^2 & a^2+3 \\
9a^4-6a^2+15 & \\
-9a^4+6a^2-15 &
\end{array}
$$

$$=\frac{2a+1}{a^2+3}\quad Rta.$$

10. $\dfrac{5x^6-10x^4+21x^3-2x+4 \quad (3)}{3x^6-6x^4+11x^3+2x-4 \quad (5)}$

$$
\begin{array}{l|l}
15x^6-30x^4+63x^3-6x+12 & 15x^6-30x^4+55x^3+10x-20\ (\div 5) \\ \hline
-15x^6+30x^4-55x^3-10x+20 & 1 \\
8x^3-16x+32\quad (\div 8) &
\end{array}
$$

$$
\begin{array}{l|l}
3x^6-6x^4+11x^3+2x-4 & x^3-2x+4 \\ \hline
-3x^6+6x^4-12x^3 & 3x^3 \\
-x^3+2x-4\quad (\div -1) &
\end{array}
$$

$$
\begin{array}{l|l}
x^3-2x+4 & x^3-2x+4 \\ \hline
-x^3+2x-4 & 1
\end{array}
\qquad mcd=x^3-2x+4
$$

$$
\begin{array}{l|l}
5x^6-10x^4+21x^3-2x+4 & x^3-2x+4 \\ \hline
-5x^6+10x^4-20x^3 & 5x^3+1 \\
x^3-2x+4 & \\
-x^3+2x-4 &
\end{array}
\qquad
\begin{array}{l|l}
3x^6-6x^4+11x^3+2x-4 & x^3-2x+4 \\ \hline
-3x^6+6x^4-12x^3 & 3x^3-1 \\
-x^3+2x-4 & \\
+x^3-2x+4 &
\end{array}
$$

$$=\frac{5x^3+1}{3x^3-1}\quad Rta.$$

11. $n^6-3n^5-n^4+3n^3+7n^2-21n$

$=n^5(n-3)-n^3(n-3)+7n(n-3)=(n-3)(n^5-n^3+7n)$

$n^6+2n^5-n^4-2n^3+7n^2+14n$

$=n^5(n+2)-n^3(n+2)+7n(n+2)=(n+2)(n^5-n^3+7n)$

$$
\begin{array}{l|l}
n^6-3n^5-n^4+3n^3+7n^2-21n & n^5-n^3+7n \\ \hline
-n^6\quad +n^4\quad -7n^2 & n-3 \\
-3n^5\quad +3n^3\quad -21n & \\
+3n^5\quad -3n^3\quad +21n &
\end{array}
\qquad
\begin{array}{l|l}
n^6+2n^5-n^4-2n^3+7n^2+14n & n^5-n^3+7n \\ \hline
-n^6\quad +n^4\quad -7n^2 & n+2 \\
2n^5\quad -2n^3\quad +14n & \\
-2n^5\quad +2n^3\quad -14n &
\end{array}
$$

$$=\frac{n-3}{n+2}\quad Rta.$$

12.

$$
\begin{array}{l|l}
a^7+2a^6-5a^5+8a^4+a^3+2a^2-5a+8 & a^6+2a^5-5a^4+10a^3+4a^2-10a+16\ (2) \\ \hline
-a^7-2a^6+5a^5-10a^4-4a^3+10a^2-16a & a \\
-2a^4-3a^3+12a^2-21a+8\quad (\div -1) &
\end{array}
$$

$$
\begin{array}{l|l}
2a^6+4a^5-10a^4+20a^3+8a^2-20a+32 & 2a^4+3a^3-12a^2+21a-8\ (a) \\ \hline
-2a^6-3a^5+12a^4-21a^3+8a^2 & a^2 \\
a^5+2a^4-a^3+16a^2-20a+32 &
\end{array}
$$

$$
\begin{array}{l|l}
2a^5+3a^4-12a^3+21a^2-8a & a^5+2a^4-a^3+16a^2-20a+32 \\ \hline
-2a^5-4a^4+2a^3-32a^2+40a-64 & 2 \\
-a^4-10a^3-11a^2+32a-64\quad (-1) &
\end{array}
$$

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12.

$$\begin{array}{l|l} a^5+2a^4-a^3+16a^2-20a+32 & a^4+10a^3+11a^2-32a+64 \quad (2) \\ \hline -a^5-10a^4-11a^3+32a^2-64a & a \\ \hline \quad -8a^4-12a^3+48a^2-84a+32 \quad (\div -4) & \end{array}$$

$$\begin{array}{l|l} 2a^4+20a^3+22a^2-64a+128 & 2a^4+3a^3-12a^2+21a-8 \\ \hline -2a^4-3a^3+12a^2-21a+8 & 1 \\ \hline \quad 17a^3+34a^2-85a+136 \quad (\div 17) & \end{array}$$

$$\begin{array}{l|l} 2a^4+3a^3-12a^2+21a-8 & a^3+2a^2-5a+8 \\ \hline -2a^4-4a^3+10a^2-16a & 2a \\ \hline \quad -a^3-2a^2+5a-8 \quad (-1) & \end{array}$$

$$\begin{array}{l|l} a^3+2a^2-5a+8 & a^3+2a^2-5a+8 \\ \hline -a^3-2a^2+5a-8 & 1 \end{array} \qquad mcd=a^3+2a^2-5a+8$$

$$\begin{array}{l|l} a^7+2a^6-5a^5+8a^4+a^3+2a^2-5a+8 & a^3+2a^2-5a+8 \\ \hline -a^7-2a^6+5a^5-8a^4 & a^4+1 \\ \hline \quad a^3+2a^2-5a+8 & \\ \quad -a^3-2a^2+5a-8 & \end{array} \qquad \begin{array}{l|l} a^6+2a^5-5a^4+10a^3+4a^2-10a+16 & a^3+2a^2-5a+8 \\ \hline -a^6-2a^5+5a^4-8a^3 & a^3+2 \\ \hline \quad 2a^3+4a^2-10a+16 & \\ \quad -2a^3-4a^2+10a-16 & \end{array}$$

$$=\frac{a^4+1}{a^3+2} \quad Rta$$

EJERCICIO 122

1. $\frac{3}{2a}\cdot\frac{2a}{2a}=\frac{6a}{4a^2}$

2. $\frac{5}{9x^2}\cdot\frac{4a}{4a}=\frac{20a}{36ax^2}$

3. $\frac{m}{ab^2}\cdot\frac{2a}{2a}=\frac{2am}{2a^2b^2}$

4. $\frac{3x}{8y}\cdot\frac{3xy^2}{3xy^2}=\frac{9x^2y^2}{24xy^3}$

5. $\frac{4m}{5n^2}\cdot\frac{n}{n}=\frac{4mn}{5n^3}$

6. $\frac{2x+7}{5}\cdot\frac{3}{3}=\frac{6x+21}{15}$

7. $\frac{2x}{x-1}\cdot\frac{x}{x}=\frac{2x^2}{x^2-x}$

8. $\frac{a^2}{a+2}\cdot\frac{2a}{2a}=\frac{2a^3}{2a^2+4a}$

9. $\frac{3a}{a+b}\cdot\frac{a+b}{a+b}=\frac{3a^2+3ab}{a^2+2ab+b^2}$

10. $\frac{x-4}{x+3}\cdot\frac{x+2}{x+2}=\frac{x^2-2x-8}{x^2+5x+6}$

11. $\frac{2a}{x+a}\cdot\frac{a^2}{a^2}=\frac{2a^3}{a^2x+a^3}$

12. $\frac{x-y}{6}\cdot\frac{2}{2}=\frac{2x-2y}{12}$

13. $\frac{5x}{a-b}\cdot\frac{a+b}{a+b}=\frac{5ax+5bx}{a^2-b^2}$

14. $\frac{x-5}{a}\cdot\frac{3x}{3x}=\frac{3x^2-15x}{3ax}$

15. $\frac{5x}{2x+y}\cdot\frac{2x+y}{2x+y}=\frac{10x^2+5xy}{4x^2+4xy+y^2}$

16. $\frac{x+3}{x+1}\cdot\frac{x-3}{x-3}=\frac{x^2-9}{x^2-2x-3}$

17. $\frac{2}{a+1}\cdot\frac{a^2-a+1}{a^2-a+1}=\frac{2a^2-2a+2}{a^3+1}$

18. $\frac{x-2y}{3x}\cdot\frac{3xy}{3xy}=\frac{3x^2y-6xy^2}{9x^2y}$

19. $\frac{x-1}{x+1}\cdot\frac{x+1}{x+1}=\frac{x^2-1}{x^2+2x+1}$

20. $\frac{a+b}{7a^2}\cdot\frac{9ab}{9ab}=\frac{9a^2b+9ab^2}{63a^3b}$

21. $\frac{x+1}{x+5}\cdot\frac{x-2}{x-2}=\frac{x^2-x-2}{x^2+3x-10}$

EJERCICIO 123

1. $\frac{6a^3-10a^2}{2a}$

$=\frac{6a^3}{2a}-\frac{10a^2}{2a}$

$=3a^2-5a$

2. $\frac{9x^3y-6x^2y^2+3xy^3}{3xy}$

$=\frac{9x^3y}{3xy}-\frac{6x^2y^2}{3xy}+\frac{3xy^3}{3xy}$

$=3x^2-2xy+y^2$

3.

$$\begin{array}{l|l} x^2+3 & x \\ \hline -x^2 & x \\ \hline \quad 3 & \end{array}$$

$=x+\frac{3}{x}$

4.

$$\begin{array}{l|l} 10a^2+15a-2 & 5a \\ \hline -10a^2 & 2a+3 \\ \hline \quad 15a-2 & \\ \quad -15a & \\ \hline \quad\quad -2 & \end{array}$$

$=2a+3-\frac{2}{5a}$

5.
$$\begin{array}{l|l} 9x^3-6x^2+3x-5 & 3x \\ \cline{2-2} -9x^3 & 3x^2-2x+1 \\ \hline -6x^2 & \\ +6x^2 & \\ \hline 3x & \\ -3x & \\ \hline -5 & \end{array}$$
$$=3x^2-2x+1-\frac{5}{3x}$$

6.
$$\begin{array}{l|l} x^2-5x-16 & x+2 \\ \cline{2-2} -x^2-2x & x-7 \\ \hline -7x-16 & \\ +7x+14 & \\ \hline -2 & \end{array}$$
$$=x-7-\frac{2}{x+2}$$

7.
$$\begin{array}{l|l} 12x^2-6x-2 & 4x-1 \\ \cline{2-2} -12x^2+3x & 3x \\ \hline -3x-2 & \end{array}$$
$$=3x+\frac{-3x-2}{4x-1}=3x-\frac{3x+2}{4x-1}$$

8.
$$\begin{array}{l|l} a^3 \qquad\qquad +3b^3 & a+2b \\ \cline{2-2} -a^3-2a^2b & a^2-2ab+4b^2 \\ \hline -2a^2b & \\ 2a^2b+4ab^2 & \\ \hline 4ab^2+3b^3 & \\ -4ab^2-8b^3 & \\ \hline -5b^3 & \end{array}$$
$$=a^2-2ab+4b^2-\frac{5b^3}{a+2b}$$

9.
$$\begin{array}{l|l} x^3-x^2-6x+1 & x^2-3 \\ \cline{2-2} -x^3 \quad +3x & x-1 \\ \hline -x^2-3x+1 & \\ x^2 \quad -3 & \\ \hline -3x-2 & \end{array}$$
$$=x-1+\frac{-3x-2}{x^2-3}=x-1-\frac{3x+2}{x^2-3}$$

10.
$$\begin{array}{l|l} 3x^3+4x^2y+2xy^2-6y^3 & 3x-2y \\ \cline{2-2} -3x^3+2x^2y & x^2+2xy+2y^2 \\ \hline 6x^2y+2xy^2 & \\ -6x^2y+4xy^2 & \\ \hline 6xy^2-6y^3 & \\ -6xy^2+4y^3 & \\ \hline -2y^3 & \end{array}$$
$$=x^2+2xy+2y^2-\frac{2y^3}{3x-2y}$$

11.
$$\begin{array}{l|l} 2x^3-7x^2+6x-8 & 2x^2-x+1 \\ \cline{2-2} -2x^3+\ x^2-x & x-3 \\ \hline -6x^2+5x-8 & \\ 6x^2-3x+3 & \\ \hline 2x-5 & \end{array}$$
$$=x-3+\frac{2x-5}{2x^2-x+1}$$

12.
$$\begin{array}{l|l} 2a^4-3a^3+a^2 & a^2-a+1 \\ \cline{2-2} -2a^4+2a^3-2a^2 & 2a^2-a-2 \\ \hline -a^3-a^2 & \\ a^3-a^2+a & \\ \hline -2a^2+a & \\ 2a^2-2a+2 & \\ \hline -a+2 & \end{array}$$
$$=2a^2-a-2-\frac{a-2}{a^2-a+1}$$

13.
$$\begin{array}{l|l} x^4-4x^2-3x & x^2-2 \\ \cline{2-2} -x^4+2x^2 & x^2-2 \\ \hline -2x^2-3x & \\ 2x^2 \quad -4 & \\ \hline -3x-4 & \end{array}$$
$$=x^2-2-\frac{3x+4}{x^2-2}$$

14.
$$\begin{array}{l|l} 10n^3-18n^2-5n+3 & 2n^2-3n+1 \\ \cline{2-2} -10n^3+15n^2-5n & 5n \\ \hline -3n^2-10n+3 & \end{array}$$
$$=5n-\frac{3n^2+10n-3}{2n^2-3n+1}$$

15.
$$\begin{array}{l|l} 8x^4 & 4x^2+5x+6 \\ \cline{2-2} -8x^4-10x^3-12x^2 & 2x^2 \\ \hline -10x^3-12x^2 & \end{array}$$
$$=2x^2-\frac{10x^3+12x^2}{4x^2+5x+6}$$

16.
$$\begin{array}{l|l} 6m^5+3m^4n & 3m^3-mn^2+n^3 \\ \cline{2-2} -6m^5 \quad +2m^3n^2-2m^2n^3 & 2m^2+mn \\ \hline 3m^4n+2m^3n^2-2m^2n^3 & \\ -3m^4n \quad +m^2n^3-mn^4 & \\ \hline 2m^3n^2-m^2n^3-mn^4 & \end{array}$$
$$=2m^2+mn+\frac{2m^3n^2-m^2n^3-mn^4}{3m^3-mn^2+n^3}$$

EJERCICIO 124

1. $a+\frac{4a}{a+2}$

$=\frac{a(a+2)+4a}{a+2}$

$=\frac{a^2+2a+4a}{a+2}=\frac{a^2+6a}{a+2}=\frac{a(a+6)}{a+2}$

2. $m-n-\frac{n^2}{m}$

$=\frac{(m-n)m-n^2}{m}=\frac{m^2-mn-n^2}{m}$

3. $x+5-\frac{3}{x-2}$

$=\frac{(x+5)(x-2)-3}{x-2}$

$=\frac{x^2+3x-10-3}{x-2}=\frac{x^2+3x-13}{x-2}$

4. $a+\frac{ab}{a+b}$

$=\frac{a(a+b)+ab}{a+b}$

$=\frac{a^2+ab+ab}{a+b}=\frac{a^2+2ab}{a+b}$

5. $\frac{1-a^2}{a}+a-3$

$=\frac{a(a-3)+1-a^2}{a}$

$=\frac{a^2-3a+1-a^2}{a}=-\frac{3a-1}{a}$

6. $1-\frac{a+x}{a-x}$

$=\frac{(a-x)-(a+x)}{a-x}$

$=\frac{a-x-a-x}{a-x}=-\frac{2x}{a-x}$

7. $\frac{2a+x}{a+x}-1$

$=\frac{2a+x-(a+x)}{a+x}$

$=\frac{2a+x-a-x}{a+x}=\frac{a}{a+x}$

8. $x+2-\frac{3}{x-1}$

$=\frac{(x+2)(x-1)-3}{x-1}$

$=\frac{x^2+x-2-3}{x-1}=\frac{x^2+x-5}{x-1}$

9. $x^2-3x-\frac{x^2-6x}{x+2}$

$=\frac{(x^2-3x)(x+2)-(x^2-6x)}{x+2}$

$=\frac{x^3-x^2-6x-x^2+6x}{x+2}$

$=\frac{x^3-2x^2}{x+2}=\frac{x^2(x-2)}{x+2}$

10. $x+y+\frac{x^2-y^2}{x-y}$

$=\frac{(x+y)(x-y)+x^2-y^2}{x-y}$

$=\frac{x^2-y^2+x^2-y^2}{x-y}$

$=\frac{2x^2-2y^2}{x-y}$

$=\frac{2(x^2-y^2)}{x-y}$

$=\frac{2(x+y)(x-y)}{x-y}$

$=2x+2y=2(x+y)$

11. $\frac{3mn}{m-n}+m-2n$

$=\frac{(m-n)(m-2n)+3mn}{m-n}$

$=\frac{m^2-3mn+2n^2+3mn}{m-n}=\frac{m^2+2n^2}{m-n}$

12. $2a-3x-\frac{5ax-6x^2}{a+2x}$

$=\frac{(2a-3x)(a+2x)-(5ax-6x^2)}{a+2x}$

$=\frac{2a^2+ax-6x^2-5ax+6x^2}{a+2x}$

$=\frac{2a^2-4ax}{a+2x}$

13. $m^2-2m+4-\frac{m^3}{m+2}$

$=\frac{(m^2-2m+4)(m+2)-m^3}{m+2}$

$=\frac{m^3+8-m^3}{m+2}=\frac{8}{m+2}$

14. $x^2-5x-\frac{3x(x+2)}{x-2}$

$=\frac{(x^2-5x)(x-2)-3x(x+2)}{x-2}$

$=\frac{x^3-7x^2+10x-3x^2-6x}{x-2}$

$=\frac{x^3-10x^2+4x}{x-2}$

15. $a^2+3ab-b^2+\frac{7ab^2-b^3}{2a-b}$

$=\frac{(a^2+3ab-b^2)(2a-b)+7ab^2-b^3}{2a-b}$

$=\frac{2a^3+5a^2b-5ab^2+b^3+7ab^2-b^3}{2a-b}$

$=\frac{2a^3+5a^2b+2ab^2}{2a-b}$

16. $\frac{x^3+2}{x^2-x+1}-(x+1)$

$=\frac{(x^3+2)+(x^2-x+1)(-x-1)}{x^2-x+1}$

$=\frac{x^3+2-x^3-1}{x^2-x+1}=\frac{1}{x^2-x+1}$

17. $x+3-\frac{x^3-2x^2+1}{x^2-4x+3}$

$=\frac{(x+3)(x^2-4x+3)-(x^3-2x^2+1)}{x^2-4x+3}$

$=\frac{x^3-x^2-9x+9-x^3+2x^2-1}{x^2-4x+3}$

$=\frac{x^2-9x+8}{x^2-4x+3}=\frac{(x-8)(x-1)}{(x-3)(x-1)}=\frac{x-8}{x-3}$

18. $3a+\frac{3a^2b+3ab^2}{a^2-b^2}$

$=\frac{3a\left(a^2-b^2\right)+3a^2b+3ab^2}{a^2-b^2}$

$=\frac{3a^3-3ab^2+3a^2b+3ab^2}{a^2-b^2}$

$=\frac{3a^3+3a^2b}{a^2-b^2}=\frac{3a^2\left(a+b\right)}{\left(a+b\right)\left(a-b\right)}$

$=\frac{3a^2}{a-b}$

19. $x-3-\frac{x^3-27}{x^2-6x+9}$

$=\frac{(x-3)(x^2-6x+9)-(x^3-27)}{x^2-6x+9}$

$=\frac{x^3-9x^2+27x-27-x^3+27}{x^2-6x+9}$

$=\frac{-9x^2+27x}{(x-3)^2}=\frac{-9x(x-3)}{(x-3)^2}$

$=\frac{-9x}{x-3}=\frac{9x}{3-x}$

20. $a^2-3a+5+\frac{2a^3-11a+9}{a^2+a-2}$

$=\frac{(a^2-3a+5)(a^2+a-2)+2a^3-11a+9}{a^2+a-2}$

$=\frac{a^4-2a^3+11a-10+2a^3-11a+9}{a^2+a-2}$

$=\frac{a^4-1}{(a+2)(a-1)}=\frac{(a^2-1)(a^2+1)}{(a+2)(a-1)}$

$=\frac{(a^2+1)(a+1)(a-1)}{(a+2)(a-1)}$

$=\frac{a^3+a^2+a+1}{a+2}$

EJERCICIO 125

1. $\frac{a}{b}$; $\frac{1}{ab}$ $\quad mcm=ab \quad ab\div b=a$

$ab\div ab=1$

$\frac{a}{b}=\frac{a\cdot a}{ab}=\frac{a^2}{ab}$

$\frac{1}{ab}=\frac{1\cdot 1}{ab}=\frac{1}{ab}\Rightarrow\frac{a^2}{ab}\ ,\ \frac{1}{ab}$ $\quad Rta.$

2. $\frac{x}{2a}\ ,\ \frac{4}{3a^2x} \quad mcm=6a^2x$

$6a^2x\div 2a=3ax$

$6a^2x\div 3a^2x=2$

$\frac{x}{2a}=\frac{x(3ax)}{6a^2x}=\frac{3ax^2}{6a^2x}$

$\frac{4}{3a^2x}=\frac{4\cdot 2}{6a^2x}=\frac{8}{6a^2x}$

$\Rightarrow\frac{3ax^2}{6a^2x}\ ,\ \frac{8}{6a^2x} \quad Rta.$

3. $\frac{1}{2x^2}\ ,\ \frac{3}{4x}\ ,\ \frac{5}{8x^3} \quad mcm=8x^3$

$8x^3\div 2x^2=4x$

$8x^3\div 4x=2x^2$

$8x^3\div 8x^3=1$

$\frac{1}{2x^2}=\frac{1\cdot 4x}{8x^3}=\frac{4x}{8x^3}$

$\frac{3}{4x}=\frac{3x\cdot 2x^2}{8x^3}=\frac{6x^2}{8x^3}$

$\frac{5}{8x^3}=\frac{5\cdot 1}{8x^3}=\frac{5}{8x^3}$

$\Rightarrow\frac{4x}{8x^3}\ ,\ \frac{6x^2}{8x^3}\ ,\ \frac{5}{8x^3} \quad Rta.$

4. $\frac{3x}{ab^2}\ ,\ \frac{x}{a^2b}\ ,\ \frac{3}{a^3} \quad mcm=a^3b^2$

$a^3b^2\div ab^2=a^2 \quad a^3b^2\div a^2b=ab \quad a^3b^2\div a^3=b^2$

$\frac{3x}{ab^2}=\frac{3x(a^2)}{a^3b^2}=\frac{3a^2x}{a^3b^2}$; $\frac{x}{a^2b}=\frac{x(ab)}{a^3b^2}=\frac{abx}{a^3b^2}$; $\frac{3}{a^3}=\frac{3(b^2)}{a^3b^2}=\frac{3b^2}{a^3b^2}$

$\Rightarrow\frac{3a^2x}{a^3b^2}\ ,\ \frac{abx}{a^3b^2}\ ,\ \frac{3b^2}{a^3b^2} \quad Rta.$

5. $\frac{7y}{6x^2}\ ,\ \frac{1}{9xy}\ ,\ \frac{5x}{12y^3} \quad mcm=36x^2y^3$

$36x^2y^3\div 6x^2=6y^3 \quad 36x^2y^3\div 9xy=4xy^2 \quad 36x^2y^3\div 12y^3=3x^2$

$\frac{7y}{6x^2}=\frac{7y(6y^3)}{36x^2y^3}=\frac{42y^4}{36x^2y^3}$; $\frac{1}{9xy}=\frac{1(4xy^2)}{36x^2y^3}=\frac{4xy^2}{36x^2y^3}$;

$\frac{5x}{12y^2}=\frac{5x(3x^2)}{36x^2y^3}=\frac{15x^3}{36x^2y^3} \quad \Rightarrow\frac{42y^4}{36x^2y^3}\ ,\ \frac{4xy^2}{36x^2y^3}\ ,\ \frac{15x^3}{36x^2y^3} \quad Rta.$

6. $\frac{a-1}{3a}\ ,\ \frac{5}{6a}\ ,\ \frac{a+2}{a^2} \quad mcm=6a^2$

$6a^2\div 3a=2a \quad 6a^2\div 6a=1 \quad 6a^2\div a^2=6$

$\frac{a-1}{3a}=\frac{(a-1)(2a)}{6a^2}=\frac{2a^2-2a}{6a^2}$; $\frac{5}{6a}=\frac{5\cdot 1}{6a^2}=\frac{5}{6a^2}$;

$\frac{a+2}{a^2}=\frac{(a+2)6}{6a^2}=\frac{6a+12}{6a^2} \quad \Rightarrow\frac{2a^2-2a}{6a^2}\ ,\ \frac{5}{6a^2}\ ,\ \frac{6a+12}{6a^2} \quad Rta.$

7. $\frac{x-y}{x^2y}\ ,\ \frac{x+y}{3xy^2}\ ,\ 5 \quad mcm=3x^2y^2$

$3x^2y^2\div x^2y=3y$

$3x^2y^2\div 3xy^2=x$

$\frac{x-y}{x^2y}=\frac{(x-y)(3y)}{3x^2y^2}=\frac{3xy-3y^2}{3x^2y^2}$; $\frac{x+y}{3xy^2}=\frac{(x+y)x}{3x^2y^2}=\frac{x^2+xy}{3x^2y^2}$;

$5=\frac{5(3x^2y^2)}{3x^2y^2}=\frac{15x^2y^2}{3x^2y^2} \quad \Rightarrow\frac{3xy-3y^2}{3x^2y^2}\ ,\ \frac{x^2+xy}{3x^2y^2}\ ,\ \frac{15x^2y^2}{3x^2y^2} \quad Rta.$

8. $\dfrac{m+n}{2m}$, $\dfrac{m-n}{5m^3n}$, $\dfrac{1}{10n^2}$ $mcm = 10m^3n^2$

$10m^3n^2 \div 2m = 5m^2n^2$

$10m^3n^2 \div 5m^3n = 2n$

$10m^3n^2 \div 10n^2 = m^3$

$$\frac{m+n}{2m}=\frac{(m+n)(5m^2n^2)}{10m^3n^2}=\frac{5m^3n^2+5m^2n^3}{10m^3n^2}$$

$$\frac{m-n}{5m^3n}=\frac{(m-n)(2n)}{10m^3n^2}=\frac{2mn-2n^2}{10m^3n^2}$$

$$\frac{1}{10n^2}=\frac{1(m^3)}{10m^3n^2}=\frac{m^3}{10m^3n^2}$$

$\Rightarrow \dfrac{5m^3n^2+5m^2n^3}{10m^3n^2}$, $\dfrac{2mn-2n^2}{10m^3n^2}$, $\dfrac{m^3}{10m^3n^2}$ *Rta.*

9. $\dfrac{a+b}{6}$, $\dfrac{a-b}{2a}$, $\dfrac{a^2+b^2}{3b^2}$ $mcm = 6ab^2$

$6ab^2 \div 6 = ab^2$

$6ab^2 \div 2a = 3b^2$

$6ab^2 \div 3b^2 = 2a$

$$\frac{a+b}{6}=\frac{(a+b)(ab^2)}{6ab^2}=\frac{a^2b^2+ab^3}{6ab^2} \ ;$$

$$\frac{a-b}{2a}=\frac{(a-b)(3b^2)}{6ab^2}=\frac{3ab^2-3b^3}{6ab^2};$$

$$\frac{a^2+b^2}{3b^2}=\frac{(a^2+b^2)(2a)}{6ab^2}=\frac{2a^3+2ab^2}{6ab^2}$$

$\Rightarrow \dfrac{a^2b^2+ab^3}{6ab^2}$, $\dfrac{3ab^2-3b^3}{6ab^2}$, $\dfrac{2a^3+2ab^2}{6ab^2}$ *Rta.*

10. $\dfrac{2a-b}{3a^2}$, $\dfrac{3b-a}{4b^2}$, $\dfrac{a-3b}{2}$ $mcm = 12a^2b^2$

$12a^2b^2 \div 3a^2 = 4b^2$

$12a^2b^2 \div 4b^2 = 3a^2$

$12a^2b^2 \div 2 = 6a^2b^2$

$$\frac{2a-b}{3a^2}=\frac{(2a-b)4b^2}{12a^2b^2}=\frac{8ab^2-4b^3}{12a^2b^2} \ ;$$

$$\frac{3b-a}{4b^2}=\frac{(3b-a)3a^2}{12a^2b^2}=\frac{9a^2b-3a^3}{12a^2b^2} \ ;$$

$$\frac{a-3b}{2}=\frac{(a-3b)6a^2b^2}{12a^2b^2}=\frac{6a^3b^2-18a^2b^3}{12a^2b^2}$$

$\Rightarrow \dfrac{8ab^2-4b^3}{12a^2b^2}$, $\dfrac{9a^2b-3a^3}{12a^2b^2}$, $\dfrac{6a^3b^2-18a^2b^3}{12a^2b^2}$ *Rta.*

11. $\dfrac{2}{5}$, $\dfrac{3}{x+1}$ $mcm = 5x+5$

$(5x+5)\div 5 = x+1$ $\quad (5x+5)\div(x+1)=5$

$$\frac{2}{5}=\frac{2(x+1)}{5x+5}=\frac{2x+2}{5x+5} \ ; \ \frac{3}{x+1}=\frac{3\cdot 5}{5x+5}=\frac{15}{5x+5}$$

$\Rightarrow \dfrac{2x+2}{5x+5}$, $\dfrac{15}{5x+5}$ *Rta.*

12. $\dfrac{a}{a+b}$, $\dfrac{b}{a^2-b^2}$ $mcm = a^2-b^2$

$(a^2-b^2)\div(a+b)=a-b$ $\quad (a^2-b^2)\div(a^2-b^2)=1$

$$\frac{a}{a+b}=\frac{a(a-b)}{a^2-b^2}=\frac{a^2-ab}{a^2-b^2} \ ;$$

$$\frac{b}{a^2-b^2}=\frac{1b}{a^2-b^2}=\frac{b}{a^2-b^2}$$

$\Rightarrow \dfrac{a^2-ab}{a^2-b^2}$, $\dfrac{b}{a^2-b^2}$ *Rta.*

13. $\dfrac{x}{x^2-1}$, $\dfrac{1}{x^2-x-2}$ $\quad x^2-1=(x+1)(x-1)$

$x^2-x-2=(x-2)(x+1)$

$mcm = (x-1)(x-2)(x+1)$

$=(x^2-1)(x-2)$

$=x^3-2x^2-x+2$

$$\begin{array}{rr|l} x^3-2x^2-x+2 & & x^2-1 \\ \hline -x^3 \quad\quad +x & & x-2 \\ \hline -2x^2 \quad +2 & & \\ 2x^2 \quad -2 & & \end{array} \qquad \begin{array}{rr|l} x^3-2x^2-x+2 & & x^2-x-2 \\ \hline -x^3+x^2+2x & & x-1 \\ \hline -x^2+x+2 & & \\ x^2-x-2 & & \end{array}$$

$$\frac{x}{x^2-1}=\frac{x(x-2)}{x^3-2x^2-x+2}=\frac{x^2-2x}{x^3-2x^2-x+2} \ ;$$

$$\frac{1}{x^2-x-2}=\frac{1(x-1)}{x^3-2x^2-x+2}=\frac{x-1}{x^3-2x^2-x+2}$$

$\Rightarrow \dfrac{x^2-2x}{x^3-2x^2-x+2}$; $\dfrac{x-1}{x^3-2x^2-x+2}$ *Rta.*

14. $\dfrac{a-3}{4(a+5)}$, $\dfrac{3a}{8}$ $mcm = 8(a+5)$

$[8(a+5)]\div[4(a+5)]=2$ $\quad 8(a+5)\div 8 = a+5$

$$\frac{a-3}{4(a+5)}=\frac{2(a-3)}{8(a+5)}=\frac{2a-6}{8(a+5)} \ ;$$

$$\frac{3a}{8}=\frac{3a(a+5)}{8(a+5)}=\frac{3a^2+15a}{8(a+5)}$$

$\Rightarrow \dfrac{2a-6}{8(a+5)}$, $\dfrac{3a^2+15a}{8(a+5)}$ *Rta.*

15. $\dfrac{x^2}{3(a-x)}$, $\dfrac{x}{6}$ $\quad mcm = 6(a-x)$

$[6(a-x)]\div[3(a-x)]=2 \qquad [6(a-x)]\div 6=a-x$

$\dfrac{x^2}{3(a-x)}=\dfrac{2x^2}{6(a-x)}$; $\dfrac{x}{6}=\dfrac{x(a-x)}{6(a-x)}=\dfrac{ax-x^2}{6(a-x)}$

$\Rightarrow \dfrac{2x^2}{6(a-x)}$, $\dfrac{ax-x^2}{6(a-x)}$ $\;Rta.$

16. $\dfrac{3}{x^2}$, $\dfrac{2}{x}$, $\dfrac{x+3}{x^2-x}$ $\quad mcm = x^2(x-1)$

$[x^2(x-1)]\div x^2=x-1 \qquad [x^2(x-1)]\div x=x(x-1)$

$[x^2(x-1)]\div(x^2-x)=x$

$\dfrac{3}{x^2}=\dfrac{3(x-1)}{x^2(x-1)}=\dfrac{3x-3}{x^2(x-1)}$;

$\dfrac{2}{x}=\dfrac{2(x^2-x)}{x^2(x-1)}=\dfrac{2x^2-2x}{x^2(x-1)}$;

$\dfrac{x+3}{x^2-x}=\dfrac{x(x+3)}{x^2(x-1)}=\dfrac{x^2+3x}{x^2(x-1)}$

$\Rightarrow \dfrac{3x-3}{x^2(x-1)}$, $\dfrac{2x^2-2x}{x^2(x-1)}$, $\dfrac{x^2+3x}{x^2(x-1)}$ $\;Rta.$

17. $\dfrac{1}{2a+2b}$, $\dfrac{a}{4a-4b}$, $\dfrac{b}{8}$ $\quad mcm = 8(a^2-b^2)$

$[8(a^2-b^2)]\div(2a+2b)=4(a-b) \qquad [8(a^2-b^2)]\div 8=a^2-b^2$

$[8(a^2-b^2)]\div(4a-4b)=2(a+b)$

$\dfrac{1}{2a+2b}=\dfrac{4(a-b)}{8(a^2-b^2)}$;

$\dfrac{a}{4a-4b}=\dfrac{2a(a+b)}{8(a^2-b^2)}=\dfrac{2a^2+2ab}{8(a^2-b^2)}$;

$\dfrac{b}{8}=\dfrac{b(a^2-b^2)}{8(a^2-b^2)}=\dfrac{a^2b-b^3}{8(a^2-b^2)}$

$\Rightarrow \dfrac{4(a-b)}{8(a^2-b^2)}$, $\dfrac{2a^2+2ab}{8(a^2-b^2)}$, $\dfrac{a^2b-b^3}{8(a^2-b^2)}$ $\;Rta.$

18. $\dfrac{x}{xy}$, $\dfrac{y}{x^2+xy}$, $\dfrac{3}{xy+y^2}$ $\quad mcm = xy(x+y)$

$[xy(x+y)]\div xy=x+y \qquad [xy(x+y)]\div(x^2+xy)=y$

$[xy(x+y)]\div(xy+y^2)=x$

$\dfrac{x}{xy}=\dfrac{x(x+y)}{xy(x+y)}=\dfrac{x^2+xy}{xy(x+y)}$

$\dfrac{y}{x^2+xy}=\dfrac{y^2}{xy(x+y)}$; $\dfrac{3}{xy+y^2}=\dfrac{3x}{xy(x+y)}$

$\Rightarrow \dfrac{x^2+xy}{xy(x+y)}$, $\dfrac{y^2}{xy(x+y)}$, $\dfrac{3x}{xy(x+y)}$ $\;Rta.$

19. $\dfrac{2}{a^2-b^2}$, $\dfrac{1}{a^2+ab}$, $\dfrac{a}{a^2-ab}$ $\quad mcm = a(a^2-b^2)$

$[a(a^2-b^2)]\div(a^2-b^2)=a$

$[a(a^2-b^2)]\div(a^2+ab)=a-b$

$[a(a^2-b^2)]\div(a^2-ab)=a+b$

$\dfrac{2}{a^2-b^2}=\dfrac{2a}{a(a^2-b^2)}$; $\dfrac{1}{a^2+ab}=\dfrac{a-b}{a(a^2-b^2)}$

$\dfrac{a}{a^2-ab}=\dfrac{a(a+b)}{a(a^2-b^2)}=\dfrac{a^2+ab}{a(a^2-b^2)}$

$\Rightarrow \dfrac{2a}{a(a^2-b^2)}$, $\dfrac{a-b}{a(a^2-b^2)}$, $\dfrac{a^2+ab}{a(a^2-b^2)}$ $\;Rta.$

20. $\dfrac{3x}{x+1}$, $\dfrac{x^2}{x-1}$, $\dfrac{x^3}{x^2-1}$ $\quad mcm = x^2-1$

$(x^2-1)\div(x+1)=x-1$

$(x^2-1)\div(x-1)=x+1 \qquad (x^2-1)\div(x^2-1)=1$

$\dfrac{3x}{x+1}=\dfrac{3x(x-1)}{x^2-1}=\dfrac{3x^2-3x}{x^2-1}$

$\dfrac{x^2}{x-1}=\dfrac{x^2(x+1)}{x^2-1}=\dfrac{x^3+x^2}{x^2-1}$; $\dfrac{x^3}{x^2-1}=\dfrac{x^3}{x^2-1}$

$\Rightarrow \dfrac{3x^2-3x}{x^2-1}$, $\dfrac{x^3+x^2}{x^2-1}$, $\dfrac{x^3}{x^2-1}$ $\;Rta.$

21. $\dfrac{1}{m^2-n^2}$, $\dfrac{m}{m^2+mn}$, $\dfrac{n}{m^2-mn}$ $\quad mcm = m(m^2-n^2)$

$[m(m^2-n^2)]\div(m^2-n^2)=m \qquad [m(m^2-n^2)]\div(m^2+mn)=m-n$

$[m(m^2-n^2)]\div(m^2-mn)=m+n$

$\dfrac{1}{m^2-n^2}=\dfrac{m}{m(m^2-n^2)}$;

$\dfrac{m}{m^2+mn}=\dfrac{m(m-n)}{m(m^2-n^2)}=\dfrac{m^2-mn}{m(m^2-n^2)}$;

$\dfrac{n}{m^2-mn}=\dfrac{n(m+n)}{m(m^2-n^2)}=\dfrac{mn+n^2}{m(m^2-n^2)}$

$\Rightarrow \dfrac{m}{m(m^2-n^2)}$, $\dfrac{m^2-mn}{m(m^2-n^2)}$, $\dfrac{n(m+n)}{m(m^2-n^2)}$ $\;Rta.$

22. $\dfrac{n+1}{n-1}$, $\dfrac{n-1}{n+1}$, $\dfrac{n^2+1}{n^2-1}$ $\quad mcm = n^2-1$

$(n^2-1)\div(n-1)=n+1$

$(n^2-1)\div(n+1)=n-1 \qquad (n^2-1)\div(n^2-1)=1$

$\dfrac{n+1}{n-1}=\dfrac{(n+1)(n+1)}{n^2-1}=\dfrac{(n+1)^2}{n^2-1}$; $\dfrac{n-1}{n+1}=\dfrac{(n-1)^2}{n^2-1}$;

$\dfrac{n^2+1}{n^2-1}=\dfrac{n^2+1}{n^2-1}$ $\Rightarrow \dfrac{(n+1)^2}{n^2-1}$, $\dfrac{(n-1)^2}{n^2-1}$, $\dfrac{n^2+1}{n^2-1}$ $\;Rta.$

23. $\dfrac{a^2-b^2}{a^2+b^2}$, $\dfrac{a^2+b^2}{a^2-b^2}$, $\dfrac{a^4+b^4}{a^4-b^4}$ $\quad mcm = a^4-b^4$

$(a^4-b^4)\div(a^2+b^2)=a^2-b^2$

$(a^4-b^4)\div(a^2-b^2)=a^2+b^2 \quad (a^4-b^4)\div(a^4-b^4)=1$

$\dfrac{a^2-b^2}{a^2+b^2}=\dfrac{(a^2-b^2)^2}{a^4-b^4}$; $\quad \dfrac{a^2+b^2}{a^2-b^2}=\dfrac{(a^2+b^2)^2}{a^4-b^4}$;

$\dfrac{a^4+b^4}{a^4-b^4}=\dfrac{a^4+b^4}{a^4-b^4}$

$\Rightarrow \dfrac{(a^2-b^2)^2}{a^4-b^4}$, $\dfrac{(a^2+b^2)^2}{a^4-b^4}$ $\quad \dfrac{a^4+b^4}{a^4-b^4}$ *Rta.*

24. $\dfrac{3x}{x-1}$, $\dfrac{x-1}{x+2}$, $\dfrac{1}{x^2+x-2}$ $\quad mcm = x^2+x-2$

$(x^2+x-2)\div(x-1)=x+2$

$(x^2+x-2)\div(x+2)=x-1$

$(x^2+x-2)\div(x^2+x-2)=1$

$\dfrac{3x}{x-1}=\dfrac{3x(x+2)}{x^2+x-2}=\dfrac{3x^2+6x}{x^2+x-2}$;

$\dfrac{x-1}{x+2}=\dfrac{(x-1)^2}{x^2+x-2}$; $\dfrac{1}{x^2+x-2}=\dfrac{1}{x^2+x-2}$

$\Rightarrow \dfrac{3x^2+6x}{x^2+x-2}$, $\dfrac{(x-1)^2}{x^2+x-2}$, $\dfrac{1}{x^2+x-2}$ *Rta.*

25. $\dfrac{x}{2}$, $\dfrac{x}{5x+15}$, $\dfrac{x-1}{10x+30}$ $\quad mcm = 10(x+3)$

$[10(x+3)]\div 2=5(x+3)$

$[10(x+3)]\div[5(x+3)]=2$

$[10(x+3)]\div[10(x+3)]=1$

$\dfrac{x}{2}=\dfrac{5x(x+3)}{10(x+3)}=\dfrac{5x^2+15}{10(x+3)}$

$\dfrac{x}{5x+15}=\dfrac{2x}{10(x+3)}$; $\dfrac{x-1}{10x+30}=\dfrac{x-1}{10(x+3)}$

$\Rightarrow \dfrac{5x^2+15}{10(x+3)}$, $\dfrac{2x}{10(x+3)}$, $\dfrac{x-1}{10(x+3)}$ *Rta.*

26. $\dfrac{2x-1}{x+4}$, $\dfrac{3x+1}{3x+12}$, $\dfrac{4x+3}{6x+24}$ $\quad mcm = 6(x+4)$

$[6(x+4)]\div(x+4)=6 \quad [6(x+4)]\div[3(x+4)]=2$

$[6(x+4)]\div[6(x+4)]=1$

$\dfrac{2x-1}{x+4}=\dfrac{6(2x-1)}{6(x+4)}=\dfrac{12x-6}{6(x+4)}$;

$\dfrac{3x+1}{3x+12}=\dfrac{2(3x+1)}{6(x+4)}=\dfrac{6x+2}{6(x+4)}$;

$\dfrac{4x+3}{6x+24}=\dfrac{4x+3}{6(x+4)}$

$\Rightarrow \dfrac{12x-6}{6(x+4)}$, $\dfrac{6x+2}{6(x+4)}$, $\dfrac{4x+3}{6(x+4)}$ *Rta.*

27. $\dfrac{a}{a+4}$, $\dfrac{2}{9a^2-25}$, $\dfrac{5}{3a-5}$ $\quad mcm = (a+4)(3a+5)(3a-5)$

$[(a+4)(3a+5)(3a-5)]\div(a+4)=(3a+5)(3a-5)$

$[(a+4)(3a+5)(3a-5)]\div(9a^2-25)=a+4$

$[(a+4)(3a+5)(3a-5)]\div(3a-5)=(a+4)(3a+5)$

$\dfrac{3}{a+4}=\dfrac{3(3a+5)(3a-5)}{(a+4)(3a+5)(3a-5)}=\dfrac{27a^2-75}{(a+4)(9a^2-25)}$;

$\dfrac{2}{9a^2-25}=\dfrac{2(a+4)}{(a+4)(3a+5)(3a-5)}=\dfrac{2a+8}{(a+4)(9a^2-25)}$;

$\dfrac{5}{3a-5}=\dfrac{5(a+4)(3a+5)}{(a+4)(3a+5)(3a-5)}=\dfrac{15a^2+85a+100}{(a+4)(9a^2-25)}$

$\Rightarrow \dfrac{27a^2-75}{(a+4)(9a^2-25)}$, $\dfrac{2a+8}{(a+4)(9a^2-25)}$, $\dfrac{15a^2+85a+100}{(a+4)(9a^2-25)}$ *Rta.*

28. $\dfrac{x+1}{x^2-4}$, $\dfrac{x+2}{x^2+x-6}$, $\dfrac{3x}{x^2+5x+6}$ $\quad mcm\ (x+2)(x^2+x-6)$

$[(x+2)(x^2+x-6)]\div(x^2-4)=x+3$

$[(x+2)(x^2+x-6)]\div(x^2+x-6)=x+2$

$[(x+2)(x^2+x-6)]\div(x^2+5x+6)=x-2$

$\dfrac{x+1}{x^2-4}=\dfrac{(x+1)(x+3)}{(x+2)(x^2+x-6)}=\dfrac{x^2+4x+3}{(x+2)(x^2+x-6)}$;

$\dfrac{x+2}{x^2+x-6}=\dfrac{(x+2)(x+2)}{(x+2)(x^2+x-6)}=\dfrac{(x+2)^2}{(x+2)(x^2+x-6)}$;

$\dfrac{3x}{x^2+5x+6}=\dfrac{3x(x-2)}{(x+2)(x^2+x-6)}=\dfrac{3x^2-6x}{(x+2)(x^2+x-6)}$

$\Rightarrow \dfrac{x^2+4x+3}{(x+2)(x^2+x-6)}$, $\dfrac{(x+2)^2}{(x+2)(x^2+x-6)}$, $\dfrac{3x^2-6x}{(x+2)(x^2+x-6)}$ *Rta.*

29. $\dfrac{a+3}{a^2+a-20}$, $\dfrac{5a}{a^2-7a+12}$, $\dfrac{a+1}{a^2+2a-15}$

$mcm=(a+5)(a^2-7a+12)$

$[(a+5)(a^2-7a+12)]\div(a^2+a-20)=a-3$

$[(a+5)(a^2-7a+12)]\div(a^2-7a+12)=a+5$

$[(a+5)(a^2-7a+12)]\div(a^2+2a-15)=a-4$

$\dfrac{a+3}{a^2+a-20}=\dfrac{(a+3)(a-3)}{(a+5)(a^2-7a+12)}=\dfrac{a^2-9}{(a+5)(a-4)(a-3)}$;

$\dfrac{5a}{a^2-7a+12}=\dfrac{5a(a+5)}{(a+5)(a^2-7a+12)}=\dfrac{5a^2+25}{(a+5)(a-4)(a-3)}$;

$\dfrac{a+1}{a^2+2a-15}=\dfrac{(a+1)(a-4)}{(a+5)(a^2-7a+12)}=\dfrac{a^2-3a-4}{(a+5)(a-4)(a-3)}$

$\Rightarrow \dfrac{a^2-9}{(a+5)(a-4)(a-3)}$, $\dfrac{5a^2+25}{(a+5)(a-4)(a-3)}$, $\dfrac{a^2-3a-4}{(a+5)(a-4)(a-3)}$ *Rta.*

30. $\dfrac{a+1}{a^3-1}\ ,\ \dfrac{2a}{a^2+a+1}\ ,\ \dfrac{1}{x-1}\qquad mcm=a^3-1$

$(a^3-1)\div(a^3-1)=1\qquad (a^3-1)\div(a^2+a+1)=a-1$

$(a^3-1)\div(a-1)=a^2+a+1$

$\dfrac{a+1}{a^3-1}=\dfrac{a+1}{a^3-1}\ ;\ \dfrac{2a}{a^2+a+1}=\dfrac{2a(a-1)}{a^3-1}=\dfrac{2a^2-2}{a^3-1}\ ;$

$\dfrac{1}{a-1}=\dfrac{a^2+a+1}{a^3-1}\Rightarrow\dfrac{a+1}{a^3-1},\dfrac{2a^2-2}{a^3-1},\dfrac{a^2+a+1}{a^3-1}\ Rta.$

31. $\dfrac{1}{x-1}\ ,\ \dfrac{1}{x^3-1}\ ,\ \dfrac{2}{3}\qquad mcm=3(x^3-1)$

$[3(x^3-1)]\div(x-1)=3(x^2+x+1)$

$[3(x^3-1)]\div(x^3-1)=3\quad [3(x^3-1)]\div 3=x^3-1$

$\dfrac{1}{x-1}=\dfrac{3(x^2+x+1)}{3(x^3-1)}=\dfrac{3x^2+3x+3}{3(x^3-1)}\ ;$

$\dfrac{1}{x^3-1}=\dfrac{3}{3(x^3-1)}\ ;\ \dfrac{2}{3}=\dfrac{2(x^3-1)}{3(x^3-1)}=\dfrac{2x^3-2}{3(x^3-1)}$

$\Rightarrow\dfrac{3x^2+3x+3}{3(x^3-1)},\dfrac{3}{3(x^3-1)},\dfrac{2x^3-2}{3(x^3-1)}\ Rta.$

32. $\dfrac{3}{2a^2+2ab}\ ,\ \dfrac{b}{a^2x+abx}\ ,\ \dfrac{1}{4ax^2-4bx^2}$

$mcm=4ax^2(a^2-b^2)$

$[4ax^2(a^2-b^2)]\div(2a^2+2ab)=2x^2(a-b)$

$[4ax^2(a^2-b^2)]\div(a^2x+abx)=4x(a-b)$

$[4ax^2(a^2-b^2)]\div(4ax^2-4bx^2)=a(a+b)$

$\dfrac{3}{2a^2+2ab}=\dfrac{3[2x^2(a-b)]}{4ax^2(a^2-b^2)}=\dfrac{6ax^2-6bx^2}{4ax^2(a^2-b^2)}\ ;$

$\dfrac{b}{a^2x+abx}=\dfrac{b[4x(a-b)]}{4ax^2(a^2-b^2)}=\dfrac{4abx-4b^2x}{4ax^2(a^2-b^2)}\ ;$

$\dfrac{1}{4ax^2-4bx^2}=\dfrac{a(a+b)}{4ax^2(a^2-b^2)}=\dfrac{a^2+ab}{4ax^2(a^2-b^2)}$

$\Rightarrow\dfrac{6ax^2-6bx^2}{4ax^2(a^2-b^2)},\dfrac{4abx-4b^2x}{4ax^2(a^2-b^2)},\dfrac{a^2+ab}{4ax^2(a^2-b^2)}Rta.$

33. $\dfrac{1}{a-1}\ ,\ \dfrac{a+1}{(a-1)^2}\ ,\ \dfrac{3(a+1)}{(a-1)^3}\quad mcm=(a-1)^3$

$(a-1)^3\div(a-1)=(a-1)^2$

$(a-1)^3\div(a-1)^2=a-1\qquad (a-1)^3\div(a-1)^3=1$

$\dfrac{1}{a-1}=\dfrac{(a-1)^2}{(a-1)^3}\ ;\ \dfrac{a+1}{(a-1)^2}=\dfrac{(a+1)(a-1)}{(a-1)^3}=\dfrac{a^2-1}{(a-1)^3}\ ;$

$\dfrac{3(a+1)}{(a-1)^3}=\dfrac{3a+3}{(a-1)^3}\ \Rightarrow\dfrac{(a-1)^2}{(a-1)^3},\dfrac{a^2-1}{(a-1)^3},\dfrac{3a+3}{(a-1)^3}\ Rta.$

34. $\dfrac{2x-3}{6x^2+7x+2}\ ,\ \dfrac{3}{2x+1}\ ,\ \dfrac{2x-1}{6x+4}\quad mcm=\ 2(3x+2)(2x+1)$

$[2(3x+2)(2x+1)]\div(6x^2+7x+2)=2$

$[2(3x+2)(2x+1)]\div(2x+1)=2(3x+2)$

$[2(3x+2)(2x+1)]\div(6x+4)=2x+1$

$\dfrac{2x-3}{6x^2+7x+2}=\dfrac{2(2x-3)}{2(3x+2)(2x+1)}=\dfrac{4x-6}{2(3x+2)(2x+1)}\ ;$

$\dfrac{3}{2x+1}=\dfrac{6(3x+2)}{2(3x+2)(2x+1)}=\dfrac{18x+12}{2(3x+2)(2x+1)}\ ;$

$\dfrac{2x-1}{6x+4}=\dfrac{(2x-1)(2x+1)}{2(3x+2)(2x+1)}=\dfrac{4x^2-1}{2(3x+2)(2x+1)}$

$\Rightarrow\dfrac{4x-6}{2(3x+2)(2x+1)},\dfrac{18x+12}{2(3x+2)(2x+1)},\dfrac{4x^2-1}{2(3x+2)(2x+1)}\ Rta.$

EJERCICIO 126

1. $\dfrac{x-2}{4}+\dfrac{3x+2}{6}=\dfrac{3(x-2)+2(3x+2)}{12}=\dfrac{3x-6+6x+4}{12}=\dfrac{9x-2}{12}$

2. $\dfrac{2}{5a^2}+\dfrac{1}{3ab}=\dfrac{3b\cdot 2+5a}{15a^2b}=\dfrac{5a+6b}{15a^2b}$

3. $\dfrac{a-2b}{15a}+\dfrac{b-a}{20b}=\dfrac{4b(a-2b)+3a(b-a)}{60ab}$

$=\dfrac{4ab-8b^2+3ab-3a^2}{60ab}=\dfrac{-3a^2+7ab-8b^2}{60ab}$

4. $\dfrac{a+3b}{3ab}+\dfrac{a^2b-4ab^2}{5a^2b^2}=\dfrac{5ab(a+3b)+3(a^2b-4ab^2)}{15a^2b^2}$

$=\dfrac{5a^2b+15ab^2+3a^2b-12ab^2}{15a^2b^2}$

$=\dfrac{8a^2b+3ab^2}{15a^2b^2}=\dfrac{ab(8a+3b)}{15a^2b^2}=\dfrac{8a+3b}{15ab}$

5. $\dfrac{a-1}{3}+\dfrac{2a}{6}+\dfrac{3a+4}{12}$

$=\dfrac{4(a-1)+2\cdot 2a+3a+4}{12}=\dfrac{4a-4+4a+3a+4}{12}=\dfrac{11a}{12}$

6. $\dfrac{n}{m^2}+\dfrac{3}{mn}+\dfrac{2}{m}=\dfrac{n\cdot n+3m+2mn}{m^2n}=\dfrac{3m+2mn+n^2}{m^2n}$

7. $\dfrac{1-x}{2x}+\dfrac{x+2}{x^2}+\dfrac{1}{3ax^2}$

$=\dfrac{3ax(1-x)+6a(x+2)+2}{6ax^2}$

$=\dfrac{3ax-3ax^2+6ax-12a+2}{6ax^2}=\dfrac{9ax+12a-3ax^2+2}{6ax^2}$

8. $\dfrac{2a-3}{3a}+\dfrac{3x+2}{10x}+\dfrac{x-a}{5ax}=\dfrac{10x(2a-3)+3a(3x+2)+6(x-a)}{30ax}$

$=\dfrac{20ax-30x+9ax+6a+6x-6a}{30ax}$

$=\dfrac{29ax-24x}{30ax}=\dfrac{x(29a-24)}{30ax}=\dfrac{29a-24}{30a}$

9. $\frac{3}{5}+\frac{x+2}{2x}+\frac{x^2+2}{6x^2}$

$=\frac{6x^2\cdot 3+15x(x+2)+5(x^2+2)}{30x^2}$

$=\frac{18x^2+15x^2+30x+5x^2+10}{30x^2}$

$=\frac{38x^2+30x+10}{30x^2}=\frac{2(19x^2+15x+5)}{30x^2}=\frac{19x^2+15x+15}{15x^2}$

10. $\frac{x-y}{12}+\frac{2x+y}{15}+\frac{y-4x}{30}$

$=\frac{5(x-y)+4(2x+y)+2(y-4x)}{60}$

$=\frac{5x-5y+8x+4y+2y-8x}{60}=\frac{5x+y}{60}$

11. $\frac{m-n}{mn}+\frac{n-a}{na}+\frac{2a-m}{am}$

$=\frac{a(m-n)+m(n-a)+n(2a-m)}{amn}$

$=\frac{am-an+mn-ma+2an-mn}{amn}=\frac{an}{amn}=\frac{1}{m}$

12. $\frac{x+2}{3x}+\frac{x^2-2}{5x^2}+\frac{2-x^3}{9x^3}$

$=\frac{15x^2(x+2)+9x(x^2-2)+5(2-x^3)}{45x^3}$

$=\frac{15x^3+30x^2+9x^3-18x+10-5x^3}{45x^3}$

$=\frac{19x^3+30x^2-18x+10}{45x^3}$

13. $\frac{1}{ab}+\frac{b^2-a^2}{ab^3}+\frac{ab+b^2}{a^2b^2}$

$=\frac{ab^2+a(b^2-a^2)+b(ab+b^2)}{a^2b^3}$

$=\frac{ab^2+ab^2-a^3+ab^2+b^3}{a^2b^3}=\frac{-a^3+3ab^2+b^3}{a^2b^3}$

14. $\frac{a+3b}{ab}+\frac{2a-3m}{am}+\frac{3}{a}$

$=\frac{m(a+3b)+b(2a-3m)+3bm}{abm}$

$=\frac{am+3bm+2ab-3bm+3bm}{abm}=\frac{am+2ab+3bm}{abm}$

EJERCICIO 127

1. $\frac{1}{a+1}+\frac{1}{a-1}=\frac{a-1+a+1}{a^2-1}=\frac{2a}{a^2-1}$

2. $\frac{2}{x+4}+\frac{1}{x-3}$

$=\frac{2(x-3)+x+4}{x^2+x-12}=\frac{2x-6+x+4}{x^2+x-12}=\frac{3x-2}{x^2+x-12}$

3. $\frac{3}{1-x}+\frac{6}{2x+5}$

$=\frac{3(2x+5)+6(1-x)}{(1-x)(2x+5)}=\frac{6x+15+6-6x}{(1-x)(2x+5)}=\frac{21}{(1-x)(2x+5)}$

4. $\frac{x}{x-y}+\frac{x}{x+y}$

$=\frac{x(x+y)+x(x-y)}{x^2-y^2}=\frac{x^2+xy+x^2-xy}{x^2-y^2}=\frac{2x^2}{x^2-y^2}$

5. $\frac{m+3}{m-3}+\frac{m+2}{m-2}$

$=\frac{(m-2)(m+3)+(m-3)(m+2)}{(m-3)(m-2)}$

$=\frac{m^2+m-6+m^2-m-6}{(m-3)(m-2)}=\frac{2m^2-12}{(m-3)(m-2)}=\frac{2(m^2-6)}{(m-3)(m-2)}$

6. $\frac{x+y}{x-y}+\frac{x-y}{x+y}=\frac{(x+y)(x+y)+(x-y)(x-y)}{x^2-y^2}$

$=\frac{(x+y)^2+(x-y)^2}{x^2-y^2}$

$=\frac{x^2+2xy+y^2+x^2-2xy+y^2}{x^2-y^2}=\frac{2x^2+2y^2}{x^2-y^2}$

7. $\frac{x}{x^2-1}+\frac{x+1}{(x-1)^2}=\frac{x(x-1)+(x+1)^2}{(x-1)^2(x+1)}$

$=\frac{x^2-x+x^2+2x+1}{(x-1)^2(x+1)}=\frac{2x^2+x+1}{(x-1)^2(x+1)}$

8. $\frac{2}{x-5}+\frac{3x}{x^2-25}=\frac{2(x+5)+3x}{x^2-25}=\frac{2x+10+3x}{x^2-25}=\frac{5x+10}{x^2-25}$

9. $\frac{1}{3x-2y}+\frac{x-y}{9x^2-4y^2}=\frac{3x+2y+x-y}{9x^2-4y^2}=\frac{4x+y}{9x^2-4y^2}$

10. $\frac{x+a}{x+3a}+\frac{3a^2-x^2}{x^2-9a^2}=\frac{(x-3a)(x+a)+3a^2-x^2}{x^2-9a^2}$

$=\frac{x^2+ax-3ax-3a^2+3a^2-x^2}{x^2-9a^2}$

$=\frac{-2ax}{x^2-9a^2}=\frac{2ax}{9a^2-x^2}$

11. $\frac{a}{1-a^2}+\frac{a}{1+a^2}$

$$=\frac{a\left(1+a^2\right)+a\left(1-a^2\right)}{\left(1-a^2\right)\left(1+a^2\right)}=\frac{a+a^3+a-a^3}{\left(1-a^2\right)\left(1+a^2\right)}=\frac{2a}{1-a^4}$$

12. $\frac{2}{a^2-ab}+\frac{2}{ab+b^2}=\frac{2b\left(a+b\right)+2a\left(a-b\right)}{ab\left(a^2-b^2\right)}$

$$=\frac{2ab+2b^2+2a^2-2ab}{ab\left(a^2-b^2\right)}=\frac{2a^2+2b^2}{ab\left(a^2-b^2\right)}$$

13. $\frac{ab}{9a^2-b^2}+\frac{a}{3a+b}$

$$=\frac{ab+a\left(3a-b\right)}{9a^2-b^2}=\frac{ab+3a^2-ab}{9a^2-b^2}=\frac{3a^2}{9a^2-b^2}$$

14. $\frac{1}{a^2-b^2}+\frac{1}{\left(a-b\right)^2}=\frac{a-b+a+b}{\left(a+b\right)\left(a-b\right)^2}=\frac{2a}{\left(a+b\right)\left(a-b\right)^2}$

15. $\frac{3}{x^2+y^2}+\frac{2}{\left(x+y\right)^2}=\frac{3\left(x+y\right)^2+2\left(x^2+y^2\right)}{\left(x^2+y^2\right)\left(x+y\right)^2}$

$$=\frac{3x^2+6xy+3y^2+2x^2+2y^2}{\left(x^2+y^2\right)\left(x+y\right)^2}$$

$$=\frac{5x^2+6xy+5y^2}{\left(x^2+y^2\right)\left(x+y\right)^2}$$

16. $\frac{x}{a^2-ax}+\frac{a+x}{ax}+\frac{a}{ax-x^2}$

$a^2-ax=a\left(a-x\right)$

$ax=ax$ $\qquad mcm=ax\left(a-x\right)$

$ax-x^2=x\left(a-x\right)$

$$=\frac{x^2+\left(a+x\right)\left(a-x\right)+a^2}{ax\left(a-x\right)}$$

$$=\frac{x^2+a^2-x^2+a^2}{ax\left(a-x\right)}=\frac{2a^2}{ax\left(a-x\right)}=\frac{2a}{x\left(a-x\right)}$$

17. $\frac{3}{2x+4}+\frac{x-1}{2x-4}+\frac{x+8}{x^2-4}$

$2x+4=2\left(x+2\right)$

$2x-4=2\left(x-2\right)$ $\qquad mcm=2\left(x^2-4\right)$

$x^2-4=\left(x-2\right)\left(x+2\right)$

$$=\frac{3\left(x-2\right)+\left(x-1\right)\left(x+2\right)+2\left(x+8\right)}{2\left(x^2-4\right)}$$

$$=\frac{3x-6+x^2+x-2+2x+16}{2\left(x^2-4\right)}$$

$$=\frac{x^2+6x+8}{2\left(x^2-4\right)}=\frac{\left(x+2\right)\left(x+4\right)}{2\left(x+2\right)\left(x-2\right)}=\frac{x+4}{2\left(x-2\right)}$$

18. $\frac{1}{x+x^2}+\frac{1}{x-x^2}+\frac{x+3}{1-x^2}$

$x+x^2=x\left(1+x\right)$

$x-x^2=x\left(1-x\right)$ $\qquad mcm=x\left(1-x^2\right)$

$1-x^2=\left(1+x\right)\left(1-x\right)$

$$=\frac{1-x+1+x+x\left(x+3\right)}{x\left(1-x^2\right)}$$

$$=\frac{2+x^2+3x}{x\left(1-x^2\right)}=\frac{\left(x+1\right)\left(x+2\right)}{x\left(1-x\right)\left(1+x\right)}=\frac{x+2}{x\left(1-x\right)}$$

19. $\frac{x-y}{x+y}+\frac{x+y}{x-y}+\frac{4xy}{x^2-y^2}$ $\qquad mcm=x^2-y^2$

$$=\frac{\left(x-y\right)\left(x-y\right)+\left(x+y\right)\left(x+y\right)+4xy}{x^2-y^2}$$

$$=\frac{x^2-2xy+y^2+x^2+2xy+y^2+4xy}{x^2-y^2}$$

$$=\frac{2x^2+2y^2+4xy}{x^2-y^2}=\frac{2\left(x^2+2xy+y^2\right)}{x^2-y^2}$$

$$=\frac{2\left(x+y\right)^2}{\left(x+y\right)\left(x-y\right)}=\frac{2\left(x+y\right)}{x-y}$$

20. $\frac{1}{a-5}+\frac{a}{a^2-4a-5}+\frac{a+5}{a^2+2a+1}$

$a-5=a-5$

$a^2-4a-5=\left(a-5\right)\left(a+1\right)$

$a^2+2a+1=\left(a+1\right)^2$ $\qquad mcm=\left(a-5\right)\left(a+1\right)^2$

$$=\frac{\left(a+1\right)^2+a\left(a+1\right)+\left(a+5\right)\left(a-5\right)}{\left(a-5\right)\left(a+1\right)^2}$$

$$=\frac{a^2+2a+1+a^2+a+a^2-25}{\left(a-5\right)\left(a+1\right)^2}$$

$$=\frac{3a^2+3a-24}{\left(a-5\right)\left(a+1\right)^2}=\frac{3\left(a^2+a-8\right)}{\left(a-5\right)\left(a+1\right)^2}$$

21. $\frac{3}{a}+\frac{2}{5a-3}+\frac{1-85a}{25a^2-9}$ $\qquad mcm=a\left(25a^2-9\right)$

$$=\frac{3\left(25a^2-9\right)+2a\left(5a+3\right)+a\left(1-85a\right)}{a\left(25a^2-9\right)}$$

$$=\frac{75a^2-27+10a^2+6a+a-85a^2}{a\left(25a^2-9\right)}=\frac{7a-27}{a\left(25a^2-9\right)}$$

22. $\frac{x+1}{10}+\frac{x-3}{5x-10}+\frac{x-2}{2}$ $\qquad mcm=10\left(x-2\right)$

$$=\frac{\left(x+1\right)\left(x-2\right)+2\left(x-3\right)+5\left(x-2\right)\left(x-2\right)}{10\left(x-2\right)}$$

$$=\frac{x^2-x-2+2x-6+5x^2-20x+20}{10\left(x-2\right)}=\frac{6x^2-19x+12}{10\left(x-2\right)}$$

23. $\dfrac{x+5}{x^2+x-12}+\dfrac{x+4}{x^2+2x-15}+\dfrac{x-3}{x^2+9x+20}$

$x^2+x-12=(x+4)(x-3)$

$x^2+2x-15=(x+5)(x-3)$

$x^2+9x+20=(x+5)(x+4)$

$mcm=(x+5)(x+4)(x-3)$

$=\dfrac{(x+5)(x+5)+(x+4)(x+4)+(x-3)(x-3)}{(x+5)(x+4)(x-3)}$

$=\dfrac{x^2+10x+25+x^2+8x+16+x^2-6x+9}{(x+5)(x+4)(x-3)}$

$=\dfrac{3x^2+12x+50}{(x+5)(x+4)(x-3)}$

24. $\dfrac{1}{x-2}+\dfrac{1-2x^2}{x^3-8}+\dfrac{x}{x^2+2x+4}$

$x^3-8=(x-2)(x^2+2x+4)\quad mcm=x^3-8$

$=\dfrac{x^2+2x+4+1-2x^2+x(x-2)}{x^3-8}$

$=\dfrac{-x^2+2x+5+x^2-2x}{x^3-8}=\dfrac{5}{x^3-8}$

25. $\dfrac{2}{a+1}+\dfrac{a}{(a+1)^2}+\dfrac{a+1}{(a+1)^3}\qquad mcm=(a+1)^3$

$=\dfrac{2(a+1)^2+a(a+1)+a+1}{(a+1)^3}$

$=\dfrac{2a^2+4a+2+a^2+a+a+1}{(a+1)^3}$

$=\dfrac{3a^2+6a+3}{(a+1)^3}=\dfrac{3(a^2+2a+1)}{(a+1)^3}=\dfrac{3(a+1)^2}{(a+1)^3}=\dfrac{3}{a+1}$

26. $\dfrac{2x}{3x^2+11x+6}+\dfrac{x+1}{x^2-9}+\dfrac{1}{3x+2}$

$3x^2+11x+6=(x+3)(3x+2)$

$x^2-9=(x+3)(x-3)$

$3x+2=3x+2\qquad mcm=(x^2-9)(3x+2)$

$=\dfrac{2x(x-3)+(x+1)(3x+2)+x^2-9}{(x^2-9)(3x+2)}$

$=\dfrac{2x^2-6x+3x^2+5x+2+x^2-9}{(x^2-9)(3x+2)}$

$=\dfrac{6x^2-x-7}{(x^2-9)(3x+2)}$

27. $\dfrac{x^2-4}{x^3+1}+\dfrac{1}{x+1}+\dfrac{3}{x^2-x+1}$

$x^3+1=(x+1)(x^2-x+1)\qquad mcm=x^3+1$

$=\dfrac{x^2-4+x^2-x+1+3x+3}{x^3+1}$

$=\dfrac{2x^2+2x}{x^3+1}=\dfrac{2x(x+1)}{(x+1)(x^2-x+1)}=\dfrac{2x}{x^2-x+1}$

28. $\dfrac{1}{x-1}+\dfrac{1}{(x-1)(x+2)}+\dfrac{x+1}{(x-1)(x+2)(x+3)}$

$mcm=(x-1)(x+2)(x+3)$

$=\dfrac{(x+2)(x+3)+x+3+x+1}{(x-1)(x+2)(x+3)}$

$=\dfrac{x^2+5x+6+2x+4}{(x-1)(x+2)(x+3)}=\dfrac{x^2+7x+10}{(x-1)(x+2)(x+3)}$

$=\dfrac{(x+5)(x+2)}{(x-1)(x+2)(x+3)}=\dfrac{x+5}{(x-1)(x+3)}$

29. $\dfrac{x-2}{2x^2-5x-3}+\dfrac{x-3}{2x^2-3x-2}+\dfrac{2x-1}{x^2-5x+6}$

$2x^2-5x-3=(x-3)(2x+1)$

$2x^2-3x-2=(x-2)(2x+1)$

$x^2-5x+6=(x-3)(x-2)$

$mcm=(x-3)(x-2)(2x+1)$

$=\dfrac{(x-2)(x-2)+(x-3)(x-3)+(2x-1)(2x+1)}{(x-3)(x-2)(2x+1)}$

$=\dfrac{x^2-4x+4+x^2-6x+9+4x^2-1}{(x-3)(x-2)(2x+1)}$

$=\dfrac{6x^2-10x+12}{(x-3)(x-2)(2x+1)}$

30. $\dfrac{a-2}{a-1}+\dfrac{a+3}{a+2}+\dfrac{a+1}{a-3}\qquad mcm=(a-1)(a+2)(a-3)$

$=\dfrac{(a+2)(a-2)(a-3)+(a-1)(a+3)(a-3)+(a-1)(a+1)(a+2)}{(a-1)(a+2)(a-3)}$

$=\dfrac{a^3-3a^2-4a+12+a^3-a^2-9a+9+a^3+2a^2-a-2}{(a-1)(a+2)(a-3)}$

$=\dfrac{3a^3-2a^2-14a+19}{(a-1)(a+2)(a-3)}$

EJERCICIO 128

1. $\frac{x-3}{4}-\frac{x+2}{8}=\frac{2(x-3)-(x+2)}{8}=\frac{2x-6-x-2}{8}=\frac{x-8}{8}$

2. $\frac{a+5b}{a^2}-\frac{b-3}{ab}$

$=\frac{b(a+5b)-a(b-3)}{a^2b}=\frac{ab+5b^2-ab+3a}{a^2b}=\frac{3a+5b^2}{a^2b}$

3. $\frac{2}{3mn^2}-\frac{1}{2m^2n}=\frac{2(2m)-(3n)}{6m^2n^2}=\frac{4m-3n}{6m^2n^2}$

4. $\frac{a-3}{5ab}-\frac{4-3ab^2}{3a^2b^3}$

$=\frac{3ab^2(a-3)-5(4-3ab^2)}{15a^2b^3}$

$=\frac{3a^2b^2-9ab^2-20+15ab^2}{15a^2b^3}=\frac{3a^2b^2+6ab^2-20}{15a^2b^3}$

5. $\frac{2a+3}{4a}-\frac{a-2}{8a}$

$=\frac{2(2a+3)-(a-2)}{8a}=\frac{4a+6-a+2}{8a}=\frac{3a+8}{8a}$

6. $\frac{y-2x}{20x}-\frac{x-3y}{24y}$

$=\frac{6y(y-2x)-5x(x-3y)}{120xy}$

$=\frac{6y^2-12xy-5x^2+15xy}{120xy}=\frac{6y^2+3xy-5x^2}{120xy}$

7. $\frac{x-1}{3}-\frac{x-2}{4}-\frac{x+3}{6}$

$=\frac{4(x-1)-3(x-2)-2(x+3)}{12}$

$=\frac{4x-4-3x+6-2x-6}{12}=\frac{-x-4}{12}=-\frac{x+4}{12}$

8. $\frac{3}{5}-\frac{2a+1}{10a}-\frac{4a^2+1}{20a^2}$

$=\frac{4a^2(3)-2a(2a+1)-(4a^2+1)}{20a^2}$

$=\frac{12a^2-4a^2-2a-4a^2-1}{20a^2}=\frac{4a^2-2a-1}{20a^2}$

9. $\frac{3}{5x}-\frac{x-1}{3x^2}-\frac{x^2+2x+3}{15x^3}$

$=\frac{3x^2(3)-5x(x-1)-(x^2+2x+3)}{15x^3}$

$=\frac{9x^2-5x^2+5x-x^2-2x-3}{15x^3}$

$=\frac{3x^2+3x-3}{15x^3}=\frac{3(x^2+x-1)}{15x^3}=\frac{x^2+x-1}{5x^3}$

10. $\frac{1}{2a}-\frac{2+b}{3ab}-\frac{5}{6a^2b^3}$

$=\frac{3ab^3-2ab^2(2+b)-5}{6a^2b^3}$

$=\frac{3ab^3-4ab^2-2ab^3-5}{6a^2b^3}=\frac{ab^3-4ab^2-5}{6a^2b^3}$

EJERCICIO 129

1. $\frac{1}{x-4}-\frac{1}{x-3}$ $\quad mcm=(x-4)(x-3)$

$=\frac{x-3-(x-4)}{(x-4)(x-3)}=\frac{x-3-x+4}{(x-4)(x-3)}=\frac{1}{(x-4)(x-3)}$

2. $\frac{m-n}{m+n}-\frac{m+n}{m-n}$ $\quad mcm=m^2-n^2$

$=\frac{(m-n)(m-n)-(m+n)(m+n)}{m^2-n^2}$

$=\frac{m^2-2mn+n^2-m^2-2mn-n^2}{m^2-n^2}=\frac{-4mn}{m^2-n^2}=\frac{4mn}{n^2-m^2}$

3. $\frac{1-x}{1+x}-\frac{1+x}{1-x}=\frac{(1-x)(1-x)-(1+x)(1+x)}{1-x^2}$

$=\frac{1-2x+x^2-1-2x-x^2}{1-x^2}=\frac{-4x}{1-x^2}=\frac{4x}{x^2-1}$

4. $\frac{a+b}{a^2+ab}-\frac{b-a}{ab+b^2}$

$a^2+ab=a(a+b)$; $ab+b^2=b(a+b)$

$mcm=ab(a+b)$

$=\frac{b(a+b)-a(b-a)}{ab(a+b)}=\frac{ab+b^2-ab+a^2}{ab(a+b)}=\frac{a^2+b^2}{ab(a+b)}$

5. $\frac{m+n}{m-n}-\frac{m^2+n^2}{m^2-n^2}=\frac{(m+n)(m+n)-(m^2+n^2)}{m^2-n^2}$

$=\frac{m^2+2mn+n^2-m^2-n^2}{m^2-n^2}=\frac{2mn}{m^2-n^2}$

6. $\frac{1}{x+x^2}-\frac{1}{x-x^2}$ $\quad x+x^2=x(x+1)$; $x-x^2=x(1-x)$

$mcm=x(1-x^2)$

$=\frac{1-x-(1+x)}{x(1-x^2)}=\frac{1-x-1-x}{x(1-x^2)}=\frac{-2x}{x(1-x^2)}=\frac{-2}{1-x^2}=\frac{2}{x^2-1}$

7. $\dfrac{a+x}{(a-x)^2}-\dfrac{x}{a^2-x^2}$

$=\dfrac{(a+x)(a+x)-x(a-x)}{(a-x)^2(a+x)}$

$=\dfrac{a^2+2ax+x^2-ax+x^2}{(a-x)^2(a+x)}=\dfrac{a^2+ax+2x^2}{(a-x)^2(a+x)}$

8. $\dfrac{a+1}{6a+3}-\dfrac{1}{12a+6}$

$6a+3=3(2a+1)$; $12a+6=6(2a+1)$

$mcm=6(2a+1)$

$=\dfrac{2(a+1)-1}{6(2a+1)}=\dfrac{2a+2-1}{6(2a+1)}=\dfrac{2a+1}{6(2a+1)}=\dfrac{1}{6}$

9. $\dfrac{a-4}{a^2-6a+9}-\dfrac{a+3}{a^2+a-12}$

$a^2-6a+9=(a-3)^2$; $a^2+a-12=(a+4)(a-3)$

$mcm=(a-3)^2(a+4)$

$=\dfrac{(a+4)(a-4)-(a-3)(a+3)}{(a+4)(a-3)^2}$

$=\dfrac{a^2-16-a^2+9}{(a+4)(a-3)^2}=\dfrac{-7}{(a+4)(a-3)^2}$

10. $\dfrac{a^2+4ab-3b^2}{a^2-9b^2}-\dfrac{b}{a+3b}$ $mcm=a^2-9b^2$

$=\dfrac{a^2+4ab-3b^2-b(a-3b)}{a^2-9b^2}$

$=\dfrac{a^2+4ab-3b^2-ab+3b^2}{a^2-9b^2}$

$=\dfrac{a^2+3ab}{a^2-9b^2}=\dfrac{a(a+3b)}{(a+3b)(a-3b)}=\dfrac{a}{a-3b}$

11. $\dfrac{x}{x^2-1}-\dfrac{x+1}{(x-1)^2}$ $mcm=(x-1)^2(x+1)$

$=\dfrac{x(x-1)-(x+1)(x+1)}{(x+1)(x-1)^2}$

$=\dfrac{x^2-x-x^2-2x-1}{(x+1)(x-1)^2}=\dfrac{-3x-1}{(x+1)(x-1)^2}=-\dfrac{3x+1}{(x+1)(x-1)^2}$

12. $\dfrac{1}{a^3-b^3}-\dfrac{1}{(a-b)^3}$ $mcm=(a-b)^3(a^2+ab+b^2)$

$=\dfrac{(a-b)^2-(a^2+ab+b^2)}{(a-b)^3(a^2+ab+b^2)}$

$=\dfrac{a^2-2ab+b^2-a^2-ab-b^2}{(a-b)^3(a^2+ab+b^2)}=\dfrac{-3ab}{(a-b)^3(a^2+ab+b^2)}$

13. $\dfrac{x+3}{6x^2+x-2}-\dfrac{1}{4x^2-4x+1}$

$6x^2+x-2=(3x+2)(2x-1)$

$4x^2-4x+1=(2x-1)^2$ $mcm=(2x-1)^2(3x+2)$

$=\dfrac{(2x-1)(x+3)-(3x+2)}{(3x+2)(2x-1)^2}$

$=\dfrac{2x^2+5x-3-3x-2}{(3x+2)(2x-1)^2}=\dfrac{2x^2+2x-5}{(3x+2)(2x-1)^2}$

14. $\dfrac{x-1}{4x+4}-\dfrac{x+2}{8x-8}$ $mcm=8(x^2-1)$

$=\dfrac{2(x-1)(x-1)-(x+2)(x+1)}{8(x^2-1)}$

$=\dfrac{2x^2-4x+2-x^2-3x-2}{8(x^2-1)}=\dfrac{x^2-7x}{8(x^2-1)}$

15. $\dfrac{x}{xy-y^2}-\dfrac{1}{y}$ $mcm=y(x-y)$

$=\dfrac{x-(x-y)}{y(x-y)}=\dfrac{x-x+y}{y(x-y)}=\dfrac{y}{y(x-y)}=\dfrac{1}{x-y}$

16. $\dfrac{b}{a^2-b^2}-\dfrac{b}{a^2+ab}$ $mcm=a(a^2-b^2)$

$=\dfrac{ab-b(a-b)}{a(a^2-b^2)}=\dfrac{ab-ab+b^2}{a(a^2-b^2)}=\dfrac{b^2}{a(a^2-b^2)}$

17. $\dfrac{2a-3}{6a+9}-\dfrac{a-1}{4a^2+12a+9}$

$6a+9=3(2a+3)$; $4a^2+12a+9=(2a+3)^2$

$mcm=3(2a+3)^2$

$=\dfrac{(2a+3)(2a-3)-3(a-1)}{3(2a+3)^2}$

$=\dfrac{4a^2-9-3a+3}{3(2a+3)^2}=\dfrac{4a^2-3a-6}{3(2a+3)^2}$

18. $\dfrac{x+1}{x^2+x+1}-\dfrac{x-1}{x^2-x+1}$ $mcm=(x^2+x+1)(x^2-x+1)$

$=\dfrac{x+1-(x-1)}{(x^2+x+1)(x^2-x+1)}$

$=\dfrac{x+1-x+1}{(x^2+x+1)(x^2-x+1)}=\dfrac{2}{(x^2+x+1)(x^2-x+1)}$

19. $\frac{a-1}{a^2+a}-\frac{1}{2a-2}-\frac{1}{2a+2}$ $\quad mcm=2a\left(a^2-1\right)$

$=\frac{2(a-1)(a-1)-a(a+1)-a(a-1)}{2a\left(a^2-1\right)}$

$=\frac{2\left(a^2-2a+1\right)-a^2-a-a^2+a}{2a\left(a^2-1\right)}$

$=\frac{2a^2-4a+2-2a^2}{2a\left(a^2-1\right)}$

$=-\frac{4a-2}{2a\left(a^2-1\right)}$

$=-\frac{2(2a-1)}{2a\left(a^2-1\right)}=-\frac{2a-1}{a\left(a^2-1\right)}=\frac{1-2a}{a\left(a^2-1\right)}$

20. $\frac{1}{4a+4}-\frac{1}{8a-8}-\frac{1}{12a^2+12}$

$mcm=24\left(a^2+1\right)\left(a^2-1\right)$

$=\frac{6\left(a^2+1\right)(a-1)-3\left(a^2+1\right)(a+1)-2\left(a^2-1\right)}{24\left(a^2+1\right)\left(a^2-1\right)}$

$=\frac{6\left(a^3-a^2+a-1\right)-3\left(a^3+a^2+a+1\right)-2a^2+2}{24\left(a^2+1\right)\left(a^2-1\right)}$

$=\frac{6a^3-6a^2+6a-6-3a^3-3a^2-3a-3-2a^2+2}{24\left(a^2+1\right)\left(a^2-1\right)}$

$=\frac{3a^3-11a^2+3a-7}{24\left(a^4-1\right)}$

21. $\frac{y}{x^2-xy}-\frac{1}{x}-\frac{1}{x-y}$ $\quad mcm=x(x-y)$

$=\frac{y-(x-y)-x}{x(x-y)}$

$=\frac{y-x+y-x}{x(x-y)}$

$=\frac{2y-2x}{x(x-y)}=\frac{2(y-x)}{x(x-y)}=-\frac{2(x-y)}{x(x-y)}=-\frac{2}{x}$

22. $\frac{a}{a^2+ab}-\frac{1}{a}-\frac{1}{a+b}$ $\quad mcm=a(a+b)$

$=\frac{a-(a+b)-a}{a(a+b)}=\frac{-a-b}{a(a+b)}=-\frac{a+b}{a(a+b)}=-\frac{1}{a}$

23. $\frac{1}{x^2-xy}-\frac{1}{x^2+xy}-\frac{2y}{x^3-xy^2}$ $\quad mcm=x\left(x^2-y^2\right)$

$=\frac{x+y-(x-y)-2y}{x\left(x^2-y^2\right)}=\frac{x-y-x+y}{x\left(x^2-y^2\right)}=\frac{0}{x\left(x^2-y^2\right)}=0$

24. $\frac{x}{x^2+x-2}-\frac{3}{x^2+2x-3}-\frac{x}{x^2+5x+6}$

$x^2+x-2=(x+2)(x-1)$

$x^2+2x-3=(x+3)(x-1)$

$x^2+5x+6=(x+3)(x+2)$

$mcm=(x+2)(x-1)(x+3)$

$=\frac{x(x+3)-3(x+2)-x(x-1)}{(x+2)(x-1)(x+3)}$

$=\frac{x^2+3x-3x-6-x^2+x}{(x+2)(x-1)(x+3)}=\frac{x-6}{(x+2)(x-1)(x+3)}$

25. $\frac{3}{x^2+x+1}-\frac{x+2}{(x-1)^2}-\frac{1-9x}{\left(x^3-1\right)(x-1)}$ $\quad mcm=\left(x^3-1\right)(x-1)$

$=\frac{3(x-1)^2-(x+2)\left(x^2+x+1\right)-(1-9x)}{\left(x^3-1\right)(x-1)}$

$=\frac{3x^2-6x+3-x^3-x^2-x-2x^2-2x-2-1+9x}{\left(x^3-1\right)(x-1)}$

$=-\frac{x^3}{\left(x^3-1\right)(x-1)}$

26. $\frac{a^2+b^2}{a^3-b^3}-\frac{a+b}{2a^2+2ab+2b^2}-\frac{1}{2a-2b}$

$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

$2a^2+2ab+2b^2=2\left(a^2+ab+b^2\right)$

$2a-2b=2(a-b)$

$mcm=2(a-b)\left(a^2+ab+b^2\right)$

$=\frac{2\left(a^2+b^2\right)-(a+b)(a-b)-\left(a^2+ab+b^2\right)}{2(a-b)\left(a^2+ab+b^2\right)}$

$=\frac{2a^2+2b^2-a^2+b^2-a^2-ab-b^2}{2(a-b)\left(a^2+ab+b^2\right)}$

$=\frac{2b^2-ab}{2(a-b)\left(a^2+ab+b^2\right)}=\frac{2b^2-ab}{2\left(a^3-b^3\right)}$

27. $\frac{3a}{2a^2-2a-4}-\frac{a-1}{4a^2+8a-32}-\frac{10a-1}{8a^2+40a+32}$

$2a^2-2a-4=2\left(a^2-a-2\right)=2(a-2)(a+1)$

$4a^2+8a-32=4\left(a^2+2a-8\right)=4(a+4)(a-2)$

$8a^2+40a+32=8\left(a^2+5a+4\right)=8(a+4)(a+1)$

$mcm=8(a+4)(a-2)(a+1)$

$=\frac{12a(a+4)-2(a-1)(a+1)-(10a-1)(a-2)}{8(a+4)(a-2)(a+1)}$

$=\frac{12a^2+48a-2a^2+2-10a^2+20a+a-2}{8(a+4)(a-2)(a+1)}$

$=\frac{69a}{8(a+4)(a-2)(a+1)}$

28. $\frac{1}{4a-12x}-\frac{a^2+9x^2}{a^3-27x^3}-\frac{a}{2\left(a^2+3ax+9a^2\right)}$

$4a-12x=4\left(a-3x\right)$

$a^3-27x^3=\left(a-3x\right)\left(a^2+3ax+9a^2\right)$

$2\left(a^2+3ax+9a^2\right)=2\left(a^2+3ax+9a^2\right)$

$mcm=4\left(a^3-27x^3\right)$

$=\frac{a^2+3ax+9x^2-4\left(a^2+9x^2\right)-2a\left(a-3x\right)}{4\left(a^3-27x^3\right)}$

$=\frac{a^2+3ax+9x^2-4a^2-36x^2-2a^2+6ax}{4\left(a^3-27x^3\right)}$

$=\frac{-5a^2+9ax-27x^2}{4\left(a^3-27x^3\right)}=-\frac{5a^2-9ax+27x^2}{4\left(a^3-27x^3\right)}$

29. $\frac{2a^2-3}{10a+10}-\frac{a+1}{50}-\frac{9a^2-14}{50a+50}$

$10a+10=10\left(a+1\right)$

$50=50$

$50a+50=50\left(a+1\right)$

$mcm=50\left(a+1\right)$

$=\frac{5\left(2a^2-3\right)-\left(a+1\right)\left(a+1\right)-\left(9a^2-14\right)}{50\left(a+1\right)}$

$=\frac{10a^2-15-a^2-2a-1-9a^2+14}{50\left(a+1\right)}$

$=\frac{-2a-2}{50\left(a+1\right)}=-\frac{2\left(a+1\right)}{50\left(a+1\right)}=-\frac{1}{25}$

EJERCICIO 130

1. $\frac{2}{x-3}+\frac{3}{x+2}-\frac{4x-7}{x^2-x-6}\qquad mcm=\left(x-3\right)\left(x+2\right)$

$=\frac{2\left(x+2\right)+3\left(x-3\right)-\left(4x-7\right)}{\left(x-3\right)\left(x+2\right)}$

$=\frac{2x+4+3x-9-4x+7}{\left(x-3\right)\left(x+2\right)}=\frac{x+2}{\left(x-3\right)\left(x+2\right)}=\frac{1}{x-3}$

2. $\frac{a}{3a+6}-\frac{1}{6a+12}+\frac{a+12}{12a+24}\qquad mcm=12\left(a+2\right)$

$=\frac{4a-2+a+12}{12\left(a+2\right)}=\frac{5a+10}{12\left(a+2\right)}=\frac{5\left(a+2\right)}{12\left(a+2\right)}=\frac{5}{12}$

3. $\frac{x}{x^2+1}+\frac{1}{3x}-\frac{1}{x^2}\qquad mcm=3x^2\left(x^2+1\right)$

$=\frac{3x^3+x\left(x^2+1\right)-3\left(x^2+1\right)}{3x^2\left(x^2+1\right)}$

$=\frac{3x^3+x^3+x-3x^2-3}{3x^2\left(x^2+1\right)}=\frac{4x^3+x-3x^2-3}{3x^2\left(x^2+1\right)}$

4. $\frac{a+3}{a^2-1}+\frac{a-1}{2a+2}+\frac{a-4}{4a-4}\qquad mcm=4\left(a^2-1\right)$

$=\frac{4\left(a+3\right)+2\left(a-1\right)\left(a-1\right)+\left(a-4\right)\left(a+1\right)}{4\left(a^2-1\right)}$

$=\frac{4a+12+2a^2-4a+2+a^2-3a-4}{4\left(a^2-1\right)}=\frac{3a^2-3a+10}{4\left(a^2-1\right)}$

5. $\frac{a-b}{a^2+ab}+\frac{a+b}{ab}-\frac{a}{ab+b^2}\qquad mcm=ab\left(a+b\right)$

$=\frac{b\left(a-b\right)+\left(a+b\right)\left(a+b\right)-a^2}{ab\left(a+b\right)}$

$=\frac{ab-b^2+a^2+2ab+b^2-a^2}{ab\left(a+b\right)}=\frac{3ab}{ab\left(a+b\right)}=\frac{3}{a+b}$

6. $\frac{x-y}{x+y}-\frac{x+y}{x-y}+\frac{4x^2}{x^2-y^2}\qquad mcm=x^2-y^2$

$=\frac{\left(x-y\right)\left(x-y\right)-\left(x+y\right)\left(x+y\right)+4x^2}{x^2-y^2}$

$=\frac{x^2-2xy+y^2-x^2-2xy-y^2+4x^2}{x^2-y^2}$

$=\frac{4x^2-4xy}{x^2-y^2}=\frac{4x\left(x-y\right)}{\left(x+y\right)\left(x-y\right)}=\frac{4x}{x+y}$

7. $\frac{x}{a^2-ax}+\frac{1}{a}+\frac{1}{x}\qquad mcm=ax\left(a-x\right)$

$=\frac{x^2+x\left(a-x\right)+a\left(a-x\right)}{ax\left(a-x\right)}$

$=\frac{x^2+ax-x^2+a^2-ax}{ax\left(a-x\right)}=\frac{a^2}{ax\left(a-x\right)}=\frac{a}{x\left(a-x\right)}$

8. $\frac{x+1}{x^2-x-20}-\frac{x+4}{x^2-4x-5}+\frac{x+5}{x^2+5x+4}$

$x^2-x-20=(x-5)(x+4)$

$x^2-4x-5=(x-5)(x+1)$

$x^2+5x+4=(x+4)(x+1)$

$mcm=(x-5)(x+4)(x+1)$

$=\frac{(x+1)(x+1)-(x+4)(x+4)+(x-5)(x+5)}{(x-5)(x+4)(x+1)}$

$=\frac{x^2+2x+1-x^2-8x-16+x^2-25}{(x-5)(x+4)(x+1)}$

$=\frac{x^2-6x-40}{(x-5)(x+4)(x+1)}$

$=\frac{(x-10)(x+4)}{(x-5)(x+4)(x+1)}=\frac{x-10}{(x-5)(x+1)}$

9. $\frac{2x+1}{12x+8}-\frac{x^2}{6x^2+x-2}+\frac{2x}{16x-8}$

$12x+8=4(3x+2)$

$6x^2+x-2=(3x+2)(2x-1)$

$16x-8=8(2x-1)$

$mcm=8(3x+2)(2x-1)$

$=\frac{2(2x+1)(2x-1)-8x^2+2x(3x+2)}{8(3x+2)(2x-1)}$

$=\frac{8x^2-2-8x^2+6x^2+4x}{8(3x+2)(2x-1)}$

$=\frac{6x^2+4x-2}{8(3x+2)(2x-1)}$

$=\frac{2(3x^2+2x-1)}{8(3x+2)(2x-1)}=\frac{3x^2+2x-1}{4(3x+2)(2x-1)}$

10. $\frac{1}{ax}-\frac{1}{a^2+ax}+\frac{1}{a+x}$ $\quad mcm=ax(a+x)$

$=\frac{a+x-x+ax}{ax(a+x)}=\frac{ax+a}{ax(a+x)}=\frac{a(x+1)}{ax(a+x)}=\frac{x+1}{x(a+x)}$

11. $\frac{1}{x+y}-\frac{1}{x-y}+\frac{2y}{x^2+y^2}$ $\quad mcm=(x^2-y^2)(x^2+y^2)$

$=\frac{(x-y)(x^2+y^2)-(x+y)(x^2+y^2)+2y(x^2-y^2)}{(x^2-y^2)(x^2+y^2)}$

$=\frac{x^3+xy^2-x^2y-y^3-x^3-xy^2-x^2y-y^3+2x^2y-2y^3}{(x^2-y^2)(x^2+y^2)}$

$=\frac{-4y^3}{x^4-y^4}=\frac{4y^3}{y^4-x^4}$

12. $\frac{a-1}{3a+3}-\frac{a-2}{6a-6}+\frac{a^2+2a-6}{9a^2-9}$ $\quad mcm=18(a^2-1)$

$=\frac{6(a-1)(a-1)-3(a-2)(a+1)+2(a^2+2a-6)}{18(a^2-1)}$

$=\frac{6a^2-12a+6-3a^2+3a+6+2a^2+4a-12}{18(a^2-1)}$

$=\frac{5a^2-5a}{18(a^2-1)}=\frac{5a(a-1)}{18(a+1)(a-1)}=\frac{5a}{18(a+1)}$

13. $\frac{1}{a^2+2a-24}+\frac{2}{a^2-2a-8}-\frac{3}{a^2+8a+12}$

$a^2+2a-24=(a+6)(a-4)$

$a^2-2a-8=(a-4)(a+2)$

$a^2+8a+12=(a+6)(a+2)$

$mcm=(a+6)(a+2)(a-4)$

$=\frac{a+2+2(a+6)-3(a-4)}{(a+6)(a+2)(a-4)}$

$=\frac{a+2+2a+12-3a+12}{(a+6)(a+2)(a-4)}=\frac{26}{(a+6)(a+2)(a-4)}$

14. $\frac{x+y}{xy}-\frac{x+2y}{xy+y^2}-\frac{y}{x^2+xy}$ $\quad mcm=xy(x+y)$

$=\frac{(x+y)(x+y)-x(x+2y)-y^2}{xy(x+y)}$

$=\frac{x^2+2xy+y^2-x^2-2xy-y^2}{xy(x+y)}=\frac{0}{xy(x+y)}=0$

15. $\frac{a^3}{a^3+1}+\frac{a+3}{a^2-a+1}-\frac{a-1}{a+1}$ $\quad mcm=a^3+1$

$=\frac{a^3+(a+1)(a+3)-(a-1)(a^2-a+1)}{a^3+1}$

$=\frac{a^3+a^2+4a+3-a^3+a^2-a+a^2-a+1}{a^3+1}=\frac{3a^2+2a+4}{a^3+1}$

16. $\frac{1}{x-1}+\frac{2x}{x^2-1}-\frac{3x^2}{x^3-1}$ $\quad mcm=(x^2-1)(x^2+x+1)$

$=\frac{(x+1)(x^2+x+1)+2x(x^2+x+1)-3x^2(x+1)}{(x^2-1)(x^2+x+1)}$

$=\frac{x^3+x^2+x+x^2+x+1+2x^3+2x^2+2x-3x^3-3x^2}{(x^2-1)(x^2+x+1)}$

$=\frac{x^2+4x+1}{(x^2-1)(x^2+x+1)}$

17. $\dfrac{a+b}{a^2-ab+b^2}-\dfrac{1}{a+b}+\dfrac{3a^2}{a^3+b^3} \qquad mcm=a^3+b^3$

$=\dfrac{(a+b)(a+b)-(a^2-ab+b^2)+3a^2}{a^3+b^3}$

$=\dfrac{a^2+2ab+b^2-a^2+ab-b^2+3a^2}{a^3+b^3}$

$=\dfrac{3a^2+3ab}{a^3+b^3}=\dfrac{3a(a+b)}{(a+b)(a^2-ab+b^2)}=\dfrac{3a}{a^2-ab+b^2}$

18. $\dfrac{2}{x-2}+\dfrac{2x+3}{x^2+2x+4}-\dfrac{6x+12}{x^3-8} \qquad mcm=x^3-8$

$=\dfrac{2(x^2+2x+4)+(2x+3)(x-2)-(6x+12)}{x^3-8}$

$=\dfrac{2x^2+4x+8+2x^2-4x+3x-6-6x-12}{x^3-8}$

$=\dfrac{4x^2-3x-10}{x^3-8}=\dfrac{(x-2)(4x+5)}{x^3-8}=\dfrac{4x+5}{x^2+2x+4}$

19. $\dfrac{3x+2}{x^2+3x-10}-\dfrac{5x+1}{x^2+4x-5}+\dfrac{4x-1}{x^2-3x+2}$

$x^2+3x-10=(x+5)(x-2)\ ;\ \ x^2+4x-5=(x+5)(x-1)$

$x^2-3x+2=(x-2)(x-1); \qquad mcm=(x+5)(x-2)(x-1)$

$=\dfrac{(3x+2)(x-1)-(5x+1)(x-2)+(4x-1)(x+5)}{(x+5)(x-2)(x-1)}$

$=\dfrac{3x^2-3x+2x-2-5x^2+10x-x+2+4x^2+20x-x-5}{(x+5)(x-2)(x-1)}$

$=\dfrac{2x^2+27x-5}{(x+5)(x-2)(x-1)}$

20. $\dfrac{1}{(n+1)^2}+\dfrac{1}{n-1}-\dfrac{1}{(n-1)^3}-\dfrac{1}{n} \qquad mcm=n(n-1)^3$

$=\dfrac{n(n-1)+n(n-1)^2-n-(n-1)^3}{n(n-1)^3}$

$=\dfrac{n^2-n+n^3-2n^2+n-n-n^3+3n^2-3n+1}{n(n-1)^3}=\dfrac{2n^2-4n+1}{n(n-1)^3}$

21. $\dfrac{1}{a^2+5}-\dfrac{a^2-5}{(a^2+5)^2}+\dfrac{a^2+5}{a^4-25} \qquad mcm=(a^4-25)(a^2+5)$

$=\dfrac{a^4-25-(a^2-5)(a^2-5)+(a^2+5)(a^2+5)}{(a^4-25)(a^2+5)}$

$=\dfrac{a^4-25-a^4+10a^2-25+a^4+10a^2+25}{(a^4-25)(a^2+5)}=\dfrac{a^4+20a^2-25}{(a^4-25)(a^2+5)}$

22. $\dfrac{1-x^2}{9-x^2}-\dfrac{x^2}{9+6x+x^2}-\dfrac{6x}{9-6x+x^2}$

$9-x^2=(3+x)(3-x) \quad ; \qquad 9+6x+x^2=(3+x)^2$

$9-6x+x^2=(3-x)^2 \quad ; \qquad mcm=(3+x)^2(3-x)^2$

$=\dfrac{(1-x^2)(9-x^2)-x^2(3-x)^2-6x(3+x)^2}{(3+x)^2(3-x)^2}$

$=\dfrac{9-x^2-9x^2+x^4-9x^2+6x^3-x^4-54x-36x^2-6x^3}{(3+x)^2(3-x)^2}$

$=\dfrac{9-54x-55x^2}{(3+x)^2(3-x)^2}$

23. $\dfrac{x}{2x+2}-\dfrac{x+1}{3x-3}+\dfrac{x-1}{6x+6}-\dfrac{5}{18x-18} \qquad mcm=18(x^2-1)$

$=\dfrac{9x(x-1)-6(x+1)(x+1)+3(x-1)(x-1)-5(x+1)}{18(x^2-1)}$

$=\dfrac{9x^2-9x-6x^2-12x-6+3x^2-6x+3-5x-5}{18(x^2-1)}$

$=\dfrac{6x^2-32x-8}{18(x^2-1)}=\dfrac{2(3x^2-16x-4)}{18(x^2-1)}=\dfrac{3x^2-16x-4}{9(x^2-1)}$

24. $\dfrac{a+2}{2a+2}-\dfrac{7a}{8a^2-8}-\dfrac{a-3}{4a-4} \qquad mcm=8(a^2-1)$

$=\dfrac{4(a+2)(a-1)-7a-2(a-3)(a+1)}{8(a^2-1)}$

$=\dfrac{4a^2+4a-8-7a-2a^2+4a+6}{8(a^2-1)}=\dfrac{2a^2+a-2}{8(a^2-1)}$

25. $\dfrac{a-3}{20a+10}+\dfrac{2a+5}{40a+20}-\dfrac{4a-1}{60a+30} \qquad mcm=120(2a+1)$

$=\dfrac{12(a-3)+6(2a+5)-4(4a-1)}{120(2a+1)}$

$=\dfrac{12a-36+12a+30-16a+4}{120(2a+1)}$

$=\dfrac{8a-2}{120(2a+1)}=\dfrac{2(4a-1)}{120(2a+1)}=\dfrac{4a-1}{60(2a+1)}$

26. $\dfrac{2}{2x^2+5x+3}-\dfrac{1}{2x^2-x-6}+\dfrac{3}{x^2-x-2}$

$2x^2+5x+3=(2x+3)(x+1)$

$2x^2-x-6=(2x+3)(x-2)$

$x^2-x-2=(x-2)(x+1)$

$mcm=(2x+3)(x+1)(x-2)$

$=\dfrac{2(x-2)-(x+1)+3(2x+3)}{(2x+3)(x+1)(x-2)}$

$=\dfrac{2x-4-x-1+6x+9}{(2x+3)(x+1)(x-2)}=\dfrac{7x+4}{(2x+3)(x+1)(x-2)}$

27. $\frac{a-1}{a-2}-\frac{a-2}{a+3}+\frac{1}{a-1}\qquad mcm=(a-2)(a+3)(a-1)$

$$=\frac{(a-1)^2(a+3)-(a-2)^2(a-1)+(a-2)(a+3)}{(a-2)(a+3)(a-1)}$$

$$=\frac{(a^2-2a+1)(a+3)-(a^2-4a+4)(a-1)+a^2+a-6}{(a-2)(a+3)(a-1)}$$

$$=\frac{a^3+3a^2-2a^2-6a+a+3-a^3+a^2+4a^2-4a-4a+4+a^2+a-6}{(a-2)(a+3)(a-1)}$$

$$=\frac{7a^2-12a+1}{(a-2)(a+3)(a-1)}$$

28. $\frac{2+3a}{2-3a}-\frac{2-3a}{2+3a}-\frac{a}{(2-3a)^2}\qquad mcm=(4-9a^2)(2-3a)$

$$=\frac{(2+3a)(4-9a^2)-(2-3a)^3-a(2+3a)}{(4-9a^2)(2-3a)}$$

$$=\frac{8-18a^2+12a-27a^3-8+36a-54a^2+27a^3-2a-3a^2}{(4-9a^2)(2-3a)}$$

$$=\frac{46a-75a^2}{(4-9a^2)(2-3a)}$$

29. $\frac{1}{5+5a}+\frac{1}{5-5a}-\frac{1}{10+10a^2}\qquad mcm=10(1+a^2)(1-a^2)$

$$=\frac{2(1+a^2)(1-a)+2(1+a^2)(1+a)-(1-a^2)}{10(1+a^2)(1-a^2)}$$

$$=\frac{2-2a+2a^2-2a^3+2+2a+2a^2+2a^3-1+a^2}{10(1+a^2)(1-a^2)}$$

$$=\frac{5a^2+3}{10\,(1-a^4)}$$

30. $\frac{1}{3-3x}-\frac{1}{3+3x}+\frac{x}{6+6x^2}-\frac{x}{2-2x^2}$

$3-3x=3(1-x)\quad;\quad 3+3x=3(1+x)$

$6+6x^2=6(1+x^2)\ ;\ 2-2x^2=2(1-x^2)$

$mcm=6(1+x^2)(1-x^2)=6(1-x^4)$

$$=\frac{2(1+x^2)(1+x)-2(1+x^2)(1-x)+x(1-x^2)-3x(1+x^2)}{6(1-x^4)}$$

$$=\frac{2+2x+2x^2+2x^3-2+2x-2x^2+2x^3+x-x^3-3x-3x^3}{6(1-x^4)}$$

$$=\frac{2x}{6(1-x^4)}=\frac{x}{3(1-x^4)}$$

EJERCICIO 131

1. $\frac{1}{m-n}+\frac{m}{n^2-m^2}=\frac{1}{m-n}-\frac{m}{m^2-n^2}\qquad mcm=m^2-n^2$

$$=\frac{m+n-m}{m^2-n^2}=\frac{n}{m^2-n^2}$$

2. $\frac{x^2}{x^2-xy}-\frac{2x}{y-x}=\frac{x^2}{x(x-y)}+\frac{2x}{x-y}=\frac{x}{x-y}+\frac{2x}{x-y}$

$mcm=x-y$

$$=\frac{x+2x}{x-y}=\frac{3x}{x-y}$$

3. $\frac{1}{2x-x^2}+\frac{x}{x^2-4}=\frac{1}{x(2-x)}-\frac{x}{4-x^2}$

$mcm=x(4-x^2)$

$$=\frac{2+x-x^2}{x(4-x^2)}=\frac{(2-x)(1+x)}{x(2-x)(x+2)}=\frac{1+x}{x(2+x)}$$

4. $\frac{a+b}{a^2-ab}+\frac{a}{b^2-a^2}=\frac{a+b}{a(a-b)}-\frac{a}{a^2-b^2}$

$mcm=a(a^2-b^2)$

$$=\frac{(a+b)(a+b)-a^2}{a(a^2-b^2)}=\frac{a^2+2ab+b^2-a^2}{a(a^2-b^2)}=\frac{2ab+b^2}{a(a^2-b^2)}$$

5. $\frac{x-4}{x^2-2x-3}-\frac{x}{6-2x}=\frac{x-4}{(x-3)(x+1)}+\frac{x}{2(x-3)}$

$mcm=2(x-3)(x+1)$

$$=\frac{2(x-4)+x(x+1)}{2(x-3)(x+1)}=\frac{2x-8+x^2+x}{2(x-3)(x+1)}=\frac{x^2+3x-8}{2(x-3)(x+1)}$$

6. $\frac{1}{x^2+2x-8}+\frac{1}{(2-x)(x+3)}=\frac{1}{(x+4)(x-2)}-\frac{1}{(x-2)(x+3)}$

$mcm=(x-2)(x+3)(x+4)$

$$=\frac{x+3-(x+4)}{(x-2)(x+3)(x+4)}$$

$$=\frac{x+3-x-4}{(x-2)(x+3)(x+4)}$$

$$=\frac{-1}{(x-2)(x+3)(x+4)}=\frac{1}{(2-x)(x+3)(x+4)}$$

7. $\frac{1}{2x+2}+\frac{2}{1-x}+\frac{7}{4x-4}=\frac{1}{2(x+1)}-\frac{2}{x-1}+\frac{7}{4(x-1)}$

$mcm=4(x^2-1)$

$$=\frac{2(x-1)-8(x+1)+7(x+1)}{4(x^2-1)}$$

$$=\frac{2x-2-8x-8+7x+7}{4(x^2-1)}=\frac{x-3}{4(x^2-1)}$$

8. $$\frac{2a}{a+3}+\frac{3a}{a-3}+\frac{2a}{9-a^2}=\frac{2a}{a+3}+\frac{3a}{a-3}-\frac{2a}{a^2-9}$$

$$mcm=a^2-9$$

$$=\frac{2a(a-3)+3a(a+3)-2a}{a^2-9}$$

$$=\frac{2a^2-6a+3a^2+9a-2a}{a^2-9}=\frac{5a^2+a}{a^2-9}$$

9. $$\frac{x+3y}{y+x}+\frac{3y^2}{x^2-y^2}-\frac{x}{y-x}=\frac{x+3y}{y+x}+\frac{3y^2}{x^2-y^2}-\frac{x}{x-y}$$

$$mcm=x^2-y^2$$

$$=\frac{(x+3y)(x-y)+3y^2+x(x+y)}{x^2-y^2}$$

$$=\frac{x^2-xy+3xy-3y^2+3y^2+x^2+xy}{x^2-y^2}=\frac{2x^2+3xy}{x^2-y^2}$$

10. $$\frac{x}{x^2+2x-3}+\frac{x-3}{(1-x)(x+2)}+\frac{1}{x+2}$$

$$=\frac{x}{(x+3)(x-1)}-\frac{x-3}{(x-1)(x+2)}+\frac{1}{x+2}$$

$$mcm=(x+3)(x+2)(x-1)$$

$$=\frac{x(x+2)-(x-3)(x+3)+(x+3)(x-1)}{(x+3)(x+2)(x-1)}$$

$$=\frac{x^2+2x-x^2+9+x^2+2x-3}{(x+3)(x+2)(x-1)}=\frac{x^2+4x+6}{(x+3)(x+2)(x-1)}$$

11. $$\frac{3}{2a+2}-\frac{1}{4a-4}-\frac{4}{8-8a^2}=\frac{3}{2(a+1)}-\frac{1}{4(a-1)}+\frac{4}{8(a^2-1)}$$

$$mcm=8(a^2-1)$$

$$=\frac{12(a-1)-2(a+1)+4}{8(a^2-1)}$$

$$=\frac{12a-12-2a-2+4}{8(a^2-1)}=\frac{10a-10}{8(a^2-1)}=\frac{10(a-1)}{8(a-1)(a+1)}=\frac{5}{4(a+1)}$$

12. $$\frac{1}{a-3}+\frac{a+1}{(3-a)(a-2)}+\frac{2}{(2-a)(1-a)}$$

$$=\frac{1}{a-3}-\frac{a+1}{(a-3)(a-2)}+\frac{2}{(a-2)(a-1)}$$

$$mcm=(a-3)(a-2)(a-1)$$

$$=\frac{(a-2)(a-1)-(a+1)(a-1)+2(a-3)}{(a-3)(a-2)(a-1)}$$

$$=\frac{a^2-3a+2-a^2+1+2a-6}{(a-3)(a-2)(a-1)}$$

$$=\frac{-a-3}{(a-3)(a-2)(a-1)}=\frac{a+3}{(3-a)(a-2)(a-1)}$$

13. $$\frac{2x}{x-1}+\frac{2x^3+2x^2}{1-x^3}+\frac{1}{x^2+x+1}$$

$$=\frac{2x}{x-1}-\frac{2x^2(x+1)}{x^3-1}+\frac{1}{x^2+x+1}$$

$$mcm=x^3-1$$

$$=\frac{2x(x^2+x+1)-2x^2(x+1)+(x-1)}{x^3-1}$$

$$=\frac{2x^3+2x^2+2x-2x^3-2x^2+x-1}{x^3-1}=\frac{3x-1}{x^3-1}$$

14. $$\frac{x+2}{3x-1}+\frac{x+1}{3-2x}+\frac{4x^2+6x+3}{6x^2-11x+3}$$

$$=\frac{x+2}{3x-1}-\frac{x+1}{2x-3}+\frac{4x^2+6x+3}{(2x-3)(3x-1)}$$

$$mcm=(2x-3)(3x-1)$$

$$=\frac{(x+2)(2x-3)-(x+1)(3x-1)+4x^2+6x+3}{(2x-3)(3x-1)}$$

$$=\frac{2x^2-3x+4x-6-3x^2+x-3x+1+4x^2+6x+3}{(2x-3)(3x-1)}$$

$$=\frac{3x^2+5x-2}{(2x-3)(3x-1)}=\frac{(x+2)(3x-1)}{(2x-3)(3x-1)}=\frac{x+2}{2x-3}$$

EJERCICIO 132

1. $$\frac{2a^2}{3b}\cdot\frac{6b^2}{4a}=\frac{12a^2b^2}{12ab}=ab$$

2. $$\frac{x^2y}{5}\cdot\frac{10a^3}{3m^2}\cdot\frac{9m}{x^3}=\frac{90x^2ya^3m}{15x^3m^2}=\frac{6a^3y}{xm}$$

3. $$\frac{5x^2}{7y^3}\cdot\frac{4y^2}{7m^3}\cdot\frac{14m}{5x^4}=\frac{14\cdot5\cdot4x^2my^2}{49\cdot5x^4m^3y^3}=\frac{8}{7x^2m^2y}$$

4. $$\frac{5}{a}\cdot\frac{2a}{b^2}\cdot\frac{3b}{10}=\frac{10\cdot3ab}{10ab^2}=\frac{3}{b}$$

5. $$\frac{2x^3}{15a^3}\cdot\frac{3a^2}{y}\cdot\frac{5x^2}{7xy^2}=\frac{15\cdot2a^2x^5}{15\cdot7a^3xy^3}=\frac{2x^4}{7ay^3}$$

6. $$\frac{7a}{6m^2}\cdot\frac{3m}{10n^2}\cdot\frac{5n^4}{14ax}=\frac{7\cdot3\cdot5amn^4}{14\cdot6\cdot10am^2n^2x}=\frac{n^2}{8mx}$$

7. $$\frac{2x^2+x}{6}+\frac{8}{4x+2}=\frac{x(2x+1)}{3}\cdot\frac{4}{2(2x+1)}=\frac{2x}{3}$$

8. $$\frac{5x+25}{14}\cdot\frac{7x+7}{10x+50}=\frac{5(x+5)}{14}\cdot\frac{7(x+1)}{10(x+5)}=\frac{x+1}{4}$$

9. $\frac{m+n}{mn-n^2}\cdot\frac{n^2}{m^2-n^2}$

$=\frac{m+n}{n(m-n)}\cdot\frac{n^2}{(m+n)(m-n)}=\frac{n}{(m-n)^2}=\frac{n}{m^2-2mn+n^2}$

10. $\frac{xy-2y^2}{x^2+xy}\cdot\frac{x^2+2xy+y^2}{x^2-2xy}$

$=\frac{y(x-2y)}{x(x+y)}\cdot\frac{(x+y)^2}{x(x-2y)}=\frac{y(x+y)}{x^2}=\frac{xy+y^2}{x^2}$

11. $\frac{x^2-4xy+4y^2}{x^2+2xy}\cdot\frac{x^2}{x^2-4y^2}$

$=\frac{(x-2y)^2}{x(x+2y)}\cdot\frac{x^2}{(x+2y)(x-2y)}=\frac{x(x-2y)}{(x+2y)^2}=\frac{x^2-2xy}{x^2+4xy+y^2}$

12. $\frac{2x^2+2x}{2x^2}\cdot\frac{x^2-3x}{x^2-2x-3}=\frac{2x(x+1)}{2x^2}\cdot\frac{x(x-3)}{(x-3)(x+1)}=1$

13. $\frac{a^2-ab+a-b}{a^2+2a+1}\cdot\frac{3}{6a^2-6ab}$

$=\frac{(a+1)(a-b)}{(a+1)(a+1)}\cdot\frac{3}{6a(a-b)}=\frac{1}{2a(a+1)}=\frac{1}{2a^2+2a}$

14. $\frac{(x-y)^3}{x^3-1}\cdot\frac{x^2+x+1}{(x-y)^2}=\frac{(x-y)}{(x-1)(x^2+x+1)}\cdot x^2+x+1=\frac{x-y}{x-1}$

15. $\frac{2a-2}{2a^2-50}\cdot\frac{a^2-4a-5}{3a+3}=\frac{2(a-1)}{2(a^2-25)}\cdot\frac{(a-5)(a+1)}{3(a+1)}$

$=\frac{(a-1)(a-5)}{3(a+5)(a-5)}=\frac{a-1}{3(a+5)}=\frac{a-1}{3a+15}$

16. $\frac{2x^2-3x-2}{6x+3}\cdot\frac{3x+6}{x^2-4}=\frac{(x-2)(2x+1)}{3(2x+1)}\cdot\frac{3(x+2)}{(x-2)(x+2)}=1$

17. $\frac{y^2+9y+18}{y-5}\cdot\frac{5y-25}{5y+15}=\frac{(y+6)(y+3)}{y-5}\cdot\frac{5(y-5)}{5(y+3)}=y+6$

18. $\frac{x^3+2x^2-3x}{4x^2+8x+3}\cdot\frac{2x^2+3x}{x^2-x}$

$=\frac{x(x^2+2x-3)}{(2x+3)(2x+1)}\cdot\frac{x(2x+3)}{x(x-1)}$

$=\frac{x(x+3)(x-1)}{(2x+1)(x-1)}=\frac{x(x+3)}{2x+1}=\frac{x^2+3x}{2x+1}$

19. $\frac{x^3-27}{a^3-1}\cdot\frac{a^2+a+1}{x^2+3x+9}$

$=\frac{(x-3)(x^2+3x+9)}{(a-1)(a^2+a+1)}\cdot\frac{a^2+a+1}{x^2+3x+9}=\frac{x-3}{a-1}$

20. $\frac{a^2+4ab+4b^2}{3}\cdot\frac{2a+4b}{(a+2b)^3}=\frac{(a+2b)^2}{3}\cdot\frac{2(a+2b)}{(a+2b)^3}=\frac{2}{3}$

21. $\frac{1-x}{a+1}\cdot\frac{a^2+a}{x-x^2}\cdot\frac{x^2}{a}=\frac{1-x}{a+1}\cdot\frac{a(a+1)}{x(1-x)}\cdot\frac{x^2}{a}=x$

22. $\frac{x^2+2x}{x^2-16}\cdot\frac{x^2-2x-8}{x^3+x^2}\cdot\frac{x^2+4x}{x^2+4x+4}$

$=\frac{x(x+2)}{(x-4)(x+4)}\cdot\frac{(x-4)(x+2)}{x^2(x+1)}\cdot\frac{x(x+4)}{(x+2)(x+2)}$

$=\frac{x^2(x+2)^2(x-4)(x+4)}{x^2(x+2)^2(x-4)(x+4)(x+1)}=\frac{1}{x+1}$

23. $\frac{(m+n)^2-x^2}{(m+x)^2-n^2}\cdot\frac{(m-n)^2-x^2}{m^2+mn-mx}$

$=\frac{(m+n+x)(m+n-x)}{(m+n+x)(m-n+x)}\cdot\frac{(m-n+x)(m-n-x)}{m(m+n-x)}$

$=\frac{m-n-x}{m}$

24. $\frac{2a^3+2ab^2}{2ax^2-2ax}\cdot\frac{x^3-x}{a^2x+b^2x}\cdot\frac{x}{x+1}$

$=\frac{2a(a^2+b^2)}{2ax(x-1)}\cdot\frac{x(x^2-1)}{x(a^2+b^2)}\cdot\frac{x}{x+1}=1$

25. $\frac{a^2-5a+6}{3a-15}\cdot\frac{6a}{a^2-a-30}\cdot\frac{a^2-25}{2a-4}$

$=\frac{(a-3)(a-2)}{3(a-5)}\cdot\frac{6a}{(a-6)(a+5)}\cdot\frac{(a+5)(a-5)}{2(a-2)}$

$=\frac{a(a-3)}{a-6}=\frac{a^2-3a}{a-6}$

26. $\frac{x^2-3xy-10y^2}{x^2-2xy-8y^2}\cdot\frac{x^2-16y^2}{x^2+4xy}\cdot\frac{x^2-6xy}{x+2y}$

$=\frac{(x-5y)(x+2y)}{(x-4y)(x+2y)}\cdot\frac{(x+4y)(x-4y)}{x(x+4y)}\cdot\frac{x(x-6y)}{x+2y}$

$=\frac{(x-5y)(x-6y)}{x+2y}=\frac{x^2-11xy+30y^2}{x+2y}$

27. $\frac{x^2+4ax+4a^2}{3ax-6a^2}\cdot\frac{2ax-4a^2}{ax+a}\cdot\frac{6a+6x}{x^2+3ax+2a^2}$

$=\frac{(x+2a)^2}{3a(x-2a)}\cdot\frac{2a(x-2a)}{a(x+1)}\cdot\frac{6(a+x)}{(x+2a)(x+a)}$

$=\frac{4(x+2a)}{a(x+1)}=\frac{4x+8a}{ax+a}$

28. $\frac{a^2-81}{2a^2+10a}\cdot\frac{a+11}{a^2-36}\cdot\frac{2a-12}{2a+18}\cdot\frac{a^3+5a^2}{2a+22}$

$=\frac{(a+9)(a-9)}{2a(a+5)}\cdot\frac{a+11}{(a+6)(a-6)}\cdot\frac{2(a-6)}{2(a+9)}\cdot\frac{a^2(a+5)}{2(a+11)}$

$=\frac{a(a-9)}{4(a+6)}=\frac{a^2-9a}{4a+24}$

29. $\dfrac{a^2+7a+10}{a^2-6a-7}\cdot\dfrac{a^2-3a-4}{a^2+2a-15}\cdot\dfrac{a^3-2a^2-3a}{a^2-2a-8}$

$=\dfrac{(a+5)(a+2)}{(a-7)(a+1)}\cdot\dfrac{(a-4)(a+1)}{(a+5)(a-3)}\cdot\dfrac{a(a^2-2a-3)}{(a-4)(a+2)}$

$=\dfrac{a(a-3)(a+1)}{(a-3)(a-7)}=\dfrac{a(a+1)}{a-7}=\dfrac{a^2+a}{a-7}$

30. $\dfrac{x^4+27x}{x^3-x^2+x}\cdot\dfrac{x^4+x}{x^4-3x^3+9x^2}\cdot\dfrac{1}{x(x+3)^2}\cdot\dfrac{x^2}{x-3}$

$=\dfrac{x(x^3+27)}{x(x^2-x+1)}\cdot\dfrac{x(x^3+1)}{x^2(x^2-3x+9)}\cdot\dfrac{x}{(x+3)^2(x-3)}$

$=\dfrac{(x+3)(x^2-3x+9)}{x^2-x+1}\cdot\dfrac{(x+1)(x^2-x+1)}{x^2-3x+9}\cdot\dfrac{1}{(x^2-9)(x+3)}$

$=\dfrac{x+1}{x^2-9}$

EJERCICIO 133

1. $\left(a+\dfrac{a}{b}\right)\left(a-\dfrac{a}{b+1}\right)$

$\Rightarrow a+\dfrac{a}{b}=\dfrac{ab+a}{b}=\dfrac{a(b+1)}{b}$

$\Rightarrow a-\dfrac{a}{b+1}=\dfrac{a(b+1)-a}{b+1}=\dfrac{ab+a-a}{b+1}=\dfrac{ab}{b+1}$

$\Rightarrow \dfrac{a(b+1)}{b}\cdot\dfrac{ab}{b+1}=a^2$

2. $\left(x-\dfrac{2}{x+1}\right)\left(x+\dfrac{1}{x+2}\right)$

$\Rightarrow x-\dfrac{2}{x+1}=\dfrac{x(x+1)-2}{x+1}=\dfrac{x^2+x-2}{x+1}=\dfrac{(x+2)(x-1)}{x+1}$

$\Rightarrow x+\dfrac{1}{x+2}=\dfrac{x(x+2)+1}{x+2}=\dfrac{x^2+2x+1}{x+2}=\dfrac{(x+1)(x+1)}{x+2}$

$\Rightarrow \dfrac{(x+2)(x-1)}{x+1}\cdot\dfrac{(x+1)^2}{x+2}=(x-1)(x+1)=x^2-1$

3. $\left(1-\dfrac{x}{a+x}\right)\left(1+\dfrac{x}{a}\right)$

$\Rightarrow 1-\dfrac{x}{a+x}=\dfrac{a+x-x}{a+x}=\dfrac{a}{a+x}$

$\Rightarrow 1+\dfrac{x}{a}=\dfrac{a+x}{a}$

$\Rightarrow \dfrac{a}{a+x}\cdot\dfrac{a+x}{a}=1$

4. $\left(a+\dfrac{ab}{a-b}\right)\left(1-\dfrac{b^2}{a^2}\right)$

$\Rightarrow a+\dfrac{ab}{a-b}=\dfrac{a(a-b)+ab}{a-b}=\dfrac{a^2-ab+ab}{a-b}=\dfrac{a^2}{a-b}$

$\Rightarrow 1-\dfrac{b^2}{a^2}=\dfrac{a^2-b^2}{a^2}$

$\Rightarrow \dfrac{a^2}{a-b}\cdot\dfrac{a^2-b^2}{a^2}=\dfrac{(a+b)(a-b)}{(a-b)}=a+b$

5. $\left(x+2-\dfrac{12}{x+1}\right)\left(x-2+\dfrac{10-3x}{x+5}\right)$

$\Rightarrow x+2-\dfrac{12}{x+1}$

$=\dfrac{(x+2)(x+1)-12}{x+1}$

$=\dfrac{x^2+3x+2-12}{x+1}=\dfrac{x^2+3x-10}{x+1}=\dfrac{(x+5)(x-2)}{x+1}$

$\Rightarrow x-2+\dfrac{10-3x}{x+5}$

$=\dfrac{(x-2)(x+5)+10-3x}{x+5}=\dfrac{x^2+3x-10+10-3x}{x+5}=\dfrac{x^2}{x+5}$

$\Rightarrow \dfrac{(x+5)(x-2)}{x+1}\cdot\dfrac{x^2}{x+5}=\dfrac{x^2(x-2)}{x+1}=\dfrac{x^3-2x^2}{x+1}$

6. $\left(1+\dfrac{x}{y}\right)\left(x-\dfrac{x^2}{x+y}\right)$

$\Rightarrow 1+\dfrac{x}{y}=\dfrac{y+x}{y}$

$\Rightarrow x-\dfrac{x^2}{x+y}=\dfrac{x(x+y)-x^2}{x+y}=\dfrac{x^2+xy-x^2}{x+y}=\dfrac{xy}{x+y}$

$\Rightarrow \dfrac{y+x}{y}\cdot\dfrac{xy}{x+y}=x$

7. $\left(a+x-\dfrac{ax+x^2}{a+2x}\right)\left(1+\dfrac{x}{a+x}\right)$

$\Rightarrow a+x-\dfrac{ax+x^2}{a+2x}$

$=\dfrac{(a+x)(a+2x)-(ax+x^2)}{a+2x}$

$=\dfrac{a^2+3ax+2x^2-ax-x^2}{a+2x}=\dfrac{a^2+2ax+x^2}{a+2x}=\dfrac{(a+x)^2}{a+2x}$

$\Rightarrow 1+\dfrac{x}{a+x}=\dfrac{a+x+x}{a+x}=\dfrac{a+2x}{a+x}$

$\Rightarrow \dfrac{(a+x)^2}{a+2x}\cdot\dfrac{a+2x}{a+x}=a+x$

8. $\left(x-\frac{x^3-6x}{x^2-25}\right)\left(x+1-\frac{8}{x+3}\right)$

$$\Rightarrow x-\frac{x^3-6x}{x^2-25}=\frac{x\left(x^2-25\right)-\left(x^3-6x\right)}{x^2-25}$$

$$=\frac{x^3-25x-x^3+6x}{x^2-25}=\frac{-19x}{x^2-25}=\frac{19x}{25-x^2}$$

$$\Rightarrow x+1-\frac{8}{x+3}=\frac{(x+1)(x+3)-8}{x+3}$$

$$=\frac{x^2+4x+3-8}{x+3}=\frac{x^2+4x-5}{x+3}=\frac{(x+5)(x-1)}{x+3}$$

$$\Rightarrow \frac{19x}{25-x^2}\cdot\frac{(x+5)(x-1)}{x+3}$$

$$=\frac{19x(x-1)}{(5-x)(x+3)}=\frac{19x^2-19x}{5x+15-x^2-3x}=\frac{19x^2-19x}{15+2x-x^2}$$

10. $\left(a+2x-\frac{14x^2}{2a+x}\right)\left(a-x+\frac{a^2+5x^2}{a+4x}\right)$

$$\Rightarrow a+2x-\frac{14x^2}{2a+x}$$

$$=\frac{(a+2x)(2a+x)-14x^2}{2a+x}$$

$$=\frac{2a^2+5ax+2x^2-14x^2}{2a+x}=\frac{2a^2+5ax-12x^2}{2a+x}=\frac{(a+4x)(2a-3x)}{2a+x}$$

$$\Rightarrow a-x+\frac{a^2+5x^2}{a+4x}$$

$$=\frac{(a-x)(a+4x)+a^2+5x^2}{a+4x}$$

$$=\frac{a^2+3ax-4x^2+a^2+5x^2}{a+4x}=\frac{2a^2+3ax+x^2}{a+4x}=\frac{(a+x)(2a+x)}{a+4x}$$

$$\Rightarrow \frac{(a+4x)(2a-3x)}{2a+x}\cdot\frac{(a+x)(2a+x)}{a+4x}$$

$$=(2a-3x)(a+x)=2a^2-ax-3x^2$$

9. $\left(m-\frac{mn}{m+n}\right)\left(1+\frac{n^3}{m^3}\right)$

$$\Rightarrow m-\frac{mn}{m+n}=\frac{m(m+n)-mn}{m+n}=\frac{m^2+mn-mn}{m+n}=\frac{m^2}{m+n}$$

$$\Rightarrow 1+\frac{n^3}{m^3}=\frac{m^3+n^3}{m^3}=\frac{(m+n)\left(m^2-mn+n^2\right)}{m^3}$$

$$\Rightarrow \frac{m^2}{m+n}\cdot\frac{(m+n)\left(m^2-mn+n^2\right)}{m^3}=\frac{m^2-mn+n^2}{m}$$

11. $\left(1+\frac{a}{b}\right)\left(1-\frac{b}{a}\right)\left(1+\frac{b^2}{a^2-b^2}\right)$

$$\Rightarrow 1+\frac{a}{b}=\frac{b+a}{b}$$

$$\Rightarrow 1-\frac{b}{a}=\frac{a-b}{a}$$

$$\Rightarrow 1+\frac{b^2}{a^2-b^2}=\frac{a^2}{a^2-b^2}$$

$$\Rightarrow \frac{b+a}{b}\cdot\frac{a-b}{a}\cdot\frac{a^2}{(a+b)(a-b)}=\frac{a^2}{ab}=\frac{a}{b}$$

12. $\left(2+\frac{2}{x+1}\right)\left(3-\frac{6}{x+2}\right)\left(1+\frac{1}{x}\right)$

$$\Rightarrow 2+\frac{2}{x+1}=\frac{2(x+1)+2}{x+1}=\frac{2x+4}{x+1}=\frac{2(x+2)}{x+1}$$

$$\Rightarrow 3-\frac{6}{x+2}=\frac{3(x+2)-6}{x+2}=\frac{3x+6-6}{x+2}=\frac{3x}{x+2}$$

$$\Rightarrow 1+\frac{1}{x}=\frac{x+1}{x}$$

$$\Rightarrow \frac{2(x+2)}{x+1}\cdot\frac{3x}{x+2}\cdot\frac{x+1}{x}=6$$

EJERCICIO 134

1. $\frac{x^2}{3y^2}\div\frac{2x}{y^3}=\frac{x^2}{3y^2}\cdot\frac{y^3}{2x}=\frac{xy}{6}$

2. $\frac{3a^2b}{5x^2}\div a^2b^3=\frac{3a^2b}{5x^2}\cdot\frac{1}{a^2b^3}=\frac{3}{5b^2x^2}$

3. $\frac{5m^2}{7n^3}\div\frac{10m^4}{14an^4}=\frac{5m^2}{7n^3}\cdot\frac{14an^4}{10m^4}=\frac{an}{m^2}$

4. $6a^2x^3\div\frac{a^2x}{5}=6a^2x^3\cdot\frac{5}{a^2x}=30x^2$

5. $\frac{15m^2}{19ax^3}\div\frac{20y^2}{38a^3x^4}=\frac{15m^2}{19ax^3}\cdot\frac{38a^3x^4}{20y^2}=\frac{3m^2a^2x}{2y^2}$

6. $\frac{11x^2y^3}{7m^2}\div 22y^4=\frac{11x^2y^3}{7m^2}\cdot\frac{1}{22y^4}=\frac{x^2}{14m^2y}$

7. $\frac{x-1}{3}\div\frac{2x-2}{6}=\frac{x-1}{3}\cdot\frac{6}{2(x-1)}=1$

8. $\frac{3a^2}{a^2+6ab+9b^2}\div\frac{5a^3}{a^2b+3ab^2}=\frac{3a^2}{(a+3b)^2}\cdot\frac{ab(a+3b)}{5a^3}=\frac{3b}{5(a+3b)}$

9. $\frac{x^3-x}{2x^2+6x}\div\frac{5x^2-5x}{2x+6}=\frac{x\left(x^2-1\right)}{2x(x+3)}\cdot\frac{2(x+3)}{5x(x-1)}=\frac{x+1}{5x}$

10. $\frac{1}{a^2-a-30}\div\frac{2}{a^2+a-42}$

$$=\frac{1}{(a-6)(a+5)}\cdot\frac{(a+7)(a-6)}{2}=\frac{a+7}{2(a+5)}$$

11. $\frac{20x^2-30x}{15x^3+15x^2}\div\frac{4x-6}{x+1}=\frac{5x(4x-6)}{15x^2(x+1)}\cdot\frac{x+1}{(4x-6)}=\frac{1}{3x}$

12. $\dfrac{a^2-6a+5}{a^2-15a+56}\div\dfrac{a^2+2a-35}{a^2-5a-24}$

$=\dfrac{(a-5)(a-1)}{(a-8)(a-7)}\cdot\dfrac{(a-8)(a+3)}{(a+7)(a-5)}$

$=\dfrac{(a-1)(a+3)}{(a-7)(a+7)}=\dfrac{a^2+2a-3}{a^2-49}$

13. $\dfrac{8x^2+26x+15}{16x^2-9}\div\dfrac{6x^2+13x-5}{9x^2-1}$

$=\dfrac{(2x+5)(4x+3)}{(4x+3)(4x-3)}\cdot\dfrac{(3x+1)(3x-1)}{(2x+5)(3x-1)}=\dfrac{3x+1}{4x-3}$

14. $\dfrac{x^3-121x}{x^2-49}\div\dfrac{x^2-11x}{x+7}$

$=\dfrac{x(x^2-121)}{(x+7)(x-7)}\cdot\dfrac{(x+7)}{x(x-11)}=\dfrac{x+11}{x-7}$

15. $\dfrac{ax^2+5}{4a^2-1}\div\dfrac{a^3x^2+5a^2}{2a-1}$

$=\dfrac{ax^2+5}{(2a+1)(2a-1)}\cdot\dfrac{2a-1}{a^2(ax^2+5)}=\dfrac{1}{a^2(2a+1)}$

16. $\dfrac{a^4-1}{a^3+a^2}\div\dfrac{a^4+4a^2+3}{3a^3+9a}$

$=\dfrac{(a^2+1)(a^2-1)}{a^2(a+1)}\cdot\dfrac{3a(a^2+3)}{(a^2+3)(a^2+1)}=\dfrac{3(a-1)}{a}$

17. $\dfrac{x^3+125}{x^2-64}\div\dfrac{x^3-5x^2+25x}{x^2+x-56}$

$=\dfrac{(x+5)(x^2-5x+25)}{(x+8)(x-8)}\cdot\dfrac{(x+8)(x-7)}{x(x^2-5x+25)}$

$=\dfrac{(x+5)(x-7)}{x(x-8)}=\dfrac{x^2-2x-35}{x^2-8x}$

18. $\dfrac{16x^2-24xy+9y^2}{16x-12y}\div\dfrac{64x^3-27y^3}{32x^2+24xy+18y^2}$

$=\dfrac{(4x-3y)^2}{4(4x-3y)}\cdot\dfrac{2(16x^2+12xy+9y^2)}{(4x-3y)(16x^2+12xy+9y^2)}=\dfrac{1}{2}$

19. $\dfrac{a^2-6a}{a^3+3a^2}\div\dfrac{a^2+3a-54}{a^2+9a}=\dfrac{a(a-6)}{a^2(a+3)}\cdot\dfrac{a(a+9)}{(a+9)(a-6)}=\dfrac{1}{a+3}$

20. $\dfrac{15x^2+7x-2}{25x^3-x}\div\dfrac{6x^2+13x+6}{25x^2+10x+1}$

$=\dfrac{(3x+2)(5x-1)}{x(5x-1)(5x+1)}\cdot\dfrac{(5x+1)^2}{(2x+3)(3x+2)}=\dfrac{5x+1}{x(2x+3)}=\dfrac{5x+1}{2x^2+3x}$

21. $\dfrac{x^3-1}{2x^2-2x+2}\div\dfrac{7x^2+7x+7}{7x^3+7}$

$=\dfrac{(x-1)(x^2+x+1)}{2(x^2-x+1)}\cdot\dfrac{7(x^3+1)}{7(x^2+x+1)}$

$=\dfrac{(x-1)}{2(x^2-x+1)}\cdot(x+1)(x^2-x+1)=\dfrac{(x-1)(x+1)}{2}=\dfrac{x^2-1}{2}$

22. $\dfrac{2mx-2my+nx-ny}{3x-3y}\div 8m+4n$

$=\dfrac{(2m+n)(x-y)}{3(x-y)}\cdot\dfrac{1}{4(2m+n)}=\dfrac{1}{12}$

23. $\dfrac{x^2-6x+9}{4x^2-1}\div\dfrac{x^2+5x-24}{2x^2+17x+8}$

$=\dfrac{(x-3)^2}{(2x+1)(2x-1)}\cdot\dfrac{(2x+1)(x+8)}{(x+8)(x-3)}=\dfrac{x-3}{2x-1}$

24. $\dfrac{2a^2+7ab-15b^2}{a^3+4a^2b}\div\dfrac{a^2-3ab-40b^2}{a^2-4ab-32b^2}$

$=\dfrac{(a+5b)(2a-3b)}{a^2(a+4b)}\cdot\dfrac{(a-8b)(a+4b)}{(a-8b)(a+5b)}=\dfrac{2a-3b}{a^2}$

EJERCICIO 135

1. $\left(1+\dfrac{a}{a+b}\right)\div\left(1+\dfrac{2a}{b}\right)$

$\Rightarrow 1+\dfrac{a}{a+b}=\dfrac{a+b+a}{a+b}=\dfrac{2a+b}{a+b}$

$\Rightarrow 1+\dfrac{2a}{b}=\dfrac{b+2a}{b}$

$\Rightarrow \dfrac{2a+b}{a+b}\div\dfrac{b+2a}{b}=\dfrac{2a+b}{a+b}\cdot\dfrac{b}{b+2a}=\dfrac{b}{a+b}$

2. $\left(x-\dfrac{2}{x+1}\right)\div\left(x-\dfrac{x}{x+1}\right)$

$\Rightarrow x-\dfrac{2}{x+1}=\dfrac{x(x+1)-2}{x+1}=\dfrac{x^2+x-2}{x+1}$

$\Rightarrow x-\dfrac{x}{x+1}=\dfrac{x(x+1)-x}{x+1}=\dfrac{x^2}{x+1}$

$\Rightarrow \dfrac{x^2+x-2}{x+1}\div\dfrac{x^2}{x+1}=\dfrac{x^2+x-2}{x+1}\cdot\dfrac{x+1}{x^2}=\dfrac{x^2+x-2}{x^2}$

3. $\left(1-a+\frac{a^2}{1+a}\right)\div\left(1+\frac{2}{a^2-1}\right)$

$$\Rightarrow 1-a+\frac{a^2}{1+a}=\frac{(1-a)(1+a)+a^2}{1+a}=\frac{1-a^2+a^2}{1+a}=\frac{1}{1+a}$$

$$\Rightarrow 1+\frac{2}{a^2-1}=\frac{a^2-1+2}{a^2-1}=\frac{a^2+1}{a^2-1}$$

$$\Rightarrow \frac{1}{1+a}\div\frac{a^2+1}{a^2-1}=\frac{1}{1+a}\cdot\frac{a^2-1}{a^2+1}=\frac{a-1}{a^2+1}$$

4. $\left(x+\frac{2}{x+3}\right)\div\left(x+\frac{3}{x+4}\right)$

$$\Rightarrow x+\frac{2}{x+3}=\frac{x(x+3)+2}{x+3}=\frac{x^2+3x+2}{x+3}=\frac{(x+1)(x+2)}{x+3}$$

$$\Rightarrow x+\frac{3}{x+4}=\frac{x(x+4)+3}{x+4}=\frac{x^2+4x+3}{x+4}=\frac{(x+3)(x+1)}{x+4}$$

$$\Rightarrow \frac{(x+1)(x+2)}{x+3}\div\frac{(x+3)(x+1)}{x+4}$$

$$=\frac{(x+1)(x+2)}{x+3}\cdot\frac{x+4}{(x+3)(x+1)}=\frac{(x+2)(x+4)}{(x+3)^2}=\frac{x^2+6x+8}{x^2+6x+9}$$

5. $\left(a+b+\frac{b^2}{a-b}\right)\div\left(1-\frac{b}{a+b}\right)$

$$\Rightarrow a+b+\frac{b^2}{a-b}=\frac{(a+b)(a-b)+b^2}{a-b}=\frac{a^2-b^2+b^2}{a-b}=\frac{a^2}{a-b}$$

$$\Rightarrow 1-\frac{b}{a+b}=\frac{a+b-b}{a+b}=\frac{a}{a+b}$$

$$\Rightarrow \frac{a^2}{a-b}\div\frac{a}{a+b}=\frac{a^2}{a-b}\cdot\frac{a+b}{a}=\frac{a(a+b)}{a-b}=\frac{a^2+ab}{a-b}$$

6. $\left(1-\frac{1}{x^3+2}\right)\div\left(x+\frac{1}{x-1}\right)$

$$\Rightarrow 1-\frac{1}{x^3+2}=\frac{x^3+2-1}{x^3+2}=\frac{x^3+1}{x^3+2}=\frac{(x+1)(x^2-x+1)}{x^3+2}$$

$$\Rightarrow x+\frac{1}{x-1}=\frac{x(x-1)+1}{x-1}=\frac{x^2-x+1}{x-1}$$

$$\Rightarrow \frac{(x+1)(x^2-x+1)}{x^3+2}\div\frac{x^2-x+1}{x-1}$$

$$=\frac{(x+1)(x^2-x+1)}{x^3+2}\cdot\frac{x-1}{x^2-x+1}=\frac{(x+1)(x-1)}{x^3-2}=\frac{x^2-1}{x^3+2}$$

7. $\left(x+\frac{1}{x+2}\right)\div\left(1+\frac{3}{x^2-4}\right)$

$$\Rightarrow x+\frac{1}{x+2}=\frac{x(x+2)+1}{x+2}=\frac{x^2+2x+1}{x+2}=\frac{(x+1)^2}{x+2}$$

$$\Rightarrow 1+\frac{3}{x^2-4}=\frac{x^2-4+3}{x^2-4}=\frac{x^2-1}{x^2-4}=\frac{(x+1)(x-1)}{(x+2)(x-2)}$$

$$\Rightarrow \frac{(x+1)^2}{x+2}\div\frac{(x+1)(x-1)}{(x+2)(x-2)}$$

$$=\frac{(x+1)^2}{x+2}\cdot\frac{(x+2)(x-2)}{(x+1)(x-1)}=\frac{(x+1)(x-2)}{x-1}=\frac{x^2-x-2}{x-1}$$

8. $\left(n-\frac{2n-1}{n^2+2}\right)\div\left(n^2+1-\frac{n-1}{n}\right)$

$$\Rightarrow n-\frac{2n-1}{n^2+2}=\frac{n(n^2+2)-(2n-1)}{n^2+2}=\frac{n^3+2n-2n+1}{n^2+2}=\frac{n^3+1}{n^2+2}$$

$$\Rightarrow n^2+1-\frac{n-1}{n}=\frac{n(n^2+1)-(n-1)}{n}=\frac{n^3+n-n+1}{n}=\frac{n^3+1}{n}$$

$$\Rightarrow \frac{n^3+1}{n^2+2}\div\frac{n^3+1}{n}=\frac{n^3+1}{n^2+2}\cdot\frac{n}{n^3+1}=\frac{n}{n^2+2}$$

EJERCICIO 136

1. $\frac{3x}{4y}\cdot\frac{8y}{9x}\div\frac{z^2}{3x^2}=\frac{2}{3}\cdot\frac{3x^2}{z^2}=\frac{2x^2}{z^2}$

2. $\frac{5a}{b}\div\left(\frac{2a}{b^2}\cdot\frac{5x}{4a^2}\right)=\frac{5a}{b}\cdot\frac{2ab^2}{5x}=\frac{2a^2b}{x}$

3. $\frac{a+1}{a-1}\cdot\frac{3a-3}{2a+2}\div\frac{a^2+a}{a^2+a-2}$

$$=\frac{a+1}{a-1}\cdot\frac{3(a-1)}{2(a+1)}\cdot\frac{(a+2)(a-1)}{a(a+1)}$$

$$=\frac{3(a+2)(a-1)}{2a(a+1)}=\frac{3a^2+3a-6}{2a^2+2a}$$

4. $\frac{64a^2-81b^2}{x^2-81}\cdot\frac{(x-9)^2}{8a-9b}\div\frac{8a^2+9ab}{(x+9)^2}$

$$=\frac{(8a+9b)(8a-9b)}{(x+9)(x-9)}\cdot\frac{(x-9)^2}{8a-9b}\cdot\frac{(x+9)^2}{a(8a+9b)}$$

$$=\frac{(x-9)(x+9)}{a}=\frac{x^2-81}{a}$$

5. $\frac{x^2-x-12}{x^2-49}\cdot\frac{x^2-x-56}{x^2+x-20}\div\frac{x^2-5x-24}{x+5}$

$$=\frac{(x-4)(x+3)}{(x+7)(x-7)}\cdot\frac{(x-8)(x+7)}{(x+5)(x-4)}\cdot\frac{x+5}{(x-8)(x+3)}=\frac{1}{x-7}$$

6. $\frac{a^2-8a+7}{a^2-11a+30}\cdot\frac{a^2-36}{a^2-1}\div\frac{a^2-a-42}{a^2-4a-5}$

$$=\frac{(a-7)(a-1)}{(a-6)(a-5)}\cdot\frac{(a+6)(a-6)}{(a+1)(a-1)}\cdot\frac{(a-5)(a+1)}{(a-7)(a+6)}=1$$

7. $\dfrac{x^4-27x}{x^2+7x-30}\cdot\dfrac{x^2+20x+100}{x^3+3x^2+9x}\div\dfrac{x^2-100}{x-3}=\dfrac{x(x-3)(x^2+3x+9)}{(x+10)(x-3)}\cdot\dfrac{(x+10)^2}{x(x^2+3x+9)}\cdot\dfrac{x-3}{(x+10)(x-10)}=\dfrac{x-3}{x-10}$

8. $\dfrac{(a^2+1)}{3a-6}\div\left(\dfrac{a^3+a}{6a-12}\cdot\dfrac{4x+8}{x-3}\right)=\dfrac{a^2+1}{3(a-2)}\div\dfrac{a(a^2+1)}{6(a-2)}\cdot\dfrac{4(x+2)}{x-3}=\dfrac{a^2+1}{3(a-2)}\cdot\dfrac{3(a-2)(x-3)}{2a(a^2+1)(x+2)}=\dfrac{x-3}{2a(x+2)}=\dfrac{x-3}{2ax+4a}$

9. $\dfrac{8x^2-10x-3}{6x^2+13x+6}\cdot\dfrac{4x^2-9}{3x^2+2x}\div\dfrac{8x^2+14x+3}{9x^2+12x+4}=\dfrac{(2x-3)(4x+1)}{(2x+3)(3x+2)}\cdot\dfrac{(2x+3)(2x-3)}{x(3x+2)}\cdot\dfrac{(3x+2)^2}{(2x+3)(4x+1)}=\dfrac{(2x-3)^2}{x(2x+3)}=\dfrac{4x^2-12x+9}{2x^2+3x}$

10. $\dfrac{(a+b)^2-c^2}{(a-b)^2-c^2}\cdot\dfrac{(a+c)^2-b^2}{a^2+ab-ac}\div\dfrac{a+b+c}{a^2}=\dfrac{(a+b+c)(a+b-c)}{(a-b+c)(a-b-c)}\cdot\dfrac{(a+b+c)(a-b+c)}{a(a+b-c)}\cdot\dfrac{a^2}{a+b+c}=\dfrac{a(a+b+c)}{a-b-c}=\dfrac{a^2+ab+ac}{a-b-c}$

11. $\dfrac{a^2-5a}{b+b^2}\div\left(\dfrac{a^2+6a-55}{b^2-1}\cdot\dfrac{ax+3a}{ab^2+11b^2}\right)=\dfrac{a(a-5)}{b(1+b)}\cdot\dfrac{b^2(b+1)(b-1)(a+11)}{a(a+11)(a-5)(x+3)}=\dfrac{b(b-1)}{x+3}=\dfrac{b^2-b}{x+3}$

12. $\dfrac{m^3+6m^2n+9mn^2}{2m^2n+7mn^2+3n^3}\cdot\dfrac{4m^2-n^2}{8m^2-2mn-n^2}\div\dfrac{m^3+27n^3}{16m^2+8mn+n^2}$

$=\dfrac{m(m+3n)^2}{n(m+3n)(2m+n)}\cdot\dfrac{(2m+n)(2m-n)}{(2m-n)(4m+n)}\cdot\dfrac{(4m+n)^2}{(m+3n)(m^2-3mn-9n^2)}=\dfrac{m(4m+n)}{n(m^2-3mn+9n^2)}=\dfrac{4m^2+mn}{m^2n-3mn^2+9n^3}$

13. $\dfrac{(a^2-ax)^2}{a^2+x^2}\cdot\dfrac{1}{a^3+a^2x}\div\left(\dfrac{a^3-a^2x}{a^2+2ax+x^2}\cdot\dfrac{a^2-x^2}{a^3+ax^2}\right)$

$=\dfrac{(a^2-ax)^2}{a^2+x^2}\cdot\dfrac{1}{a^2(a+x)}\cdot\dfrac{a(a+x)^2(a^2+x^2)}{a(a^2-ax)(a+x)(a-x)}=\dfrac{a^2-ax}{a^2(a-x)}=\dfrac{a(a-x)}{a^2(a-x)}=\dfrac{1}{a}$

14. $\dfrac{(a^2-3a)^2}{9-a^2}\cdot\dfrac{27-a^3}{(a+3)^2-3a}\div\dfrac{a^4-9a^2}{(a^2+3a)^2}$

$=\dfrac{(a^2-3a)^2}{(3+a)(3-a)}\cdot\dfrac{(3-a)(9+3a+a^2)}{a^2+3a+9}\cdot\dfrac{(a^2+3a)^2}{(a^2-3a)(a^2+3a)}=\dfrac{(a^2-3a)(a^2+3a)}{3+a}=\dfrac{a(a^2-3a)(a+3)}{a+3}=a^3-3a^2$

EJERCICIO 137

1. $\dfrac{a-\frac{a}{b}}{b-\frac{1}{b}}=\dfrac{\frac{ab-a}{b}}{\frac{b^2-1}{b}}=\dfrac{a(b-1)}{(b+1)(b-1)}=\dfrac{a}{b+1}$

2. $\dfrac{x^2-\frac{1}{x}}{1-\frac{1}{x}}=\dfrac{\frac{x^3-1}{x}}{\frac{x-1}{x}}=\dfrac{(x-1)(x^2+x+1)}{x-1}=x^2+x+1$

3. $\dfrac{\frac{a}{b}-\frac{b}{a}}{1+\frac{b}{a}}=\dfrac{\frac{a^2-b^2}{ab}}{\frac{a+b}{a}}=\dfrac{(a+b)(a-b)}{b(a+b)}=\dfrac{a-b}{b}$

4. $\dfrac{\frac{1}{m}+\frac{1}{n}}{\frac{1}{m}-\frac{1}{n}}=\dfrac{\frac{m+n}{mn}}{\frac{n-m}{mn}}=\dfrac{m+n}{n-m}$

5. $\dfrac{x+\frac{x}{2}}{x-\frac{x}{4}}=\dfrac{\frac{2x+x}{2}}{\frac{4x-x}{4}}=\dfrac{2(3x)}{3x}=2$

6. $\dfrac{\frac{x}{y}-\frac{y}{x}}{1+\frac{y}{x}}=\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{x}}=\dfrac{(x+y)(x-y)}{y(x+y)}=\dfrac{x-y}{y}$

7. $\dfrac{x+4+\frac{3}{x}}{x-4-\frac{5}{x}}=\dfrac{\frac{x^2+4x+3}{x}}{\frac{x^2-4x-5}{x}}=\dfrac{(x+1)(x+3)}{(x-5)(x+1)}=\dfrac{x+3}{x-5}$

8. $\dfrac{a-4+\frac{4}{a}}{1-\frac{2}{a}}=\dfrac{\frac{a^2-4a+4}{a}}{\frac{a-2}{a}}=\dfrac{(a-2)(a-2)}{(a-2)}=a-2$

9. $$\frac{\frac{2a^2-b^2}{a}-b}{\frac{4a^2+b^2}{4ab}+1}=\frac{\frac{2a^2-b^2-ab}{a}}{\frac{4a^2+b^2+4ab}{4ab}}=\frac{4b\left(2a^2-b^2-ab\right)}{4a^2+b^2+4ab}=\frac{4b(a-b)(2a+b)}{(2a+b)^2}=\frac{4b(a-b)}{2a+b}=\frac{4ab-4b^2}{2a+b}$$

10. $$\frac{2+\frac{3a}{5b}}{a+\frac{10b}{3}}=\frac{\frac{10b+3a}{5b}}{\frac{3a+10b}{3}}=\frac{3}{5b}$$

11. $$\frac{a-x+\frac{x^2}{a+x}}{a^2-\frac{a^2}{a+x}}=\frac{\frac{a^2-x^2+x^2}{a+x}}{\frac{a^3+a^2x-a^2}{a+x}}=\frac{a^2}{a^2(a+x-1)}=\frac{1}{a+x-1}$$

12. $$\frac{a+5-\frac{14}{a}}{1+\frac{8}{a}+\frac{7}{a^2}}=\frac{\frac{a^2+5a-14}{a}}{\frac{a^2+8a+7}{a^2}}=\frac{a(a+7)(a-2)}{(a+7)(a+1)}=\frac{a(a-2)}{a+1}=\frac{a^2-2a}{a+1}$$

13. $$\frac{\frac{1}{a}-\frac{9}{a^2}+\frac{20}{a^3}}{\frac{16}{a}-a}=\frac{\frac{a^2-9a+20}{a^3}}{\frac{16-a^2}{a}}=\frac{(a-5)(a-4)}{a^2(4+a)(4-a)}=-\frac{a-5}{a^2(a+4)}=-\frac{a-5}{a^3+4a^2}=\frac{5-a}{a^3+4a^2}$$

14. $$\frac{\frac{20x^2+7x-6}{x}}{\frac{4}{x^2}-25}=\frac{\frac{(4x+3)(5x-2)}{x}}{\frac{4-25x^2}{x^2}}=\frac{x(4x+3)(5x-2)}{(2+5x)(2-5x)}=-\frac{4x^2+3x}{5x+2}$$

15. $$\frac{1+\frac{1}{x-1}}{1+\frac{1}{x^2-1}}=\frac{\frac{x-1+1}{x-1}}{\frac{x^2-1+1}{x^2-1}}=\frac{x(x+1)}{x^2}=\frac{x+1}{x}$$

16. $$\frac{a-\frac{ab}{a+b}}{a+\frac{ab}{a-b}}=\frac{\frac{a^2+ab-ab}{a+b}}{\frac{a^2-ab+ab}{a-b}}=\frac{a^2(a-b)}{a^2(a+b)}=\frac{a-b}{a+b}$$

17. $$\frac{x-1-\frac{5}{x+3}}{x+5-\frac{35}{x+3}}=\frac{\frac{x^2+2x-8}{x+3}}{\frac{x^2+8x-20}{x+3}}=\frac{(x+4)(x-2)}{(x+10)(x-2)}=\frac{x+4}{x+10}$$

18. $$\frac{a+2-\frac{7a+9}{a+3}}{a-4+\frac{5a-11}{a+1}}=\frac{\frac{a^2+5a+6-7a-9}{a+3}}{\frac{a^2-3a-4+5a-11}{a+1}}=\frac{(a+1)\left(a^2-2a-3\right)}{(a+3)\left(a^2+2a-15\right)}=\frac{(a+1)(a-3)(a+1)}{(a+3)(a+5)(a-3)}=\frac{(a+1)^2}{(a+3)(a+5)}=\frac{a^2+2a+1}{a^2+8a+15}$$

EJERCICIO 138

1. $$\frac{1+\frac{x+1}{x-1}}{\frac{1}{x-1}-\frac{1}{x+1}}=\frac{\frac{x-1+x+1}{x-1}}{\frac{x+1-(x-1)}{x^2-1}}=\frac{2x(x+1)}{2}=x^2+x$$

2. $$\frac{\frac{1}{x-1}+\frac{2}{x+1}}{\frac{x-2}{x}+\frac{2x+6}{x+1}}=\frac{\frac{x+1+2(x-1)}{x^2-1}}{\frac{\left(x^2-x-2\right)+x(2x+6)}{x(x+1)}}=\frac{x\,(3x-1)}{(x-1)\left(3x^2+5x-2\right)}=\frac{x\,(3x-1)}{(x-1)(3x-1)(x+2)}=\frac{x}{x^2+x-2}$$

3. $$\frac{\frac{a}{a-b}-\frac{b}{a+b}}{\frac{a+b}{a-b}+\frac{a}{b}}=\frac{\frac{a(a+b)-b(a-b)}{(a-b)(a+b)}}{\frac{b(a+b)+a(a-b)}{b(a-b)}}=\frac{\frac{a^2+ab-ab+b^2}{a+b}}{\frac{ab+b^2+a^2-ab}{b}}=\frac{\frac{a^2+b^2}{a+b}}{\frac{a^2+b^2}{b}}=\frac{b}{a+b}$$

4. $$\frac{\frac{x+3}{x+4}-\frac{x+1}{x+2}}{\frac{x-1}{x+2}-\frac{x-3}{x+4}}=\frac{\frac{(x+3)(x+2)-(x+1)(x+4)}{(x+4)(x+2)}}{\frac{(x-1)(x+4)-(x-3)(x+2)}{(x+2)(x+4)}}=\frac{x^2+5x+6-x^2-5x-4}{x^2+3x-4-x^2+x+6}=\frac{2}{4x+2}=\frac{2}{2(2x+1)}=\frac{1}{2x+1}$$

5. $$\frac{\frac{m^2}{n}+\frac{m^2-n^2}{m+n}}{\frac{m-n}{n}+\frac{n}{m}}$$

$$=\frac{\frac{m^2\left(m+n\right)-n\left(m^2-n^2\right)}{n\left(m+n\right)}}{\frac{m\left(m-n\right)+n^2}{nm}}$$

$$=\frac{\frac{m^3+m^2n-m^2n+n^3}{m+n}}{\frac{m^2-mn+n^2}{m}}$$

$$=\frac{\frac{m^3+n^3}{m+n}}{\frac{m^2-mn+n^2}{m}}$$

$$=\frac{\frac{\left(m+n\right)\left(m^2-mn+n^2\right)}{m+n}}{\frac{m^2-mn+n^2}{m}}$$

$$=m^2-mn+n^2\cdot\frac{m}{m^2-mn+n^2}=m$$

6. $$\frac{\frac{a^2}{b^3}+\frac{1}{a}}{\frac{a}{b}-\frac{b-a}{a-b}}$$

$$=\frac{\frac{a^3+b^3}{ab^3}}{\frac{a\left(a-b\right)-b\left(b-a\right)}{b\left(a-b\right)}}$$

$$=\frac{\frac{\left(a+b\right)\left(a^2-ab+b^2\right)}{ab^3}}{\frac{a^2-ab-b^2+ab}{b\left(a-b\right)}}$$

$$=\frac{\frac{\left(a+b\right)\left(a^2-ab+b^2\right)}{ab^2}}{\frac{a^2-b^2}{a-b}}$$

$$=\frac{\left(a+b\right)\left(a^2-ab+b^2\right)}{ab^2}\cdot\frac{a-b}{\left(a+b\right)\left(a-b\right)}$$

$$=\frac{a^2-ab+b^2}{ab^2}$$

7. $$\frac{1+\frac{2x}{1+x^2}}{2x+\frac{2x^5+2}{1-x^4}}$$

$$=\frac{\frac{1+x^2+2x}{1+x^2}}{\frac{2x\left(1-x^4\right)+2x^5+2}{\left(1+x^2\right)\left(1-x^2\right)}}$$

$$=\frac{x^2+2x+1}{\frac{2x-2x^5+2x^5+2}{1-x^2}}$$

$$=\frac{\left(x+1\right)^2}{\frac{2\left(x+1\right)}{\left(x+1\right)\left(1-x\right)}}$$

$$=\frac{\left(x+1\right)^2\cdot\left(x+1\right)\left(1-x\right)}{2\left(x+1\right)}$$

$$=\frac{\left(x+1\right)^2\left(1-x\right)}{2}$$

$$=\frac{\left(x^2+2x+1\right)\left(1-x\right)}{2}$$

$$=\frac{x^2-x^3+2x-2x^2+1-x}{2}$$

$$=\frac{1+x-x^2-x^3}{2}$$

8. $$\frac{\frac{x+y}{x-y}-\frac{x-y}{x+y}}{\frac{x+y}{x}-\frac{x+2y}{x+y}}$$

$$=\frac{\frac{\left(x+y\right)^2-\left(x-y\right)^2}{\left(x-y\right)\left(x+y\right)}}{\frac{\left(x+y\right)^2-x\left(x+2y\right)}{x\left(x+y\right)}}$$

$$=\frac{\frac{x^2+2xy+y^2-x^2+2xy-y^2}{x-y}}{\frac{x^2+2xy+y^2-x^2-2xy}{x}}$$

$$=\frac{\frac{4xy}{x-y}}{\frac{y^2}{x}}$$

$$=\frac{4xy}{x-y}\cdot\frac{x}{y^2}=\frac{4x^2}{y\left(x-y\right)}=\frac{4x^2}{xy-y^2}$$

9. $$\frac{\frac{a+x}{a-x}-\frac{b+x}{b-x}}{\frac{2}{a-x}-\frac{2}{b-x}}$$

$$=\frac{\frac{\left(a+x\right)\left(b-x\right)-\left(b+x\right)\left(a-x\right)}{\left(a-x\right)\left(b-x\right)}}{\frac{2\left(b-x\right)-2\left(a-x\right)}{\left(a-x\right)\left(b-x\right)}}$$

$$=\frac{ab-ax+bx-x^2-ab+bx-ax+x^2}{2b-2x-2a+2x}$$

$$=\frac{2bx-2ax}{2b-2a}=\frac{2x\left(b-a\right)}{2\left(b-a\right)}=x$$

10. $$\frac{\frac{a}{a+x}-\frac{a}{2a+2x}}{\frac{a}{a-x}+\frac{a}{a+x}}=\frac{\frac{2a-a}{2\left(a+x\right)}}{\frac{a\left(a+x\right)+a\left(a-x\right)}{\left(a-x\right)\left(a+x\right)}}$$

$$=\frac{\frac{a}{2}}{\frac{a^2+ax+a^2-ax}{a-x}}$$

$$=\frac{\frac{a}{2}}{\frac{2a^2}{a-x}}=\frac{a}{2}\cdot\frac{a-x}{2a^2}=\frac{a-x}{4a}$$

11. $$\frac{\frac{a+2b}{a-b}+\frac{b}{a}}{\frac{a+b}{a}+\frac{3b}{a-b}}=\frac{\frac{a\left(a+2b\right)+b\left(a-b\right)}{a\left(a-b\right)}}{\frac{\left(a+b\right)\left(a-b\right)+3ab}{a\left(a-b\right)}}$$

$$=\frac{a^2+2ab+ab-b^2}{a^2-b^2+3ab}$$

$$=\frac{a^2+3ab-b^2}{a^2+3ab-b^2}=1$$

12. $$\frac{1-\frac{7}{x}+\frac{12}{x^2}}{x-\frac{16}{x}}$$

$$=\frac{\frac{x^2-7x+12}{x^2}}{\frac{x^2-16}{x}}$$

$$=\frac{\frac{\left(x-4\right)\left(x-3\right)}{x}}{\left(x-4\right)\left(x+4\right)}$$

$$=\frac{\left(x-4\right)\left(x-3\right)}{x}\cdot\frac{1}{\left(x-4\right)\left(x+4\right)}$$

$$=\frac{x-3}{x\left(x+4\right)}=\frac{x-3}{x^2+4x}$$

13. $$\frac{\frac{a^2}{b}-\frac{b^2}{a}}{\frac{1}{b}+\frac{1}{a}+\frac{b}{a^2}}$$
$$=\frac{\frac{a^3-b^3}{ab}}{\frac{a^2+ab+b^2}{a^2b}}$$
$$=\frac{a^3-b^3}{\frac{a^2+ab+b^2}{a}}$$
$$=(a-b)(a^2+ab+b^2)\cdot\frac{a}{a^2+ab+b^2}$$
$$=a(a-b)=a^2-ab$$

14. $$\frac{x-2y-\frac{4y^2}{x+y}}{x-3y-\frac{5y^2}{x+y}}$$
$$=\frac{\frac{(x-2y)(x+y)-4y^2}{x+y}}{\frac{(x-3y)(x+y)-5y^2}{x+y}}$$
$$=\frac{x^2-xy-6y^2}{x^2-2xy-8y^2}$$
$$=\frac{(x-3y)(x+2y)}{(x-4y)(x+2y)}=\frac{x-3y}{x-4y}$$

15. $$\frac{\frac{2}{1-a}+\frac{2}{1+a}}{\frac{2}{1+a}-\frac{2}{1-a}}$$
$$=\frac{\frac{2(1+a)+2(1-a)}{(1-a)(1+a)}}{\frac{2(1-a)-2(1+a)}{(1+a)(1-a)}}$$
$$=\frac{2+2a+2-2a}{2-2a-2-2a}=-\frac{4}{4a}=-\frac{1}{a}$$

16. $$\frac{\frac{1}{x+y+z}-\frac{1}{x-y+z}}{\frac{1}{x-y+z}-\frac{1}{x+y+z}}$$
$$=\frac{\frac{x-y+z-x-y-z}{(x+y+z)(x-y+z)}}{\frac{x+y+z-x+y-z}{(x-y+z)(x+y+z)}}=\frac{-2y}{2y}=-1$$

17. $$\frac{1+\frac{2b+c}{a-b-c}}{1-\frac{c-2b}{a-b+c}}$$
$$=\frac{\frac{a-b-c+2b+c}{a-b-c}}{\frac{a-b+c-c+2b}{a-b+c}}$$
$$=\frac{\frac{a+b}{a-b-c}}{\frac{a+b}{a-b+c}}$$
$$=\frac{a+b}{a-b-c}\cdot\frac{a-b+c}{a+b}=\frac{a-b+c}{a-b-c}$$

18. $$\frac{\frac{a}{1-a}+\frac{1-a}{a}}{\frac{1-a}{a}-\frac{a}{1-a}}$$
$$=\frac{\frac{a^2+(1-a)^2}{a(1-a)}}{\frac{(1-a)^2-a^2}{a(1-a)}}$$
$$=\frac{a^2+1-2a+a^2}{1-2a+a^2-a^2}=\frac{2a^2-2a+1}{1-2a}$$

19. $$\frac{\frac{x+1-\frac{6x+12}{x+2}}{x-5}}{\frac{x-4+\frac{11x-22}{x-2}}{x+7}}$$
$$=\frac{\frac{x+1-\frac{6(x+2)}{x+2}}{x-5}}{\frac{x-4+\frac{11(x-2)}{x-2}}{x+7}}=\frac{\frac{x-5}{x-5}}{\frac{x+7}{x+7}}=1$$

20. $$\frac{1}{1+\frac{1}{x}}=\frac{1}{\frac{x+1}{x}}=1\cdot\frac{x}{x+1}=\frac{x}{x+1}$$

21. $$\frac{1}{1+\frac{1}{1-\frac{1}{x}}}=\frac{1}{1+\frac{1}{\frac{x-1}{x}}}$$
$$=\frac{1}{1+\frac{x}{x-1}}$$
$$=\frac{1}{\frac{x-1+x}{x-1}}$$
$$=\frac{1}{\frac{2x-1}{x-1}}=1\cdot\frac{x-1}{2x-1}=\frac{x-1}{2x-1}$$

22. $$1-\frac{1}{2+\frac{1}{\frac{x}{3}-1}}$$
$$=1-\frac{1}{2+\frac{1}{\frac{x-3}{3}}}$$
$$=1-\frac{1}{2+\frac{3}{x-3}}$$
$$=1-\frac{1}{\frac{2x-3}{x-3}}$$
$$=1-\frac{x-3}{2x-3}$$
$$=\frac{2x-3-x+3}{2x-3}=\frac{x}{2x-3}$$

23. $$\frac{2}{1+\frac{2}{1+\frac{2}{x}}}$$
$$=\frac{2}{1+\frac{2}{\frac{x+2}{x}}}$$
$$=\frac{2}{1+\frac{2x}{x+2}}$$
$$=\frac{2}{\frac{x+2+2x}{x+2}}$$
$$=\frac{2}{\frac{3x+2}{x+2}}=2\cdot\frac{x+2}{3x+2}=\frac{2x+4}{3x+2}$$

24. $$\frac{1}{x-\frac{x}{x-\frac{x^2}{x+1}}}$$
$$=\frac{1}{x-\frac{x}{\frac{x^2+x-x^2}{x+1}}}$$
$$=\frac{1}{x-\frac{x}{\frac{x}{x+1}}}$$
$$=\frac{1}{x-\frac{x^2+x}{x}}$$
$$=\frac{1}{\frac{x^2-x^2-x}{x}}=\frac{1}{\frac{-x}{x}}=-1$$

25. $\dfrac{1}{a+2-\dfrac{a+1}{a-\dfrac{1}{a}}}$

$=\dfrac{1}{a+2-\dfrac{a+1}{\dfrac{a^2-1}{a}}}=\dfrac{1}{a+2-\dfrac{a(a+1)}{(a+1)(a-1)}}=\dfrac{1}{a+2-\dfrac{a}{a-1}}=\dfrac{1}{\dfrac{(a+2)(a-1)-a}{a-1}}=\dfrac{1}{\dfrac{a^2+a-2-a}{a-1}}=1\cdot\dfrac{a-1}{a^2-2}=\dfrac{a-1}{a^2-2}$

26. $\dfrac{x-1}{x+2-\dfrac{x^2+2}{x-\dfrac{x-2}{x+1}}}=\dfrac{x-1}{x+2-\dfrac{x^2+2}{\dfrac{x^2+x-x+2}{x+1}}}=\dfrac{x-1}{x+2-\dfrac{x^2+2}{\dfrac{x^2+2}{x+1}}}=\dfrac{x-1}{x+2-\dfrac{(x^2+2)(x+1)}{x^2+2}}=\dfrac{x-1}{x+2-x-1}=x-1$

EJERCICIO 139

1. $\dfrac{x-2}{x+3}$ *Para* $x\to 2$ $\quad \dfrac{2-2}{2+3}=\dfrac{0}{5}=0$

2. $\dfrac{x-2}{x-3}$ *Para* $x\to 3$ $\quad \dfrac{3-2}{3-3}=\dfrac{1}{0}=\infty$

3. $\dfrac{x^2-a^2}{x^2+a^2}$ *Para* $x\to a$ $\quad \dfrac{a^2-a^2}{a^2+a^2}=\dfrac{0}{2a^2}=0$

4. $\dfrac{x^2+y^2}{x^2-y^2}$ *Para* $x\to y$ $\quad \dfrac{y^2+y^2}{y^2-y^2}=\dfrac{2y^2}{0}=\infty$

5. $\dfrac{\dfrac{x-1}{3}}{x-2}$ *Para* $x\to 2$ $\quad \dfrac{\dfrac{2-1}{3}}{2-2}=\dfrac{\dfrac{1}{3}}{0}=\dfrac{1}{\infty}=0$

6. $\dfrac{x^2-9}{x^2+x-12}$ *Para* $x\to 3$

$\dfrac{(x+3)(x-3)}{(x+4)(x-3)}=\dfrac{x+3}{x+4}=\dfrac{3+3}{3+4}=\dfrac{6}{7}$

7. $\dfrac{a^2-a-6}{a^2+2a-15}$ *Para* $a\to 3$

$\dfrac{(a-3)(a+2)}{(a+5)(a-3)}=\dfrac{a+2}{a+5}=\dfrac{3+2}{3+5}=\dfrac{5}{8}$

8. $\dfrac{x^2-7x+10}{x^3-2x^2-x+2}$ *Para* $x\to 2$

$\dfrac{(x-5)(x-2)}{x^2(x-2)-(x-2)}$

$=\dfrac{(x-5)(x-2)}{(x^2-1)(x-2)}=\dfrac{x-5}{x^2-1}=\dfrac{2-5}{4-1}=\dfrac{-3}{3}=-1$

9. $\dfrac{x^2-2x+1}{x^3-2x^2-x+2}$ *Para* $x\to 1$

$\dfrac{(x-1)^2}{x^2(x-2)-(x-2)}$

$=\dfrac{(x-1)^2}{(x^2-1)(x-2)}$

$=\dfrac{(x-1)^2}{(x-1)(x+1)(x-2)}=\dfrac{x-1}{x^2-x-2}=\dfrac{1-1}{1^2-1-2}=\dfrac{0}{-2}=0$

10. $\dfrac{a^3-8}{a^2+11a-26}$ *Para* $a\to 2$

$\dfrac{(a-2)(a^2+2a+4)}{(a+13)(a-2)}=\dfrac{a^2+2a+4}{a+13}=\dfrac{2^2+2\cdot 2+4}{2+13}=\dfrac{12}{15}=\dfrac{4}{5}$

11. $\dfrac{x^2-7x+6}{x^2-2x+1}$ *Para* $x\to 1$ $\quad \dfrac{(x-6)(x-1)}{(x-1)^2}=\dfrac{x-6}{x-1}=\dfrac{1-5}{1-1}=\dfrac{-4}{0}=\infty$

12. $\dfrac{x^3-3x-2}{x^3-7x+6}$ *Para* $x\to 2$

$x^3-3x-2=$

1	0	−3	−2	2
	2	4	2	
1	2	1	0	

$\Rightarrow (x-2)(x^2+2x+1)$

$x^3-7x+6=$

1	0	−7	6	2
	2	4	−6	
1	2	−3	0	

$\Rightarrow (x-2)(x^2+2x-3)$

$\dfrac{(x-2)(x+1)^2}{(x-2)(x^2+2x-3)}=\dfrac{(x+1)^2}{x^2+2x-3}=\dfrac{(2+1)^2}{2^2+2\cdot 2-3}=\dfrac{9}{5}$

13. $\dfrac{x^2-16}{x^3-4x^2-x+4}$ *Para* $x\to 4$

$$\frac{(x-4)(x+4)}{x^2(x-4)-(x-4)}=\frac{(x-4)(x+4)}{(x^2-1)(x-4)}=\frac{x+4}{x^2-1}=\frac{4+4}{4^2-1}=\frac{8}{15}$$

14. $\dfrac{4x^2-4x+1}{4x^2+8x-5}$ *Para* $x\to\dfrac{1}{2}$

$$\frac{(2x-1)^2}{(2x+5)(2x-1)}=\frac{2x-1}{2x+5}=\frac{2\cdot\frac{1}{2}-1}{2\cdot\frac{1}{2}+5}=\frac{1-1}{1+5}=\frac{0}{6}=0$$

15. $\dfrac{8x^2-6x+1}{4x^3+12x^2-15x+4}$ *Para* $x\to\dfrac{1}{2}$

$\Rightarrow 8x^2-6x+1=(8x)^2-6(8x)+8$

$=(8x-4)(8x-2)=(2x-1)(4x-1)$

$\Rightarrow 4x^3+12x^2-15x+4=$

$$\begin{array}{rrrr|l} 4 & 12 & -15 & 4 & -4 \\ & -16 & 16 & -4 & \\ \hline 4 & -4 & 1 & 0 & \end{array}$$

$=(x+4)(4x^2-4x+1)=(x+4)(2x-1)^2$

$$\Rightarrow\frac{(2x-1)(4x-1)}{(2x-1)^2(x+4)}=\frac{4x-1}{(2x-1)(x+4)}=\frac{4\cdot\frac{1}{2}-1}{\left(2\cdot\frac{1}{2}-1\right)\left(\frac{1}{2}+4\right)}=\frac{1}{0}=\infty$$

16. $\dfrac{x^3-9x+10}{x^4-x^3-11x^2+9x+18}$ *Para* $x\to 2$

$\Rightarrow x^3-9x+10$

$$\begin{array}{rrrr|l} 1 & 0 & -9 & 10 & 2 \\ & 2 & 4 & -10 & \\ \hline 1 & 2 & -5 & 0 & \end{array}$$

$=(x-2)(x^2+2x-5)$

$\Rightarrow x^4-x^3-11x^2+9x+18$

$$\begin{array}{rrrrr|l} 1 & -1 & -11 & 9 & 18 & 2 \\ & 2 & 2 & -18 & -18 & \\ \hline 1 & 1 & -9 & -9 & 0 & \end{array}$$

$=(x-2)(x^3+x^2-9x-9)$

$$\Rightarrow\frac{(x-2)(x^2+2x-5)}{(x-2)(x^3+x^2-9x-9)}$$

$$=\frac{x^2+2x-5}{x^3+x^2-9x-9}=\frac{2^2+2\cdot2-5}{2^3+2^2-9\cdot2-9}=\frac{3}{-15}=-\frac{1}{5}$$

17. $\dfrac{x^3-a^3}{x-a}$ *Para* $x\to a$

$$\frac{(x-a)(x^2+ax+a^2)}{x-a}=x^2+ax+a^2=a^2+a^2+a^2=3a^2$$

18. $\dfrac{a^2-2ab+b^2}{a^2-ab}$ *Para* $b\to a$ $\quad\dfrac{(a-b)^2}{a(a-b)}=\dfrac{a-b}{a}=\dfrac{a-a}{a}=0$

19. $\dfrac{x^2-y^2}{xy-y^2}$ *Para* $y\to x$

$$\frac{(x+y)(x-y)}{y(x-y)}=\frac{x+y}{y}=\frac{x+x}{x}=\frac{2x}{x}=2$$

20. $\dfrac{x^3-a^3}{a^2x-a^3}$ *Para* $x\to a$

$$\frac{(x-a)(x^2+ax+a^2)}{a^2(x-a)}$$

$$=\frac{x^2+ax+a^2}{a^2}=\frac{a^2+a^2+a^2}{a^2}=\frac{3a^2}{a^2}=3$$

21. $\dfrac{x^3-3x+2}{2x^3-6x^2+6x-2}$ *Para* $x\to 1$

$\Rightarrow x^3-3x+2$

$$\begin{array}{rrrr|l} 1 & 0 & -3 & 2 & 1 \\ & 1 & 1 & -2 & \\ \hline 1 & 1 & -2 & 0 & \end{array}$$

$=(x-1)(x^2+x-2)$

$=(x-1)(x+2)(x-1)$

$=(x-1)^2(x+2)$

$\Rightarrow 2x^3-6x^2+6x-2$

$$\begin{array}{rrrr|l} 2 & -6 & 6 & -2 & 1 \\ & 2 & -4 & 2 & \\ \hline 2 & -4 & 2 & 0 & \end{array}$$

$=(x-1)(2x^2-4x+2)$

$=(x-1)(x-1)(2x-2)$

$=(x-1)^2(2x-2)$

$$\Rightarrow\frac{(x-1)^2(x+2)}{(x-1)^2(2x-2)}=\frac{x+2}{2x-2}=\frac{1+2}{2\cdot1-2}=\frac{3}{0}=\infty$$

22. $\dfrac{x^4-x^3-7x^2+x+6}{x^4-3x^3-3x^2+11x-6}$ *Para* $x\to 3$

$\Rightarrow x^4-x^3-7x^2+x+6$

$$\begin{array}{rrrrr|l} 1 & -1 & -7 & 1 & 6 & -2 \\ & -2 & 6 & 2 & -6 & \\ \hline 1 & -3 & -1 & 3 & 0 & \end{array}$$

$(x+2)(x^3-3x^2-x+3)$

$$\begin{array}{rrrr|l} 1 & -3 & -1 & 3 & 1 \\ & 1 & -2 & -3 & \\ \hline 1 & -2 & -3 & 0 & \end{array}$$

$=(x+2)(x-1)(x^2-2x-3)$

$=(x+2)(x-1)(x-3)(x+1)$

Continúa

Continuación

22. $\Rightarrow x^4-3x^3-3x^2+11x-6$

$$\begin{array}{rrrrr|r} 1 & -3 & -3 & 11 & -6 & -2 \\ & -2 & 10 & -14 & 6 & \\ \hline 1 & -5 & 7 & -3 & 0 & \end{array}$$

$=(x+2)(x^3-5x^2+7x-3)$

$$\begin{array}{rrrr|r} 1 & -5 & 7 & -3 & 1 \\ & 1 & -4 & 3 & \\ \hline 1 & -4 & 3 & 0 & \end{array}$$

$=(x+2)(x-1)(x^2-4x+3)$

$=(x+2)(x-1)(x-3)(x-1)$

$$\Rightarrow \frac{(x+2)(x-1)(x-3)(x+1)}{(x+2)(x-1)(x-3)(x-1)}$$

$$=\frac{x+1}{x-1}=\frac{3+1}{3-1}=\frac{4}{2}=2$$

23. $\dfrac{3x^3-5x^2-4x+4}{x^4+2x^3-3x^2-8x-4}$ *Para* $x\to 2$

$\Rightarrow 3x^3-5x^2-4x+4$

$$\begin{array}{rrrr|r} 3 & -5 & -4 & 4 & 2 \\ & 6 & 2 & -4 & \\ \hline 3 & 1 & -2 & 0 & \end{array}$$

$=(x-2)(3x^2+x-2)$

$\Rightarrow x^4+2x^3-3x^2-8x-4$

$$\begin{array}{rrrrr|r} 1 & 2 & -3 & -8 & -4 & 2 \\ & 2 & 8 & 10 & 4 & \\ \hline 1 & 4 & 5 & 2 & 0 & \end{array}$$

$(x-2)(x^3+4x^2+5x+2)$

$$\begin{array}{rrrr|r} 1 & 4 & 5 & 2 & -2 \\ & -2 & -4 & -2 & \\ \hline 1 & 2 & 1 & 0 & \end{array}$$

$=(x-2)(x+2)(x^2+2x+1)$

$$\Rightarrow \frac{(x-2)(3x^2+x-2)}{(x-2)(x+2)(x+1)^2}$$

$$=\frac{3x^2+x-2}{(x+2)(x+1)^2}$$

$$=\frac{3\cdot 2^2+2-2}{(2+2)(2+1)^2}=\frac{12}{36}=\frac{1}{3}$$

24. $\dfrac{x^2-5x+4}{x^4-2x^3-9x^2+2x+8}$ *Para* $x\to 1$

$\Rightarrow x^2-5x+4=(x-1)(x-4)$

$\Rightarrow x^4-2x^3-9x^2+2x+8$

$$\begin{array}{rrrrr|r} 1 & -2 & -9 & 2 & 8 & 1 \\ & 1 & -1 & -10 & -8 & \\ \hline 1 & -1 & -10 & -8 & 0 & \end{array}$$

Continúa

Continuación

24. $(x-1)(x^3-x^2-10x-8)$

$$\begin{array}{rrrr|r} 1 & -1 & -10 & -8 & 4 \\ & 4 & 12 & 8 & \\ \hline 1 & 3 & 2 & 0 & \end{array}$$

$=(x-1)(x-4)(x^2+3x+2)$

$$\Rightarrow \frac{(x-1)(x-4)}{(x-1)(x-4)(x^2+3x+2)}$$

$$=\frac{1}{x^2+3x+2}=\frac{1}{1^2+3\cdot 1+2}=\frac{1}{6}$$

25. $\dfrac{x^5-4x^3+8x^2-32}{x^5-3x^3+10x^2-4x-40}$ *Para* $x\to 2$

$\Rightarrow x^5-4x^3+8x^2-32$

$$\begin{array}{rrrrrr|r} 1 & 0 & -4 & 8 & 0 & -32 & 2 \\ & 2 & 4 & 0 & 16 & 32 & \\ \hline 1 & 2 & 0 & 8 & 16 & 0 & \end{array}$$

$(x-2)(x^4+2x^3+8x+16)$

$$\begin{array}{rrrrr|r} 1 & 2 & 0 & 8 & 16 & -2 \\ & -2 & 0 & 0 & -16 & \\ \hline 1 & 0 & 0 & 8 & 0 & \end{array}$$

$=(x-2)(x+2)(x^3+8)$

$\Rightarrow x^5-3x^3+10x^2-4x-40$

$$\begin{array}{rrrrrr|r} 1 & 0 & -3 & 10 & -4 & -40 & 2 \\ & 2 & 4 & 2 & 24 & 40 & \\ \hline 1 & 2 & 1 & 12 & 20 & 0 & \end{array}$$

$(x-2)(x^4+2x^3+x^2+12x+20)$

$$\begin{array}{rrrrr|r} 1 & 2 & 1 & 12 & 20 & -2 \\ & -2 & 0 & -2 & -20 & \\ \hline 1 & 0 & 1 & 10 & 0 & \end{array}$$

$(x-2)(x+2)(x^3+x+10)$

$$\begin{array}{rrrr|r} 1 & 0 & 1 & 10 & -2 \\ & -2 & 4 & -10 & \\ \hline 1 & -2 & 5 & 0 & \end{array}$$

$=(x-2)(x+2)^2(x^2-2x+5)$

$$\Rightarrow \frac{(x-2)(x+2)(x^3+8)}{(x-2)(x+2)^2(x^2-2x+5)}$$

$$=\frac{x^3+8}{(x+2)(x^2-2x+5)}$$

$$=\frac{2^3+8}{(2+2)(2^2-2\cdot 2+5)}=\frac{16}{20}=\frac{4}{5}$$

26. $\dfrac{8x^2+6x-9}{12x^2-13x+3}$ *Para* $x\to \dfrac{3}{4}$

$$\Rightarrow \frac{(8x)^2+6(8x)-72}{(12x)^2-13(12x)+36}$$

$$=\frac{(8x+12)(8x-6)}{(12x-9)(12x-4)}$$

$$=\frac{(2x+3)(4x-3)}{(4x-3)(3x-1)}$$

$$=\frac{2x+3}{3x-1}=\frac{2\cdot\frac{3}{4}+3}{3\cdot\frac{3}{4}-1}=\frac{\frac{9}{2}}{\frac{5}{4}}=\frac{36}{10}=\frac{18}{5}$$

27. $\dfrac{x^3+6x^2+12x+8}{x^4-8x^2+16}$ *Para* $x\to -2$

$\Rightarrow x^3+6x^2+12x+8$

$$\begin{array}{rrrr|r} 1 & 6 & 12 & 8 & -2 \\ & -2 & -8 & -8 & \\ \hline 1 & 4 & 4 & 0 & \end{array}$$

$=(x+2)(x^2+4x+4)=(x+2)^3$

$\Rightarrow x^4-8x^2+16$

$$\begin{array}{rrrrr|r} 1 & 0 & -8 & 0 & 16 & -2 \\ & -2 & 4 & 8 & -16 & \\ \hline 1 & -2 & -4 & 8 & 0 & \end{array}$$

$(x+2)(x^3-2x^2-4x+8)$

$$\begin{array}{rrrr|r} 1 & -2 & -4 & 8 & -2 \\ & -2 & 8 & -8 & \\ \hline 1 & -4 & 4 & 0 & \end{array}$$

$=(x+2)^2(x-4x+4)$

$=(x+2)^2(x-2)^2$

$$\Rightarrow \frac{(x+2)^3}{(x+2)^2(x-2)^2}$$

$$=\frac{x+2}{(x-2)^2}=\frac{-2+2}{(-2-2)^2}=\frac{0}{16}=0$$

28. $\dfrac{9x^3+3x^2+3x+1}{27x^3+1}$ *Para* $x\to -\dfrac{1}{3}$

$$=\frac{3x^2(3x+1)+(3x+1)}{27x^3+1}$$

$$=\frac{(3x^2+1)(3x+1)}{(3x+1)(9x^2-3x+1)}$$

$$=\frac{3x^2+1}{9x^2-3x+1}$$

$$=\frac{3\left(-\frac{1}{3}\right)^2+1}{9\left(-\frac{1}{3}\right)^2-3\left(-\frac{1}{3}\right)+1}=\frac{\frac{4}{3}}{3}=\frac{4}{9}$$

29. $\dfrac{1}{x-1}-\dfrac{3}{x^3-1}\quad Para\, x\to 1\quad \dfrac{x^2+x+1-3}{x^3-1}=\dfrac{x^2+x-2}{x^3-1}=\dfrac{(x-1)(x+2)}{(x-1)(x^2+x+1)}=\dfrac{x+2}{x^2+x+1}=\dfrac{1+2}{1^2+1+1}=\dfrac{3}{3}=1$

30. $\left(x^2+3x-10\right)\left(1+\dfrac{1}{x-2}\right)\quad Para\, x\to 2\qquad (x+5)(x-2)\left(\dfrac{x-1}{x-2}\right)=(x+5)(x-1)\Rightarrow(2+5)(2-1)=7$

EJERCICIO 140

1. $\dfrac{12x^2+31x+20}{18x^2+21x-4}$

$=\dfrac{(12x)^2+31(12x)+240}{(18x)^2+21(18x)-72}$

$=\dfrac{(12x+16)(12x+15)}{(18x+24)(18x-3)}=\dfrac{(3x+4)(4x+5)}{(3x+4)(6x-1)}=\dfrac{4x+5}{6x-1}$

2. $\left(\dfrac{1}{a}+\dfrac{2}{a^2}+\dfrac{1}{a^3}\right)\div\left(a+2-\dfrac{2a+1}{a}\right)$

$=\left(\dfrac{a^2+2a+1}{a^3}\right)\div\left(\dfrac{a^2+2a-2a-1}{a}\right)$

$=\dfrac{(a+1)^2}{a^3}\cdot\dfrac{a}{(a+1)(a-1)}=\dfrac{a+1}{a^2(a-1)}=\dfrac{a+1}{a^3-a^2}$

3. $\dfrac{x^3+3x^2+9x}{x^5-27x^2}$

$=\dfrac{x\left(x^2+3x+9\right)}{x^2\left(x^3-27\right)}=\dfrac{x^2+3x+9}{x(x-3)\left(x^2+3x+9\right)}=\dfrac{1}{x(x-3)}$

4. $\dfrac{(x+y)^2}{y}-\dfrac{x(x-y)^2}{xy}$

$=\dfrac{x(x+y)^2-x(x-y)^2}{xy}$

$=\dfrac{x^3+2x^2y+xy^2-x^3+2x^2y-xy^2}{xy}=\dfrac{4x^2y}{xy}=4x$

5. $\dfrac{a^4-2b^3+a^2b(b-2)}{a^4-a^2b-2b^2}$

$=\dfrac{a^4-2b^3+a^2b^2-2a^2b}{\left(a^2-2b\right)\left(a^2+b\right)}$

$=\dfrac{a^2\left(a^2+b^2\right)-2b\left(b^2+a^2\right)}{\left(a^2-2b\right)\left(a^2+b\right)}=\dfrac{\left(a^2-2b\right)\left(a^2+b^2\right)}{\left(a^2-2b\right)\left(a^2+b\right)}=\dfrac{a^2+b^2}{a^2+b}$

6. $a+\dfrac{1+5a}{a^2-5}=\dfrac{a^3-5a+1+5a}{a^2-5}=\dfrac{a^3+1}{a^2-5}=\dfrac{(a+1)\left(a^2-a+1\right)}{a^2-5}$

$a-\dfrac{a+5}{a+1}=\dfrac{a^2+a-a-5}{a+1}=\dfrac{a^2-5}{a+1}$

$\Rightarrow\dfrac{(a+1)\left(a^2-a+1\right)}{a^2-5}\cdot\dfrac{a^2-5}{a+1}=a^2-a+1$

7. $x^2+5x-4-\dfrac{x^3-29}{x-5}$

$=\dfrac{(x-5)\left(x^2+5x-4\right)-x^3+29}{x-5}$

$=\dfrac{x^3-29x+20-x^3+29}{x-5}=\dfrac{49-29x}{x-5}$

$x+34+\dfrac{170-x^2}{x-5}$

$=\dfrac{x^2+29x-170+170-x^2}{x-5}=\dfrac{29x}{x-5}$

$\Rightarrow\dfrac{49-29x}{x-5}\div\dfrac{29x}{x-5}=\dfrac{49-29x}{x-5}\cdot\dfrac{x-5}{29x}=\dfrac{49-29x}{29x}$

8. $\dfrac{4x^2-5xy+y^2}{3x}=\dfrac{4x^2}{3x}-\dfrac{5xy}{3x}+\dfrac{y^2}{3x}=\dfrac{4x}{3}-\dfrac{5y}{3}+\dfrac{y^2}{3x}$

9. $\dfrac{m-n-x}{mnx}=\dfrac{m}{mnx}-\dfrac{n}{mnx}-\dfrac{x}{mnx}=\dfrac{1}{nx}-\dfrac{1}{mx}-\dfrac{1}{mn}$

10. $\dfrac{x^3-xy^2}{x-y}=x^2+xy$

$\Rightarrow\dfrac{x\left(x^2-y^2\right)}{x-y}=x(x+y)$

$\Rightarrow\dfrac{x(x+y)(x-y)}{x-y}=x(x+y)\Rightarrow x(x+y)=x(x+y)$

11. $x^2-2x+1-\dfrac{9x-3x^2}{x-3}=\dfrac{x^3-1}{x-1}$

$\Rightarrow\dfrac{(x-1)^2(x-3)-3x(3-x)}{x-3}=\dfrac{(x-1)\left(x^2+x+1\right)}{x-1}$

$\Rightarrow\dfrac{x^3-5x^2+7x-3-9x+3x^2}{x-3}=x^2+x+1$

$\Rightarrow\dfrac{x^3-2x^2-2x-3}{x-3}=x^2+x+1$

$\Rightarrow\dfrac{(x-3)\left(x^2+x+1\right)}{x-3}=x^2+x+1\Rightarrow\ x^2+x+1=x^2+x+1$

12. $\dfrac{a^4-5a^2+4}{a^3+a^2-4a-4}=a-3+\dfrac{2+4a}{2a+1}$

$\Rightarrow\dfrac{(a-2)(a-1)(a+2)(a+1)}{(a-2)(a+1)(a+2)}=\dfrac{2a^2-5a-3+2+4a}{2a+1}$

$\Rightarrow a-1=\dfrac{2a^2-a-1}{2a+1}\Rightarrow a-1=\dfrac{(2a+1)(a-1)}{2a+1}\Rightarrow a-1=a-1$

13. $\dfrac{1}{a-b}+\dfrac{1}{a+b}+\dfrac{2a}{a^2-ab+b^2}$

$$=\frac{a^3+b^3+(a-b)(a^2-ab+b^2)+2a(a^2-b^2)}{(a^3+b^3)(a-b)}$$

$$=\frac{a^3+b^3+a^3-2a^2b+2ab^2-b^3+2a^3-2ab^2}{(a^3+b^3)(a-b)}$$

$$=\frac{4a^3-2a^2b}{(a^3+b^3)(a-b)}$$

14. $\left(\dfrac{a^2}{1-a^2}-\dfrac{a^4}{1-a^4}\right)\cdot\left(1-a+\dfrac{1+a^3}{a^2}\right)$

$$=\left(\frac{a^2(1+a^2)-a^4}{1-a^4}\right)\cdot\left(\frac{a^2(1-a)+1+a^3}{a^2}\right)$$

$$=\left(\frac{a^2+a^4-a^4}{1-a^4}\right)\cdot\left(\frac{a^2-a^3+1+a^3}{a^2}\right)$$

$$=\frac{a^2}{1-a^4}\cdot\frac{a^2+1}{a^2}=\frac{1}{1-a^2}$$

15. $\left(\dfrac{x^2-9}{x^2-x-12}\div\dfrac{x-3}{x^2+3x}\right)\cdot\dfrac{a^2x^2-16a^2}{2x^2+7x+3}\cdot\left(\dfrac{2}{a^2x}+\dfrac{1}{a^2x^2}\right)$

$$=\left(\frac{(x+3)(x-3)}{(x-4)(x+3)}\cdot\frac{x(x+3)}{x-3}\right)\cdot\frac{a^2(x-4)(x+4)}{(x+3)(2x+1)}\cdot\left(\frac{2x+1}{a^2x^2}\right)=\frac{x+4}{x}$$

16. $\dfrac{3x^3-x^2-12x+4}{6x^4+x^3-25x^2-4x+4}$

$\Rightarrow 3x^3-x^2-12x+4$

$$\begin{array}{cccc|c} 3 & -1 & -12 & 4 & -2 \\ & -6 & 14 & -4 & \\ \hline 3 & -7 & 2 & 0 & \end{array}$$

$=(x+2)(3x^2-7x+2)=(x+2)(x-2)(3x-1)$

$6x^4+x^3-25x^2-4x+4$

$$\begin{array}{ccccc|c} 6 & 1 & -25 & -4 & 4 & -2 \\ & -12 & 22 & 6 & -4 & \\ \hline 6 & -11 & -3 & 2 & 0 & \end{array}$$

$(x+2)(6x^3-11x^2-3x+2)$

$$\begin{array}{cccc|c} 6 & -11 & -3 & 2 & 2 \\ & 12 & 2 & -2 & \\ \hline 6 & 1 & -1 & 0 & \end{array}$$

$=(x-2)(x+2)(6x^2+x-1)$

$=(x-2)(x+2)(2x+1)(3x-1)$

$$\Rightarrow\frac{(x+2)(x-2)(3x-1)}{(x-2)(x+2)(2x+1)(3x-1)}=\frac{1}{2x+1}$$

17. $\dfrac{16-81x^2}{72x^2-5x-12}$

$$=\frac{(4-9x)(4+9x)}{(72x)^2-5(72x)-864}$$

$$=\frac{(4-9x)(4+9x)}{(72x-32)(72x+27)}$$

$$=\frac{(4-9x)(4+9x)}{(9x-4)(8x+3)}=-\frac{(4-9x)(4+9x)}{(4-9x)(8x+3)}=-\frac{9x+4}{8x+3}$$

18. $\left(\dfrac{1}{x}-\dfrac{2}{x+2}+\dfrac{3}{x+3}\right)\div\left(\dfrac{x}{x+2}+\dfrac{x}{x+3}+\dfrac{6}{x^2+5x+6}\right)$

$$=\frac{x^2+5x+6-2x(x+3)+3x(x+2)}{x(x^2+5x+6)}\div\frac{x(x+3)+x(x+2)+6}{x^2+5x+6}$$

$$=\frac{x^2+5x+6-2x^2-6x+3x^2+6x}{x(x^2+5x+6)}\cdot\frac{x^2+5x+6}{x^2+3x+x^2+2x+6}$$

$$=\frac{2x^2+5x+6}{x}\cdot\frac{1}{2x^2+5x+6}=\frac{1}{x}$$

19. $\dfrac{\frac{b}{a}}{1-\frac{b^2}{a^2}}+\dfrac{1+\frac{b}{a-b}}{2-\frac{a-3b}{a-b}}$

$$=\frac{\frac{b}{a}}{\frac{a^2-b^2}{a^2}}+\frac{\frac{a-b+b}{a-b}}{\frac{2a-2b-a+3b}{a-b}}$$

$$=\frac{a^2b}{a(a^2-b^2)}+\frac{\frac{a}{a-b}}{\frac{a+b}{a-b}}$$

$$=\frac{a^2b}{a(a^2-b^2)}+\frac{a}{a+b}$$

$$=\frac{a^2b+a^2(a-b)}{a(a^2-b^2)}=\frac{a^2b+a^3-a^2b}{a(a^2-b^2)}$$

$$=\frac{a^3}{a(a^2-b^2)}=\frac{a^2}{a^2-b^2}$$

20. $\dfrac{1}{3}\left(\dfrac{x^2-36}{x}\div\dfrac{x}{x^2-4}\right)\cdot\dfrac{1}{x-\frac{36}{x}}\cdot\dfrac{1}{x-\frac{4}{x}}$

$$=\frac{1}{3}\left(\frac{(x+6)(x-6)}{x}\cdot\frac{(x+2)(x-2)}{x}\right)\cdot\frac{1}{\frac{x^2-36}{x}}\cdot\frac{1}{\frac{x^2-4}{x}}$$

$$=\frac{(x+6)(x-6)}{3x}\cdot\frac{(x+2)(x-2)}{x}\cdot\frac{x}{x^2-36}\cdot\frac{x}{x^2-4}$$

$$=\frac{x^2(x+6)(x-6)(x+2)(x-2)}{3x^2(x+6)(x-6)(x+2)(x-2)}=\frac{1}{3}$$

21. $$\frac{\dfrac{3a}{(a-2b)^2}+\dfrac{5}{a-5b}+\dfrac{1}{a-2b}}{\dfrac{3a^2-14ab+10b^2}{a^2-4ab+4b^2}}$$

$$=\frac{\dfrac{3a(a-5b)+5(a-2b)^2+(a-2b)(a-5b)}{(a-2b)^2(a-5b)}}{\dfrac{3a^2-14ab+10b^2}{(a-2b)^2}}$$

$$=\frac{3a^2-15ab+5a^2-20ab+20b^2+a^2-7ab+10b^2}{(a-5b)(3a^2-14ab+10b^2)}$$

$$=\frac{9a^2-42ab+30b^2}{(a-5b)(3a^2-14ab+10b^2)}=\frac{3(3a^2-14ab+10b^2)}{(a-5b)(3a^2-14ab+10b^2)}=\frac{3}{a-5b}$$

22. $$\frac{\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}}{\dfrac{x-1}{x+1}+\dfrac{x+1}{x-1}}\cdot\frac{x^2+1}{2a^2-2b}\div\frac{2x}{a^2-b}$$

$$=\frac{\dfrac{(x+1)^2-(x-1)^2}{x^2-1}}{\dfrac{(x-1)^2+(x+1)^2}{x^2-1}}\cdot\frac{x^2+1}{2(a^2-b)}\cdot\frac{a^2-b}{2x}$$

$$=\frac{x^2+2x+1-x^2+2x-1}{x^2-2x+1+x^2+2x+1}\cdot\frac{x^2+1}{4x}=\frac{4x}{2(x^2+1)}\cdot\frac{x^2+1}{4x}=\frac{1}{2}$$

23. $$\frac{1}{3x-9}-\frac{1}{6x+12}-\frac{1}{2(x-3)^2}+\frac{1}{x-6+\dfrac{9}{x}}$$

$$=\frac{1}{3(x-3)}-\frac{1}{6(x+2)}-\frac{1}{2(x-3)^2}+\frac{x}{x^2-6x+9}$$

$$=\frac{2(x-3)(x+2)-(x-3)^2-3(x+2)+6(x+2)}{6(x-3)^2(x+2)}$$

$$=\frac{2x^2-2x-12-x^2+6x-9-3x-6+6x^2+12x}{6(x-3)^2(x+2)}$$

$$=\frac{7x^2+13x-27}{6(x-3)^2(x+2)}$$

24. $$\frac{a-b+\dfrac{a^2+b^2}{a+b}}{a+b-\dfrac{a^2-2b^2}{a-b}}\cdot\frac{b+\dfrac{b^2}{a}}{a-b}\cdot\frac{1}{1+\dfrac{2a-b}{b}}$$

$$=\frac{\dfrac{a^2-b^2+a^2+b^2}{a+b}}{\dfrac{a^2-b^2-a^2+2b^2}{a-b}}\cdot\frac{\dfrac{ab+b^2}{a}}{a-b}\cdot\frac{1}{\dfrac{b+2a-b}{b}}$$

$$=\frac{\dfrac{2a^2}{a+b}}{\dfrac{b^2}{a-b}}\cdot\frac{\dfrac{b(a+b)}{a}}{a-b}\cdot\frac{1}{\dfrac{2a}{b}}=\frac{2a^2(a-b)}{b^2(a+b)}\cdot\frac{b(a+b)}{a(a-b)}\cdot\frac{b}{2a}=1$$

EJERCICIO 141

1. $\dfrac{x}{6}+5=\dfrac{1}{3}-x$

$\Rightarrow \dfrac{x+30}{6}=\dfrac{1-3x}{3}$

$x+30=2(1-3x)$

$x+30=2-6x$

$x+6x=2-30$

$7x=-28$

$x=\dfrac{-28}{7}$

$x=-4$

2. $\dfrac{3x}{5}-\dfrac{2x}{3}+\dfrac{1}{5}=0 \quad mcm=15$

$9x-10x+3=0$

$-x=-3$

$x=3$

3. $\dfrac{1}{2x}+\dfrac{1}{4}-\dfrac{1}{10x}=\dfrac{1}{5} \quad mcm=20x$

$10+5x-2=4x$

$5x-4x=-10+2$

$x=-8$

4. $\dfrac{x}{2}+2-\dfrac{x}{12}=\dfrac{x}{6}-\dfrac{5}{4} \quad mcm=12$

$6x+24-x=2x-15$

$6x-x-2x=-15-24$

$3x=-39$

$x=\dfrac{-39}{3}$

$x=-13$

5. $\dfrac{3x}{4}-\dfrac{1}{5}+2x=\dfrac{5}{4}-\dfrac{3x}{20} \quad mcm=20$

$15x-4+40x=25-3x$

$55x+3x=25+4$

$58x=29$

$x=\dfrac{29}{58}$

$x=\dfrac{1}{2}$

6. $\dfrac{2}{3x}-\dfrac{5}{x}=\dfrac{7}{10}-\dfrac{3}{2x}+1 \quad mcm=30x$

$20-150=21x-45+30x$

$-130+45=51x$

$\dfrac{-85}{51}=x$

$\dfrac{-5}{3}=x$

7. $\dfrac{x-4}{3}-5=0 \quad mcm=3$

$x-4-15=0$

$x-19=0$

$x=19$

8. $x-\dfrac{x+2}{12}=\dfrac{5x}{2} \quad mcm=12$

$12x-x-2=30x$

$11x-30x=2$

$-19x=2$

$x=-\dfrac{2}{19}$

9. $x-\dfrac{5x-1}{3}=4x-\dfrac{3}{5} \quad mcm=15$

$15x-25x+5=60x-9$

$-10x-60x=-9-5$

$-70x=-14$

$x=\dfrac{-14}{-70}$

$x=\dfrac{1}{5}$

10. $10x-\frac{8x-3}{4}=2\left(x-3\right) \quad mcm=4$

$$40x-8x+3=8\left(x-3\right)$$
$$40x-8x=8x-24-3$$
$$40x-16x=-27$$
$$x=\frac{-27}{24}$$
$$x=-\frac{9}{8}$$

11. $\frac{x-2}{3}-\frac{x-3}{4}=\frac{x-4}{5} \quad mcm=60$

$$20\left(x-2\right)-15\left(x-3\right)=12\left(x-4\right)$$
$$20x-40-15x+45=12x-48$$
$$5x-12x=-48-5$$
$$-7x=-53$$
$$x=\frac{53}{7}$$

12. $\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}=-\frac{x-5}{5}$

$mcm=60$

$$30\left(x-1\right)-20\left(x-2\right)-15\left(x-3\right)=-12\left(x-5\right)$$
$$30x-30-20x+40-15x+45=-12x+60$$
$$-5x+55=-12x+60$$
$$-5x+12x=60-55$$
$$7x=5$$
$$x=\frac{5}{7}$$

13. $x-\left(5x-1\right)-\frac{7-5x}{10}=1 \quad mcm=10$

$$10x-10\left(5x-1\right)-\left(7-5x\right)=10$$
$$10x-50x+10-7+5x=10$$
$$-35x+3=10$$
$$-35x=7$$
$$x=\frac{-7}{35}$$
$$x=-\frac{1}{5}$$

14. $2x-\frac{5x-6}{4}+\frac{1}{3}\left(x-5\right)=-5x$

$mcm=12$

$$24x-3\left(5x-6\right)+4\left(x-5\right)=-60x$$
$$24x-15x+18+4x-20=-60x$$
$$13x-2=-60x$$
$$13x+60x=2$$
$$x=\frac{2}{73}$$

15. $4-\frac{10x+1}{6}=4x-\frac{16x+3}{4} \quad mcm=12$

$$48-2\left(10x+1\right)=48x-3\left(16x+3\right)$$
$$48-20x-2=48x-48x-9$$
$$-20x+46=-9$$
$$-20x=-55$$
$$x=\frac{-55}{-20}$$
$$x=\frac{11}{4}$$

16. $\frac{1}{2}\left(x-1\right)-\left(x-3\right)=\frac{1}{3}\left(x+3\right)+\frac{1}{6} \quad mcm=6$

$$3\left(x-1\right)-6\left(x-3\right)=2\left(x+3\right)+1$$
$$3x-3-6x+18=2x+6+1$$
$$-3x+15=2x+7$$
$$-5x=-8$$
$$x=\frac{8}{5}$$

17. $\frac{6x+1}{3}-\frac{11x-2}{9}-\frac{1}{4}\left(5x-2\right)=\frac{5}{6}\left(6x+1\right) \quad mcm=36$

$$12\left(6x+1\right)-4\left(11x-2\right)-9\left(5x-2\right)=30\left(6x+1\right)$$
$$72x+12-44x+8-45x+18=180x+30$$
$$-17x+38=180x+30$$
$$-197x=-8$$
$$x=\frac{8}{197}$$

18. $\frac{4x+1}{3}=\frac{1}{3}\left(4x-1\right)-\frac{13+2x}{6}-\frac{1}{2}\left(x-3\right) \quad mcm=6$

$$2\left(4x+1\right)=2\left(4x-1\right)-\left(13+2x\right)-3\left(x-3\right)$$
$$8x+2=8x-2-13-2x-3x+9$$
$$2=-6-5x$$
$$8=-5x$$
$$-\frac{8}{5}=x$$

19. $\frac{2}{5}\left(5x-1\right)+\frac{3}{10}\left(10x-3\right)=-\frac{1}{2}\left(x-2\right)-\frac{6}{5} \quad mcm=10$

$$4\left(5x-1\right)+3\left(10x-3\right)=-5\left(x-2\right)-12$$
$$20x-4+30x-9=-5x+10-12$$
$$50x-13=-5x+10-12$$
$$50x-13=-5x-2$$
$$55x=11$$
$$x=\frac{1}{5}$$

20. $\frac{3x-1}{2}-\frac{5x+4}{3}-\frac{x+2}{8}=\frac{2x-3}{5}-\frac{1}{10}$

$mcm=120$

$60(3x-1)-40(5x+4)-15(x+2)=24(2x-3)-12$

$180x-60-200x-160-15x-30=48x-72-12$

$-35x-250=48x-84$

$-83x=166$

$x=-2$

21. $\frac{7x-1}{3}-\frac{5-2x}{2x}=\frac{4x-3}{4}+\frac{1+4x^2}{3x}$

$mcm=12x$

$4x(7x-1)-6(5-2x)=3x(4x-3)+4(1+4x^2)$

$28x^2-4x-30+12x=12x^2-9x+4+16x^2$

$8x-30=-9x+4$

$17x=34$

$x=2$

22. $\frac{2x+7}{3}-\frac{2(x^2-4)}{5x}-\frac{4x^2-6}{15x}=\frac{7x^2+6}{3x^2}$

$mcm=15x^2$

$5x^2(2x+7)-6x(x^2-4)-x(4x^2-6)=5(7x^2+6)$

$10x^3+35x^2-6x^3+24x-4x^3+6x=35x^2+30$

$30x=30$

$x=\frac{30}{30}$

$x=1$

23. $\frac{2}{3}\left(\frac{x+1}{5}\right)=\frac{3}{4}\left(\frac{x-6}{3}\right)$ $\quad mcm=60$

$8(x+1)=15(x-6)$

$8x+8=15x-90$

$8x-15x=-90-8$

$-7x=-98$

$x=\frac{-98}{-7}$

$x=14$

24. $\frac{3}{5}\left(\frac{2x-1}{6}\right)-\frac{4}{3}\left(\frac{3x+2}{4}\right)-\frac{1}{5}\left(\frac{x-2}{3}\right)+\frac{1}{5}=0$

$mcm=60$

$6(2x-1)-20(3x+2)-4(x-2)+12=0$

$12x-6-60x-40-4x+8+12=0$

$-52x-26=0$

$x=-\frac{26}{52}=-\frac{1}{2}$

25. $10-\frac{3x+5}{6}=3\tfrac{11}{12}-\frac{\frac{x}{2}}{4}$

$\Rightarrow \frac{60-3x-5}{6}=\frac{47}{12}-\frac{x}{8}$ $\quad mcm=24$

$4(55-3x)=94-3x$

$220-12x=94-3x$

$-12x+3x=94-220$

$-9x=-126$

$x=\frac{-126}{-9}$

$x=14$

26. $9x-2-7x\left(\frac{1}{x}-\frac{1}{2}\right)=\frac{1+\frac{x}{2}}{2}+2\tfrac{3}{4}$

$\Rightarrow 9x-2-7x\left(\frac{2-x}{2}\right)=\frac{x+2}{4}+\frac{11}{4}$

$\Rightarrow 9x-2-7\left(\frac{2-x}{2}\right)=\frac{x+13}{4}$ $\quad mcm=4$

$36x-8-14(2-x)=x+13$

$36x-8-28+14x=x+13$

$50x-36=x+13$

$49x=49$

$x=1$

27. $\frac{3x}{8}-\frac{7}{10}-\frac{12x-5}{16}-\frac{2x-3}{20}+\frac{4x+9}{4}+\frac{7}{80}=0$ $\quad mcm=160$

$60x-112-10(12x-5)-8(2x-3)+40(4x+9)+14=0$

$60x-112-120x+50-16x+24+160x+360+14=0$

$84x+336=0$

$84x=-336$

$x=\frac{-336}{84}$

$x=-4$

28. $\frac{5x}{4}-\frac{3}{17}(x-20)-(2x-1)=\frac{x+24}{34}$ $\quad mcm=68$

$85x-12(x-20)-68(2x-1)=2(x+24)$

$85x-12x+240-136x+68=2x+48$

$-63x+308=2x+48$

$-65x=-260$

$x=\frac{-260}{-65}$

$x=4$

29. $5+\frac{x}{4}=\frac{1}{3}\left(2-\frac{x}{2}\right)-\frac{2}{3}+\frac{1}{4}\left(10-\frac{5x}{3}\right)$

$\Rightarrow \frac{20+x}{4}=\frac{1}{3}\left(\frac{4-x}{2}\right)-\frac{2}{3}+\frac{1}{4}\left(\frac{30-5x}{3}\right)$

$\Rightarrow \frac{20+x}{4}=\frac{4-x}{6}-\frac{2}{3}+\frac{30-5x}{12}$

$mcm=12$

$3(20+x)=2(4-x)-8+30-5x$

$60+3x=8-2x-8+30-5x$

$60+3x=-7x+30$

$10x=-30$

$x=-3$

30. $\frac{5(x+2)}{12}+\frac{4}{9}-\frac{22-x}{36}=3x-20-\frac{8-x}{12}-\frac{20-3x}{18}$ $mcm=36$

$15(x+2)+16-22+x=108x-720-3(8-x)-2(20-3x)$

$15x+30-6+x=108x-720-24+3x-40+6x$

$16x+24=117x-784$

$-101x=-808$

$x=8$

31. $\left(3-\frac{x}{2}\right)-\left(1-\frac{x}{3}\right)=7-\left(x-\frac{x}{2}\right)$

$\Rightarrow \frac{6-x}{2}-\frac{3-x}{3}=7-\frac{x}{2}$ $mcm=6$

$3(6-x)-2(3-x)=42-3x$

$18-3x-6+2x=42-3x$

$12-x=42-3x$

$2x=30$

$x=15$

32. $(x+3)(x-3)-x^2-\frac{5}{4}=\left(x-\frac{x}{5}\right)-\left(3x-\frac{3}{4}\right)$

$\Rightarrow x^2-9-x^2-\frac{5}{4}=\frac{4x}{5}-\frac{12x-3}{4}$

$\Rightarrow -\frac{41}{4}=\frac{16x-60x+15}{20}$

$\Rightarrow -\frac{41}{4}=\frac{-44x+15}{20}$ $mcm=20$

$-205=-44x+15$

$-220=-44x$

$5=x$

33. $2x-\left(2x-\frac{3x-1}{8}\right)=\frac{2}{3}\left(\frac{x+2}{6}\right)-\frac{1}{4}$

$\Rightarrow 2x-\frac{16x-3x+1}{8}=\frac{2x+4}{18}-\frac{1}{4}$ $mcm=72$

$144x-9(13x+1)=4(2x+4)-18$

$144x-117x-9=8x+16-18$

$27x-9=8x-2$

$19x=7$

$x=\frac{7}{19}$

EJERCICIO 142

1. $\frac{3}{5}+\frac{3}{2x-1}=0$ $mcm=5(2x-1)$

$3(2x-1)+15=0$

$6x-3+15=0$

$6x=-12$

$x=-2$

2. $\frac{2}{4x-1}=\frac{3}{4x+1}$ $mcm=16x^2-1$

$2(4x+1)=3(4x-1)$

$8x+2=12x-3$

$-4x=-5$

$x=\frac{5}{4}$

3. $\frac{5}{x^2-1}=\frac{1}{x-1}$ $mcm=x^2-1$

$5=x+1$

$4=x$

4. $\frac{3}{x+1}-\frac{1}{x^2-1}=0$ $mcm=x^2-1$

$3(x-1)-1=0$

$3x-3-1=0$

$3x=4$

$x=\frac{4}{3}$

5. $\frac{5x+8}{3x+4}=\frac{5x+2}{3x-4}$ $mcm=(3x+4)(3x-4)$

$(3x-4)(5x+8)=(5x+2)(3x+4)$

$15x^2+4x-32=15x^2+26x+8$

$-26x+4x=8+32$

$-22x=40$

$x=-\frac{40}{22}$

$x=-\frac{20}{11}$

6. $\frac{10x^2-5x+8}{5x^2+9x-19}=2$ $mcm=5x^2+9x-19$

$10x^2-5x+8=2(5x^2+9x-19)$

$10x^2-5x+8=10x^2+18x-38$

$-5x-18x=-38-8$

$-23x=-46$

$x=\frac{-46}{-23}$

$x=2$

7. $\frac{1}{3x-3}+\frac{1}{4x+4}=\frac{1}{12x-12}$ $mcm=12\left(x^2-1\right)$

$4\left(x+1\right)+3\left(x-1\right)=x+1$

$4x+4+3x-3=x+1$

$7x-x=0$

$6x=0$

$x=0$

8. $\frac{x}{4}-\frac{x^2-8x}{4x-5}=\frac{7}{4}$ $mcm=4\left(4x-5\right)$

$x\left(4x-5\right)-4\left(x^2-8x\right)=7\left(4x-5\right)$

$4x^2-5x-4x^2+32x=28x-35$

$27x=28x-35$

$-x=-35$

$x=35$

9. $\frac{2x-9}{10}+\frac{2x-3}{2x-1}=\frac{x}{5}$ $mcm=10\left(2x-1\right)$

$\left(2x-9\right)\left(2x-1\right)+10\left(2x-3\right)=2x\left(2x-1\right)$

$4x^2-20x+9+20x-30=4x^2-2x$

$-21=-2x$

$\frac{-21}{-2}=x$

$10\tfrac{1}{2}=x$

10. $\frac{\left(3x-1\right)^2}{x-1}=\frac{18x-1}{2}$ $mcm=2\left(x-1\right)$

$2\left(9x^2-6x+1\right)=\left(x-1\right)\left(18x-1\right)$

$18x^2-12x+2=18x^2-19x+1$

$-12x+19x=1-2$

$7x=-1$

$x=-\frac{1}{7}$

11. $\frac{2x+7}{5x+2}-\frac{2x-1}{5x-4}=0$ $mcm=\left(5x+2\right)\left(5x-4\right)$

$\left(2x+7\right)\left(5x-4\right)-\left(2x-1\right)\left(5x+2\right)=0$

$10x^2+27x-28-10x^2+x+2=0$

$28x=26$

$x=\frac{13}{14}$

12. $\frac{\left(5x-2\right)\left(7x+3\right)}{7x\left(5x-1\right)}-1=0$ $mcm=7x\left(5x-1\right)$

$\left(5x-2\right)\left(7x+3\right)-7x\left(5x-1\right)=0$

$35x^2+x-6-35x^2+7x=0$

$8x=6$

$x=\frac{3}{4}$

13. $\frac{3}{x-4}=\frac{2}{x-3}+\frac{8}{x^2-7x+12}$ $mcm=x^2-7x+12$

$3\left(x-3\right)=2\left(x-4\right)+8$

$3x-9=2x-8+8$

$3x-2x=9$

$x=9$

14. $\frac{6x-1}{18}-\frac{3\left(x+2\right)}{5x-6}=\frac{1+3x}{9}$ $mcm=18\left(5x-6\right)$

$\left(6x-1\right)\left(5x-6\right)-54\left(x+2\right)=2\left(1+3x\right)\left(5x-6\right)$

$30x^2-41x+6-54x-108=-26x-12+30x^2$

$-95x-102=-26x-12$

$-69x=90$

$x=-\frac{90}{69}$

$x=-1\tfrac{7}{23}$

15. $\frac{5}{1+x}-\frac{3}{1-x}-\frac{6}{1-x^2}=0$ $mcm=1-x^2$

$5\left(1-x\right)-3\left(1+x\right)-6=0$

$5-5x-3-3x-6=0$

$-8x-4=0$

$-8x=4$

$x=-\frac{1}{2}$

16. $\frac{1+2x}{1+3x}-\frac{1-2x}{1-3x}=-\frac{3x-14}{1-9x^2}$ $mcm=1-9x^2$

$\left(1+2x\right)\left(1-3x\right)-\left(1-2x\right)\left(1+3x\right)=-3x+14$

$1-x-6x^2-1-x+6x^2=-3x+14$

$-2x=-3x+14$

$x=14$

17. $\frac{3x-1}{x^2+7x+12}=\frac{1}{2x+6}+\frac{7}{6x+24}$

$\Rightarrow x^2+7x+12=\left(x+3\right)\left(x+4\right)$

$\Rightarrow 2x+6=2\left(x+3\right)$

$\Rightarrow 6x+24=6\left(x+4\right)$ $mcm=6\left(x+3\right)\left(x+4\right)$

$6\left(3x-1\right)=3\left(x+4\right)+7\left(x+3\right)$

$18x-6=3x+12+7x+21$

$18x-6=10x+33$

$8x=39$

$x=\frac{39}{8}$

$x=4\tfrac{7}{8}$

18. $\frac{1}{(x-1)^2}-\frac{3}{2x-2}=-\frac{3}{2x+2}\quad mcm=2(x-1)^2(x+1)$

$$2(x+1)-3(x-1)(x+1)=-3(x-1)^2$$
$$2x+2-3x^2+3=-3x^2+6x-3$$
$$2x+5=6x-3$$
$$-4x=-8$$
$$x=2$$

19. $\frac{5x+13}{15}-\frac{4x+5}{5x-15}=\frac{x}{3}\quad mcm=15(x-3)$

$$(5x+13)(x-3)-3(4x+5)=5x(x-3)$$
$$5x^2-2x-39-12x-15=5x^2-15x$$
$$-14x-54=-15x$$
$$x=54$$

20. $\frac{2x-1}{2x+1}-\frac{x-4}{3x-2}=\frac{2}{3}\quad mcm=3(2x+1)(3x-2)$

$$3(2x-1)(3x-2)-3(x-4)(2x+1)=2(2x+1)(3x-2)$$
$$18x^2-21x+6-6x^2+21x+12=12x^2-2x-4$$
$$18=-2x-4$$
$$22=-2x$$
$$-11=x$$

21. $\frac{4x+3}{2x-5}-\frac{3x+8}{3x-7}=1\quad mcm=(2x-5)(3x-7)$

$$(4x+3)(3x-7)-(3x+8)(2x-5)=(2x-5)(3x-7)$$
$$12x^2-19x-21-6x^2-x+40=6x^2-29x+35$$
$$-20x+19=-29x+35$$
$$9x=16$$
$$x=\frac{16}{9}$$
$$x=1\tfrac{7}{9}$$

22. $\frac{10x-7}{15x+3}=\frac{3x+8}{12}-\frac{5x^2-4}{20x+4}\quad mcm=12(5x+1)$

$$4(10x-7)=(3x+8)(5x+1)-3(5x^2-4)$$
$$40x-28=15x^2+43x+8-15x^2+12$$
$$40x-28=43x+20$$
$$-3x=48$$
$$x=-16$$

23. $\frac{4x-1}{5}+\frac{x-2}{2x-7}=\frac{8x-3}{10}-1\tfrac{3}{10}\quad mcm=10(2x-7)$

$$2(4x-1)(2x-7)+10(x-2)=(8x-3)(2x-7)-13(2x-7)$$
$$16x^2-60x+14+10x-20=16x^2-62x+21-26x+91$$
$$-50x-6=-88x+112$$
$$38x=118$$
$$x=\frac{118}{38}$$
$$x=3\tfrac{2}{19}$$

24. $\frac{1}{x-1}-\frac{2}{x-2}=\frac{3}{2x-2}-\frac{2\frac{1}{3}}{2x-4}$

$$\frac{1}{x-1}-\frac{2}{x-2}=\frac{3}{2x-2}-\frac{\frac{7}{3}}{2x-4}$$
$$\frac{1}{x-1}-\frac{2}{x-2}=\frac{3}{2x-2}-\frac{7}{6x-12}$$
$$mcm=6(x-1)(x-2)$$
$$6(x-2)-12(x-1)=9(x-2)-7(x-1)$$
$$6x-12-12x+12=9x-18-7x+7$$
$$-6x=2x-11$$
$$-8x=-11$$
$$x=\frac{-11}{-8}=1\tfrac{3}{8}$$

25. $\frac{1}{x+3}-\frac{2}{5x-20}=\frac{1\frac{1}{2}}{3x-12}-\frac{2}{x+3}$

$$mcm=15(x-4)(x+3)$$
$$15(x-4)-6(x+3)=\frac{15(x+3)}{2}-30(x-4)$$
$$15x-60-6x-18=\frac{15x+45}{2}-30x-120$$
$$2(9x-78)=15x+45-2(30x-120)$$
$$18x-156=15x+45-60x+240$$
$$18x-156=-45x+285$$
$$63x=441$$
$$x=7$$

26. $\frac{1}{6-2x}-\frac{4}{5-5x}=\frac{10}{12-4x}-\frac{3}{10-10x}$

$$mcm=20(1-x)(3-x)$$
$$10(1-x)-16(3-x)=50(1-x)-6(3-x)$$
$$10-10x-48+16x=50-50x-18+6x$$
$$6x-38=-44x+32$$
$$50x=70$$
$$x=\frac{7}{5}$$
$$x=1\tfrac{2}{5}$$

27. $\frac{2}{3}-\frac{6x^2}{9x^2-1}=\frac{2}{3x-1}\quad mcm=3(9x^2-1)$

$$2(9x^2-1)-18x^2=6(3x+1)$$
$$18x^2-2-18x^2=18x+6$$
$$-2=18x+6$$
$$-8=18x$$
$$-\frac{4}{9}=x$$

28. $\frac{5x^2-27x}{5x+3}-\frac{1}{x}=x-6 \quad mcm=x(5x+3)$

$x(5x^2-27x)-(5x+3)=x(5x+3)(x-6)$

$5x^3-27x^2-5x-3=5x^3-27x^2-18x$

$-5x-3=-18x$

$13x=3$

$x=\frac{3}{13}$

29. $\frac{4x+1}{4x-1}-\frac{6}{16x^2-1}=\frac{4x-1}{4x+1} \quad mcm=16x^2-1$

$(4x+1)^2-6=(4x-1)^2$

$16x^2+8x+1-6=16x^2-8x+1$

$16x-5=1$

$16x=6$

$x=\frac{3}{8}$

30. $3\left(\frac{x-1}{x+1}\right)+2\left(\frac{x+1}{x-4}\right)=\frac{5x(x-1)}{x^2-3x-4}$

$mcm=x^2-3x-4$

$3(x-1)(x-4)+2(x+1)^2=5x(x-1)$

$3x^2-15x+12+2x^2+4x+2=5x^2-5x$

$-11x+14=-5x$

$-6x=-14$

$x=\frac{-7}{-3}$

$x=2\tfrac{1}{3}$

31. $2\left(\frac{x+2}{x-2}\right)-3\left(\frac{x-2}{2x+3}\right)=\frac{x^2+78}{2x^2-x-6}$

$mcm=(x-2)(2x+3)$

$2(x+2)(2x+3)-3(x-2)^2=x^2+78$

$4x^2+14x+12-3x^2+12x-12=x^2+78$

$26x=78$

$x=3$

32. $\frac{1}{x^2+3x-28}-\frac{1}{x^2+12x+35}=\frac{3}{x^2+x-20}$

$\Rightarrow x^2+3x-28=(x+7)(x-4)$

$\Rightarrow x^2+12x+35=(x+7)(x+5)$

$\Rightarrow x^2+x-20=(x+5)(x-4)$

$mcm=(x+7)(x+5)(x-4)$

$x+5-(x-4)=3(x+7)$

$x+5-x+4=3x+21$

$9=3x+21$

$-12=3x$

$-4=x$

33. $\frac{x-2}{x^2+8x+7}=\frac{2x-5}{x^2-49}-\frac{x-2}{x^2-6x-7}$

$\Rightarrow x^2+8x+7=(x+7)(x+1)$

$\Rightarrow x^2-49=(x+7)(x-7)$

$\Rightarrow x^2-6x-7=(x-7)(x+1)$

$mcm=(x+7)(x-7)(x+1)=(x^2-49)(x+1)$

$(x-2)(x-7)=(2x-5)(x+1)-(x-2)(x+7)$

$x^2-9x+14=2x^2-3x-5-x^2-5x+14$

$-9x+14=-8x+9$

$-x=-5$

$x=5$

34. $\frac{4x+5}{15x^2+7x-2}-\frac{2x+3}{12x^2-7x-10}-\frac{2x-5}{20x^2-29x+5}=0$

$\Rightarrow 15x^2+7x-2=(3x+2)(5x-1)$

$\Rightarrow 12x^2-7x-10=(4x-5)(3x+2)$

$\Rightarrow 20x^2-29x+5=(4x-5)(5x-1)$

$mcm=(3x+2)(5x-1)(4x-5)$

$(4x+5)(4x-5)-(2x+3)(5x-1)-(2x-5)(3x+2)=0$

$16x^2-25-10x^2-13x+3-6x^2+11x+10=0$

$-2x-12=0$

$-2x=12$

$x=-6$

35. $\frac{7}{2x+1}-\frac{3}{x+4}=\frac{2}{x+1}-\frac{3(x+1)}{2x^2+9x+4}$

$mcm=(2x+1)(x+4)(x+1)$

$7(x+4)(x+1)-3(2x+1)(x+1)=2(2x+1)(x+4)-3(x+1)^2$

$7x^2+35x+28-6x^2-9x-3=4x^2+18x+8-3x^2-6x-3$

$26x+25=12x+5$

$14x=-20$

$x=-\frac{10}{7}$

$x=-1\tfrac{3}{7}$

36. $\frac{(x+3)^2}{(x-3)^2}=\frac{x-1}{x+1}+\frac{2(7x+1)}{x^2-2x-3}$

$mcm=(x-3)^2(x+1)$

$(x+3)^2(x+1)=(x-3)^2(x-1)+2(7x+1)(x-3)$

$x^3+7x^2+15x+9=x^3-7x^2+15x-9+14x^2-40x-6$

$9=-15-40x$

$24=-40x$

$-\frac{3}{5}=x$

37. $\frac{x-4}{x+5}-\frac{x+1}{x-2}=-\frac{12(x+3)}{(x+5)^2}$ $\quad mcm=(x+5)^2(x-2)$

$$(x+5)(x-2)(x-4)-(x+5)^2(x+1)=-12(x+3)(x-2)$$
$$x^3-x^2-22x+40-x^3-11x^2-35x-25=-12x^2-12x+72$$
$$-57x+15=-12x+72$$
$$-45x=57$$
$$x=-\frac{57}{45}$$
$$x=-1\tfrac{4}{15}$$

38. $\frac{x-3}{x-4}-\frac{x-2}{x-3}=\frac{x+2}{x+1}-\frac{x+3}{x+2}$ $\quad mcm=(x-4)(x-3)(x+1)(x+2)$

$$(x-3)^2(x+1)(x+2)-(x^2-4)(x-4)(x+1)=(x+2)^2(x-4)(x-3)-(x^2-9)(x-4)(x+1)$$
$$x^4-3x^3-7x^2+15x+18-x^4+3x^3+8x^2-12x-16=x^4-3x^3-12x^2+20x+48-x^4+3x^3+13x^2-27x-36$$
$$3x+2=-7x+12$$
$$10x=10$$
$$x=1$$

39. $\frac{x+6}{x+2}-\frac{x+1}{x-3}=\frac{x-5}{x-1}-\frac{x}{x+4}$ $\quad mcm=(x+2)(x-3)(x-1)(x+4)$

$$(x+6)(x-3)(x-1)(x+4)-(x^2-1)(x+2)(x+4)=(x-5)(x+2)(x-3)(x+4)-x(x+2)(x-3)(x-1)$$
$$x^4+6x^3-13x^2-66x+72-x^4-6x^3-7x^2+6x+8=x^4-2x^3-25x^2+26x+120-x^4+2x^3+5x^2-6x$$
$$-60x+80=20x+120$$
$$-80x=40$$
$$x=-\frac{1}{2}$$

EJERCICIO 143

1. $a(x+1)=1$

$$x+1=\frac{1}{a}$$
$$-1+x+1=\frac{1}{a}-1$$
$$x=\frac{1-a}{a}$$

2. $ax-4=bx-2$

$$ax-bx=4-2$$
$$x(a-b)=2$$
$$x=\frac{2}{a-b}$$

3. $ax+b^2=a^2-bx$

$$ax+bx=a^2-b^2$$
$$x(a+b)=(a+b)(a-b)$$
$$x=a-b$$

4. $3(2a-x)+ax=a^2+9$

$$6a-3x+ax=a^2+9$$
$$x(a-3)=a^2+9-6a$$
$$x(a-3)=(a-3)^2$$
$$x=a-3$$

5. $a(x+b)+x(b-a)=2b(2a-x)$

$$ax+ba+xb-ax=4ab-2bx$$
$$xb+2bx=4ab-ab$$
$$3bx=3ab$$
$$x=\frac{3ab}{3b}$$
$$x=a$$

6. $(x-a)^2-(x+a)^2=a(a-7x)$

$$x^2-2ax+a^2-x^2-2ax-a^2=a^2-7ax$$
$$-4ax+7ax=a^2$$
$$3ax=a^2$$
$$x=\frac{a^2}{3a}$$
$$x=\frac{a}{3}$$

7. $ax-a(a+b)=-x-(1+ab)$

$$ax-a^2-ab=-x-1-ab$$
$$ax+x=a^2-1$$
$$x(a+1)=(a+1)(a-1)$$
$$x=a-1$$

8. $a^2(a-x)-b^2(x-b)=b^2(x-b)$

$$a^3-a^2x-b^2x+b^3=b^2x-b^3$$
$$-a^2x-2b^2x=-a^3-2b^3$$
$$a^2x+2b^2x=a^3+2b^3$$
$$x(a^2+2b^2)=a^3+2b^3$$
$$x=\frac{a^3+2b^3}{a^2+2b^2}$$

9. $(x+a)(x-b)-(x+b)(x-2a)=b(a-2)+3a$

$$x^2-bx+ax-ab-x^2+2ax-bx+2ab=ab-2b+3a$$
$$-2bx+3ax+ab=ab-2b+3a$$
$$x(3a-2b)=3a-2b$$
$$x=1$$

10. $x^2+a^2=(a+x)^2-a(a-1)$

$x^2+a^2=a^2+2ax+x^2-a^2+a$

$a^2=2ax+a$

$\dfrac{a(a-1)}{2a}=x$

$\dfrac{a-1}{2}=x$

11. $m(n-x)-m(n-1)=m(mx-a)$

$mn-mx-mn+m=m^2x-am$

$-mx-m^2x=-am-m$

$-xm(1+m)=-m(a+1)$

$x=\dfrac{a+1}{m+1}$

12. $x-a+2=2ax-3(a+x)-2(a-5)$

$x-a+2=2ax-3a-3x-2a+10$

$x-a=2ax-5a-3x+8$

$x+4a-8=2ax-3x$

$4(a-2)=2ax-4x$

$4(a-2)=2x(a-2)$

$2=x$

13. $a(x-a)-2bx=b(b-2a-x)$

$ax-a^2-2bx=b^2-2ab-bx$

$ax-bx=b^2-2ab+a^2$

$x(a-b)=(b-a)(b-a)$

$x=-\dfrac{(a-b)(b-a)}{a-b}$

$x=-(b-a)$

$x=a-b$

14. $ax+bx=(x+a-b)^2-(x-2b)(x+2a)$

$x(a+b)=x^2+2ax-2bx+a^2-2ab+b^2-x^2-2ax+2bx+4ab$

$x(a+b)=a^2+2ab+b^2$

$x(a+b)=(a+b)^2$

$x=a+b$

15. $x(a+b)-3-a(a-2)=2(x-1)-x(a-b)$

$ax+bx-3-a^2+2a=2x-2-ax+bx$

$2ax-2x=a^2-2a+1$

$2x(a-1)=(a-1)^2$

$x=\dfrac{a-1}{2}$

16. $(m+4x)(3m+x)=(2x-m)^2+m(15x-m)$

$3m^2+13mx+4x^2=4x^2-4mx+m^2+15mx-m^2$

$2mx=-3m^2$

$x=-\dfrac{3m^2}{2m}$

$x=-\dfrac{3m}{2}$

17. $a^2(a-x)-a^2(a+1)-b^2(b-x)-b(1-b^2)+a(1+a)=0$

$a^3-a^2x-a^3-a^2-b^3+b^2x-b+b^3+a+a^2=0$

$-a^2x+b^2x-b+a=0$

$x(b^2-a^2)-(b-a)=0$

$x(b-a)(a+b)=b-a$

$x(a+b)=1$

$x=\dfrac{1}{a+b}$

18. $(ax-b)^2=(bx-a)(a-x)-x^2(b-a^2)+a^2+b(1-2b)$

$a^2x^2-2abx+b^2=abx+bx^2-a^2-ax-bx^2+a^2x^2+a^2+b-2b^2$

$ax(1-3b)=b(1-3b)$

$x=\dfrac{b}{a}$

19. $(x+b)^2-(x-a)^2-(a+b)^2=0$

$x^2+2bx+b^2-x^2+2ax-a^2-a^2-2ab-b^2=0$

$2bx+2ax-2a^2-2ab=0$

$2x(b+a)-2a(a+b)=0$

$2(x-a)(a+b)=0$

$2x=2a$

$x=a$

20. $(x+m)^3-12m^3=-(x-m)^3+2x^3$

$x^3+3x^2m+3xm^2+m^3-12m^3=-x^3+3x^2m-3xm^2+m^3+2x^3$

$2x^3+6xm^2-2x^3=12m^3$

$6xm^2=12m^3$

$x=\dfrac{12m^3}{6m^2}$

$x=2m$

EJERCICIO 144

1. $\frac{m}{x}-\frac{1}{m}=\frac{2}{m} \qquad mcm=mx$

$m^2-x=2x$

$m^2=3x$

$\frac{m^2}{3}=x$

2. $\frac{a}{x}+\frac{b}{2}=\frac{4a}{x} \qquad mcm=2x$

$2a+bx=8a$

$bx=6a$

$x=\frac{6a}{b}$

3. $\frac{x}{2a}-\frac{1-x}{a^2}=\frac{1}{2a} \qquad mcm=2a^2$

$ax-2(1-x)=a$

$ax-2+2x=a$

$x(a+2)=a+2$

$x=\frac{a+2}{a+2}$

$x=1$

4. $\frac{m}{x}+\frac{n}{m}=\frac{n}{x}+1 \qquad mcm=mx$

$m^2+nx=mn+mx$

$m^2-mn=mx-nx$

$m(m-n)=x(m-n)$

$m=x$

5. $\frac{a-1}{a}+\frac{1}{2}=\frac{3a-2}{x}$

$mcm=2ax$

$2x(a-1)+ax=2a(3a-2)$

$2ax-2x+ax=6a^2-4a$

$3ax-2x=2a(3a-2)$

$x(3a-2)=2a(3a-2)$

$x=2a$

6. $\frac{a-x}{a}-\frac{b-x}{b}=\frac{2(a-b)}{ab}$

$mcm=ab$

$b(a-x)-a(b-x)=2(a-b)$

$ab-bx-ab+ax=2(a-b)$

$x(a-b)=2(a-b)$

$x=2$

7. $\frac{x-3a}{a^2}-\frac{2a-x}{ab}=-\frac{1}{a} \qquad mcm=a^2b$

$b(x-3a)-a(2a-x)=-ab$

$bx-3ab-2a^2+ax=-ab$

$bx+ax=2ab+2a^2$

$x(a+b)=2a(a+b)$

$x=2a$

8. $\frac{x+m}{m}-\frac{x+n}{n}=\frac{m^2+n^2}{mn}-2 \qquad mcm=mn$

$n(x+m)-m(x+n)=m^2+n^2-2mn$

$xn+mn-mx-mn=m^2-2mn+n^2$

$x(n-m)=(m-n)^2$

$x(n-m)=-(n-m)(m-n)$

$x=-(m-n)$

$x=n-m$

9. $\frac{x-b}{a}=2-\frac{x-a}{b} \qquad mcm=ab$

$b(x-b)=2ab-a(x-a)$

$bx-b^2=2ab-ax+a^2$

$bx+ax=a^2+2ab+b^2$

$x(a+b)=(a+b)^2$

$x=a+b$

10. $\frac{4x}{2a+b}-3=-\frac{3}{2} \qquad mcm=2(2a+b)$

$8x-6(2a+b)=-3(2a+b)$

$8x-12a-6b=-6a-3b$

$8x=6a+3b$

$x=\frac{3(2a+b)}{8}$

11. $\frac{2a+3x}{x+a}=\frac{2(6x-a)}{4x+a} \qquad mcm=(x+a)(4x+a)$

$(2a+3x)(a+4x)=(2x+2a)(6x-a)$

$2a^2+11ax+12x^2=12x^2+10ax-2a^2$

$ax=-4a^2$

$x=-4a$

12. $\frac{2(x-c)}{4x-b}=\frac{2x+c}{4(x-b)} \qquad mcm=4(4x-b)(x-b)$

$8(x-c)(x-b)=(2x+c)(4x-b)$

$8x^2-8xb-8xc+8bc=8x^2-2xb+4xc-bc$

$-6xb-12xc=-9bc$

$-6x(b+2c)=-9bc$

$x=\frac{3bc}{2(b+2c)}$

13. $\frac{1}{n}-\frac{m}{x}=\frac{1}{mn}-\frac{1}{x} \qquad mcm=xmn$

$$xm-m^2n=x-mn$$
$$xm-x=m^2n-mn$$
$$x\left(m-1\right)=mn\left(m-1\right)$$
$$x=mn$$

14. $\frac{\left(x-2b\right)\left(2x+a\right)}{\left(x-a\right)\left(a-2b+x\right)}=2 \qquad mcm=\left(x-a\right)\left(a-2b+x\right)$

$$\left(x-2b\right)\left(2x+a\right)=2\left(x-a\right)\left(a-2b+x\right)$$
$$2x^2+ax-4bx-2ab=-4bx+2x^2-2a^2+4ab$$
$$ax=6ab-2a^2$$
$$ax=2a\left(3b-a\right)$$
$$x=2\left(3b-a\right)$$

15. $\frac{x+m}{x-n}=\frac{n+x}{m+x} \qquad mcm=\left(m+x\right)\left(x-n\right)$

$$\left(x+m\right)^2=\left(x+n\right)\left(x-n\right)$$
$$x^2+2mx+m^2=x^2-n^2$$
$$2mx=-m^2-n^2$$
$$x=-\frac{m^2+n^2}{2m}$$

16. $\frac{x\left(2x+3b\right)\left(x+b\right)}{x+3b}=2x^2-bx+b^2 \qquad mcm=x+3b$

$$x\left(2x+3b\right)\left(x+b\right)=\left(x+3b\right)\left(2x^2-bx+b^2\right)$$
$$2x^3+5x^2b+3b^2x=2x^3+5bx^2-2b^2x+3b^3$$
$$3b^2x+2b^2x=3b^3$$
$$x=\frac{3b}{5}$$

17. $\frac{3}{4}\left(\frac{x}{b}+\frac{x}{a}\right)=\frac{1}{3}\left(\frac{x}{b}-\frac{x}{a}\right)+\frac{5a+13b}{12a}$

$$\Rightarrow \frac{3}{4}\left(\frac{ax+bx}{ab}\right)=\frac{1}{3}\left(\frac{ax-bx}{ab}\right)+\frac{5a+13b}{12a}$$
$$\Rightarrow \frac{3ax+3bx}{4ab}=\frac{ax-bx}{3ab}+\frac{5a+13b}{12a} \qquad mcm=12ab$$
$$3\left(3ax+3bx\right)=4\left(ax-bx\right)+b\left(5a+13b\right)$$
$$9ax+9bx=4ax-4bx+5ab+13b^2$$
$$5ax+13bx=5ab+13b^2$$
$$x\left(5a+13b\right)=b\left(5a+13b\right)$$
$$x=b$$

18. $\frac{x+a}{3}=\frac{\left(x-b\right)^2}{3x-a}+\frac{3ab-3b^2}{9x-3a} \qquad mcm=3\left(3x-a\right)$

$$\left(x+a\right)\left(3x-a\right)=3\left(x-b\right)^2+3ab-3b^2$$
$$3x^2+2ax-a^2=3x^2-6bx+3b^2+3ab-3b^2$$
$$2ax+6bx=a^2+3ab$$
$$2x\left(a+3b\right)=a\left(a+3b\right)$$
$$x=\frac{a}{2}$$

19. $\frac{5x+a}{3x+b}=\frac{5x-b}{3x-a} \qquad mcm=\left(3x+b\right)\left(3x-a\right)$

$$\left(3x-a\right)\left(5x+a\right)=\left(5x-b\right)\left(3x+b\right)$$
$$15x^2-2ax-a^2=15x^2+2bx-b^2$$
$$-2ax-2bx=a^2-b^2$$
$$-2x\left(a+b\right)=\left(a+b\right)\left(a-b\right)$$
$$x=\frac{b-a}{2}$$

20. $\frac{x+a}{x-a}-\frac{x-a}{x+a}=\frac{a\left(2x+ab\right)}{x^2-a^2} \qquad mcm=\left(x-a\right)\left(x+a\right)$

$$\left(x+a\right)^2-\left(x-a\right)^2=a\left(2x+ab\right)$$
$$x^2+2ax+a^2-x^2+2ax-a^2=2ax+a^2b$$
$$4ax=2ax+a^2b$$
$$2ax=a^2b$$
$$x=\frac{ab}{2}$$

21. $\frac{2x-3a}{x+4a}-2=\frac{11a}{x^2-16a^2} \qquad mcm=x^2-16a^2$

$$\left(2x-3a\right)\left(x-4a\right)-2\left(x^2-16a^2\right)=11a$$
$$2x^2-11ax+12a^2-2x^2+32a^2=11a$$
$$-11ax+44a^2=11a$$
$$-11ax=11a-44a^2$$
$$-11ax=11a\left(1-4a\right)$$
$$x=-\left(1-4a\right)$$
$$x=4a-1$$

22. $\frac{1}{x+a}+\frac{x^2}{a^2+ax}=\frac{x+a}{a} \qquad mcm=a\left(x+a\right)$

$$a+x^2=\left(x+a\right)^2$$
$$a+x^2=x^2+2ax+a^2$$
$$a-a^2=2ax$$
$$\frac{a\left(1-a\right)}{2a}=x$$
$$\frac{1-a}{2}=x$$

23. $\frac{2\left(a+x\right)}{b}-\frac{3\left(b+x\right)}{a}=\frac{6\left(a^2-2b^2\right)}{ab} \qquad mcm=ab$

$$2a\left(a+x\right)-3b\left(b+x\right)=6\left(a^2-2b^2\right)$$
$$2a^2+2ax-3b^2-3bx=6a^2-12b^2$$
$$2ax-3bx=4a^2-9b^2$$
$$x\left(2a-3b\right)=\left(2a-3b\right)\left(2a+3b\right)$$
$$x=2a+3b$$

24. $m(n-x)-(m-n)(m+x)=n^2-\frac{1}{n}(2mn^2-3m^2n) \qquad mcm=n$

$$nm(n-x)-n(m-n)(m+x)=n^3-(2mn^2-3m^2n)$$
$$mn^2-mnx-m^2n+mn^2-mnx+n^2x=n^3-2mn^2+3m^2n$$
$$-2mnx+n^2x=-4mn^2+4m^2n+n^3$$
$$nx(n-2m)=n(n^2-4mn+4m^2)$$
$$x(n-2m)=(n-2m)^2$$
$$x=n-2m$$

EJERCICIO 145

1. $x \to \quad N^o\, buscado$

$$x-\frac{3x}{8}=2x-11$$
$$8x-3x=8(2x-11)$$
$$5x=16x-88$$
$$-11x=-88$$
$$x=\frac{-88}{-11}$$
$$x=8$$

2. $x \to \quad N^o\, buscado$

$$x+\frac{5x}{6}=3x-14$$
$$6x+5x=6(3x-14)$$
$$11x=18x-84$$
$$-7x=-84$$
$$x=\frac{-84}{-7}$$
$$x=12$$

3. $x \to \quad N^o\ que\ se\ resta$

$$22-x=11+\frac{6x}{5}$$
$$5(22-x)=55+6x$$
$$110-5x=55+6x$$
$$-11x=-55$$
$$x=5$$

4. $x \to \quad N^o\ que\ tiene\ diferencia$

$$\frac{5}{4}x-\frac{7}{8}x=30 \qquad mcm=8$$
$$10x-7x=240$$
$$3x=240$$
$$x=80$$

5. $x \to \quad N^o\ buscado$

$$x-17=\frac{3}{5}x-\frac{1}{6}x \qquad mcm=30$$
$$30x-510=18x-5x$$
$$-510=-17x$$
$$30=x$$

6. $x \to \quad N^o\ buscado$

$$\frac{x}{5}+\frac{3}{8}x-49=2\left(\frac{1}{6}x-\frac{1}{12}x\right)$$
$$\Rightarrow \frac{x}{5}+\frac{3}{8}x-49=\frac{x}{6} \qquad mcm=120$$
$$24x+45x-5.880=20x$$
$$49x=5.880$$
$$x=120$$

7. $x \to \quad Edad\ de\ A$

$$\frac{3x}{5} \to Edad\ de\ B$$
$$x+\frac{3x}{5}-4=2\left(\frac{3x}{5}\right) \qquad mcm=5$$
$$5x+3x-20=6x$$
$$2x=20$$
$$x=10\,años \to \quad Edad\ de\ A$$
$$\Rightarrow \frac{3\cdot 10}{5}=\frac{30}{5}=6\,años \to \quad Edad\ de\ B$$

8. $x \to \quad Lo\ que\ tiene\ A$

$$\frac{7x}{8} \to \quad Lo\ que\ tiene\ B$$
$$x+90=2\left(\frac{7x}{8}\right) \qquad mcm=8$$
$$8x+720=14x$$
$$-6x=-720$$
$$x=\$120 \to \quad Tiene\ A$$
$$\Rightarrow \frac{7\cdot 120}{8}=\$105 \to \quad Tiene\ B$$

9. $x \to \quad Long.pieza$

$$x-\frac{3x}{5}=40 \qquad mcm=5$$
$$5x-3x=200$$
$$2x=200$$
$$x=100m$$

10. $x \to \quad Lo\ que\ tenia$

$$x-\left(\frac{1}{3}x+\frac{1}{8}x\right)=39 \qquad mcm=24$$
$$24x-8x-3x=936$$
$$13x=936$$
$$x=72\,bs.$$

11. $x \to \quad N^o\ buscado$

$$3x-48=\frac{x}{3} \qquad mcm=3$$
$$9x-144=x$$
$$8x=144$$
$$x=18$$

12. $x \to \quad N^o\ buscado$

$$4x-19=\frac{x}{2}+30 \qquad mcm=2$$
$$8x-38=x+60$$
$$7x=98$$
$$x=14$$

13. $x \to \quad N^o\ buscado$

$$80-\frac{x}{2}=x-10 \qquad mcm=2$$
$$160-x=2x-20$$
$$-3x=-180$$
$$x=60$$

14. $x \to \quad N^o\ buscado$

$$\frac{7x}{8}-2=\frac{4x}{5} \qquad mcm=40$$
$$35x-80=32x$$
$$3x=80$$
$$x=\frac{80}{3}$$
$$x=26\tfrac{2}{3}$$

15. $x \to \quad Ancho\ del\ buque$

$$800-744=\frac{8x}{9}$$
$$56=\frac{8x}{9}$$
$$504=8x$$
$$\frac{504}{8}=x$$
$$63\,Pies=x$$

EJERCICIO 146

1.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow \; N^o\ mayor$
$$\frac{4(x+1)}{5}=x-4$$
$$\frac{4x+4}{5}=x-4$$
$$4x+4=5(x-4)$$
$$4x+4=5x-20$$
$$-x=-24$$
$$x=24 \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1$$
$$=24+1=25 \rightarrow \; N^o\ mayor$$

2.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow \; N^o\ mayor$
$$\frac{7x}{8}-17=\frac{3(x+1)}{5} \quad mcm=40$$
$$35x-680=24(x+1)$$
$$35x-680=24x+24$$
$$11x=704$$
$$x=64 \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1$$
$$=64+1=65 \rightarrow \; N^o\ mayor$$

3.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow \; N^o\ mayor$
$$x-81=\frac{3x}{4}-\frac{2(x+1)}{5} \quad mcm=20$$
$$20x-1.620=15x-8(x+1)$$
$$20x-1.620=15x-8x-8$$
$$13x=1.612$$
$$x=124 \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1$$
$$=124+1=125 \quad x \rightarrow \; N^o\ mayor$$

4.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow N^o\ mayor$
$$\frac{1}{5}(x+1)+\frac{1}{33}x-8=\frac{3(x+1)}{20} \quad mcm=660$$
$$132(x+1)+20x-5.280=99(x+1)$$
$$132x+132+20x-5.280=99x+99$$
$$152x-99x=5.148+99$$
$$53x=5.247$$
$$x=99 \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1=99+1=100 \rightarrow \; N^o\ mayor$$

5.
$2x \rightarrow \; N^o\ menor$
$2x+2 \rightarrow N^o\ mayor$
$$(2x+2)^2-(2x)^2=324$$
$$4x^2+8x+4-4x^2=324$$
$$8x=320$$
$$x=40$$
$$\Rightarrow 2x=2\cdot 40=80 \rightarrow \; N^o\ menor$$
$$\Rightarrow 2x+2=80+2=82 \rightarrow \; N^o\ mayor$$

6.
$x \rightarrow \; Tiene\ B$
$x+1 \rightarrow Tiene\ A$
$$x-8=\frac{4(x+1)}{5}-4 \quad mcm=5$$
$$5x-40=4x+4-20$$
$$5x-4x=40-16$$
$$x=\$24 \rightarrow \; Tiene\ B$$
$$\Rightarrow x+1=24+1=\$25 \Rightarrow Tiene\ A$$

7.
$x \rightarrow \; Gane\ ayer$
$x+1 \rightarrow Gane\ hoy$
$$x+x+1-25=\frac{2x}{5}$$
$$5(2x-24)=2x$$
$$10x-120=2x$$
$$8x=120$$
$$x=\$15 \rightarrow \; Gane\ ayer$$
$$\Rightarrow x+1=15+1=\$16 \rightarrow \; Gane\ hoy$$

8.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow N^o medio$
$x+2 \rightarrow N^o mayor$
$$\frac{x}{20}+\frac{x+1}{27}+\frac{x+2}{41}=9 \quad mcm=22.140$$
$$1.107x+820(x+1)+540(x+2)=199.260$$
$$1.107x+820x+820+540x+1.080=199.260$$
$$2.467x=197.360$$
$$x=80 \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1=80+1=81 \rightarrow \; N^o\ medio$$
$$\Rightarrow x+2=80+2=82 \rightarrow \; N^o\ mayor$$

9.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow N^o medio$
$x+2 \rightarrow N^o mayor$
$$\frac{3x}{5}+\frac{5(x+2)}{6}-31=x+1 \quad mcm=30$$
$$18x+25(x+2)-930=30x+30$$
$$18x+25x+50-930=30x+30$$
$$43x-30x=880+30$$
$$13x=910$$
$$x=70 \;\; x \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1=70+1=71 \quad \rightarrow \; N^o\ medio$$
$$\Rightarrow x+2=70+2=72 \quad \rightarrow \; N^o\ mayor$$

10.
$x \rightarrow \; N^o\ menor$
$x+1 \rightarrow N^o medio$
$x+2 \rightarrow N^o mayor$
$$\frac{3(x+1)}{7}-\frac{3x}{10}-1=\frac{1}{11}(x+2) \quad mcm=770$$
$$330(x+1)-231x-770=70(x+2)$$
$$330x+330-231x-770=70x+140$$
$$29x=580$$
$$x=20 \rightarrow \; N^o\ menor$$
$$\Rightarrow x+1=20+1=21 \rightarrow N^o medio$$
$$\Rightarrow x+2=20+2=22 \rightarrow N^o mayor$$

11. $x \rightarrow Edad\ B$

$x+2 \rightarrow Edad\ A$

$x-2 \rightarrow Edad\ C$

$$x+x-2-12=\frac{7(x+2)}{8}$$

$$8(2x-14)=7(x+2)$$

$$16x-112=7x+14$$

$$9x=126$$

$$x=14 \rightarrow Edad\ B$$

$$\Rightarrow x+2=14+2=16 \rightarrow Edad\ A$$

$$\Rightarrow x-2=14-2=12 \rightarrow Edad\ C$$

12. $x \rightarrow Edad\ B$

$x-1 \rightarrow Edad\ A$

$x+1 \rightarrow Edad\ C$

$$(x+1)^2-x^2=\frac{17(x-1)}{5}-4 \quad mcm=5$$

$$5(x^2+2x+1)-5x^2=17(x-1)-20$$

$$5x^2+10x+5-5x^2=17x-17-20$$

$$-7x=-42$$

$$x=6 \rightarrow Edad\ B$$

$$\Rightarrow x-1=6-1=5 \rightarrow Edad\ A$$

$$\Rightarrow x+1=6+1=7 \rightarrow Edad\ C$$

EJERCICIO 147

1. $x \rightarrow N^o\ mayor$

$59-x \rightarrow N^o\ menor$

$$\frac{x-5}{59-x}=2$$

$$x-5=2(59-x)$$

$$x-5=118-2x$$

$$3x=123$$

$$x=41 \rightarrow N^o\ mayor$$

$$\Rightarrow 59-x=59-41=18 \rightarrow N^o\ menor$$

2. $x \rightarrow N^o\ mayor$

$436-x \rightarrow N^o\ menor$

$$\frac{x-73}{436-x}=2$$

$$x-73=2(436-x)$$

$$x-73=872-2x$$

$$3x=945$$

$$x=315 \rightarrow N^o\ mayor$$

$$\Rightarrow 436-x=436-315=121 \rightarrow N^o\ menor$$

3. $x \rightarrow N^o\ mayor$

$x-44 \rightarrow N^o\ menor$

$$\frac{x-2}{x-44}=3$$

$$x-2=3(x-44)$$

$$x-2=3x-132$$

$$-2x=-130$$

$$x=65 \rightarrow N^o\ mayor$$

$$\Rightarrow x-44=65-44=21 \rightarrow N^o\ menor$$

4. $x \rightarrow N^o\ mayor$

$x-56 \rightarrow N^o\ menor$

$$\frac{x-8}{x-56}=3$$

$$x-8=3(x-56)$$

$$x-8=3x-168$$

$$-2x=-160$$

$$x=80 \rightarrow N^o\ mayor$$

$$\Rightarrow x-56=80-56=24 \rightarrow N^o\ menor$$

5. $x \rightarrow Parte\ mayor$

$260-x \rightarrow Parte\ menor$

$$\frac{2x-40}{3(260-x)}=2$$

$$2x-40=6(260-x)$$

$$2x-40=1.560-6x$$

$$8x=1.600$$

$$x=200 \rightarrow Parte\ mayor$$

$$\Rightarrow 260-x$$

$$=260-200=60 \rightarrow Parte\ menor$$

6. $x \rightarrow Parte\ de\ A$

$196-x \rightarrow Parte\ de\ B$

$$\frac{\frac{3x}{8}-16}{\frac{196-x}{5}}=1$$

$$\frac{3x-128}{8}\cdot\frac{5}{196-x}=1$$

$$5(3x-128)=8(196-x)$$

$$15x-640=1.568-8x$$

$$23x=2.208$$

$$x=96\, soles \rightarrow Parte\ de\ A$$

$$\Rightarrow 196-x$$

$$=196-96=100\, soles \rightarrow Parte\ de\ B$$

EJERCICIO 148

1. $x \rightarrow$ *Ganó el 1er día*

$\frac{x}{2} \rightarrow$ *Ganó el 2º día*

$\frac{x}{4} \rightarrow$ *Ganó el 3er día*

$$x+\frac{x}{2}+\frac{x}{4}=175 \quad mcm=4$$

$$4x+2x+x=700$$

$$7x=700$$

$$x=\$100 \rightarrow Ganó\ el\ 1^{er}\ día$$

$$\Rightarrow \frac{x}{2}=\frac{100}{2}=\$50 \rightarrow Ganó\ el\ 2^o\ día$$

$$\Rightarrow \frac{x}{4}=\frac{100}{4}=\$25 \rightarrow Ganó\ el\ 3^{er}\ día$$

2. $x \rightarrow$ *Perdió miercoles*

$\frac{3}{5}x \rightarrow$ *Perdió jueves*

$\frac{1}{2}x \rightarrow$ *Perdió viernes*

$$x+\frac{3}{5}x+\frac{1}{2}x=252 \quad mcm=10$$

$$10x+6x+5x=2.520$$

$$21x=2.520$$

$$x=\$120 \rightarrow Perdió\ miercoles$$

$$\Rightarrow \frac{3}{5}\cdot 120=\$72 \rightarrow Perdió\ jueves$$

$$\Rightarrow \frac{1}{2}\cdot 120=\$60 \rightarrow Perdió\ viernes$$

3. $x \rightarrow$ *Tiene A*

$\frac{2}{3}x \rightarrow$ *Tiene B*

$\frac{2}{5}x \rightarrow$ *Tiene C*

$$x+\frac{2x}{3}+\frac{2x}{5}=248 \quad mcm=15$$

$$15x+10x+6x=3.720$$

$$31x=3.720$$

$$x=120\ suc. \rightarrow Tiene\ A$$

$$\Rightarrow \frac{2}{3}\cdot 120=80\, sucres \rightarrow Tiene\ B$$

$$\Rightarrow \frac{2}{5}\cdot 120=48\, sucres \rightarrow Tiene\ C$$

4. $x \rightarrow Edad\ A$

$\frac{3}{5}x \rightarrow Edad\ B$

$\frac{9}{40}x \rightarrow Edad\ C$

$x+\frac{3x}{5}+\frac{9x}{40}=73 \quad mcm=40$

$40x+24x+9x=2.920$

$73x=2.920$

$x=40$

$40\ años \rightarrow edad\ de\ A$

$\Rightarrow \frac{3}{5}\cdot 40=24\ años \rightarrow edad\ de\ B$

$\Rightarrow \frac{9}{40}\cdot 40=9\ años \rightarrow edad\ de\ C$

5. $x \rightarrow 1^{er}\ día$

$\frac{1}{3}x \rightarrow 2^{o}\ día$

$\frac{1}{9}x \rightarrow 3^{o}\ día$

$\frac{1}{27}x \rightarrow 4^{o}\ día$

$x+\frac{1}{3}x+\frac{1}{9}x+\frac{1}{27}x=120 \quad mcm=27$

$27x+9x+3x+x=3.240$

$40x=3.240$

$x=81\,Km.$

$81\,Km. \rightarrow 1^{er}\ día$

$\Rightarrow \frac{1}{3}\cdot 81=27\,Km. \rightarrow 2^{o}\ día$

$\Rightarrow \frac{1}{9}\cdot 81=9\,Km. \rightarrow 3^{o}\ día$

$\Rightarrow \frac{1}{27}\cdot 81=3\,Km. \rightarrow 4^{o}\ día$

6. $x \rightarrow 1^{a}\ semana$

$\frac{11}{10}x \rightarrow 2^{a}\ semana$

$\frac{121}{100}x \rightarrow 3^{a}\ semana$

$\frac{1.331}{1.000} \rightarrow 4^{a}\ semana$

$x+\frac{11}{10}x+\frac{121}{100}x+\frac{1.331}{1.000}x=4.641 \quad mcm=1.000$

$1.000x+1.100x+1.210x+1.331x=4'641.000$

$4.641x=4'641.000$

$x=1.000\ Km.$

Continúa

Continuación

6. $1.000\,Km. \rightarrow 1^{a}\ semana$

$\Rightarrow \frac{11}{10}\cdot 1.000=1.100\,Km. \rightarrow 2^{a}\ semana$

$\Rightarrow \frac{121}{100}\cdot 1.000=1.210\,Km. \rightarrow 3^{a}\ semana$

$\Rightarrow \frac{1.331}{1.000}\cdot 1.000=1.331\,Km. \rightarrow 4^{a}\ semana$

7. $x \rightarrow 1^{a}\ Persona \quad ; \quad \frac{x}{2} \rightarrow 2^{a}\ Persona$

$\frac{x}{8} \rightarrow 3^{a}\ Persona \quad ; \quad \frac{x}{40} \rightarrow 4^{a}\ Persona$

$\frac{x}{400} \rightarrow 5^{a}\ Persona$

$x+\frac{x}{2}+\frac{x}{8}+\frac{x}{40}+\frac{x}{400}=330.500 \quad mcm=400$

$400x+200x+50x+10x+x=132'200.000$

$661x=132'200.000$

$x=200.000\ colones$

$200.000\ colones \rightarrow 1^{a}\ Persona$

$\Rightarrow \frac{x}{2}=\frac{200.000}{2}=100.000\,colones \rightarrow 2^{a}\ Persona$

$\Rightarrow \frac{x}{8}=\frac{200.000}{8}=25.000\,colones \rightarrow 3^{a}\ Persona$

$\Rightarrow \frac{x}{40}=\frac{200.000}{40}=5.000\,colones \rightarrow 4^{a}\ Persona$

$\Rightarrow \frac{x}{400}=\frac{200.000}{400}=500\,colones \rightarrow 5^{a}\ Persona$

8. $x \rightarrow Rec.\ en\ barco$

$\frac{4}{9}x \rightarrow Rec.\ en\ tren$

$\frac{5}{18}x \rightarrow Rec.\ en\ avión$

$x+\frac{4x}{9}+\frac{5x}{18}=9.362 \quad mcm=18$

$18x+8x+5x=168.516$

$31x=168.516$

$x=5.436\ Km.$

$5.436\ Km. \rightarrow Rec.\ en\ barco$

$\Rightarrow \frac{4}{9}x=\frac{4}{9}\cdot 5.436=2.416\,Km. \rightarrow Rec.\ en\ tren$

$\Rightarrow \frac{5}{18}x=\frac{5}{18}\cdot 5.436=1.510\,Km. \rightarrow Rec.\ en\ avión$

EJERCICIO 149

1. $x \to Tenía\ al\ principio$

$x-20-\frac{2(x-20)}{3}=10 \quad mcm=3$

$3x-60-2(x-20)=30$

$3x-60-2x+40=30$

$x-20=30$

$x=50$

$\$50 \to Tenía\ al\ principio$

2. $x \to Tenía\ al\ principio$

$\frac{x}{2}-\frac{x}{4}=21 \quad mcm=4$

$2x-x=84$

$x=84$

$84\ Q. \to Tenía\ al\ principio$

3. $x \to Tengo\ ahora$

$x+7-\frac{4(x+7)}{5}=20 \quad mcm=5$

$5x+35-4(x+7)=100$

$5x+35-4x-28=100$

$x+7=100$

$x=93$

$\$\ 93 \to Tengo\ ahora$

4. $x \to Tenía\ al\ principio$

$x-\frac{2x}{5}=\frac{3}{5}x \to quedó$

$\frac{5}{6}\cdot\frac{3}{5}x=\frac{1}{2}x \to\ prestó$

Luego

$x-\frac{2}{5}x-\frac{1}{2}x=500 \quad mcm=10$

$10x-4x-5x=5.000$

$x=5.000$

$5.000\ bs. \to Tenía\ al\ principio$

5. $x \to Aves\ en\ la\ granja$

$\frac{4}{5}x \to Palomas$

$x-\frac{4}{5}x=\frac{x}{5} \quad Resto$

$\frac{x}{5}\cdot\frac{3}{4}=\frac{3}{20}x \quad \to Gallinas$

$x-\frac{4x}{5}-\frac{3x}{20}=4 \quad mcm=20$

$20x-16x-3x=80$

$x=80$

$80 \to Aves\ en\ la\ granja$

$\Rightarrow \frac{4}{5}x=\frac{4}{5}\cdot 80=64 \ \to Palomas$

$\Rightarrow \frac{3}{20}x=\frac{3}{20}\cdot 80=12 \ \to Gallinas$

$4 \ \to Gallos$

6. $x \to Tenía\ al\ principio$

$x-\frac{4}{5}x=\frac{x}{5} \quad \to quedó$

$\frac{2}{3}\cdot\frac{x}{5}=\frac{2}{15}x \ \to Perdí$

$x-\frac{4}{5}x-\frac{2}{15}x-8=0 \quad mcm=15$

$15x-12x-2x-120=0$

$x=120$

$120\ soles \to Tenía\ al\ principio$

7. $x \to Tenía\ al\ principio$

$x-\frac{5}{12}x+42=x+2 \quad mcm=12$

$12x-5x+504=12x+24$

$-5x=-480$

$x=96$

$\$\ 96 \ \to Tenía\ al\ principio$

8. $x \to Tenía\ al\ principio$

$x-\frac{x}{2}-15=30 \quad mcm=2$

$2x-x-30=60$

$x=90$

$\$\ 90 \to Tenía\ al\ principio$

9. $x \to Tenía\ al\ principio$

$x-\frac{3}{4}x+1.300=x+100$

$mcm=4$

$4x-3x+5.200=4x+400$

$-3x=-4.800$

$x=1.600$

$1.600\, suc. \to Tenía\ al\ principio$

10. $x \to Tenía\ al\ principio$

$x-\frac{3}{4}x=\frac{x}{4} \quad \to quedó$

$\frac{2}{3}\cdot\frac{x}{4}=\frac{x}{6} \quad \to Libros$

$x-\frac{3}{4}x-\frac{x}{6}=\frac{2}{5}x-38$

$mcm=60$

$60x-45x-10x=24x-2.280$

$-19x=-2.280$

$x=120$

$\$120 \ \to Tenía\ al\ principio$

EJERCICIO 150

1. $x \to\ Edad\ actual\ A$

$3x \to\ Edad\ actual\ B$

$x-15=\frac{1}{6}(3x-15) \quad mcm=6$

$6x-90=3x-15$

$3x=75$

$x=25\ \ años\ \to\ Edad\ actual\ A$

$\Rightarrow 3x=25\cdot 3=75 años \to\ Edad\ actual\ B$

2. $x \to\ Edad\ actual\ B$

$3x \to\ Edad\ actual\ A$

$3x+20=2(x+20)$

$3x+20=2x+40$

$x=20\ \ años\ \to\ Edad\ actual\ B$

$\Rightarrow 3x=3\cdot 20=60 años \to\ Edad\ actual\ A$

3. $x \to Edad\ actual\ A$

$$x-5=\frac{9}{11}(x+5) \quad mcm=11$$
$$11x-55=9x+45$$
$$2x=100$$
$$x=50$$
$50\ años \to Edad\ actual\ A$

4. $x \to Edad\ actual\ A$

$$2(x-6)=x+24$$
$$2x-12=x+24$$
$$x=36$$
$36\ años \to Edad\ actual\ A$

5. $x \to Edad\ hijo$

$3x \to Edad\ padre$
$$2(x+16)=3x+16$$
$$2x+32=3x+16$$
$$16=x$$
$16\ años \to Edad\ hijo$

$\Rightarrow 3x=3\cdot 16=48\ años \to Edad\ padre$

6. $x \to Edad\ padre$

$\frac{2}{5}x \to Edad\ hijo$
$$\frac{2(x-8)}{7}=\frac{2x}{5}-8 \quad mcm=35$$
$$10x-80=14x-280$$
$$200=4x$$
$$50=x$$
$50\ años \to Edad\ padre$

$\Rightarrow \frac{2}{5}x=\frac{2}{5}\cdot 50=20\ años \to Edad\ hijo$

7. $x \to Edad\ actual\ de\ A$

$65-x \to Edad\ actual\ de\ B$
$$65-x+10=\frac{5(x+10)}{12} \quad mcm=12$$
$$900-12x=5x+50$$
$$850=17x$$
$$50=x$$
$50\ años \to Edad\ actual\ de\ A$

$\Rightarrow 65-x=65-50=15 años \to Edad\ actual\ de\ B$

8. $x \to Edad\ padre$

$x-25 \to Edad\ hijo$
$$x-25-15=\frac{3(x-15)}{8} \quad mcm=8$$
$$8x-320=3x-45$$
$$5x=275$$
$$x=55$$
$55\ años \to Edad\ padre$

$\Rightarrow x-25=55-25=30 años \to Edad\ hijo$

9. $2x \to Edad\ padre\ hace\ 10\ años$

$x \to Edad\ hijo\ hace\ 10\ años$

$2x+10 \to Edad\ actual\ padre$

$x+10 \to Edad\ actual\ hijo$
$$2x+10+10=\frac{3(x+10+10)}{2} \quad mcm=2$$
$$4x+40=3x+60$$
$$x=20$$
$\Rightarrow 2x+10=2\cdot 20+10=50\ años \to Ed.\ act.\ padre$

$\Rightarrow x+10=20+10=30\ años \to Ed.\ act. hijo$

10. $x+18 \to Edad\ de\ A$

$x \to Edad\ de\ B$
$$x+18-18=\frac{5(x-18)}{2} \quad mcm=2$$
$$2x=5x-90$$
$$90=3x$$
$$30=x$$
$30\ años \to Edad\ de\ B$

$\Rightarrow x+18=30+18=48\ años \to Edad\ A$

11. $3x \to Edad\ A$

$x \to Edad\ B$
$$3x-4+x-4=x+16$$
$$4x-8=x+16$$
$$3x=24$$
$$x=8$$
$8\ años \to Edad\ B$

$\Rightarrow 3x=3\cdot 8=24\ años \to Edad\ A$

EJERCICIO 151

1. $x \to Tiene\ B$

$2x \to Tiene\ A$

$2x-20=\frac{4(x+20)}{5}\quad mcm=5$

$10x-100=4x+80$

$6x=180$

$x=30$

$30\ soles \to Tiene\ B$

$\Rightarrow 2x=2\cdot 30=60\ soles \to Tiene\ A$

2. $x \to Tiene\ B$

$\frac{x}{2} \to Tiene\ A$

$x-24=\frac{x}{2}+24\quad mcm=2$

$2x-48=x+48$

$x=96$

$96\ colones \to Tiene\ B$

$\Rightarrow \frac{x}{2}=\frac{96}{2}=48\ colones \to Tiene\ A$

3. $x \to Tiene\ A$

$2x \to Tiene\ B$

$\frac{3}{5}(2x-6)=x+6\quad mcm=5$

$6x-18=5x+30$

$x=48$

$\$48 \to Tiene\ A$

$\Rightarrow 2x=2\cdot 48=\$96 \to Tiene\ B$

4. $x \to Tiene\ A$

$\frac{3}{5}x \to Tiene\ B$

$\frac{3}{5}x+30=\frac{9}{5}(x-30)\ mcm=5$

$3x+150=9x-270$

$420=6x$

$70=x$

$\$70 \to Tiene\ A$

$\Rightarrow \frac{3}{5}x=\frac{3}{5}\cdot 70=\$42 \to Tiene\ B$

5. $x \to Tiene\ A\ y B$

$x-30=\frac{x+30}{2}\ mcm=2$

$2x-60=x+30$

$x=90$

$90\ suc. \to Tiene\ A\ y B$

6. $x \to Tiene\ A$

$\frac{2}{3}x \to Tiene\ B$

$\frac{2}{3}x+22=\frac{7}{5}(x-22)\quad mcm=15$

$10x+330=21x-462$

$792=11x$

$72=x\quad \$72 \to Tiene\ A$

$\Rightarrow \frac{2}{3}x=\frac{2}{3}\cdot 72=\$48 \to Tiene\ B$

7. $x \to Tiene\ B$

$\frac{4}{5}x \to Tiene\ A$

$\frac{4}{5}x+13=x-5\quad mcm=5$

$4x+65=5x-25$

$90=x\quad \$90 \to Tiene\ B$

$\Rightarrow \frac{4}{5}x=\frac{4}{5}\cdot 90=\$72 \to Tiene\ A$

8. $x \to Tiene\ A$

$\frac{x}{2} \to Tiene\ B$

$\frac{x}{2}+\frac{1}{3}x=x+5\quad mcm=6$

$3x+2x=6x+30$

$5x=6x+30$

$30=x\quad \$30 \to Tiene\ A$

$\Rightarrow \frac{x}{2}=\frac{30}{2}=\$15 \to Tiene\ B$

9. $x \to Empezaron\ A\ y\ B$

$x-\frac{3}{5}x=x-24\quad mcm=5$

$5x-3x=5x-120$

$-3x=-120$

$x=40$

$40\ balboas \to Empezaron\ A\ y\ B$

10. $x \to Empezaron\ A\ y\ B$

$x-\frac{3}{4}x=\frac{x}{4} \to Le\ queda\ \ B$

$\frac{x}{12} \to 3^a\ parte\ de\ lo\ queda\ B$

$\frac{x}{12}+24=\frac{x+288}{12} \to ha\ ganado\ A$

$\frac{x}{4}=x-\left(\frac{x+288}{12}\right)\ mcm=12$

$3x=12x-x-288$

$3x=11x-288$

$-8x=-288$

$x=36$

$36\ soles \to Empezaron\ A\ y\ B$

EJERCICIO 152

1. $x \to N^o\ años\ que\ pasan$

$28+x=\frac{3}{4}(x+38)\ mcm=4$

$112+4x=3x+114$

$x=2$

$2 \to años\ pasan$

2. $x \to N^o\ años\ que\ pasan$

$30+x=\frac{7}{6}(x+25)\quad mcm=6$

$180+6x=7x+175$

$5=x$

$5 \to años\ pasan$

3. $x \to N^o\ años\ hace$

$48-x=\frac{9}{10}(52-x)$

$mcm=10$

$480-10x=468-9x$

$12=x$

$12 \to años\ hace$

4. $x \to N^o\ años\ hace$

$18-x=\frac{1}{4}(27-x)$

$mcm=4$

$72-4x=27-x$

$45=3x$

$15=x$

$15 \to años\ hace$

5. $x \to La\ suma\ de\ dinero$

$\frac{3}{5}(50+x)=x+22\quad mcm=5$

$150+3x=5x+110$

$40=2x$

$20=x$

$\$20 \to La\ suma\ de\ dinero$

6. $x \to Gastó\ cada\ uno$

$\frac{3}{11}(90-x)=50-x\quad mcm=11$

$270-3x=550-11x$

$8x=280$

$x=35$

$35Q. \to Gastó\ cada\ uno$

7. $x \to$ *Edad actual hermano*

$\frac{3}{4}x \to$ *Edad actual persona*

$(x+x)+\frac{3}{4}x+x=75$

$3x+\frac{3}{4}x=75 \quad mcm=4$

$12x+3x=300$

$x=20$

20 *años* $\to$ *Edad actual hermano*

$\Rightarrow \frac{3}{4}x=\frac{3}{4}\cdot 20=15$ *años* $\to$ *Edad actual persona*

8. $x \to$ *ganó cada uno*

$54+x+32+x=4x+66$

$2x+86=4x+66$

$20=2x$

$10=x$

$\$\,10 \to$ *ganó cada uno*

9. $x \to$ *Le dió A a B*

$153-x=\frac{1}{4}(12+x) \quad mcm=4$

$612-4x=12+x$

$600=5x$

$120=x$

$120bs. \to$ *Le dió A a B*

EJERCICIO 153

1. $x+3 \to$ *Long. rect.*

$x \to$ *Ancho rect.*

$(x+4)(x+1)-22=x(x+3)$

$x^2+5x+4-22=x^2+3x$

$2x=18$

$x=9$

$9m \to$ *Ancho rect.*

$\Rightarrow x+3=9+3=12m \to$ *Long. rect.*

2. $x \to$ *Una de las dimensiones*

$2x \to$ *La otra dimensión*

$(x+5)(2x+5)-160=2x^2$

$2x^2+15x+25-160=2x^2$

$15x=135$

$x=9$

$9m\cdot 18m \to$ *Dimensiones*

3. $x+2 \to$ *Una dimensión*

$x \to$ *Otra dimensión*

$(x-5)(x-3)+115=x(x+2)$

$x^2-8x+15+115=x^2+2x$

$-10x=-130$

$x=13$

$15m\cdot 13m \to$ *Dimensiones*

4. $\frac{x}{2}+24 \to$ *Long.del rect.*

$\frac{x}{2}-12 \to$ *Ancho del rect.*

$\left(\frac{x}{2}+24\right)\left(\frac{x}{2}-12\right)=x\left(\frac{x}{2}-12\right)$

Continúa

Continuación

4. $\frac{x}{2}+24=x \quad mcm=2$

$x+48=2x$

$48=x$

$48m\cdot 12m \to$ *Dimensiones*

5. $x \to$ *Long. del cuadrado Equiv. al rect.*

$x+7 \to$ *Long. del rect.*

$x-6 \to$ *Ancho del rect.*

$(x+7)(x-6)=x^2$

$x^2+x-42=x^2$

$x=42$

$\Rightarrow x+7=42+7=49 \to$ *Long. del rect.*

$\Rightarrow x-6=42-6=36 \to$ *Ancho del rect.*

$49m\cdot 36m \to$ *Dimensiones*

6. $x \to$ *Ancho*

$x+30 \to$ *Longitud*

$(x+30-20)(x+15)+150=x(x+30)$

$(x+10)(x+15)+150=x^2+30x$

$x^2+25x+150+150=x^2+30x$

$300=5x$

$60=x$

$60m \to$ *Ancho*

$\Rightarrow x+30=60+30=90m \to$ *Long.*

$90m\cdot 60m \to$ *Dimensiones*

7. $x \to$ *Ancho*

$x+10 \to$ *Longitud*

$(x+8)(x+1)=x(x+10)$

$x^2+9x+8=x^2+10x$

$8=x \to$ *Ancho*

$\Rightarrow x+10=8+10=18 \to$ *Long.*

$18m\cdot 8m \to$ *Dimensiones*

EJERCICIO 154

1. $x \to$ *Numerador*

$x-2 \to$ *Denominador*

$\frac{x}{x-2+7}=\frac{1}{2}$

$\frac{x}{x+5}=\frac{1}{2}$

$2x=x+5$

$x=5$

$5 \to$ *Numerador*

$5-2=3 \to$ *Denominador*

$\frac{5}{3} \to$ *fracción*

2. $x \to$ *Numerador*

$x+1 \to$ *Denominador*

$\frac{x}{x+1+15}=\frac{1}{3}$

$\frac{x}{x+16}=\frac{1}{3}$

$3x=x+16$

$2x=16$

$x=8$

$8 \to$ *Numerador*

$8+1=9 \to$ *Denominador*

$\frac{8}{9} \to$ *Fracción*

3. $x-8 \to$ *Numerador*

$x \to$ *Denominador*

$\frac{x-8+1}{x+1}=\frac{3}{4}$

$\frac{x-7}{x+1}=\frac{3}{4}$

$4(x-7)=3(x+1)$

$4x-28=3x+3$

$x=31$

$31-8=23 \to$ *Numerador*

$31 \to$ *Denominador*

$\frac{23}{31} \to$ *Fracción*

4. $x \to$ *Numerador*

$2x+1 \to$ *Denominador*

$$\frac{x-4}{2x+1}=\frac{1}{3}$$

$$3(x-4)=2x+1$$

$$3x-12=2x+1$$

$$x=13$$

$13 \to$ *Numerador*

$2\cdot 13+1=27 \to$ *Denominador*

$\frac{13}{27} \to$ *Fracción*

5. $x \to$ *Numerador*

$2x+6 \to$ *Denominador*

$$\frac{x+15}{2x+6-1}=\frac{4}{3}$$

$$\frac{x+15}{2x+5}=\frac{4}{3}$$

$$3(x+15)=4(2x+5)$$

$$3x+45=8x+20$$

$$25=5x$$

$$5=x$$

$5 \to$ *Numerador*

$2\cdot 5+6=16 \to$ *Denominador*

$\frac{5}{16} \to$ *Fracción*

6. $x \to$ *Numerador*

$x+1 \to$ *Denominador*

$$\frac{x}{x+1+4}=3\left(\frac{x}{x+1}\right)-2$$

$$\frac{x}{x+5}=\frac{3x}{x+1}-2 \quad mcm=(x+5)(x+1)$$

$$x(x+1)=3x(x+5)-2(x+5)(x+1)$$

$$x^2+x=3x^2+15x-2x^2-12x-10$$

$$x=3x-10$$

$$10=2x$$

$$5=x$$

$5 \to$ *Numerador*

$5+1=6 \to$ *Denominador*

$\frac{5}{6} \to$ *Fracción*

7. $x \to$ *Numerador*

$3x-1 \to$ *Denominador*

$$\frac{x+8}{3x-1+4}=\frac{11}{12}$$

$$\frac{x+8}{3x+3}=\frac{11}{12}$$

$$12(x+8)=33(x+1)$$

$$12x+96=33x+33$$

$$63=21x$$

$$3=x$$

$3 \to$ *Numerador*

$3\cdot 3-1=8 \to$ *Denominador*

$\frac{3}{8} \to$ *Fracción*

8. $x \to$ *Numerador*

$x-22 \to$ *Denominador*

$$\frac{x}{x-22}-\frac{x-15}{x-22}=3 \quad mcm=x-22$$

$$x-(x-15)=3(x-22)$$

$$15=3x-66$$

$$81=3x$$

$$27=x$$

$27 \to$ *Numerador*

$27-22=5 \to$ *Denominador*

$\frac{27}{5} \to$ *Fracción*

EJERCICIO 155

1. $x \to$ *cifra de unidades*

$x+2 \to$ *cifra de decenas*

$$10(x+2)+x$$

$$10x+20+x$$

$11x+20 \to$ *El Número*

$$\frac{11x+20}{2(x+1)}=7$$

$$11x+20=14(x+1)$$

$$11x+20=14x+14$$

$$6=3x$$

$$2=x$$

$2 \to$ *cifra de unidades*

$2+2=4 \to$ *cifra de decenas*

$42 \to$ *N° buscado*

2. $x \to$ *cifra de unidades*

$x-4 \to$ *cifra de decenas*

$$10(x-4)+x$$

$$10x-40+x$$

$11x-40 \to$ *El Número*

$$\frac{11x-40}{2(x-2)}=4$$

$$11x-40=8x-16$$

$$3x=24$$

$$x=8$$

$8 \to$ *cifra de unidades*

$8-4=4 \to$ *cifra de decenas*

$48 \to$ *N° buscado*

3. $x \to$ *cifra de unidades*

$2x \to$ *cifra de decenas*

$$10(2x)+x-9$$

$21x-9 \to$ *N° disminuido en 9*

$$\frac{21x-9}{3x}=6$$

$$21x-9=18x$$

$$3x=9$$

$$x=3$$

$63 \to$ *N° buscado*

4. $x \rightarrow$ *cifra de unidades*

$x+1 \rightarrow$ *cifra de decenas*

$10(x+1)+x$

$3(11x+10)$

$33x+30 \rightarrow N^o$ *multip.*$*3$

$33x+30=21(2x+1)$

$33x+30=42x+21$

$9=9x$

$1=x$

$1 \rightarrow$ *Cifra de unidades*

$1+1=2 \rightarrow$ *Cifra de decenas*

$21 \rightarrow N^o$ *buscado*

5. $x \rightarrow$ *cifra de unidades*

$7-x \rightarrow$ *cifra de decenas*

$10(7-x)+x+8$

$78-9x \rightarrow N^o$ *aumentado en* 8

$\frac{78-9x}{2(7-x)}=6$

$78-9x=12(7-x)$

$78-9x=84-12x$

$3x=6$

$x=2$

$52 \rightarrow N^o$ *buscado*

6. $x \rightarrow$ *cifra de unidades*

$x+2 \rightarrow$ *cifra de decenas*

$10(x+2)+x$

$11x+20 \rightarrow$ *El Número*

$11x+20-27=10x$

$11x-7=10x$

$x=7$

$7 \rightarrow$ *cifra de unidades*

$7+2=9 \rightarrow$ *cifra de decenas*

$97 \rightarrow N^o$ *buscado*

7. $x \rightarrow$ *cifra de unidades*

$2x \rightarrow$ *cifra de decenas*

$10(2x)+x-4$

$21x-4 \rightarrow N^o$ *dis min en* 4

$\frac{21x-4}{2x-x}=20$

$21x-4=20x$

$x=4$

$84 \rightarrow N^o$ *buscado*

EJERCICIO 156

1. $x \rightarrow$ *Días trabaj. juntos*

$\frac{1}{3}+\frac{1}{6}=\frac{1}{x}$ $\quad mcm=6x$

$2x+x=6$

$3x=6$

$x=2$

2 *días* $\rightarrow$ *Hacen la obra*

2. $x \rightarrow$ *Tiempo en llenar depósito*

$\frac{1}{10}+\frac{1}{20}=\frac{1}{x}$ $\quad mcm=20x$

$2x+x=20$

$3x=20$

$x=\frac{20}{3}$

$x=6\frac{2}{3}$

$6\frac{2}{3}$min. $\rightarrow$ *En llenar depósito*

3. $x \rightarrow$ *Hacen obra juntos*

$\frac{1}{4}+\frac{1}{6}+\frac{1}{12}=\frac{1}{x}$ $\quad mcm=12x$

$3x+2x+x=12$

$6x=12$

$x=2$

2 *días* $\rightarrow$ *Hacen obra juntos*

4. $x \rightarrow$ *Hacen obra juntos*

$\frac{1}{1\frac{1}{2}}+\frac{1}{6}+\frac{1}{2\frac{2}{5}}=\frac{1}{x}$

$\frac{2}{3}+\frac{1}{6}+\frac{5}{12}=\frac{1}{x}$ $\quad mcm=12x$

Continúa

Continuación

4. $8x+2x+5x=12$

$15x=12$

$x=\frac{12}{15}$

$x=\frac{4}{5}$

$\frac{4}{5}$ *de día hacen la obra*

5. $x \rightarrow$ *Tiempo en llenar depósito*

$\frac{1}{5}+\frac{1}{6}+\frac{1}{12}=\frac{1}{x}$ $\quad mcm=60x$

$12x+10x+5x=60$

$27x=60$

$x=\frac{60}{27}$

$x=2\frac{2}{9}$

$2\frac{2}{9}$ min $\rightarrow$ *Llenar el depósito*

6. $x \rightarrow$ *Tiempo en llenar depósito*

$\frac{1}{4}+\frac{1}{8}-\frac{1}{20}=\frac{1}{x}$ $\quad mcm=40x$

$10x+5x-2x=40$

$13x=40$

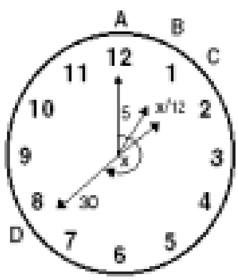
$x=\frac{40}{13}$

$x=3\frac{1}{13}$

$3\frac{1}{13}$ min. $\rightarrow$ *Llenar el depósito*

EJERCICIO 157

1.



$ABCD=x$

$AB=5$

$BC=\frac{X}{12}$

$CD=30$

$x=5+30+\frac{x}{12}$

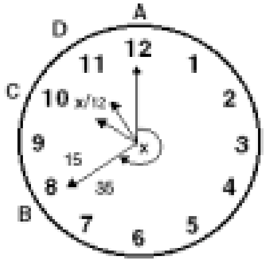
$x=35+\frac{x}{12}$

$12x=420+x$

$11x=420$

$x=\frac{420}{11}=38\frac{2}{11}$

2.



$AB=x$

$BD=15$

$AC=50$

$CD=\frac{X}{12}$

$AB+BD=AC+CD$

$x+15=50+\frac{x}{12} \quad mcm=12$

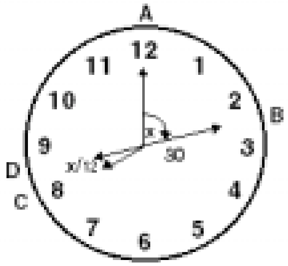
$12x+180=600+x$

$11x=420$

$x=\frac{420}{11}$

$x=38\frac{2}{11}$

3.



$AB=x$

$BD=30$

$AC=40$

$CD=\frac{x}{12}$

$AB+BD=AC+CD$

$x+30=40+\frac{x}{12} \quad mcm=12$

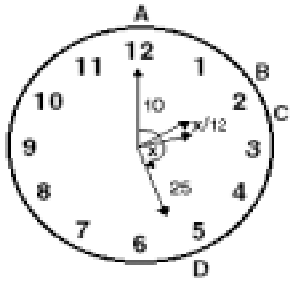
$12x+360=480+x$

$11x=120$

$x=\frac{120}{11}$

$x=10\frac{10}{11}$

5.



$ABCD=x$

$AB=10$

$BC=\frac{x}{12}$

$CD=15$

$ABCD=AB+BC+CD$

$x=10+\frac{x}{12}+15$

$x=25+\frac{x}{12}$

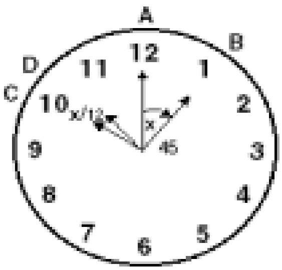
$mcm=12$

$12x=300+x$

$11x=300$

$x=\frac{300}{11}$

$x=27\frac{3}{11}$



$AB=x$

$BD=45$

$AC=50$

$CD=\frac{x}{12}$

$AC+CD=AB+BD$

$50+\frac{x}{12}=x+45 \quad mcm=12$

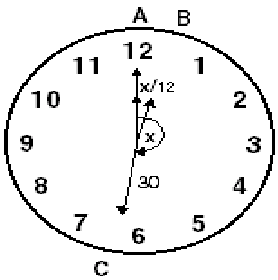
$600+x=12x+540$

$60=11x$

$\frac{60}{11}=x$

$5\frac{5}{11}=x$

4.



$AB=\frac{x}{12}$

$BC=30$

$ABC=x$

$ABC=AB+BC$

$x=\frac{x}{12}+30 \quad mcm=12$

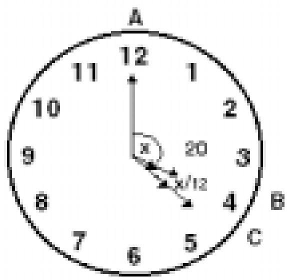
$12x=x+360$

$11x=360$

$x=\frac{360}{11}$

$x=32\frac{8}{11}$

6.



$ABC=x$

$AB=20$

$BC=\frac{x}{12}$

$ABC=AB+BC$

$x=20+\frac{x}{12} \quad mcm=12$

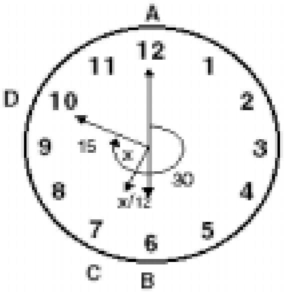
$12x=240+x$

$11x=240$

$x=\frac{240}{11}$

$x=21\frac{9}{11}$

7.



$ABCD = x$

$AB = 30$

$BC = \frac{x}{12}$

$CD = 15$

$ABCD = AB + BC + CD$

$x = 30 + \frac{x}{12} + 15 \quad mcm = 12$

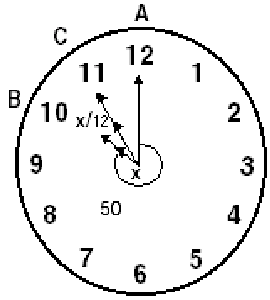
$12x = 360 + x + 180$

$11x = 540$

$x = \frac{540}{11}$

$x = 49\frac{1}{11}$

8.



$ABC = x$

$AB = 50$

$BC = \frac{x}{12}$

$ABC = AB + BC$

$x = 50 + \frac{x}{12} \quad mcm = 12$

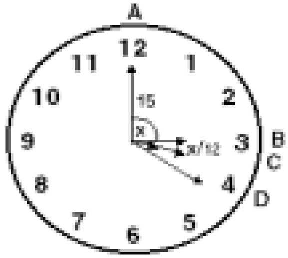
$12x = 600 + x$

$11x = 600$

$x = \frac{600}{11}$

$x = 54\frac{6}{11}$

10.



$ABCD = x$

$AB = 15$

$BC = \frac{x}{12}$

$CD = 5$

$ABCD = AB + BC + CD$

$x = 15 + \frac{x}{12} + 5$

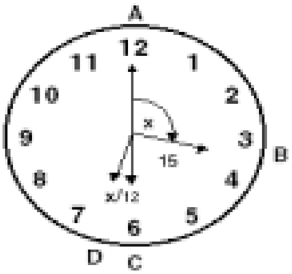
$x = 20 + \frac{x}{12} \quad mcm = 12$

$12x = 240 + x$

$11x = 240$

$x = \frac{240}{11}$

$x = 21\frac{9}{11}$



$AB = x$

$BD = 15$

$AC = 30$

$CD = \frac{x}{12}$

$AB + BD = AC + CD$

$x + 15 = 30 + \frac{x}{12}$

$x - \frac{x}{12} = 15 \quad mcm = 12$

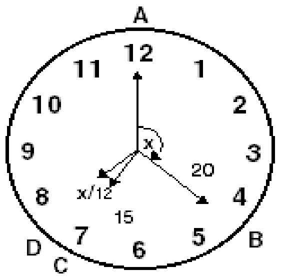
$12x - x = 180$

$11x = 180$

$x = \frac{180}{11}$

$x = 16\frac{4}{11}$

9.



$AB = x$

$BD = 15$

$AC = 35$

$CD = \frac{x}{12}$

$AB + BD = AC + CD$

$x + 15 = 35 + \frac{x}{12}$

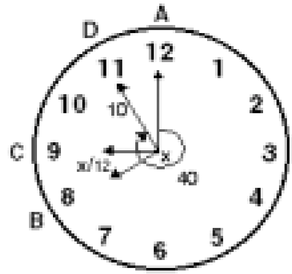
$12x + 180 = 420 + x$

$11x = 240$

$x = \frac{240}{11}$

$x = 21\frac{9}{11}$

11.



$ABCD = x$

$AB = 40$

$BC = \frac{x}{12}$

$CD = 10$

$ABCD = AB + BC + CD$

$x = 40 + \frac{x}{12} + 10$

$x = 50 + \frac{x}{12} \quad mcm = 12$

$12x = 600 + x$

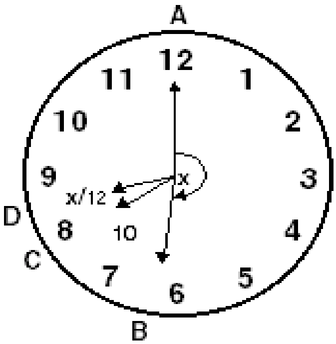
$11x = 600$

$x = \frac{600}{11}$

$x = 54\frac{6}{11}$

Continúa

11. Continuación



$AB = x$

$BD = 10$

$AC = 40$

$CD = \frac{x}{12}$

$AB + BD = AC + CD$

$x + 10 = 40 + \frac{x}{12}$

$x - \frac{x}{12} = 30$

$mcm = 12$

$12x - x = 360$

$11x = 360$

$x = \frac{360}{11}$

$x = 32\frac{8}{11}$

EJERCICIO 158

1. $x \rightarrow N^{o}\ mayor$

$x - 6 \rightarrow N^{o}\ menor$

$\frac{x}{2} - 10 = \frac{3}{8}(x - 6)\quad mcm = 8$

$4x - 80 = 3x - 18$

$x = 62 \rightarrow N^{o}\ mayor$

$62 - 6 = 56 \rightarrow N^{o}\ menor$

2. $x \rightarrow Le\,dió\ A\ a\ B$

$\frac{11}{10}(120 - x) = 90 + x\quad mcm = 10$

$1.320 - 11x = 900 + 10x$

$420 = 21x$

$20 = x$

$\$\ 20 \rightarrow Le\,dió\ A\ a\ B$

3. $x \rightarrow N^{o}\ buscado$

$\frac{x+6}{8} + 5$

$\frac{x+46}{8} \rightarrow Cociente\ sumado\ 5$

$\frac{\frac{x+46}{8}}{2} = \frac{x+46}{16} \rightarrow Nueva\ sum \div en 2$

$\frac{x+46}{16} = 4$

$x + 46 = 64$

$x = 18 \quad \rightarrow N^{o}\ buscado$

4. $x \rightarrow Persona\ favorecida$

$\frac{5}{7}x \rightarrow Persona\ que\ recibió\ menos$

$x + \frac{5}{7}x = 48.000 \qquad mcm = 7$

$7x + 5x = 336.000$

$12x = 336.000$

$x = 28.000$

$28.000\ \ soles\ \rightarrow Pers.\ favorecida$

$\frac{5}{7} \cdot 28.000 = 20.000$

$20.000\, soles \rightarrow Pers.\ que\ rec.men.$

5. $x \to$ *Parte mayor*

$84 - x \to$ *Parte menor*

$\frac{1}{10}x = \frac{1}{4}(84 - x) \quad mcm = 20$

$2x = 5(84 - x)$

$2x = 420 - 5x$

$7x = 420$

$x = 60$

$60 \to$ *Parte mayor*

$84 - 60 = 24 \to$ *Parte menor*

6. $x \to$ *Parte mayor*

$120 - x \to$ *Parte menor*

$120 - x = \frac{3}{5}x \quad mcm = 5$

$600 - 5x = 3x$

$600 = 8x$

$75 = x \to$ *Parte mayor*

$120 - 75 = 45 \to$ *Parte menor*

7. $x \to$ *Sueldo mensual*

$\frac{x}{2} + \frac{3}{8}x = \frac{7}{8}x \to$ *mensual*

$15x - 15\left(\frac{7}{8}x\right) = 300 \quad mcm = 8$

$120x - 105x = 2.400$

$15x = 2.400$

$x = 160$

$\$160 \to$ *Sueldo mensual*

8. $x \to$ *Lo que tenía*

$\frac{1}{5}x \to$ *Gastó en ropa*

$\frac{3}{8}x \to$ *Gastó en libros*

$x - \frac{1}{5}x - \frac{3}{8}x - 102 = 0 \quad mcm = 40$

$40x - 8x - 15x - 4.080 = 0$

$17x = 4.080$

$x = 240$

$\frac{1}{5} \cdot 240 = \$48 \to$ *Gastó en ropa*

$\frac{3}{8} \cdot 240 = \$90 \to$ *Gastó en libros*

9. $x \to$ *Edad de A*

$\frac{2}{5}x \to$ *Edad de B*

$\frac{2}{3} \cdot \frac{2}{5}x = \frac{4}{15}x \to$ *Edad de C*

$x + \frac{2}{5}x + \frac{4}{15}x = 25 \quad mcm = 15$

$15x + 6x + 4x = 375$

$25x = 375$

$x = 15$

15 *años* $\to$ *Edad de A*

$\frac{2}{5} \cdot 15 = 6$ *años* $\to$ *Edad de B*

$\frac{4}{15} \cdot 15 = 4$ *años* $\to$ *Edad de C*

10. $x \to$ *Costo auto*

$8.000 + \frac{x}{3} = x + 2.000 \quad mcm = 3$

$24.000 + x = 3x + 6.000$

$18.000 = 2x$

$9.000 = x$

$9.000\, bs \to$ *Costo auto*

11. $x \to$ *Libros compre*

$\frac{7x}{2} - \frac{5x}{2} = 8 \quad mcm = 2$

$7x - 5x = 16$

$2x = 16$

$x = 8 \to$ *Lib.compre*

12. $x \to$ *Cierto N° de libros*

$\frac{3x}{4} \to$ *Vr.cierto N° de libros*

$\frac{3x}{4}\left(\frac{7}{10}\right) = \frac{21x}{40} \to Vr.\frac{3}{4}$ *del N° de lib. ant.*

$\frac{3x}{4} + \frac{21x}{40} = \frac{51x}{40} \to$ *Vr.total de lib.compré*

$x + \frac{3x}{4} = \frac{7x}{4} \to$ *total lib. comp.*

Continúa

12. Continuación

$\frac{7x}{4} \cdot \frac{3}{2} = \frac{21x}{8} \to$ *Vr. total vta. de todos los lib.*

Luego

$\frac{21x}{8} - \frac{51x}{40} = 54 \quad mcm = 40$

$105x - 51x = 2.160$

$54x = 2.160$

$x = 40$

$\frac{7x}{4} = \frac{7 \cdot 40}{4} = 70$

$70 \to$ *Total libros comprados*

13. $x \to 1^a$ *parte*

$\frac{5x}{6} \to 2^a$ *parte*

$\frac{3}{5} \cdot \frac{5x}{6} = \frac{1}{2}x \to 3^a$ *parte*

$\frac{1}{3} \cdot \frac{1}{2}x = \frac{1}{6}x \to 4^a$ *parte*

$x + \frac{5x}{6} + \frac{1}{2}x + \frac{1}{6}x = 150$

$mcm = 6$

$6x + 5x + 3x + x = 900$

$15x = 900$

$x = 60$

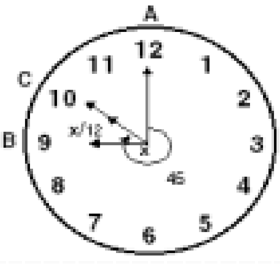
$60 \to 1^a$ *parte*

$\frac{5}{6} \cdot 60 = 50 \to 2^a$ *parte*

$\frac{1}{2} \cdot 60 = 30 \to 3^a$ *parte*

$\frac{1}{6} \cdot 60 = 10 \to 4^a$ *parte*

14.



$ABC = x$

$AB = 45$

$BC = \frac{x}{12}$

$ABC = AB + BC$

$x = 45 + \frac{x}{12} \quad mcm = 12$

$12x = 540 + x$

$11x = 540$

$x = \frac{540}{11}$

$x = 49\frac{1}{11}$

15. $x \rightarrow$ *Edad A*

$x - 10 \rightarrow$ *Edad B*

$x - 10 - 15 = \frac{3}{4}(x - 15) \; mcm = 4$

$4x - 40 - 60 = 3x - 45$

$4x - 100 = 3x - 45$

$x = 55$

55 *años* $\rightarrow$ *Edad A*

$55 - 10 = 45$ *años* $\rightarrow$ *Edad B*

16. $x \rightarrow$ *Puede hacerla A*

$\frac{1}{10} + \frac{1}{x} = \frac{1}{6} \quad mcm = 60x$

$6x + 60 = 10x$

$60 = 4x$

$15 = x$

15 *días* $\rightarrow$ *Puede hacerla A*

17. $x \rightarrow$ *Parte mayor*

$650 - x \rightarrow$ *Parte menor*

$\frac{x}{5} = 650 - x - 50$

$\frac{x}{5} = 600 - x \quad mcm = 5$

$x = 3.000 - 5x$

$6x = 3.000$

$x = 500$

$500 \rightarrow$ *Parte mayor*

$\Rightarrow 650 - 500 = 150 \rightarrow$ *Parte menor*

18. $x \rightarrow$ *Edad actual B*

$\frac{x}{4} \rightarrow$ *Edad actual A*

$\frac{x}{4} - 10 = \frac{1}{10}(x - 10) \quad mcm = 40$

$10x - 400 = 4x - 40$

$6x = 360$

$x = 60$

60 *años* $\rightarrow$ *Edad actual B*

$\frac{60}{4} = 15$ *años* $\rightarrow$ *Edad actual A*

19. $x \rightarrow$ *Nº mayor*

$x - 1 \rightarrow$ *Nº menor*

$x^2 - (x-1)^2 - 43 = \frac{1}{11}(x - 1)$

$x^2 - x^2 + 2x - 1 - 43 = \frac{1}{11}(x - 1)$

$2x - 44 = \frac{1}{11}(x - 1)$

$11(2x - 44) = x - 1$

$22x - 484 = x - 1$

$21x = 483$

$x = 23$

$23 \rightarrow$ *Nº mayor*

$23 - 1 = 22 \rightarrow$ *Nº menor*

20. $x \rightarrow$ *Vr. sortija*

$7\left(\frac{3.000 + x}{12}\right) = 1.500 + x \quad mcm = 12$

$21.000 + 7x = 18.000 + 12x$

$3.000 = 5x$

$600 = x$

600 *sucres* $\rightarrow$ *Vr. sortija*

21. $x \rightarrow$ *Nº personas*

$x + \frac{1}{5}x = \frac{6x}{5} \rightarrow$ *Nº más de pers.*

$\frac{120}{\frac{6x}{5}} = \frac{100}{x}$

$\frac{100}{x} = \frac{120}{x} - 2 \quad mcm = x$

$100 = 120 - 2x$

$2x = 20$

$x = 10$

10 *personas* $\rightarrow$ *Se repartió $*

22. $x \rightarrow$ *libros que compró*

$x + \frac{x}{4} = \frac{5x}{4} \rightarrow$ *Libros más*

$\frac{400}{\frac{5x}{4}} = \frac{320}{x}$

$\frac{320}{x} = \frac{400}{x} - 2 \quad mcm = x$

$320 = 400 - 2x$

$2x = 80$

$x = 40$

40 *libros* $\rightarrow$ *compró*

$\frac{400}{40} = \$10 \rightarrow$ *Pagó por c/u*

23. $x \rightarrow$ *Suma repartida*

$\frac{x}{2} - 30 \rightarrow$ *Recibió A*

$\frac{3x}{7} + 20 \rightarrow$ *Recibió B*

$\frac{x}{2} - 30 + \frac{3x}{7} + 20 + 30 = x$

$mcm = 14$

$7x + 6x + 280 = 14x$

$280 = x$

$\frac{280}{2} - 30 = \$110 \rightarrow$ *Rec. A*

$\frac{3(280)}{7} + 20 = \$140 \rightarrow$ *Rec. B*

24. $x \rightarrow$ *Lib.comprad.*

$\frac{6x}{5} \rightarrow$ *Vr. de x lib.*

$x - \frac{x}{3} = \frac{2x}{3} \rightarrow$ *Resto*

$\frac{2x}{3} \cdot \frac{9}{4} = \frac{3x}{2} \rightarrow$ *Vr.vta resto*

$\frac{3x}{2} - \frac{6x}{5} = 9 \quad mcm = 10$

$15x - 12x = 90$

$3x = 90$

$x = 30$

$30 \rightarrow$ *Libros comprados*

25. $x \rightarrow$ *Fortuna*

$x - \frac{x}{2} - \frac{x}{4} - \frac{x}{6} = 2.500$

$mcm = 12$

$12x - 6x - 3x - 2x = 30.000$

$x = 30.000$

30.000 *colones* $\rightarrow$ *Fortuna*

26. $x \rightarrow$ *sueldo anual*

$x-\frac{3x}{5}-\frac{x}{8}-\frac{x}{20}=810$

$mcm=40$

$40x-24x-5x-2x=32.400$

$9x=32.400$

$x=3.600$

3.600 *balboas* $\rightarrow$ *sueldo anual*

27. $x \rightarrow$ *Ahorros*

$x-\frac{3x}{8}-\frac{5x}{12}=\frac{5x}{24}\ \rightarrow$ *Le quedaba*

$\frac{3}{5}\cdot\frac{5x}{24}=\frac{x}{8}$

$\frac{5x}{24}-\frac{x}{8}=400 \quad mcm=24$

$5x-3x=9.600$

$2x=9.600$

$x=4.800$

$\$4.800 \rightarrow$ *ahorros*

28. $x \rightarrow$ *Parte mayor*

$350-x \rightarrow$ *Parte menor*

$350-x-\frac{3x}{5}=x-\frac{17}{15}(350-x)$

$mcm=15$

$5.250-15x-9x=15x-5.950+17x$

$-24x-32x=-11.200$

$-56x=-11.200$

$x=200$

$200 \rightarrow$ *Parte mayor*

$350-200=150 \rightarrow$ *Parte menor*

29. $x \rightarrow$ *Suma repartida*

$\$15 \rightarrow$ Recibió A

$x-15=B+C$

$x-15-C=B$

$C=A+B$

$C=15+(x-15-C)$

$C=x-C$

$2C=x$

$C=\frac{x}{2}$

$15+15+\frac{2}{3}\cdot\frac{x}{2}+\frac{x}{2}=x$

$30+\frac{x}{3}+\frac{x}{2}=x \quad mcm=6$

$180+2x+3x=6x$

$180=x$

$\$180 \rightarrow$ *Suma recibida*

30. $x \rightarrow$ *Pesos*

$\frac{3}{4}x \rightarrow N^o$ *piezas de* 20 *ctvs*

$\frac{2}{3}\cdot\frac{3x}{4}=\frac{x}{2} \rightarrow N^o$ *piezas de* 10 *ctvs*

$x+\frac{3}{4}(0,20x)+\frac{0,10x}{2}=9,60$

$mcm=4$

$4x+0,6x+0,2x=38,4$

$4,8x=38,4$

$x=8$

$8 \rightarrow$ *Pesos*

$\frac{3}{4}\cdot 8=6 \rightarrow$ *piezas de* 20 *ctvs*

$\frac{8}{2}=4 \rightarrow$ *piezas de* 10 *ctvs*

31. $x \rightarrow$ *Capital primitivo*

$\frac{1}{5}x \rightarrow 1^{er}$ *año*

$x-\frac{x}{5}=\frac{4x}{5} \rightarrow$ *Le quedaba*

$\frac{3}{10}\cdot\frac{4x}{5}=\frac{6x}{25} \rightarrow 2^o$ *año*

$\frac{4x}{5}+\frac{6x}{25}=\frac{26x}{25} \rightarrow$ *Tenia al term. el 2º año*

$\frac{3}{5}\cdot\frac{26x}{25}=\frac{78x}{125} \rightarrow 3^{er}$ *año*

$x-\frac{x}{5}+\frac{6x}{25}+\frac{78x}{125}=13.312$

$125x-25x+30x+78x=1'664.000$

$208x=1'664.000$

$x=8.000$

8.000 *Q.* $\rightarrow$ *Capital primitivo*

32. $x \rightarrow$ *Edad de A*

$x-10=\frac{2}{3}(x+5) \quad mcm=3$

$3x-30=2x+10$

$x=40$

40 *años* $\rightarrow$ *Edad de A*

33. $x \rightarrow$ *Cant. hombres en el lado del* 1er *cuadrado*

$x^2+36 \rightarrow$ *Hombres en la tropa*

$(x+1)^2=x^2+36+75$

Continúa

Continuación

33. $x^2+2x+1=x^2+111$

$2x=110$

$x=55$

$55 \rightarrow$ *hombres en el lado del* 1er *cuadrado*

$(55)^2+36$

$=3.061 \rightarrow$ *Homb. en la tropa*

34. $x \rightarrow$ *Lo que tenía*

$x-\frac{5x}{8}-20=\frac{x}{4}+16$

$mcm=8$

$8x-5x-160=2x+128$

$x=288$

$\$288 \rightarrow$ *tenía*

35. $x \rightarrow$ *Empezó a jugar*

$2x-60 \rightarrow$ *Le quedaba*

$\frac{3}{10}(2x-60)=\frac{6x-180}{10}$

$x+x-60-\left(\frac{6x-180}{10}\right)-\frac{7x}{8}=0$

$mcm=40$

$80x-2.400-24x+720-35x=0$

$21x=1.680$

$x=80$

80 *Lempiras* $\rightarrow$ *Empezó a jugar*

36. $x \rightarrow$ *Cifra unidades*

$x+5 \rightarrow$ *Cifra decenas*

$10(x+5)+x-18=6(x+5+x)$

$10x+50+x-18=12x+30$

$2=x$

$2 \rightarrow$ *Cifra unidades*

$2+5=7 \rightarrow$ *Cifra decenas*

$72 \rightarrow N^o$ *buscado*

37. $x \rightarrow$ *Unidades*

$9-x \rightarrow$ *Decenas*

$10(9-x)+x-27=10x+9-x$

$90-10x+x-27=10x+9-x$

$54=18x$

$3=x$

$3 \rightarrow$ *Unidades*

$9-3=6 \rightarrow$ *Decenas*

$63 \rightarrow N^o$ *buscado*

38. $x \to$ *Mangos que había*

$x-\frac{x}{3}-4=\frac{2x-12}{3} \to$ *Quedaban*

$\frac{1}{3}\left(\frac{2x-12}{3}\right)+6=\frac{2x+42}{9}$

$\frac{\frac{2x+42}{9}}{2}+9=\frac{x+102}{9}$

$x-\frac{x}{3}-4-\left(\frac{2x+42}{9}\right)-\left(\frac{x+102}{9}\right)=0$

$mcm=9$

$9x-3x-36-2x-42-x-102=0$

$3x=180$

$x=60$

$60 \to$ *Mangos había*

39. $x \to$ *Ganó c/u*

$\frac{7}{10}(80+x)=50+x \quad mcm=10$

$560+7x=500+10x$

$60=3x$

$20=x$

$\$20 \to$ *Ganó c/u*

40. $x \to$ *Pluma fuente*

$\frac{3}{5}x \to$ *Lapicero*

$\frac{5}{6}(x-0{,}20)=\frac{3}{5}x+0{,}30$

$mcm=30$

$25x-5=18x+9$

$7x=14$

$x=2$

$\$2 \to$ *Costo la pluma*

$\frac{3\cdot 2}{5}=\$1{,}20 \to$ *Costo lapicero*

41. $x \to$ *Tenía el lunes antes de*

$\frac{x}{2}+2=\frac{x+4}{2} \to$ *Lunes*

$x-\left(\frac{x+4}{2}\right)=\frac{x-4}{2} \to$ *Quedaba*

$\frac{\frac{x-4}{2}}{2}+2=\frac{x+4}{4} \to$ *Martes*

$\frac{x-4}{2}-\left(\frac{x+4}{4}\right)=\frac{x-12}{4} \to$ *Quedaba*

$\frac{\frac{x-12}{4}}{2}+2=\frac{x+4}{8} \to$ *Miercoles*

Continúa

Continuación

41. $\frac{x-12}{4}-\left(\frac{x+4}{8}\right)=0 \qquad mcm=8$

$2x-24-x-4=0$

$x-28=0$

$x=28$

$\Rightarrow \$28 \to$ *Tenía el lunes antes de*

42. $x \to$ *Capital primitivo*

$x+\frac{x}{2}-6.000=\frac{3x-12.000}{2} \to$ *Tenía*

$\frac{\frac{3x-12.000}{2}}{2}-6.000=\frac{3x-36.000}{4} \to$ *2º año*

$\frac{3x-12.000}{2}+\frac{3x-36.000}{4}=\frac{9x-60.000}{4} \to$ *Tenía*

$\frac{\frac{9x-60.000}{4}}{2}-6.000=\frac{9x-108.000}{8} \to$ *3º año*

$\frac{9x-60.000}{4}+\frac{9x-108.000}{8}=32.250 \qquad mcm=8$

$18x-120.000+9x-108.000=258.000$

$27x=486.000$

$x=18.000$

$\$18.000 \to$ *Capital primitivo*

43. $x \to$ *Precio traje*

$\$15 \to$ *Precio bastón*

$s+15=\frac{3x}{4}$

$x+15-5=2s$

$\frac{x+10}{2} \to$ *Precio sombrero*

$\frac{x+10}{2}+15=\frac{3x}{4} \qquad mcm=4$

$2x+20+60=3x$

$80=x$

$\$80 \to$ *Precio traje*

$\frac{80+10}{2}=\$45 \to$ *Precio sombrero*

44. $x \to$ *Espacio recorrido por perro y conejo*

	Perro	Conejo
supuesto	2	5
Pregunta	?	50

Luego: $\frac{2\cdot 50}{5} \to$ *Saltos del perro con respecto a la ventaja*

	Perro	Conejo
supuesto	3	8
Pregunta	x	?

Luego: $\frac{8x}{3} \to$ *Saltos que avanza el conejo al ser alcanzado*

Continúa

Continuación

44.

	Perro	*Conejo*
sup*uesto*	2	5
Pr*egunta*	x	?

Luego: $\frac{5x}{2} \rightarrow$ *Saltos que da el perro para alcanzar al conejo*

Entonces:

$$\frac{8x}{3} - \frac{2 \cdot 50}{5} = \frac{5x}{2} \qquad mcm = 6$$

$$16x - 120 = 15x$$

$$x = 120$$

$$\frac{5 \cdot 120}{2} = 300 \rightarrow \textit{Saltos del perro para alcanzar el conejo}$$

45. $x \rightarrow$ *Espacio recorrido por perro y liebre*

	Perro	*Liebre*
sup*uesto*	3	4
Pr*egunta*	?	60

Luego: $\frac{3 \cdot 60}{4} \rightarrow$ *Saltos del perro con respecto a la ventaja*

	Perro	*Liebre*
sup*uesto*	5	8
Pr*egunta*	x	?

Luego: $\frac{8x}{5} \rightarrow$ *Saltos que avanza la liebre al ser alcanzada*

	Perro	*Liebre*
sup*uesto*	3	4
Pr*egunta*	x	?

Luego: $\frac{4x}{3} \rightarrow$ *Saltos que da el perro para alcanzar a la liebre*

Entonces:

$$\frac{8x}{5} - \frac{3 \cdot 60}{4} = \frac{4x}{3}$$

$$\frac{8x}{5} - 45 = \frac{4x}{3} \qquad mcm = 15$$

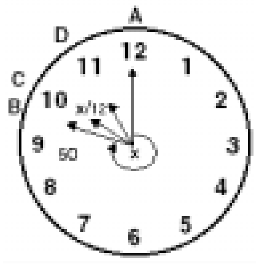
$$24x - 675 = 20x$$

$$4x = 675$$

$$x = 168{,}75$$

$$\frac{4}{3} \cdot 168{,}75 = 225 \rightarrow \textit{Saltos del perro para alcanzar la liebre}$$

46.



$$AB = x$$

$$AC = 50$$

$$CD = \frac{x}{12}$$

$$BD = 6$$

$$AB + BD = AC + CD$$

$$x + 6 = 50 + \frac{x}{12}$$

$$x - \frac{x}{12} = 44 \qquad mcm = 12$$

$$12x - x = 528$$

$$11x = 528$$

$$x = \frac{528}{11}$$

$$x = 48$$

47. $x \rightarrow$ *Aporta A*

$$\frac{3}{4}x \rightarrow \textit{Aporta B}$$

$$x - \frac{x}{5} = \frac{4x}{5} \rightarrow 1^{er} \textit{ año } A$$

$$\frac{3x}{4} + 3.000 = \frac{3x + 12.000}{4} \rightarrow 1^{er} \textit{ año } B$$

$$\frac{4x}{5} + 1.600 = \frac{4x + 8.000}{5} \rightarrow 2^{o} \textit{ año } A$$

$$\frac{3x + 12.000}{4} - \frac{1}{9}\left(\frac{3x + 12.000}{4}\right) \qquad mcm = 36$$

$$\frac{27x + 108.000 - 3x - 12.000}{36}$$

$$\frac{24x + 96.000}{36} = \frac{3(8x + 32.000)}{36}$$

$$\frac{8x + 32.000}{12} \rightarrow 2^{o} \textit{ año } B$$

Entonces:

$$\frac{4x + 8.000}{5} = \frac{8x + 32.000}{12} \qquad mcm = 60$$

$$48x + 96.000 = 40x + 160.000$$

$$8x = 64.000$$

$$x = 8.000$$

$$8.000\,bs \rightarrow \textit{Aportó A}$$

$$\frac{3 \cdot 8.000}{4} = 6.000\ bs \rightarrow \textit{Aportó B}$$

48. $x \to$ *Años para que la edad del padre sea = a la suma de sus hijos*

$60+x=16+x+14+x$

$60+x=30+2x$

$30=x$

30 años $\to$ *Edad del padre sea igual a la suma de sus hijos*

49. $x \to$ *Km. recorridos*

$\frac{x}{50}+\frac{x}{10}=12 \quad mcm=50$

$x+5x=600$

$6x=600$

$x=100$

$100 \to$ *Km. recorridos*

50. *Buey* $\to \$80$

$P+80=2C$

$C+80=\frac{13P}{2}$

Luego:

$\frac{P+80}{2}=C$

$\frac{P+80}{2}+80=\frac{13P}{2} \quad mcm=2$

$P+80+160=13P$

$240=12P$

$20=P$

$\$20 \to$ *Costo el perro*

$\frac{20+80}{2}=C \quad mcm=2$

$20+80=2C$

$100=2C$

$50=C$

$\$\ 50 \to$ *Costo el caballo*

EJERCICIO 159

1. $a=30m$

$v=8\,m/s$

$v'=5\,m/s$

$x=\frac{30\cdot 8}{8-5}$

$x=\frac{240}{3}$

$x=80m$

2. $a=160\,Km$

$v=50\,Km/h$

$v'=-30\,Km/h$

$x=\frac{160\cdot 50}{50-(-30)}$

$x=\frac{8.000}{80}$

$x=100\ Km.$

3. $a=200Km$

$v=90\,Km/h$

$v'=40\,Km/h$

$x=\frac{200\cdot 90}{90-40}$

Continúa

Continuación

3. $x=\frac{18.000}{50}$

$x=360\ Km$ *de A*

De B.

Como la distancia entre A y B es 200 Km

Entonces B recorrió

$360\ Km-200\ Km=160\ Km$

4. $a=80Km$

$v=90\,Km/h$

$v'=70\,Km/h$

$x=\frac{80\cdot 90}{90-70}$

$x=\frac{7.200}{20}$

$x=360\ Km$

El tiempo empleado en encontrarse será.

$\frac{360Km}{90\,Km/h}=4\,horas$

5. $a=550\,Km$

$v=100\,Km/h$

$v'=-120\,Km/h$

De A.

$x=\frac{550\cdot 100}{100-(-120)}$

$x=\frac{55.000}{220}$

$x=250\ Km$

Como los trenes pasan por A y B a las 8am

Entonces De A.

$x=250\,Km$

$v=100\,Km/h$

Tiempo empleado para los 250 Km. es

$\frac{250\,Km}{100\,Km/h}=2,5\,horas$

$2,5\,horas \to 2\,h\,30\,\text{min.}$

Luego:

$8am+2\,h\,30\,\text{min}=10:30$

$10:30\,am \to$ *Se encuentran*

6. $a=70\,Km$

$v=9\,{}^{Km}\!/\!_{h}$

$v'=-5\,{}^{Km}\!/\!_{h}$

$x\rightarrow$ *Distancia recorrida por A al encontrarse*

$x=\dfrac{70\cdot 9}{9-(-5)}$

$x=\dfrac{630}{14}$

$x=45\ Km\rightarrow\ A$

Como A recorrió $45\,Km$

Entonces B

$70\ Km-45\ Km=25\,Km\ \rightarrow\ B$

9. $a=186\,Km$

$v=v$

$v'=-v$

$x=\dfrac{186v}{v-(-v)}$

$x=\dfrac{186v}{2v}$

$x=93\,Km.$

7. *A y B distantes* $29\tfrac{1}{2}\,Km$

$\Rightarrow 29\tfrac{1}{2}\,Km-2\tfrac{1}{2}\,Km=27\,Km$

$a=27\,Km$

$v=5\,{}^{Km}\!/\!_{h}$

$v'=-4\,{}^{Km}\!/\!_{h}$

$x=\dfrac{27\cdot 5}{5-(-4)}$

$x=\dfrac{135}{9}$

$x=15\ Km$

Luego A recorre

$15\,Km+2\tfrac{1}{2}\,Km=17\tfrac{1}{2}\,Km$

B recorre

$29\tfrac{1}{2}\,Km-17\tfrac{1}{2}\,Km=12\,Km$

8. *Recorrido tren de carga en 3 horas es*

$42\dfrac{Km}{h}(3h)=126\ Km$

$a=126\,Km$

$v=60\,{}^{Km}\!/\!_{h}$

$v'=42\,{}^{Km}\!/\!_{h}$

$x=\dfrac{126\cdot 60}{60-42}$

$x=\dfrac{7.560}{18}$

$x=420\ Km$

$420\,Km\rightarrow$ *Alcanza el tren de pasajeros al de carga*

El tiempo empleado es:

$\dfrac{420\ Km}{60\ {}^{Km}\!/\!_{h}}=7\,horas$

EJERCICIO 162

1. $b=10cm$

$h=8cm$

$A=\dfrac{b\cdot h}{2}$

$A=\dfrac{10cm\cdot 8cm}{2}$

$A=40cm^2$

2. $d=8m$

$A=\dfrac{d^{\,2}}{2}$

$A=\dfrac{(8m)^2}{2}$

$A=32\,m^2$

3. $t=15s$

$v=9\,{}^{m}\!/\!_{s}$

$e=v\cdot t$

$e=(15s)(9\,{}^{m}\!/\!_{s})$

$e=135m$

4. $v=9\,{}^{m}\!/\!_{s}$

$e=108m$

$t=\dfrac{e}{v}\Rightarrow t=\dfrac{108m}{9\,{}^{m}\!/\!_{s}}$

$t=12s$

5. $b=4m$

$c=3m$

$a^2=b^2+c^2$

$a^2=(4m)^2+(3m)^2$

$a^2=16m^2+9m^2$

$a^2=25m^2$

$a=\sqrt{25m^2}$

$a=5m$

6. $a=13m$

$c=5m$

$b^2=a^2-c^2$

$b^2=(13m)^2-(5m)^2$

$b^2=169m^2-25m^2$

$b^2=144m^2$

$b=\sqrt{144m^2}$

$b=12m$

7. $r=5m$

$\pi=3{,}14$

$A=\pi\cdot r^2$

$A=3{,}14(5m)^2$

$A=3{,}14\cdot 25m^2$

$A=78{,}5m^2$

8. $r=5m$

$\pi=3{,}14$

$C=2\pi\cdot r$

$C=2\cdot 3{,}14\cdot 5m$

$C=10m\cdot 3{,}14$

$C=31{,}4m$

9. $h=9m$

$r=2m$

$\pi=3{,}14$

$v=\dfrac{h\cdot\pi\cdot r^2}{3}$

$v=\dfrac{9m\cdot 3{,}14(2m)^2}{3}$

$v=12m^2\cdot 3{,}14$

$v=37{,}68m^2$

10. $P=8{,}24g$

$V=8cm^3$

$D=\dfrac{p}{v}$

$D=\dfrac{8{,}24g}{8cm^3}$

$D=1{,}03\,{}^{g}\!/\!_{cm^3}$

11. $L=4m$

$A=\dfrac{L^2\sqrt{3}}{4}$

$A=\dfrac{(4m)^2\sqrt{3}}{4}$

$A=4m^2\sqrt{3}$

$A=6{,}92m^2$

12. $N=6$

$S=180^\circ(6-2)$

$S=180^\circ(4)$

$S=720^\circ$

EJERCICIO 163

1. $e=v\cdot t$

$v=\dfrac{e}{t}$ Rta

$t=\dfrac{e}{v}$ Rta

2. $A=h\left(\dfrac{b+b'}{2}\right)$

$2A=h(b+b')$

$\dfrac{2A}{b+b'}=h$ Rta

3. $e=\dfrac{1}{2}a\cdot t^2$

$2e=a\cdot t^2$

$\dfrac{2e}{t^2}=a$ Rta

4. $A=\dfrac{1}{2}a\cdot l\cdot n$

$2A=a\cdot l\cdot n$

$\dfrac{2A}{l\cdot n}=a$ Rta

$\dfrac{2A}{n\cdot a}=l$ Rta

$\dfrac{2A}{l\cdot a}=n$ Rta

5. $A=\pi\cdot r^2$

$\dfrac{A}{\pi}=r^2$

$\sqrt{\dfrac{A}{\pi}}=r$ Rta

6. $a^2=b^2+c^2-2bx$

$a^2-b^2-c^2=-2bx$

$-\dfrac{a^2-b^2-c^2}{2b}=x$

$\dfrac{b^2+c^2-a^2}{2b}=x$ Rta

7. $\overline{V}=V_o+a\cdot t$

$\overline{V}-a\cdot t=V_0$ Rta

$\overline{V}-V_0=a\cdot t$

$\dfrac{\overline{V}-V_0}{t}=a$ Rta

$\dfrac{\overline{V}-V_0}{a}=t$ Rta

8. $\overline{V}=V_o-a\cdot t$

$\overline{V}+a\cdot t=V_0$ Rta

$a\cdot t=V_0-\overline{V}$

$a=\dfrac{V_0-\overline{V}}{t}$ Rta

$t=\dfrac{V_0-\overline{V}}{a}$ Rta

9. $D=\dfrac{P}{V}$

$DV=P$

$V=\dfrac{P}{D}$ Rta

$P=VD$ Rta

10. $a^2=b^2+c^2$

$a^2-c^2=b^2$

$\sqrt{a^2-c^2}=b$ Rta

$a^2-b^2=c^2$

$\sqrt{a^2-b^2}=c$ Rta

11. $V=a\cdot t$

$\dfrac{V}{a}=t$ Rta

$a=\dfrac{V}{t}$ Rta

12. $\dfrac{1}{f}=\dfrac{1}{p'}-\dfrac{1}{p}$

$\dfrac{p'\cdot p}{f\cdot p'\cdot p}=\dfrac{f\cdot p-f\cdot p'}{f\cdot p'\cdot p}$

$p'\cdot p=f\,(p-p')$

$p'=\dfrac{f\,(p-p')}{p}$

$p'=\dfrac{f\cdot p}{p}-\dfrac{f\cdot p'}{p}$

$p'=f-\dfrac{f\cdot p'}{p}$

$p'+\dfrac{f\cdot p'}{p}=f$

$\dfrac{p'\cdot p+f\cdot p'}{p}=f$

$p'(p+f)=p\cdot f$

$p'=\dfrac{p\cdot f}{p+f}$ Rta

Continúa

Continuación

12. $\dfrac{p'\cdot p+f\cdot p'}{p}=f$

$\dfrac{p'\cdot p}{p}+\dfrac{f\cdot p'}{p}=f$

$p'+\dfrac{f\cdot p'}{p}=f$

$\dfrac{f\cdot p'}{p}=f-p'$

$f\cdot p'=p(f-p')$

$\dfrac{f\cdot p'}{f-p'}=p$ Rta

13. $v=\sqrt{\dfrac{e}{d}}$

$v^2=\dfrac{e}{d}$

$v^2\cdot d=e$ Rta

$d=\dfrac{e}{v^2}$ Rta

14. $e=V_0\cdot t+\dfrac{1}{2}a\cdot t^2$

$2e=2V_0\cdot t+a\cdot t^2$

$2e-a\cdot t^2=2V_0\cdot t$

$\dfrac{2e-a\cdot t^2}{2\cdot t}=V_0$ Rta

15. $e=V_0\cdot t-\dfrac{1}{2}a\cdot t^2$

$2e=2V_0\cdot t-a\cdot t^2$

$2e+a\cdot t^2=2V_0\cdot t$

$\dfrac{2e+a\cdot t^2}{2\cdot t}=V_0$ Rta

$a\cdot t^2=2V_0\cdot t-2e$

$a=\dfrac{2V_0\cdot t-2e}{t^2}$

$a=\dfrac{2(V_0\cdot t-e)}{t^2}$ Rta

16. $V=\dfrac{h\cdot\pi\cdot r^2}{3}$

$\dfrac{3V}{\pi\cdot r^2}=h$ Rta

$\dfrac{3V}{\pi\cdot h}=r^2$

$\sqrt{\dfrac{3v}{\pi\cdot h}}=r$ Rta

17. $I=\dfrac{c\cdot t\cdot r}{100}$

$100I=c\cdot t\cdot r$

$\dfrac{100I}{t\cdot r}=c$ Rta

$\dfrac{100I}{r\cdot c}=t$ Rta

$\dfrac{100I}{c\cdot t}=r$ Rta

18. $E=I\cdot R$

$\dfrac{E}{I}=R$ Rta

$\dfrac{E}{R}=I$ Rta

19. $e=\dfrac{V^2}{2a}$

$2ae=V^2$

$\sqrt{2ae}=V$ Rta

20. $u=a+(n-1)r$

$u-(n-1)r=a$ Rta

$u-a=(n-1)r$

$\dfrac{u-a}{r}=n-1$

$\dfrac{u-a}{r}+1=n$

$\dfrac{u-a+r}{r}=n$ Rta

$u-a+r=nr$

$u-a=nr-r$

$u-a=r(n-1)$

$\dfrac{u-a}{n-1}=r$ Rta

21. $u=a\cdot r^{\,n-1}$

$\dfrac{u}{r^{\,n-1}}=a$ Rta

$\dfrac{u}{a}=r^{\,n-1}$

$\sqrt[n-1]{\dfrac{u}{a}}=r$ Rta

22. $I=\dfrac{Q}{t}$

$Q=I\cdot t$ Rta

$\dfrac{Q}{I}=t$ Rta

EJERCICIO 164

1. $x-5<2x-6$
$-5+6<2x-x$
$1<x$
$1\rightarrow$ *Lim.* inf*erior de x*

2. $5x-12>3x-4$
$5x-3x>12-4$
$2x>8$
$x>4$
$4\rightarrow$ *Lim.* inf*eror de x*

3. $x-6>21-8x$
$x+8x>21+6$
$9x>27$
$x>3$
$3\rightarrow$ *Lim.* inf*erior de x*

4. $3x-14<7x-2$
$-14+2<7x-3x$
$-12<4x$
$-3<x$

5. $2x-\frac{5}{3}>\frac{x}{3}+10$
$6x-5>x+30$
$6x-x>30+5$
$5x>35$
$x>7$

6. $3x-4+\frac{x}{4}<\frac{5x}{2}+2$
$12x-16+x<10x+8$
$12x+x-10x<16+8$
$3x<24$
$x<8$

7. $(x-1)^2-7>(x-2)^2$
$x^2-2x+1-7>x^2-4x+4$
$-6-4>-4x+2x$
$-10>-2x$
$10<2x$
$5<x$

8. $(x+2)(x-1)+26<(x+4)(x+5)$
$x^2+x-2+26<x^2+9x+20$
$24-20<9x-x$
$4<8x$
$\frac{1}{2}<x$

9. $3(x-2)+2x(x+3)>(2x-1)(x+4)$
$3x-6+2x^2+6x>2x^2+7x-4$
$9x-6>7x-4$
$9x-7x>-4+6$
$2x>2$
$x>1$

10. $6(x^2+1)-(2x-4)(3x+2)<3(5x+21)$
$6x^2+6-6x^2+8x+8<15x+63$
$8x+14<15x+63$
$8x-15x<63-14$
$-7x<49$
$7x>-49$
$x>-7$

11. $(x-4)(x+5)<(x-3)(x-2)$
$x^2+x-20<x^2-5x+6$
$5x+x<20+6$
$6x<26$
$x<\frac{26}{6}$
$x<\frac{13}{3}$

12. $(2x-3)^2+4x^2(x-7)<4(x-2)^3$
$4x^2-12x+9+4x^3-28x^2<4x^3-24x^2+48x-32$
$-12x-48x<-9-32$
$-60x<-41$
$60x>41$
$x>\frac{41}{60}$

13. $\frac{2x+1}{3x-1}>\frac{2x+5}{3x+2}$
$(3x+2)(2x+1)>(2x+5)(3x-1)$
$6x^2+7x+2>6x^2+13x-5$
$2+5>13x-7x$
$7>6x$
$\frac{7}{6}>x$

14. $\frac{x+3}{3}-\frac{4}{x+2}>\frac{x}{3}$
$(x+2)(x+3)-12>x(x+2)$
$x^2+5x+6-12>x^2+2x$
$5x-2x>6$
$3x>6$
$x>2$

15. $\frac{5}{3x+1}-\frac{20}{9x^2-1}<\frac{2}{3x-1}$
$5(3x-1)-20<2(3x+1)$
$15x-5-20<6x+2$
$15x-6x<25+2$
$9x<27$
$x<3$

16. $\frac{1}{x^2+x}>\frac{1}{x^2-x}-\frac{1}{x^2-1}$
$x-1>x+1-x$
$x>2$

17. $x\rightarrow$ *Nºs. enteros*
$\frac{x}{3}+15>\frac{x}{2}+1$
$2x+90>3x+6$
$90-6>3x-2x$
$84>x$
Nºs. enteros menores que 84 *Rta*

EJERCICIO 165

1. $x-3>5$ *y* $2x+5>17$
$x-3>5$ $\quad 2x+5>17$
$x>5+3$ $\quad 2x>12$
$x>8$ $\quad x>6$
$x>8$ *Rta*

2. $5-x>-6$ *y* $2x+9>3x$
$-x>-6-5$ $\quad -x>-9$
$x<11$ $\quad x<9$
$x<9$ *Rta*

3. $6x+5>4x+11$ $\quad 4-2x>10-5x$
$2x>6$ $\quad 3x>6$
$x>3$ $\quad x>2$
$x>3$ *Rta*

4. $5x-4>7x-16 \quad y \quad 8-7x<16-15x$

$12>2x \qquad 8x<8$

$6>x \qquad x<1$

$x<1$ *Rta*

5. $\frac{x}{2}-3>\frac{x}{4}+2 \quad y \quad 2x+\frac{3}{5}<6x-23\frac{2}{5}$

$2x-12>x+8 \qquad 10x+3<30x-117$

$x>20 \qquad 120<20x$

$6<x$

$x>20$ *Rta*

6. $2x-3<x+10 \quad y \quad 6x-4>5x+6$

$2x-x<10+3 \qquad 6x-5x>6+4$

$x<13 \qquad x>10$

$10<x<13$ *Rta*

7. $\frac{x}{4}-1>\frac{x}{3}-1\frac{1}{2} \quad y \quad 2x-3\frac{3}{5}>x+\frac{2}{5}$

$3x-12>4x-18 \qquad 10x-18>5x+2$

$18-12>4x-3x \qquad 5x>20$

$6>x \qquad x>4$

$4<x<6$ *Rta*

8. $(x-1)(x+2)<(x+2)(x-3) \quad y \quad (x+3)(x+5)>(x+4)(x+3)$

$x^2+x-2<x^2-x-6 \qquad x^2+8x+15>x^2+7x+12$

$2x<-4 \qquad x>-3$

$x<-2$

$-3<x<-2$ *Rta*

9. $\frac{x+2}{x+8}>\frac{x-2}{x+3} \quad y \quad \frac{x-1}{x+4}<\frac{x-5}{x-1}$

$(x+2)(x+3)>(x-2)(x+8) \qquad (x-1)^2<(x-5)(x+4)$

$x^2+5x+6>x^2+6x-16 \qquad x^2-2x+1<x^2-x-20$

$22>x \qquad 21<x$

$21<x<22$ *Rta*

10. $x \rightarrow$ *Nºs. enteros*

$3x-6>\frac{x}{2}+4 \quad y \quad 4x+8<3x+15$

$6x-12>x+8 \qquad x<7$

$5x>20$

$x>4$

$4<x<7$ *Luego son* 5 *y* 6 *Rta*

EJERCICIO 166

1. $x=Ky$

$9=K6$

$\frac{9}{6}=K$

$x=\frac{3}{2}\cdot 8$

$x=12$

2. $x=Ky$

$2=K3$

$\frac{2}{3}=K$

$24=\frac{2}{3}y$

$\frac{3\cdot 24}{2}=y$

$36=y$

3. $A=KBC$

$30=K(2)(5)$

$\frac{30}{10}=K$

$3=K$

$A=3(7)(4)$

$A=84$

4. $x=KYZ$

$4=K(3)(6)$

$\frac{4}{18}=K$

$\frac{2}{9}=K$

$10=Ky9$

$10=\frac{2}{9}y9$

$10=2y$

$5=y$

5. $A=\frac{K}{B}$

$3=\frac{K}{5}$

$15=K$

$A=\frac{K}{7}$

$A=\frac{15}{7}=2\frac{1}{7}$

6. $B=\frac{K}{A}$

$\frac{1}{3}=\frac{K}{\frac{1}{2}}$

$\frac{1}{3}=2K$

$\frac{1}{6}=K$

$\frac{1}{12}=\frac{\frac{1}{6}}{A} \Rightarrow \frac{1}{12}=\frac{1}{6A}$

$\Rightarrow 6A=12 \Rightarrow A=2$

7. $A=\frac{KB}{C}$

$8=\frac{K12}{3}$

$8=4K$

$2=K$

$A=\frac{2\cdot 7}{14}$

$A=1$

8. $x=\frac{Ky}{Z}$

$3=\frac{4K}{8}$

$24=4K$

$6=K$

$10=\frac{7\cdot 6}{Z}$

$Z=\frac{42}{10}$

$Z=4\frac{1}{5}$

9. $x=K(y^2-1)$

$48=K(5^2-1)$

$48=K(25-1)$

$\frac{48}{24}=K$

$2=K$

$x=2(7^2-1)$

$x=2\cdot 48$

$x=96$

10. $x=\frac{K}{y^2-1}$

$9=\frac{K}{(3^2-1)}$

$9=\frac{K}{8}$

$72=K$

$x=\frac{72}{(5^2-1)}$

$x=\frac{72}{24}$

$x=3$

11. $A \rightarrow$ *Area del cuadrado*

$d \rightarrow$ *Diagonal*

$A=Kd^2$

$18m^2=K(6m)^2$

$\frac{18m^2}{36m^2}=K$

$\frac{1}{2}=K$

$A=\frac{1}{2}(10m)^2$

$A=\frac{100m^2}{2}$

$A=50m^2$

12. $Ap \rightarrow$ *Area piramide*

$a \rightarrow$ *apotema*

$pb \rightarrow$ *perim. de la b.*

$Ap=Kapb$

$480m^2=K(12m)(80m)$

$480m^2=K(960m^2)$

$\frac{480m^2}{960m^2}=K$

$\frac{1}{2}=K$

$Ap=\frac{1}{2}(6m)(40m)$

$Ap=\frac{240m^2}{2}$

$Ap=120m^2$

13. $Vp \rightarrow Vol.\ piramide$
$h \rightarrow Altura$
$Ab \rightarrow Area\ \ base$
$Vp = KhAb$
$96m^3 = K \cdot 8m(36m^2)$
$96m^3 = K(288m^3)$
$\frac{1}{3} = K$
$Vp = \frac{1}{3} \cdot 12m(64m^2)$
$Vp = \frac{768m^3}{3}$
$Vp = 256m^3$

14. $Ao \rightarrow Area\ circulo$
$r \rightarrow Radio$
$Ao = Kr^2$
$616cm^2 = K(14cm)^2$
$\frac{616cm^2}{196cm^2} = K$
$\frac{22}{7} = K$
$Ao = \frac{22}{7}(7cm)^2$
$Ao = 22 \cdot 7cm^2$
$Ao = 154cm^2$

15. $Lo \rightarrow Long.\ circunf.$
$r \rightarrow Radio$
$Lo = Kr$
$44cm = K \cdot 7cm$
$\frac{44}{7} = K$
$66cm = \frac{44r}{7}$
$462cm = 44r$
$\frac{462}{44}cm = r$
$10\frac{1}{2}cm = r$

16. $x = \frac{K}{y^2}$
$4 = \frac{K}{6^2}$
$36 \cdot 4 = K$
$144 = K$
$9 = \frac{144}{y^2}$
$y^2 = \frac{144}{9}$
$y = \pm\sqrt{\frac{144}{9}}$
$y = \pm\frac{12}{3}$
$y = \pm 4$

EJERCICIO 167

1. $A = KB$
$10 = K5$
$2 = K \Rightarrow A = 2B$

2. $e \rightarrow Espacio$
$V \rightarrow Velocidad$
$t \rightarrow Tiempo$
$e = KVt$
$Si\ \ K = 1 \Rightarrow e = Vt$

3. $Ar \rightarrow Area\ rombo$
$D \rightarrow Diag\ mayor$
$D' \rightarrow Diag\ menor$
$A = KDD'$
$24cm^2 = K \cdot 8cm \cdot 6cm$
$24cm^2 = K \cdot 48cm^2$
$\frac{1}{2} = K$
$A = \frac{1}{2}DD'$

4. $A = \frac{KB}{C}$
$K = 3 \ \Rightarrow A = \frac{3B}{C}$

5. $C = Kr$
$132cm = K \cdot 21cm$
$\frac{132cm}{21cm} = K$
$\frac{44}{7} = K$
$C = \frac{44}{7}r$

6. $C \rightarrow Espacio\ (h)$
$t \rightarrow Tiempo$
$h \rightarrow Altura\ (e)$
$e = K \cdot t^2$
$19,6m = K(2s)^2$
$\frac{19,6m}{4s^2} = K$
$4,9\frac{m}{s^2} = K$
$e = 4,9t^2$

7. $F = \frac{KmV^2}{r}$

8. $y \rightarrow Función$
$x \rightarrow Vble.\ indep.$
$y = 2x + 3$

9. $L \rightarrow$ Lado de un cuad. inscrito en un circulo
$r \rightarrow Radio$
$L = K \cdot r$
$K = \sqrt{2}$
$L = \sqrt{2} \cdot r$

10. $y \rightarrow Función$
$x \rightarrow Vble.\ indep.$
$y = \frac{x^2}{2} + 2$

11. $y = \frac{5 - 2x}{3}$

12. $F \rightarrow$ Fuerza de atracc.
$m \rightarrow$ Masa cuerpo1
$m' \rightarrow$ Masa cuerpo2
$d \rightarrow$ Distancia
$F = \frac{K \cdot m \cdot m'}{d^2}$

13. $h \rightarrow Altura\ \Delta$
$A \rightarrow Area\ del\ \Delta$
$B \rightarrow Base$
$h = \frac{K \cdot A}{B}$
$10cm = \frac{K \cdot 20cm^2}{4cm}$
$10cm = K \cdot 5cm$
$2 = K$
$h = \frac{2A}{B}$

14. $W \rightarrow$ Energ. cinetica
$m \rightarrow Masa$
$V \rightarrow Velocidad$
$W = KmV^2$
$K = \frac{1}{2}$
$W = \frac{1}{2}mV^2$

15. $B \rightarrow$ *Area base piramide*
$V \rightarrow$ *Volumen*
$h \rightarrow$ *Altura*

$$B = \frac{KV}{h}$$
$$100 = \frac{K \cdot 400}{12}$$
$$1.200 = K \cdot 400$$
$$3 = K$$
$$B = \frac{3V}{h}$$

16. $x = \frac{K}{y}$

$$2 = \frac{K}{5}$$
$$10 = K$$
$$x = \frac{10}{y}$$

17. $x = \frac{K}{y^2}$

$$3 = \frac{K}{2^2}$$
$$12 = K$$
$$x = \frac{12}{y^2}$$

18. $A = \frac{KB}{C}$

$$3 = \frac{K \cdot 24}{4}$$
$$12 = K \cdot 24$$
$$\frac{1}{2} = K$$
$$A = \frac{\frac{1}{2}B}{C}$$
$$A = \frac{B}{2C}$$

EJERCICIO 173

1. $x + y = 5$

$y = 5 - x$

$5 - x \rightarrow$ *Entero* $y +$

Para que y sea entero

$y +$ *el mayor valor que*

podemos dar a x es 4.

Por tanto las soluciones

enteras y + son:

$x = 1 \quad y = 4$

$x = 2 \quad y = 3$

$x = 3 \quad y = 2$

$x = 4 \quad y = 1$

2. $2x + 3y = 37$

$$2x = 37 - 3y$$
$$x = \frac{37 - 3y}{2}$$
$$x = \frac{36}{2} + \frac{1}{2} - \frac{2y}{2} - \frac{y}{2}$$
$$x = 18 - y + \frac{1 - y}{2}$$
$$x - 18 + y = \frac{1 - y}{2}$$
$$\frac{1 - y}{2} \rightarrow \textit{Entero}$$
$$\frac{3 - 3y}{2} \rightarrow \textit{Entero}$$
$$\frac{2}{2} + \frac{1}{2} - \frac{2y}{2} - \frac{y}{2}$$
$$1 - y + \frac{1 - y}{2} \quad \rightarrow \textit{Entero}$$

Continúa

2. Continuación

$$\frac{1 - y}{2} = m$$
$$1 - y = 2m$$
$$-y = 2m - 1$$
$$y = 1 - 2m$$
$$\Rightarrow 2x + 3(1 - 2m) = 37$$
$$2x + 3 - 6m = 37$$
$$2x = 34 + 6m$$
$$x = \frac{2(17 + 3m)}{2}$$
$$x = 17 + 3m$$

$y = 1 - 2m \quad \rightarrow$ *m entero*

$x = 17 + 3m$

$m = 0 \quad y = 1 \quad x = 17 \quad$ *sol*

$m = 1 \quad y = -1 \quad \rightarrow$ *no*

$m = -1 \quad y = 3 \quad x = 14 \quad$ *sol*

$m = -2 \quad y = 5 \quad x = 11 \quad$ *sol*

$m = -3 \quad y = 7 \quad x = 8 \quad$ *sol*

$m = -4 \quad y = 9 \quad x = 5 \quad$ *sol*

$m = -5 \quad y = 11 \quad x = 2 \quad$ *sol*

$m = -6 \quad y = 13 \quad x = -1 \quad \rightarrow$ *no*

3. $3x + 5y = 43$

$$3x = 43 - 5y$$
$$x = \frac{43 - 5y}{3}$$
$$x = \frac{42}{3} + \frac{1}{3} - \frac{3y}{3} - \frac{2y}{3}$$
$$x = 14 - y + \frac{1 - 2y}{3}$$

Continúa

3. Continuación

$$x - 14 + y = \frac{1 - 2y}{3}$$
$$\frac{1 - 2y}{3} \rightarrow \textit{Entero}$$
$$\frac{2 - 4y}{3} \rightarrow \textit{Entero}$$
$$\frac{2}{3} - \frac{3y}{3} - \frac{1y}{3}$$
$$-y + \frac{2 - y}{3}$$
$$\frac{2 - y}{3} = m$$
$$2 - y = 3m$$
$$-y = 3m - 2$$
$$y = 2 - 3m$$
$$3x + 5(2 - 3m) = 43$$
$$3x + 10 - 15m = 43$$
$$3x = 33 + 15m$$
$$x = \frac{3(11 + 5m)}{3}$$
$$x = 11 + 5m$$

$y = 2 - 3m$

$x = 11 + 5m$

$m = 0 \quad x = 11 \quad y = 2 \quad$ *sol*

$m = 1 \quad x = 16 \quad y = -1 \rightarrow$ *No*

$m = -1 \quad x = 6 \quad y = 5 \quad$ *sol*

$m = -2 \quad x = 1 \quad y = 8 \quad$ *sol*

$m = -3 \quad x = -4 \quad \rightarrow$ *No*

4. $x+3y=9$

$x=9-3y$

$9-3y \rightarrow Entero$

$3-y \rightarrow Entero$

$3-y=m$

$-y=m-3$

$y=3-m$

$x+3(3-m)=9$

$x+9-3m=9$

$x=3m$

$y=3-m$

$x=3m$

$m=0 \quad x=0 \quad y=3 \rightarrow No$

$m=1 \quad x=3 \quad y=2 \quad sol$

$m=2 \quad x=6 \quad y=1 \quad sol$

$m=3 \quad x=9 \quad y=0 \rightarrow No$

$m=4 \quad x=12 \quad y=-1 \rightarrow No$

5. $7x+8y=115$

$7x=115-8y$

$$x=\frac{115-8y}{7}$$

$$x=\frac{112}{7}+\frac{3}{7}-\frac{7y}{7}-\frac{y}{7}$$

$$x=16-y+\frac{3-y}{7}$$

$\frac{3-y}{7} \rightarrow Entero$

$\frac{24-8y}{7} \rightarrow Entero$

$\frac{24}{7}-\frac{7y}{7}-\frac{y}{7}$

$-y+\frac{24-y}{7}$

$\frac{24-y}{7}=m$

$24-y=7m$

$-y=7m-24$

$y=24-7m$

$7x+8(24-7m)=115$

$7x+192-56m=115$

$7x-56m=-77$

$$x=\frac{7(8m-11)}{7}$$

$x=8m-11$

Continúa

5. Continuación

$y=24-7m$

$x=8m-11$

$m=0 \quad x=-11 \rightarrow No$

$m=1 \quad x=-3 \rightarrow No$

$m=2 \quad x=5 \quad y=10 \quad sol$

$m=3 \quad x=13 \quad y=3 \quad sol$

$m=4 \quad x=21 \quad y=-4 \rightarrow No$

$m=-1 \quad x=-19 \rightarrow No$

6. $15x+7y=136$

$7y=136-15x$

$$y=\frac{136-15x}{7}$$

$$y=\frac{133}{7}+\frac{3}{7}-\frac{14x}{7}-\frac{x}{7}$$

$$y=19-2x+\frac{3-x}{7}$$

$y-19+2x=\frac{3-x}{7}$

$\frac{3-x}{7} \rightarrow Entero$

$\frac{24-8x}{7} \rightarrow Entero$

$\frac{24}{7}-\frac{7x}{7}-\frac{x}{7}$

$-x+\frac{24-x}{7}$

$\frac{24-x}{7}=m$

$24-x=7m$

$-x=7m-24$

$x=24-7m$

$15(24-7m)+7y=136$

$360-105m+7y=136$

$-105m+7y=-224$

$$y=\frac{7(15m-32)}{7}$$

$y=15m-32$

$x=24-7m$

$y=15m-32$

$m=0 \quad x=24 \quad y=-32 \rightarrow No$

$m=1 \quad x=17 \quad y=-17 \rightarrow No$

$m=2 \quad x=10 \quad y=-2 \rightarrow No$

$m=3 \quad x=3 \quad y=13 \quad sol$

$m=4 \quad x=-4 \rightarrow No$

$m=-1 \quad x=-31 \rightarrow No$

$x=31 \quad y=-47 \rightarrow No$

7. $x+5y=24$

$x=24-5y$

$x=24+1-1-5y$

$$x=\frac{25}{5}-\frac{1}{5}-\frac{5y}{5}$$

$x+\frac{1}{5}=5-y$

$5-y=m$

$-y=m-5$

$y=5-m$

$x+5(5-m)=24$

$x+25-5m=24$

$x=5m-1$

$y=5-m$

$x=5m-1$

$m=0 \quad x=-1 \rightarrow No$

$m=1 \quad x=4 \quad y=4 \quad sol$

$m=2 \quad x=9 \quad y=3 \quad sol$

$m=3 \quad x=14 \quad y=2 \quad sol$

$m=4 \quad x=19 \quad y=1 \quad sol$

$m=5 \quad x=24 \quad y=0 \rightarrow No$

$m=6 \quad x=29 \quad y=-1 \rightarrow No$

$m=-1 \quad x=-6 \rightarrow No$

8. $9x+11y=203$

$9x=203-11y$

$$x=\frac{203-11y}{9}$$

$$x=\frac{198}{9}+\frac{5}{9}-\frac{9y}{9}-\frac{2y}{9}$$

$$x=22-y+\frac{5-2y}{9}$$

$x-22+y=\frac{5-2y}{9}$

$\frac{5-2y}{9} \rightarrow Entero$

$\frac{25-10y}{9} \rightarrow Entero$

$\frac{18}{9}+\frac{7}{9}-\frac{9y}{9}-\frac{y}{9}$

$2-y+\frac{7-y}{9}$

$\frac{7-y}{9}=m$

$7-y=9m$

$-y=9m-7$

$y=7-9m$

Continúa

8. Continuación

$9x+11(7-9m)=203$

$9x+77-99m=203$

$x=\frac{9(14+11m)}{9}$

$x=14+11m$

$y=7-9m$

$x=14+11m$

$m=0 \quad x=14 \quad y=7 \quad sol$

$m=1 \quad x=25 \quad y=-2 \rightarrow No$

$m=-1 \quad x=3 \quad y=16 \quad sol$

$m=-2 \quad x=-8 \rightarrow No$

9. $5x+2y=73$

$2y=73-5x$

$y=\frac{73-5x}{2}$

$y=\frac{72}{2}+\frac{1}{2}-\frac{4x}{2}-\frac{x}{2}$

$y=36-2x+\frac{1-x}{2}$

$y-36+2x=\frac{1-x}{2}$

$\frac{1-x}{2}=m$

$1-x=2m$

$-x=2m-1$

$x=1-2m$

$5(1-2m)+2y=73$

$5-10m+2y=73$

$y=\frac{2(34+5m)}{2}$

$y=34+5m$

$x=1-2m$

$y=34+5m$

$m=0 \quad x=1 \quad y=34 \quad sol$

$m=1 \quad x=-1 \rightarrow No$

$m=-1 \quad x=3 \quad y=29 \quad sol$

$m=-2 \quad x=5 \quad y=24 \quad sol$

$m=-3 \quad x=7 \quad y=19 \quad sol$

$m=-4 \quad x=9 \quad y=14 \quad sol$

$m=-5 \quad x=11 \quad y=9 \quad sol$

$m=-6 \quad x=13 \quad y=4 \quad sol$

$m=-7 \quad x=15 \quad y=-1 \rightarrow No$

10. $8x+13y=162$

$8x=162-13y$

$x=\frac{162-13y}{8}$

$x=\frac{160}{8}+\frac{2}{8}-\frac{8y}{8}-\frac{5y}{8}$

$x=20-y+\frac{2-5y}{8}$

$x-20+y=\frac{2-5y}{8}$

$\frac{2-5y}{8} \rightarrow Entero$

$\frac{10-25y}{8} \rightarrow Entero$

$\frac{10}{8}-\frac{24y}{8}-\frac{y}{8}$

$-3y+\frac{10-y}{8}$

$\frac{10-y}{8}=m$

$10-y=8m$

$-y=8m-10$

$y=10-8m$

$8x+13(10-8m)=162$

$8x+130-104m=162$

$x=\frac{8(4+13m)}{8}$

$x=4+13m$

$y=10-8m$

$x=4+13m$

$m=0 \quad x=4 \quad y=10 \quad sol$

$m=1 \quad x=17 \quad y=2 \quad sol$

$m=2 \quad x=30 \quad y=-6 \rightarrow No$

$m=-1 \quad x=-9 \rightarrow No$

11. $7x+5y=104$

$5y=104-7x$

$y=\frac{104-7x}{5}$

$y=\frac{100}{5}+\frac{4}{5}-\frac{5x}{5}-\frac{2x}{5}$

$y=20-x+\frac{4-2x}{5}$

$y-20+x=\frac{4-2x}{5}$

$\frac{4-2x}{5} \rightarrow Entero$

$\frac{12-6x}{5} \rightarrow Entero$

Continúa

11. Continuación

$\frac{10}{5}+\frac{2}{5}-\frac{5x}{5}-\frac{x}{5}$

$2-x+\frac{2-x}{5}$

$2-x+\frac{2-x}{5}$

$\frac{2-x}{5}=m$

$2-x=5m$

$-x=5m-2$

$x=2-5m$

$7(2-5m)+5y=104$

$14-35m+5y=104$

$y=\frac{5(18+7m)}{5}$

$y=18+7m$

$x=12-5m$

$y=18+7m$

$m=0 \quad x=2 \quad y=18 \quad sol$

$m=1 \quad x=-3 \rightarrow No$

$m=-1 \quad x=7 \quad y=11 \quad sol$

$m=-2 \quad x=12 \quad y=4 \quad sol$

$m=-3 \quad x=17 \quad y=-3 \rightarrow No$

12. $10x+y=32$

$y=32-10x$

$y=32+2-2-10x$

$y=\frac{30}{10}+\frac{2}{10}-\frac{10x}{10}$

$y=3+\frac{2}{10}-x$

$y-\frac{2}{10}=3-x$

$3-x=m$

$-x=m-3$

$x=3-m$

$10(3-m)+y=32$

$30-10m+y=32$

$y=2+10m$

$m=0 \quad x=3 \quad y=2 \quad sol$

$m=1 \quad x=2 \quad y=12 \quad sol$

$m=2 \quad x=1 \quad y=22 \quad sol$

$m=3 \quad x=0 \quad y=32 \rightarrow No$

$m=4 \quad x=-1 \rightarrow No$

$m=-1 \quad x=4 \quad y=-8 \rightarrow No$

13. $9x+4y=86$

$$4y=86-9x$$
$$y=\frac{86-9x}{4}$$
$$y=\frac{84}{4}+\frac{2}{4}-\frac{8x}{4}-\frac{x}{4}$$
$$y=21-2x+\frac{2-x}{4}$$
$$y-21+2x=\frac{2-x}{4}$$
$$\frac{2-x}{4}=m$$
$$2-x=4m$$
$$-x=4m-2$$
$$x=2-4m$$
$$9(2-4m)+4y=86$$
$$18-36m+4y=86$$
$$y=\frac{4(17+9m)}{4}$$
$$y=17+9m$$
$$x=2-4m$$

$m=0 \quad x=2 \quad y=17 \quad sol$

$m=1 \quad x=-2 \quad \rightarrow No$

$m=-1 \quad x=6 \quad y=8 \quad sol$

$m=-2 \quad x=10 \quad y=-1 \rightarrow No$

14. $9x+11y=207$

$$9x=207-11y$$
$$x=\frac{207-11y}{9}$$
$$x=\frac{198}{9}+\frac{9}{9}-\frac{9y}{9}-\frac{2y}{9}$$
$$x=22-y+\frac{9-2y}{9}$$
$$x-22+y=\frac{9-2y}{9}$$

$\frac{9-2y}{9} \rightarrow Entero$

$\frac{45-10y}{9} \rightarrow Entero$

$$\frac{45}{9}-\frac{9y}{9}-\frac{y}{9}$$
$$\frac{45-y}{9}=m$$
$$45-y=9m$$
$$-y=9m-45$$
$$y=45-9m$$

Continúa

14. Continuación

$$9x+11(45-9m)=207$$
$$9x+495-99m=207$$
$$x=\frac{9(11m-32)}{9}$$
$$x=11m-32$$
$$y=45-9m$$
$$x=11m-32$$

$m=0 \quad x=-32 \quad \rightarrow No$

$m=1 \quad x=-21 \quad \rightarrow No$

$m=2 \quad x=-10 \quad \rightarrow No$

$m=3 \quad x=1 \quad y=18 \quad sol$

$m=4 \quad x=12 \quad y=9 \quad sol$

$m=5 \quad x=23 \quad y=0 \quad \rightarrow No$

$m=6 \quad x=34 \quad y=-9 \quad \rightarrow No$

$m=-1 \quad x=-43 \quad \rightarrow No$

15. $11x+12y=354$

$$11x=354-12y$$
$$x=\frac{354-12y}{11}$$
$$x=\frac{352}{11}+\frac{2}{11}-\frac{11y}{11}-\frac{y}{11}$$
$$x=32-y+\frac{2-y}{11}$$
$$x-32+y=\frac{2-y}{11}$$
$$\frac{2-y}{11}=m$$
$$2-y=11m$$
$$-y=11m-2$$
$$y=2-11m$$
$$11x+12(2-11m)=354$$
$$11x+24-132m=354$$
$$x=\frac{11(30+12m)}{11}$$
$$x=30+12m$$
$$y=2-11m$$

$m=0 \quad x=30 \quad y=2 \quad sol$

$m=1 \quad x=42 \quad y=-9 \quad \rightarrow No$

$m=-1 \quad x=18 \quad y=13 \quad sol$

$m=-2 \quad x=6 \quad y=24 \quad sol$

$m=-3 \quad x=-6 \quad \rightarrow No$

16. $10x+13y=294$

$$10x=294-13y$$
$$x=\frac{294-13y}{10}$$
$$x=\frac{290}{10}+\frac{4}{10}-\frac{10y}{10}-\frac{3y}{10}$$
$$x-29+y=\frac{4-3y}{10}$$

$\frac{4-3y}{10} \rightarrow Entero$

$\frac{28-21y}{10} \rightarrow Entero$

$$\frac{28}{10}-\frac{20y}{10}-\frac{y}{10}$$
$$-2y+\frac{28-y}{10}$$
$$\frac{28-y}{10}=m$$
$$28-y=10m$$
$$-y=10m-28$$
$$y=28-10m$$
$$10x+13(28-10m)=294$$
$$10x+364-130m=294$$
$$x=\frac{10(13m-7)}{10}$$
$$x=13m-7$$
$$y=28-10m$$

$m=0 \quad x=-7 \quad \rightarrow No$

$m=1 \quad x=6 \quad y=18 \quad sol$

$m=2 \quad x=19 \quad y=8 \quad sol$

$m=3 \quad x=32 \quad y=-2 \quad \rightarrow No$

$m=-1 \quad x=-20 \quad \rightarrow No$

17. $11x+8y=300$

$$8y=300-11x$$
$$y=\frac{300-11x}{8}$$
$$y=\frac{296}{8}+\frac{4}{8}-\frac{3x}{8}-\frac{8x}{8}$$
$$y=37-x+\frac{4-3x}{8}$$
$$y-37+x=\frac{4-3x}{8}$$

$\frac{4-3x}{8} \rightarrow Entero$

$\frac{12-9x}{8} \rightarrow Entero$

Continúa

Continuación

17. $\frac{12}{8}-\frac{8x}{8}-\frac{x}{8}$

$\frac{12-x}{8}=m$

$12-x=8m$

$-x=8m-12$

$x=12-8m$

$11(12-8m)+8y=300$

$132-88m+8y=300$

$y=\frac{8(21+11m)}{8}$

$y=21+11m$

$x=12-8m$

$m=0 \quad x=12 \quad y=21 \quad sol$

$m=1 \quad x=4 \quad y=32 \quad sol$

$m=2 \quad x=-4 \quad \rightarrow No$

$m=-1 \quad x=20 \quad y=10 \quad sol$

$m=-2 \quad x=28 \quad y=-1 \quad \rightarrow No$

18. $21x+25y=705$

$21x=705-25y$

$x=\frac{705-25y}{21}$

$x=\frac{693}{21}+\frac{12}{21}-\frac{21y}{21}-\frac{4y}{21}$

$x=33-y+\frac{12-4y}{21}$

$\frac{12-4y}{21} \rightarrow Entero$

$\frac{192-64y}{21} \rightarrow Entero$

$\frac{192}{21}-\frac{63y}{21}-\frac{y}{21}$

$-3y+\frac{192-y}{21}$

$\frac{192-y}{21}=m$

$192-y=21m$

$-y=21m-192$

$y=192-21m$

$21x+25(192-21m)=705$

$21x+4.800-525m=705$

$x=\frac{21(25m-195)}{21}$

$x=25m-195$

Continúa

18. Continuación

$y=192-21m$

$m=0 \quad x=-195 \rightarrow No$

$m=7 \quad x=-20 \rightarrow No$

$m=8 \quad x=5 \quad y=24 \quad sol$

$m=9 \quad x=30 \quad y=3 \quad sol$

$m=10 \quad x=55 \quad y=-18 \rightarrow No$

$m=-1 \quad x=-220 \rightarrow No$

19. $3x-4y=5$

$3x=5+4y$

$x=\frac{5+4y}{3}$

$x=\frac{2}{3}+\frac{3}{3}+\frac{3y}{3}+\frac{y}{3}$

$x=1+y+\frac{2+y}{3}$

$x-1-y=\frac{2+y}{3}$

$\frac{2+y}{3}=m$

$2+y=3m$

$y=3m-2$

$3x-4(3m-2)=5$

$3x-12m+8=5$

$x=\frac{3(4m-1)}{3}$

$x=4m-1$

$y=3m-2$

$m=1 \quad x=3 \quad y=1 \quad sol$

$m=2 \quad x=7 \quad y=4 \quad sol$

$m=3 \quad x=11 \quad y=7 \quad sol$

20. $5x-8y=1$

$5x=1+8y$

$x=\frac{1+8y}{5}$

$\frac{1+8y}{5} \rightarrow Entero$

$\frac{2+16y}{5} \rightarrow Entero$

$\frac{2}{5}+\frac{15y}{5}+\frac{y}{5}$

$3y+\frac{2+y}{5}$

$\frac{2+y}{5}=m$

Continúa

20. Continuación

$2+y=5m$

$y=5m-2$

$5x-8(5m-2)=1$

$5x-40m+16=1$

$x=\frac{5(8m-3)}{5}$

$x=8m-3$

$y=5m-2$

$m=1 \quad x=5 \quad y=3 \quad sol$

$m=2 \quad x=13 \quad y=8 \quad sol$

$m=3 \quad x=21 \quad y=13 \quad sol$

21. $7x-13y=43$

$7x=43+13y$

$x=\frac{43+13y}{7}$

$x=\frac{42}{7}+\frac{1}{7}+\frac{7y}{7}+\frac{6y}{7}$

$x=6+y+\frac{1+6y}{7}$

$x-6-y=\frac{1+6y}{7}$

$\frac{1+6y}{7} \rightarrow Entero$

$\frac{6+36y}{7} \rightarrow Entero$

$\frac{6}{7}+\frac{35y}{7}+\frac{y}{7}$

$5y+\frac{6+y}{7}$

$\frac{6+y}{7}=m$

$6+y=7m$

$y=7m-6$

$7x-13(7m-6)=43$

$7x-91m+78=43$

$x=\frac{7(13m-5)}{7}$

$x=13m-5$

$y=7m-6$

$m=1 \quad x=8 \quad y=1 \quad sol$

$m=2 \quad x=21 \quad y=8 \quad sol$

$m=3 \quad x=34 \quad y=15 \quad sol$

22. $11x-12y=0$

$$11x=12y$$
$$x=\frac{12y}{11}$$
$$x=\frac{11y}{11}+\frac{y}{11}$$
$$x-y=\frac{y}{11}$$
$$\frac{y}{11}=m$$
$$y=11m$$
$$11x-12(11m)=0$$
$$11x-132m=0$$
$$x=12m$$
$$y=11m$$

$m=1 \quad x=12 \quad y=11 \quad sol$

$m=2 \quad x=24 \quad y=22 \quad sol$

$m=3 \quad x=36 \quad y=33 \quad sol$

23. $14x-17y=32$

$$14x=32+17y$$
$$x=\frac{32+17y}{14}$$
$$x=\frac{28}{14}+\frac{4}{14}+\frac{14y}{14}+\frac{3y}{14}$$
$$x=2+y+\frac{4+3y}{14}$$
$$\frac{4+3y}{14} \to Entero$$
$$\frac{20+15y}{14} \to Entero$$
$$\frac{14}{14}+\frac{6}{14}+\frac{14y}{14}+\frac{y}{14}$$
$$1+y+\frac{6+y}{14}$$
$$\frac{6+y}{14}=m$$
$$6+y=14m$$
$$y=14m-6$$
$$14x-17(14m-6)=32$$
$$14x-238m+102=32$$
$$x=\frac{14(17m-5)}{14}$$
$$x=17m-5$$
$$y=14m-6$$

$m=1 \quad x=12 \quad y=8 \quad sol$

$m=2 \quad x=29 \quad y=22 \quad sol$

$m=3 \quad x=46 \quad y=36 \quad sol$

24. $7x-11y=83$

$$7x=83+11y$$
$$x=\frac{83+11y}{7}$$
$$x=\frac{77}{7}+\frac{6}{7}+\frac{7y}{7}+\frac{4y}{7}$$
$$x=11+y+\frac{6+4y}{7}$$
$$x-11-y=\frac{6+4y}{7}$$
$$\frac{6+4y}{7} \to Entero$$
$$\frac{12+8y}{7} \to Entero$$
$$\frac{7}{7}+\frac{5}{7}+\frac{7y}{7}+\frac{y}{7}$$
$$1+y+\frac{5+y}{7}$$
$$\frac{5+y}{7}=m$$
$$5+y=7m$$
$$y=7m-5$$
$$7x-11(7m-5)=83$$
$$7x-77m+55=83$$
$$x=\frac{7(11m+4)}{7}$$
$$x=11m+4$$
$$y=7m-5$$

$m=1 \quad x=15 \quad y=2 \quad sol$

$m=2 \quad x=26 \quad y=9 \quad sol$

$m=3 \quad x=37 \quad y=16 \quad sol$

25. $8x-13y=407$

$$8x=407+13y$$
$$x=\frac{407+13y}{8}$$
$$x=\frac{400}{8}+\frac{7}{8}+\frac{8y}{8}+\frac{5y}{8}$$
$$x=50+y+\frac{7+5y}{8}$$
$$x-50-y=\frac{7+5y}{8}$$
$$\frac{7+5y}{8} \to Entero$$
$$\frac{35+25y}{8} \to Entero$$
$$\frac{32}{8}+\frac{3}{8}+\frac{24y}{8}+\frac{y}{8}$$

Continúa

25. Continuación

$$4+3y+\frac{3+y}{8}$$
$$\frac{3+y}{8}=m$$
$$3+y=8m$$
$$y=8m-3$$
$$8x-13(8m-3)=407$$
$$8x-104m+39=407$$
$$x=\frac{8(13m+46)}{8}$$
$$x=13m+46$$
$$y=8m-3$$

$m=1 \quad x=59 \quad y=5 \quad sol$

$m=2 \quad x=72 \quad y=13 \quad sol$

$m=3 \quad x=85 \quad y=21 \quad sol$

26. $20y-23x=411$

$$20y=411+23x$$
$$y=\frac{411+23x}{20}$$
$$y=\frac{400}{20}+\frac{11}{20}+\frac{20x}{20}+\frac{3x}{20}$$
$$y=20+x+\frac{11+3x}{20}$$
$$y-20-x=\frac{11+3x}{20}$$
$$\frac{11+3x}{20} \to Entero$$
$$\frac{77+21x}{20} \to Entero$$
$$\frac{60}{20}+\frac{17}{20}+\frac{20x}{20}+\frac{x}{20}$$
$$3+x+\frac{17+x}{20}$$
$$\frac{17+x}{20}=m$$
$$17+x=20m$$
$$x=20m-17$$
$$20y-23(20m-17)=411$$
$$20y-460m+391=411$$
$$y=\frac{20(23m+1)}{20}$$
$$x=20m-17$$
$$y=23m+1$$

$m=1 \quad x=3 \quad y=24 \quad sol$

$m=2 \quad x=23 \quad y=47 \quad sol$

$m=3 \quad x=43 \quad y=70 \quad sol$

27. $5y-7x=312$

$5y=312+7x$

$y=\frac{312+7x}{5}$

$y=\frac{310}{5}+\frac{2}{5}+\frac{5x}{5}+\frac{2x}{5}$

$y=62+x+\frac{2+2x}{5}$

$y-62-x=\frac{2+2x}{5}$

$\frac{2+2x}{5}\rightarrow \textit{Entero}$

$\frac{6+6x}{5}\rightarrow \textit{Entero}$

$\frac{5}{5}+\frac{1}{5}+\frac{5x}{5}+\frac{x}{5}$

$1+x+\frac{1+x}{5}$

$\frac{1+x}{5}=m$

$1+x=5m$

$x=5m-1$

$5y-7(5m-1)=312$

$5y-35m+7=312$

$y=\frac{5(7m+61)}{5}$

$y=7m+61$

$x=5m-1$

$m=1 \quad x=4 \quad y=68$

$m=2 \quad x=9 \quad y=75$

$m=3 \quad x=14 \quad y=82$

EJERCICIO 174

1. $x\rightarrow$ *Billetes* $2

$y\rightarrow$ *Billetes* $5

$2x+5y=42$

$2x=42-5y$

$x=\frac{42-5y}{2}$

$x=\frac{40}{2}+\frac{2}{2}-\frac{3y}{2}-\frac{2y}{2}$

$x=20-y+\frac{2-3y}{2}$

$x-20+y=\frac{2-3y}{2}$

$\frac{2-3y}{2}\rightarrow \textit{Entero}$

Continúa

1. Continuación

$\frac{2}{2}-\frac{2y}{2}-\frac{y}{2}$

$-y+\frac{2-y}{2}$

$\frac{2-y}{2}=m$

$2-y=2m$

$-y=2m-2$

$y=2-2m$

$2x+5(2-2m)=42$

$2x+10-10m=42$

$x=\frac{2(5m+16)}{2}$

$x=5m+16$

$y=2-2m$

$m=0 \quad x=16 \quad y=2 \quad sol$

$m=1 \quad x=21 \quad y=0 \quad \rightarrow No$

$m=2 \quad x=26 \quad y=-2 \quad \rightarrow No$

$m=-1 \quad x=11 \quad y=4 \quad sol$

$m=-2 \quad x=6 \quad y=6 \quad sol$

$m=-3 \quad x=1 \quad y=8 \quad sol$

$m=-4 \quad x=-4 \quad \rightarrow No$

1 *de* $2 *y* 8 *de* $5

6 *de* $2 *y* 6 *de* $5

11 *de* $2 *y* 4 *de* $5 *ó*

16 *de* $2 *y* 2 *de* $5

2. $x\rightarrow$ *Monedas de* $5

$y\rightarrow$ *Monedas de* $10

$5x+10y=45$

$5x=45-10y$

$x=\frac{45-10y}{5}$

$x=\frac{40}{5}+\frac{5}{5}-\frac{5y}{5}-\frac{5y}{5}$

$x=8-y+\frac{5-5y}{5}$

$x-8+y=1-y$

$1-y=m$

$-y=m-1$

$y=1-m$

$5x+10(1-m)=45$

$5x+10-10m=45$

$x=\frac{5(2m+7)}{5}$

$x=2m+7$

Continúa

2. Continuación

$y=1-m$

$m=0 \quad x=7 \quad y=1 \quad sol$

$m=1 \quad x=9 \quad y=0 \quad \rightarrow No$

$m=-1 \quad x=5 \quad y=2 \quad sol$

$m=-2 \quad x=3 \quad y=3 \quad sol$

$m=-3 \quad x=1 \quad y=4 \quad sol$

$m=-4 \quad x=-1 \quad \rightarrow No$

1 *de* $5 *y* 4 *de* $10

3 *de* $5 *y* 3 *de* $10

5 *de* $5 *y* 2 *de* $10

7 *de* $5 *y* 1 *de* $10

3. $5x+3y=62$

$3y=62-5x$

$y=\frac{62-5x}{3}$

$y=\frac{60}{3}+\frac{2}{3}-\frac{3x}{3}-\frac{2x}{3}$

$y=20-x+\frac{2-2x}{3}$

$y-20+x=\frac{2-2x}{3}$

$\frac{2-2x}{3}\rightarrow \textit{Entero}$

$\frac{10-10x}{3}\rightarrow \textit{Entero}$

$\frac{9}{3}+\frac{1}{3}-\frac{9x}{3}-\frac{x}{3}$

$3-3x+\frac{1-x}{3}$

$\frac{1-x}{3}=m$

$1-x=3m$

$-x=3m-1$

$x=1-3m$

$5(1-3m)+3y=62$

$5-15m+3y=62$

$y=\frac{3(5m+19)}{3}$

$y=5m+19$

$x=1-3m$

$m=0 \quad x=1 \quad y=19 \quad sol$

$m=1 \quad x=-2 \quad \rightarrow No$

$m=-1 \quad x=4 \quad y=14 \quad sol$

$m=-2 \quad x=7 \quad y=9 \quad sol$

$m=-3 \quad x=10 \quad y=4 \quad sol$

1 *y* 19 ; 4 *y* 14 ; 7 *y* 9 ; 10 *y* 4

4. $x \rightarrow Sombreros$

$y \rightarrow Pares\ de\ zapatos$

$$8x+15y=340$$
$$8x=340-15y$$
$$x=\frac{340-15y}{8}$$
$$x=\frac{336}{8}+\frac{4}{8}-\frac{8y}{8}-\frac{7y}{8}$$
$$x=42-y+\frac{4-7y}{8}$$
$$x-42+y=\frac{4-7y}{8}$$
$$\frac{4-7y}{8} \rightarrow Entero$$
$$\frac{28-49y}{8} \rightarrow Entero$$
$$\frac{24}{8}+\frac{4}{8}-\frac{48y}{8}-\frac{y}{8}$$
$$3-6y+\frac{4-y}{8}$$
$$\frac{4-y}{8}=m$$
$$4-y=8m$$
$$-y=8m-4$$
$$y=4-8m$$
$$8x+15(4-8m)=340$$
$$8x+60-120m=340$$
$$x=\frac{8(15m+35)}{8}$$
$$x=15m+35$$
$$y=4-8m$$

$m=0 \quad x=35 \quad y=4 \quad sol$

$m=1 \quad x=50 \quad y=-4 \quad \rightarrow No$

$m=-1 \quad x=20 \quad y=12 \quad sol$

$m=-2 \quad x=5 \quad y=20 \quad sol$

$m=-3 \quad x=-10 \quad \rightarrow No$

5 *sombreros y* 20 *pares de zap.*

20 *sombreros y* 12 *pares de zap.*

35 *sombreros y* 4 *pares de zap.*

5. $x \rightarrow Metros\ de\ lana$

$y \rightarrow Metros\ de\ seda$

$$1{,}50x+2{,}50y=42$$

$\rightarrow$ *Si multiplico por 2 la ecuación inicial no se altera. Luego*

$$3x+5y=84$$
$$3x=84-5y$$
$$x=\frac{84-5y}{3}$$
$$x=\frac{81}{3}+\frac{3}{3}-\frac{3y}{3}-\frac{2y}{3}$$
$$x=27-y+\frac{3-2y}{3}$$
$$x-27+y=\frac{3-2y}{3}$$
$$\frac{3-2y}{3} \rightarrow Entero$$
$$\frac{15-10y}{3} \rightarrow Entero$$
$$\frac{12}{3}+\frac{3}{3}-\frac{9y}{3}-\frac{y}{3}$$
$$4-3y+\frac{3-y}{3}$$
$$\frac{3-y}{3}=m$$
$$3-y=3m$$
$$-y=3m-3$$
$$y=3-3m$$
$$3x+5(3-3m)=84$$
$$3x+15-15m=84$$
$$x=\frac{3(5m+23)}{3}$$
$$x=5m+23$$
$$y=3-3m$$

$m=0 \quad x=23 \quad y=3 \quad sol$

$m=1 \quad x=28 \quad y=0 \quad \rightarrow No$

$m=-1 \quad x=18 \quad y=6 \quad sol$

$m=-2 \quad x=13 \quad y=9 \quad sol$

$m=-3 \quad x=8 \quad y=12 \quad sol$

$m=-4 \quad x=3 \quad y=15 \quad sol$

$m=-5 \quad x=-2 \quad \rightarrow No$

3*m de lana y* 15*m de seda*

8*m de lana y* 12*m de seda*

13*m de lana y* 9*m de seda*

18*m de lana y* 6*m de seda*

23*m de lana y* 3*m de seda*

6. $x \rightarrow niños$

$y \rightarrow adultos$

$$0{,}45x+y=17 \quad (20)$$
$$9x+20y=340$$
$$9x=340-20y$$
$$x=\frac{340-20y}{9}$$
$$x=\frac{333}{9}+\frac{7}{9}-\frac{18y}{9}-\frac{2y}{9}$$
$$x=37-2y+\frac{7-2y}{9}$$
$$x-37+2y=\frac{7-2y}{9}$$
$$\frac{7-2y}{9} \rightarrow Entero$$
$$\frac{98-28y}{9} \rightarrow Entero$$
$$\frac{90}{9}+\frac{8}{9}-\frac{27y}{9}-\frac{y}{9}$$
$$10-3y+\frac{8-y}{9}$$
$$\frac{8-y}{9}=m$$
$$8-y=9m$$
$$-y=9m-8$$
$$y=8-9m$$
$$9x+20(8-9m)=340$$
$$9x+160-180m=340$$
$$x=\frac{9(20m+20)}{9}$$
$$x=20m+20$$
$$y=8-9m$$

$m=0 \quad x=20 \quad y=8 \quad sol$

$m=1 \quad x=0 \quad y=-1 \quad \rightarrow No$

$m=-1 \quad x=0 \quad y=17 \quad \rightarrow No$

$m=-2 \quad x=-20 \quad \rightarrow No$

20 *niños y* 8 *adultos*

7. $x \rightarrow Caballos$

$y \rightarrow Vacas$

$$460x+440y=41.000 \ (\div 20)$$
$$23x+22y=2.050$$
$$22y=2.050-23x$$
$$y=\frac{2.050-23x}{22}$$
$$y=\frac{2.046}{22}+\frac{4}{22}-\frac{22x}{22}-\frac{x}{22}$$

Continúa

7. Continuación

$y=93-x+\frac{4-x}{22}$

$\frac{4-x}{22}=m$

$4-x=22m$

$-x=22m-4$

$x=4-22m$

$23(4-22m)+22y=2.050$

$92-506m+22y=2.050$

$y=\frac{22(23m+89)}{22}$

$y=23m+89$

$x=4-22m$

$m=0 \quad x=4 \quad y=89 \quad sol$

$m=1 \quad x=-18 \quad \rightarrow No$

$m=-1 \quad x=26 \quad y=66 \quad sol$

$m=-2 \quad x=48 \quad y=43 \quad sol$

$m=-3 \quad x=70 \quad y=20 \quad sol$

$m=-4 \quad x=92 \quad y=-3 \rightarrow No$

4 *caballos* *y* 89 *vacas*

26 *caballos* *y* 66 *vacas*

48 *caballos* *y* 43 *vacas*

70 *caballos* *y* 20 *vacas*

8. $3x+3=5y+5$

$3x-5y=2$

$3x=2+5y$

$x=\frac{2+5y}{3}$

$\frac{2+5y}{3} \rightarrow Entero$

$\frac{4+10y}{3} \rightarrow Entero$

$\frac{3}{3}+\frac{1}{3}+\frac{9y}{3}+\frac{y}{3}$

$1+3y+\frac{1+y}{3}$

$\frac{1+y}{3}=m$

$1+y=3m$

$y=3m-1$

$3x-5(3m-1)=2$

$3x-15m+5=2$

$x=\frac{3(5m-1)}{3}$

$x=5m-1$

Continúa

8. Continuación

$y=3m-1$

$m=0 \quad x=-1 \quad \rightarrow No$

$m=1 \quad x=4 \quad y=2 \quad sol$

Los Nºs son 4 *y* 2

9. $x \rightarrow$ *Monedas de* 0,25

$y \rightarrow$ *Monedas de* 0,10

$0,25x+0,10y=2,10 \ (100)$

$25x+10y=210$

$10y=210-25x$

$y=\frac{210-25x}{10}$

$y=\frac{200}{10}+\frac{10}{10}-\frac{20x}{10}-\frac{5x}{10}$

$y=20-2x+\frac{10-5x}{10}$

$y-20+2x=\frac{2-x}{2}$

$\frac{2-x}{2}=m$

$2-x=2m$

$-x=2m-2$

$x=2-2m$

$25(2-2m)+10y=210$

$50-50m+10y=210$

$y=\frac{10(5m+16)}{10}$

$y=5m+16$

$x=2-2m$

$m=0 \quad x=2 \quad y=16 \quad sol$

$m=1 \quad x=0 \quad \rightarrow No$

$m=-1 \quad x=4 \quad y=11 \quad sol$

$m=-2 \quad x=6 \quad y=6 \quad sol$

$m=-3 \quad x=8 \quad y=1 \quad sol$

$m=-4 \quad x=10 \quad y=-4 \quad \rightarrow No$

2 *mon. de* 0,25 *y* 16 *mon. de* 0,10

4 *mon. de* 0,25 *y* 11 *mon. de* 0,10

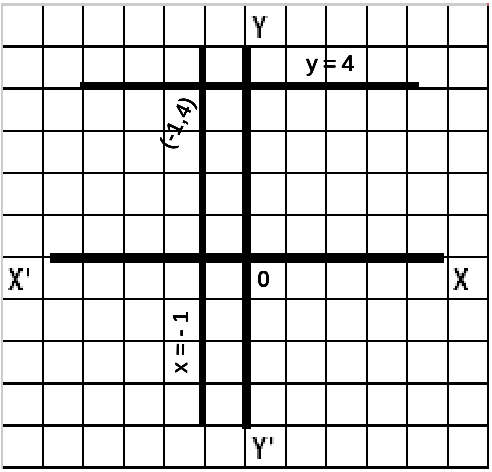
6 *mon. de* 0,25 *y* 6 *mon. de* 0,10

8 *mon. de* 0,25 *y* 1 *mon. de* 0,10

EJERCICIO 175

21. $x+1=0 \qquad y-4=0$

$x=-1 \qquad y=4$



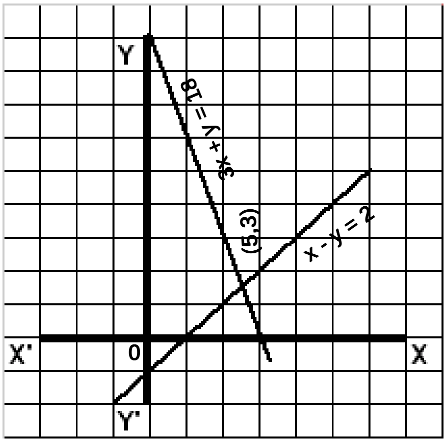
Intersección $\rightarrow (-1\,,\,4)$

23. $x-y=2$ *con* $3x+y=18$

$x=0 \quad y=-2 \qquad x=0 \quad y=18$

$x=2 \quad y=0 \qquad x=6 \quad y=0$

Escala 1:2

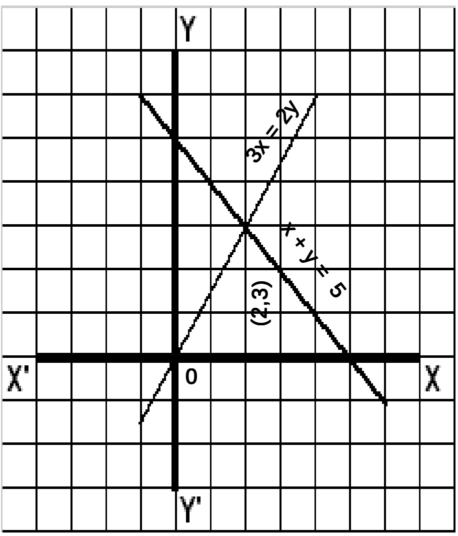


Intersección $\rightarrow (5\,,\,3)$

22. $3x=2y$ *con* $x+y=5$

$3x-2y=0 \qquad x=0 \quad y=5$

$x=1 \quad y=1\frac{1}{2} \qquad x=5 \quad y=0$

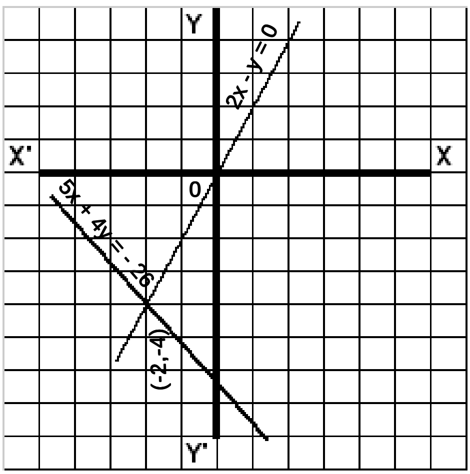


Intersección $\rightarrow (2\,,\,3)$

24. $2x-y=0$ *con* $5x+4y=-26$

$x=1 \quad y=2 \qquad x=0 \quad y=-6\frac{1}{2}$

$x=-2 \quad y=-4$

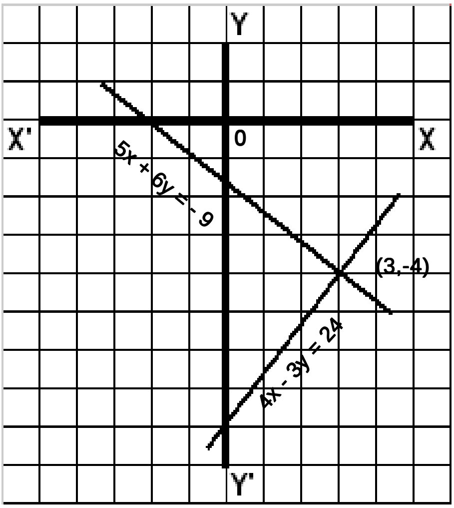


Intersección $\rightarrow (-2\,,\,-4)$

25. $5x+6y=-9$ *con* $4x-3y=24$

$x=0 \quad y=-1\frac{1}{2}$ $\qquad$ $x=0 \quad y=-8$

$x=3 \quad y=-4$ $\qquad$ $x=3 \quad y=-4$

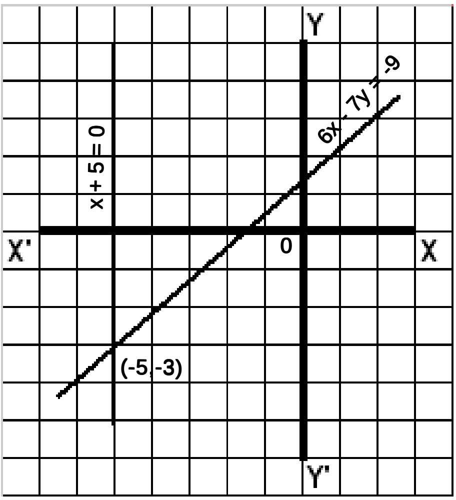


Intersección $\rightarrow$ $(3\,,-4)$

26. $x+5=0$ *con* $6x-7y=-9$

$x=-5$ $\qquad$ $x=2 \quad y=3$

$x=-5 \quad y=-3$



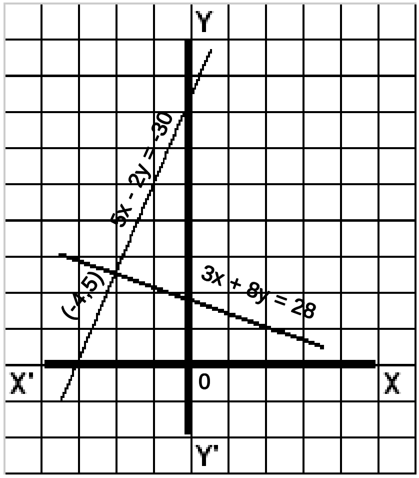
Intersección $\rightarrow$ $(-5\,,-3)$

27. $3x+8y=28$ *con* $5x-2y=-30$

$x=4 \quad y=2$ $\qquad$ $x=0 \quad y=15$

$x=0 \quad y=3\frac{1}{2}$ $\qquad$ $x=-2 \quad y=10$

Escala 1:2

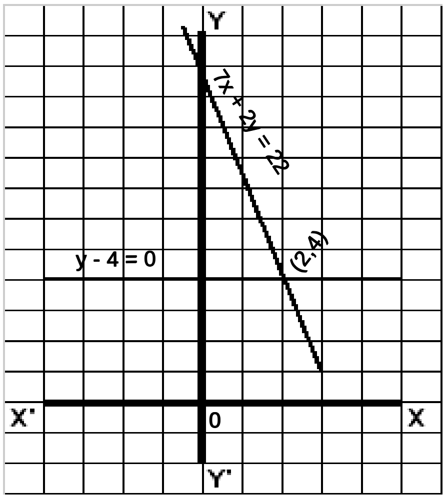


Intersección $\rightarrow$ $(-4\,,5)$

28. $y-4=0$ *con* $7x+2y=22$

$y=4$ $\qquad$ $x=0 \quad y=11$

$x=2 \quad y=4$



Intersección $\rightarrow$ $(2\,,4)$

29. $6x=-5y$ *con* $4x-3y=-38$

$6x+5y=0$ $\quad x=-2 \quad y=10$

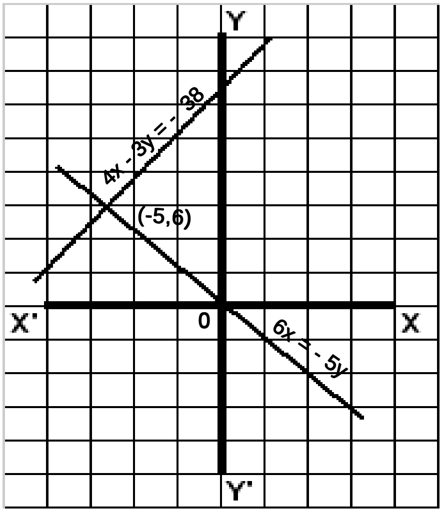
$x=5 \quad y=-6$ $\quad x=1 \quad y=14$

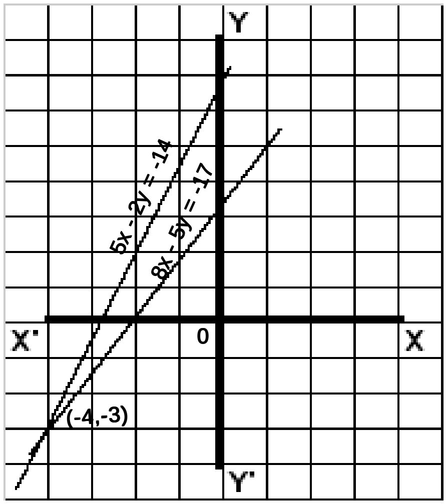
30. $5x-2y=-14$ *con* $8x-5y=-17$

$x=0 \quad y=7$ $\quad x=1 \quad y=5$

$x=-2 \quad y=2$ $\quad x=-4 \quad y=-3$

Escala 1:2





Intersección $\rightarrow (-5\,,\,6)$

Intersección $\rightarrow (-4\,,\,-3)$

EJERCICIO 176

1. $x+6y=27 \;\; ; \;\; 7x-3y=9$

$x=27-6y \qquad 7x=9+3y$

$$x=\frac{9+3y}{7}$$

$$27-6y=\frac{9+3y}{7}$$

$$7(27-6y)=9+3y$$

$$189-42y=9+3y$$

$$180=45y$$

$$4=y$$

$$x+6(4)=27$$

$$x+24=27$$

$$x=3$$

$sol: x=3 \quad y=4$

2. $3x-2y=-2 \;\; ; \;\; 5x+8y=-60$

$3x=-2+2y \qquad 5x=-60-8y$

$$x=-\frac{2-2y}{3} \qquad x=-\frac{60+8y}{5}$$

$$-\frac{2-2y}{3}=-\frac{60+8y}{5}$$

$$5(2-2y)=3(60+8y)$$

$$10-10y=180+24y$$

$$-170=34y$$

$$-5=y$$

$$3x-2(-5)=-2$$

$$3x+10=-2$$

$$3x=-12$$

$$x=-4$$

$sol: x=-4 \quad y=-5$

3. $3x+5y=7 \;\; ; \;\; 2x-y=-4$

$5y=7-3x \qquad -y=-4-2x$

$$y=\frac{7-3x}{5} \qquad y=4+2x$$

$$\frac{7-3x}{5}=4+2x$$

$$7-3x=5(4+2x)$$

$$7-3x=20+10x$$

$$-13=13x$$

$$-1=x$$

$$2(-1)-y=-4$$

$$-2-y=-4$$

$$-y=-2$$

$$y=2$$

$sol: x=-1 \quad y=2$

4. $7x-4y=5 \quad ; \quad 9x+8y=13$

$7x=5+4y \qquad 9x=13-8y$

$x=\frac{5+4y}{7} \qquad x=\frac{13-8y}{9}$

$9(5+4y)=7(13-8y)$

$45+36y=91-56y$

$92y=46$

$y=\frac{46}{92}=\frac{1}{2}$

$9x+8\left(\frac{1}{2}\right)=13$

$9x+4=13$

$9x=9$

$x=1$

$sol: x=1 \quad y=\frac{1}{2}$

5. $9x+16y=7 \quad ; \quad 4y-3x=0$

$16y=7-9x \qquad 4y=3x$

$y=\frac{7-9x}{16} \qquad y=\frac{3x}{4}$

$4(7-9x)=16(3x)$

$28-36x=48x$

$28=84x$

$x=\frac{28}{84}=\frac{1}{3}$

$4y-1=0$

$4y=1$

$y=\frac{1}{4}$

$sol: x=\frac{1}{3} \quad y=\frac{1}{4}$

6. $14x-11y=-29 \quad ; \quad 13y-8x=30$

$14x=11y-29 \qquad -8x=30-13y$

$x=\frac{11y-29}{14} \qquad x=\frac{13y-30}{8}$

$8(11y-29)=14(13y-30)$

$88y-232=182y-420$

$188=94y$

$2=y$

$13(2)-8x=30$

$26-8x=30$

$-8x=4$

$x=-\frac{1}{2}$

$sol: \quad x=-\frac{1}{2} \qquad y=2$

7. $15x-11y=-87$

$15x=11y-87$

$x=\frac{11y-87}{15}$

$-12x-5y=-27$

$12x=27-5y$

$x=\frac{27-5y}{12}$

$12(11y-87)=15(27-5y)$

$132y-1.044=405-75y$

$207y=1.449$

$y=\frac{1.449}{207}=7$

$12x+5(7)=27$

$12x+35=27$

$12x=-8$

$x=-\frac{8}{12}=-\frac{2}{3}$

$sol: \quad x=-\frac{2}{3} \qquad y=7$

8. $7x+9y=42$

$9y=42-7x$

$y=\frac{42-7x}{9}$

$12x+10y=-4$

$10y=-4-12x$

$y=-\frac{4+12x}{10}$

$10(42-7x)=-9(4+12x)$

$420-70x=-36-108x$

$38x=-456$

$x=-12$

$12(-12)+10y=-4$

$-144+10y=-4$

$10y=140$

$y=14$

$sol: x=-12 \quad y=14$

9. $6x-18y=-85$; $24x-5y=-5$

$-18y=-6x-85$ $\qquad -5y=-5-24x$

$y=\frac{6x+85}{18}$ $\qquad y=\frac{5+24x}{5}$

$5(6x+85)=18(5+24x)$

$30x+425=90+432x$

$335=402x$

$x=\frac{335}{402}=\frac{5}{6}$

$6\left(\frac{5}{6}\right)-18y=-85$

$5-18y=-85$

$-18y=-90$

$y=5$

$sol: \; x=\frac{5}{6} \quad y=5$

EJERCICIO 177

1. $x+3y=6$; $5x-2y=13$

$x=6-3y$

$5(6-3y)-2y=13$

$30-15y-2y=13$

$-17y=-17$

$y=1$

$x+3(1)=6$

$x+3=6$

$x=3$

$sol: x=3 \quad y=1$

2. $5x+7y=-1$; $-3x+4y=-24$

$5x=-1-7y$

$x=-\frac{1+7y}{5}$

$3\left(\frac{1+7y}{5}\right)+4y=-24$

$\frac{3+21y}{5}+4y=-24$

$3+21y+20y=-120$

$41y=-123$

$y=-3$

$-3x+4(-3)=-24$

$-3x-12=-24$

$-3x=-12$

$x=4$

$sol: x=4 \quad y=-3$

3. $3x+4y=8$; $8x-9y=-77$

$4y=8-3x$

$y=\frac{8-3x}{4}$

$8x-9\left(\frac{8-3x}{4}\right)=-77$

$32x-9(8-3x)=-308$

$32x-72+27x=-308$

$59x=-236$

$x=-4$

$3(-4)+4y=8$

$-12+4y=8$

$4y=20$

$y=5$

$sol: \quad x=-4 \quad y=5$

4. $x-5y=8$; $-7x+8y=25$

$8y=25+7x$

$y=\frac{25+7x}{8}$

$x-5\left(\frac{25+7x}{8}\right)=8$

$8x-5(25+7x)=64$

$8x-125-35x=64$

$-27x=189$

$x=-7$

$-7-5y=8$

$-5y=15$

$y=-3$

$sol: \; x=-7 \quad y=-3$

5. $15x+11y=32$; $7y-9x=8$

$7y=8+9x$

$y=\frac{8+9x}{7}$

$15x+11\left(\frac{8+9x}{7}\right)=32$

$105x+11(8+9x)=224$

$105x+88+99x=224$

$204x=136$

$x=\frac{136}{204}=\frac{2}{3}$

$15\left(\frac{2}{3}\right)+11y=32$

$10+11y=32$

$11y=22$

$y=2$

$sol: \quad x=\frac{2}{3} \quad y=2$

6. $10x+18y=-11$; $16x-9y=-5$

$10x=-11-18y$

$x=-\frac{11+18y}{10}$

$-16\left(\frac{11+18y}{10}\right)-9y=-5$

$-16(11+18y)-90y=-50$

$-176-288y-90y=-50$

$-378y=126$

$y=-\frac{126}{378}=-\frac{1}{3}$

$10x+18\left(-\frac{1}{3}\right)=-11$

$10x-6=-11$

$10x=-5$

$x=-\frac{1}{2}$

$sol: \quad x=-\frac{1}{2} \quad y=-\frac{1}{3}$

7. $4x+5y=5\ ;\ -4x-10y=-7$

$$10y=7-4x$$
$$y=\frac{7-4x}{10}$$
$$4x+5\left(\frac{7-4x}{10}\right)=5$$
$$8x+7-4x=10$$
$$4x=3$$
$$x=\frac{3}{4}$$
$$4\left(\frac{3}{4}\right)+5y=5$$
$$3+5y=5$$
$$5y=2$$
$$y=\frac{2}{5}$$

$sol:\ x=\frac{3}{4}\quad y=\frac{2}{5}$

8. $32x-25y=13\ ;\ 16x+15y=1$

$$16x=1-15y$$
$$x=\frac{1-15y}{16}$$
$$32\left(\frac{1-15y}{16}\right)-25y=13$$
$$2(1-15y)-25y=13$$
$$2-30y-25y=13$$
$$2-55y=13$$
$$-55y=11$$
$$y=-\frac{1}{5}$$
$$16x+15\left(-\frac{1}{5}\right)=1$$
$$16x-3=1$$
$$16x=4$$
$$x=\frac{1}{4}$$

$sol:\quad x=\frac{1}{4}\quad y=-\frac{1}{5}$

9. $11x-13y=-163\qquad ;\ -8x+7y=94$

$$11x=-163+13y$$
$$x=-\frac{163-13y}{11}$$
$$8\left(\frac{163-13y}{11}\right)+7y=94$$
$$8(163-13y)+77y=1.034$$
$$1.304-104y+77y=1.034$$
$$-27y=-270$$
$$y=10$$
$$11x-13(10)=-163$$
$$11x-130=-163$$
$$11x=-33$$
$$x=-3$$

$sol:\quad x=-3\quad y=10$

EJERCICIO 178

1. $6x-5y=-9\quad(2)$

$4x+3y=13\quad(-3)$

$$12x-10y=-18$$
$$-12x-9y=-39$$
$$-19y=-57$$
$$y=3$$
$$4x+3(3)=13$$
$$4x+9=13$$
$$4x=4$$
$$x=1$$

$sol:\ x=1\quad y=3$

2. $7x-15y=1$

$-x-6y=8\quad(7)$

$$7x-15y=1$$
$$-7x-42y=56$$
$$-57y=57$$
$$y=-1$$
$$7x-15(-1)=1$$
$$7x+15=1$$
$$7x=-14$$
$$x=-2$$

$sol:\ x=-2\quad y=-1$

3. $3x-4y=41\quad(3)$

$11x+6y=47\quad(2)$

$$9x-12y=123$$
$$22x+12y=94$$
$$31x=217$$
$$x=7$$
$$3(7)-4y=41$$
$$21-4y=41$$
$$-4y=20$$
$$y=-5$$

$sol:\ x=7\quad y=-5$

4. $9x+11y=-14\quad(4)$

$6x-5y=-34\quad(-6)$

$$36x+44y=-56$$
$$-36x+30y=204$$
$$74y=148$$
$$y=2$$
$$6x-5(2)=-34$$
$$6x-10=-34$$
$$6x=-24$$
$$x=-4$$

$sol:\ x=-4\quad y=2$

5. $10x-3y=36$

$2x+5y=-4\quad(-5)$

$$10x-3y=36$$
$$-10x-25y=20$$
$$-28y=56$$
$$y=-2$$
$$10x-3(-2)=36$$
$$10x=30$$
$$x=3$$

$sol:\ x=3\quad y=-2$

6. $11x-9y=2\quad(5)$

$13x-15y=-2\quad(-3)$

$$55x-45y=10$$
$$-39x+45y=6$$
$$16x=16$$
$$x=1$$
$$11(1)-9y=2$$
$$11-9y=2$$
$$-9y=-9$$
$$y=1$$

$sol:\ x=1\quad y=1$

7. $18x+5y=-11 \quad (2)$

$12x+11y=31 \quad (-3)$

$36x+10y=-22$

$-36x-33y=-93$

$-23y=-115$

$y=5$

$18x+5(5)=-11$

$18x+25=-11$

$18x=-36$

$x=-2$

$sol: \; x=-2 \quad y=5$

8. $9x+7y=-4 \quad (-11)$

$11x-13y=-48 \quad (9)$

$-99x-77y=44$

$99x-117y=-432$

$-194y=-388$

$y=2$

$9x+7(2)=-4$

$9x+14=-4$

$9x=-18$

$x=-2$

$sol: x=-2 \quad y=2$

9. $12x-14y=20 \quad (7)$

$-14x+12y=-19 \quad (6)$

$84x-98y=140$

$-84x+72y=-114$

$-26y=26$

$y=-1$

$12x-14(-1)=20$

$12x+14=20$

$12x=6$

$x=\frac{1}{2}$

$sol: \quad x=\frac{1}{2} \quad y=-1$

10. $15x-y=40 \quad (8)$

$19x+8y=236$

$120x-8y=320$

$19x+8y=236$

$139x = 556$

$x=4$

$15(4)-y=40$

$60-y=40$

$-y=-20$

$y=20$

$sol: \quad x=4 \quad y=20$

11. $36x-11y=-14 \quad (2)$

$24x-17y=10 \quad (-3)$

$72x-22y=-28$

$-72x+51y=-30$

$29y=-58$

$y=-2$

$36x-11(-2)=-14$

$36x+22=-14$

$36x=-36$

$x=-1$

$sol: \; x=-1 \quad y=-2$

12. $12x-17y=104 \quad (5)$

$15x+19y=-31 \quad (-4)$

$60x-85y=520$

$-60x-76y=124$

$-161y=644$

$y=-4$

$12x-17(-4)=104$

$12x+68=104$

$12x=36$

$x=3$

$sol: \quad x=3 \quad y=-4$

EJERCICIO 179

1. $8x-5=7y-9$

$8x-7y=-9+5$

$8x-7y=-4$

$6x=3y+6$

$6x-3y=6$

$8x-7y=-4 \quad (3)$

$6x-3y=6 \quad (-4)$

$24x-21y=-12$

$-24x+12y=-24$

$-9y=-36$

$y=4$

$6x-3(4)=6$

$6x-12=6$

$6x=18$

$x=3$

$sol: \; x=3 \quad y=4$

2. $x-1=y+1$

$x-y=2$

$x-3=3y-7$

$x-3y=-4$

$x-y=2$

$x-3y=-4 \quad (-1)$

$x-y=2$

$-x+3y=4$

$2y=6$

$y=3$

$x-3=2$

$x=5$

$sol: x=5 \quad y=3$

3. $3(x+2)=2y \qquad 2(y+5)=7x$

$3x+6=2y \qquad 2y+10=7x$

$3x-2y=-6 \qquad 7x-2y=10$

$3x-2y=-6 \; ; \qquad 7x-2y=10$

$3x=-6+2y \qquad 7x=10+2y$

$x=-\frac{6-2y}{3} \qquad x=\frac{10+2y}{7}$

$-7(6-2y)=3(10+2y)$

$-42+14y=30+6y$

$8y=72$

$y=9$

$7x-2(9)=10$

$7x-18=10$

$7x=28$

$x=4$

$sol: \; x=4 \quad y=9$

4. $x-1=2(y+6)$

$x-1=2y+12$

$x-2y=13$

$x=13+2y$

$x+6=3(1-2y)$

$x+6=3-6y$

$x+6y=-3$

$x=-3-6y$

$13+2y=-3-6y$

$-3-6y-2y=13$

$-8y=16$

$y=-2$

$x-2(-2)=13$

$x+4=13$

$x=9$

$sol: x=9 \quad y=-2$

5. $30-(8-x)=2y+30$

$30-8+x=2y+30$

$x-2y=8$

$5x-29=x-(5-4y)$

$5x-29=x-5+4y$

$4x-4y=24$

$x-y=6$

$$\begin{array}{rl} x-2y=8 & \\ x-y=6 & (-1) \\ \hline x-2y=8 & \\ -x+y=-6 & \\ \hline -y=2 & \end{array}$$

$y=-2$

$x-2(-2)=8$

$x+4=8$

$x=4$

$sol: \ x=4 \quad y=-2$

6. $3x-(9x+y)=5y-(2x+9y)$

$3x-9x-y=5y-2x-9y$

$-4x+3y=0$

$4x-(3y+7)=5y-47$

$4x-3y-7=5y-47$

$4x-8y=-40$

$x-2y=-10$

$$\begin{array}{rl} -4x+3y=0 & \\ x-2y=-10 & (4) \\ \hline -4x+3y=0 & \\ 4x-8y=-40 & \\ \hline \end{array}$$

Continúa

6. Continuación

$-5y=-40$

$y=8$

$x-2(8)=-10$

$x-16=-10$

$x=6$

$sol: \ x=6 \quad y=8$

7. $(x-y)-(6x+8y)=-(10x+5y+3)$

$x-y-6x-8y=-10x-5y-3$

$-5x-9y=-10x-5y-3$

$5x-4y=-3$

$(x+y)-(9y-11x)=2y-2x$

$x+y-9y+11x=2y-2x$

$12x-8y=2y-2x$

$14x-10y=0$

$$\begin{array}{rl} 5x-4y=-3 & (5) \\ 14x-10y=0 & (-2) \\ \hline 25x-20y=-15 & \\ -28x+20y=0 & \\ \hline -3x \quad =-15 & \end{array}$$

$x=5$

$5(5)-4y=-3$

$25-4y=-3$

$-4y=-28$

$y=7$

$sol: \ x=5 \quad y=7$

8. $5(x+3y)-(7x+8y)=-6$

$5x+15y-7x-8y=-6$

$-2x+7y=-6$

$7x-9y-2(x-18y)=0$

$7x-9y-2x+36y=0$

$5x+27y=0$

$$\begin{array}{rl} -2x+7y=-6 & (5) \\ 5x+27y=0 & (2) \\ \hline -10x+35y=-30 & \\ 10x+54y=0 & \\ \hline 89y=-30 & \end{array}$$

$y=-\frac{30}{89}$

Continúa

8. Continuación

$-2x+7\left(-\frac{30}{89}\right)=-6$

$-178x-210=-534$

$-178x=-324$

$x=\frac{324}{178}=1\frac{73}{89}$

$sol: \ x=1\frac{73}{89} \quad y=-\frac{30}{89}$

9. $2(x+5)=4(y-4x)$

$2x+10=4y-16x$

$18x-4y=-10$

$10(y-x)=11y-12x$

$10y-10x=11y-12x$

$2x-y=0$

$$\begin{array}{rl} 18x-4y=-10 & \\ 2x-y=0 & (-9) \\ \hline 18x-4y=-10 & \\ -18x+9y=0 & \\ \hline 5y=-10 & \end{array}$$

$y=-2$

$2x-(-2)=0$

$2x+2=0$

$2x=-2$

$x=-1$

$sol: x=-1 \quad y=-2$

10. $3x-4y-2(2x-7)=0$

$3x-4y-4x+14=0$

$-x-4y=-14$

$x+4y=14$

$5(x-1)-(2y-1)=0$

$5x-5-2y+1=0$

$5x-2y=4$

$$\begin{array}{rl} x+4y=14 & \\ 5x-2y=4 & (2) \\ \hline x+4y=14 & \\ 10x-4y=8 & \\ \hline 11x \quad =22 & \end{array}$$

$x=2$

$2+4y=14$

$4y=12$

$y=3$

$sol: \ x=2 \quad y=3$

11. $12(x+2y)-8(2x+y)=2(5x-6y)$

$12x+24y-16x-8y=10x-12y$

$-4x+16y=10x-12y$

$-14x+28y=0$

$-x+2y=0$

$20(x-4y)=-10$

$20x-80y=-10$

$2x-8y=-1$

$-x+2y=0 \quad (2)$

$2x-8y=-1$

$-2x+4y=0$

$2x-8y=-1$

$-4y=-1$

$y=\frac{1}{4}$

$-x+2\left(\frac{1}{4}\right)=0$

$-4x+2=0$

$-4x=-2$

$x=\frac{1}{2}$

$sol: \ x=\frac{1}{2} \quad y=\frac{1}{4}$

12. $x(y-2)-y(x-3)=-14$

$xy-2x-xy+3y=-14$

$-2x+3y=-14$

$y(x-6)-x(y+9)=54$

$xy-6y-xy-9x=54$

$-9x-6y=54$

$3x+2y=-18$

$-2x+3y=-14 \quad (3)$

$3x+2y=-18 \quad (2)$

$-6x+9y=-42$

$6x+4y=-36$

$13y=-78$

$y=-6$

$-2x+3(-6)=-14$

$-2x-18=-14$

$-2x=4$

$x=-2$

$sol: \quad x=-2 \quad y=-6$

EJERCICIO 180

1. $\frac{3x}{2}+y=11$

$3x+2y=22$

$x+\frac{y}{2}=7$

$2x+y=14$

$3x+2y=22 \quad ; \quad 2x+y=14$

$3x=22-2y \qquad 2x=14-y$

$x=\frac{22-2y}{3} \qquad x=\frac{14-y}{2}$

$2(22-2y)=3(14-y)$

$44-4y=42-3y$

$-y=-2$

$y=2$

$2x+2=14$

$2x=12$

$x=6$

$sol: \quad x=6 \quad y=2$

2. $\frac{5x}{12}-y=9 \quad ; \quad x-\frac{3y}{4}=15$

$5x-12y=108 \quad ; \quad 4x-3y=60$

$5x=108+12y$

$x=\frac{108+12y}{5}$

$4\left(\frac{108+12y}{5}\right)-3y=60$

$4(108+12y)-15y=300$

$432+48y-15y=300$

$33y=-132$

$y=-4$

$4x-3(-4)=60$

$4x+12=60$

$4x=48$

$x=12$

$sol: \quad x=12 \quad y=-4$

3. $\frac{x}{7}+\frac{y}{3}=5 \quad ; \quad 3y-\frac{x}{14}=26$

$3x+7y=105 \quad ; \quad 42y-x=364$

$3x+7y=105$

$-x+42y=364 \quad (3)$

$3x+7y=105$

$-3x+126y=1.092$

$133y=1.197$

$y=9$

$3x+7(9)=105$

$3x+63=105$

$3x=42$

$x=14$

$sol: \quad x=14 \quad y=9$

4. $\frac{x}{5}=\frac{y}{4}$; $\frac{y}{3}=\frac{x}{3}-1$

$4x=5y$ $\quad y=x-3$

$4x-5y=0 \quad x-y=3$

$4x-5y=0$

$x-y=3 \quad (-4)$

$4x-5y=0$

$-4x+4y=-12$

$-y=-12$

$y=12$

$x-12=3$

$x=15$

$sol: \ x=15 \quad y=12$

5. $\frac{3x}{5}-\frac{1}{4}y=2$; $2x=\frac{5y}{2}$

$12x-5y=40 \quad 4x=5y$

$4x-5y=0$

$4x-5y=0$

$12x-5y=40 \quad (-1)$

$4x-5y=0$

$-12x+5y=-40$

$-8x=-40$

$x=5$

$4(5)-5y=0$

$20-5y=0$

$-5y=-20$

$y=4$

$sol: \ x=5 \quad y=4$

6. $\frac{2x}{3}-\frac{3y}{4}=1$; $\frac{1}{8}y-\frac{5x}{6}=2$

$8x-9y=12$; $3y-20x=48$

$8x-9y=12$

$-20x+3y=48 \quad (3)$

$8x-9y=12$

$-60x+9y=144$

$-52x=156$

$x=-3$

$8(-3)-9y=12$

$-24-9y=12$

$-9y=36$

$y=-4$

$sol: \ x=-3 \quad y=-4$

7. $\frac{x}{8}-\frac{y}{5}=-\frac{11}{10}$; $\frac{x}{5}+\frac{y}{4}=\frac{-59}{40}$

$5x-8y=-44$; $8x+10y=-59$

$5x-8y=-44 \quad (5)$

$8x+10y=-59 \quad (4)$

$25x-40y=-220$

$32x+40y=-236$

$57x=-456$

$x=-8$

$5(-8)-8y=-44$

$-40-8y=-44$

$-8y=-4$

$y=\frac{1}{2}$

$sol: \ x=-8 \quad y=\frac{1}{2}$

8. $\frac{x}{7}+\frac{y}{8}=0$; $\frac{1}{7}x-\frac{3y}{4}=7$

$8x+7y=0$; $4x-21y=196$

$8x=-7y \quad 4x=196+21y$

$x=\frac{-7y}{8} \quad x=\frac{196+21y}{4}$

$-28y=8(196+21y)$

$-28y=1.568+168y$

$-196y=1.568$

$y=-8$

$x=\frac{-7(-8)}{8}$

$x=7$

$sol: \ x=7 \quad y=-8$

9. $\frac{2x+1}{5}=\frac{y}{4}$; $2x-3y=-8$

$4(2x+1)=5y$

$8x+4=5y$

$8x-5y=-4$

$2x-3y=-8 \quad (-4)$

$8x-5y=-4$

$-8x+12y=32$

$8x-5y=-4$

$7y=28$

$y=4$

Continúa

9. Continuación

$8x-5(4)=-4$

$8x-20=-4$

$8x=16$

$x=2$

$sol: \ x=2 \quad y=4$

10. $12x+5y+6=0$

$12x+5y=-6$

$\frac{5x}{3}-\frac{7y}{6}=-12$

$10x-7y=-72$

$12x+5y=-6 \quad (5)$

$10x-7y=-72 \quad (-6)$

$60x+25y=-30$

$-60x+42y=432$

$67y=402$

$y=6$

$10x-7(6)=-72$

$10x-42=-72$

$10x=-30$

$x=-3$

$sol: \ x=-3 \quad y=6$

11. $\frac{x}{5}=3(y+2)$

$x=15(y+2)$

$x=15y+30$

$\frac{y}{5}+3x=\frac{224}{5}$

$y+15x=224$

$15x=224-y$

$x=\frac{224-y}{15}$

$15(15y+30)=224-y$

$225y+450=224-y$

$226y=-226$

$y=-1$

$x=15(-1)+30$

$x=-15+30$

$x=15$

$sol: \ x=15 \quad y=-1$

12. $\frac{x}{5}-\frac{y}{6}=-\frac{1}{30}$; $\frac{x}{3}-\frac{y}{20}=\frac{13}{12}$

$6x-5y=-1$; $20x-3y=65$

$6x - 5y=-1 \quad (3)$

$20x - 3y=65 \quad (-5)$

$18x-15y=-3$

$-100x+15y=-325$

$-82x=-328$

$x=4$

$6(4)-5y=-1$

$-5y=-25$

$y=5$

$sol: x=4 \quad y=5$

13. $\frac{x-3}{3}-\frac{y-4}{4}=0$; $\frac{x-4}{2}+\frac{y+2}{5}=3$

$4(x-3)-3(y-4)=0$; $5(x-4)+2(y+2)=30$

$4x-12-3y+12=0$; $5x-20+2y+4=30$

$4x-3y=0 \qquad 5x+2y=46$

$x=\frac{3y}{4}$

$5\left(\frac{3y}{4}\right)+2y=46$

$15y+8y=184$

$23y=184$

$y=8$

$x=\frac{3(8)}{4}$

$x=6$

$sol: \quad x=6 \quad y=8$

14. $\frac{x-1}{2}-\frac{y-1}{3}=-\frac{13}{36}$; $\frac{x+1}{3}-\frac{y+1}{2}=-\frac{2}{3}$

$18(x-1)-12(y-1)=-13$; $2x+2-3y-3=-4$

$18x-12y=-7 \qquad 2x-3y=-3$

$18x-12y=-7$

$2x-3y=-3 \quad (-4)$

$18x-12y=-7$

$-8x+12y=12$

$10x=5$

$x=\frac{1}{2}$

$2\left(\frac{1}{2}\right)-3y=-3$

$-3y=-4$

$y=\frac{4}{3}$

$sol: \quad x=\frac{1}{2} \quad y=\frac{4}{3}$

15. $\frac{x+1}{10}=\frac{y-4}{5}$; $\frac{x-4}{5}=\frac{y-2}{10}$

$x+1=2(y-4)$; $2(x-4)=y-2$

$x+1=2y-8$; $2x-8=y-2$

$x-2y=-9 \qquad 2x-y=6$

$x=2y-9$

$2(2y-9)-y=6$

$4y-18-y=6$

$3y=24$

$y=8$

$x=2(8)-9$

$x=7$

$sol: x=7 \quad y=8$

16. $x=-\frac{3y+3}{4}$; $y=-\frac{1+5x}{4}$

$4x=-3y-3$; $4y=-1-5x$

$4x+3y=-3 \qquad 4y+5x=-1$

$4x+3y=-3 \quad (4)$

$5x+4y=-1 \quad (-3)$

$16x+12y=-12$

$-15x-12y=3$

$x=-9$

$4(-9)+3y=-3$

$-36+3y=-3$

$3y=33$

$y=11$

$sol: x=-9 \quad y=11$

17. $\frac{x+y}{6}=\frac{x-y}{12}$; $\frac{2x}{3}=y+3$

$2(x+y)=x-y$; $2x=3(y+3)$

$2x+2y=x-y$; $2x=3y+9$

$x+3y=0$; $2x-3y=9$

$x=-3y$

$2(-3y)-3y=9$

$-6y-3y=9$

$-9y=9$

$y=-1$

$x=-3(-1)$

$x=3$

$sol: x=3 \quad y=-1$

18. $3x-\frac{y-3}{5}=6$

$15x-y+3=30$

$15x-y=27$

$15x=27+y$

$x=\frac{27+y}{15}$

$3y-\frac{x-2}{7}=9$

$21y-x+2=63$

$-x+21y=61$

$-x=61-21y$

$x=21y-61$

$27+y=15(21y-61)$

$27+y=315y-915$

$-314y=-942$

$y=3$

$x=\frac{27+3}{15}$

$x=2$

$sol:\quad x=2\quad y=3$

19. $\frac{x+y}{6}-\frac{y-x}{3}=\frac{7}{24}$

$4(x+y)-8(y-x)=7$

$4x+4y-8y+8x=7$

$12x-4y=7$

$\frac{x}{2}+\frac{x-y}{6}=\frac{5}{12}$

$6x+2(x-y)=5$

$6x+2x-2y=5$

$8x-2y=5$

$12x-4y=7$

$8x-2y=5\quad(-2)$

$12x-4y=7$

$-16x+4y=-10$

$-4x=-3$

$x=\frac{3}{4}$

$12\left(\frac{3}{4}\right)-4y=7$

$9-4y=7$

$-4y=-2$

$y=\frac{1}{2}$

$sol:\quad x=\frac{3}{4}\quad y=\frac{1}{2}$

20. $\frac{x-2}{4}-\frac{y-x}{2}=x-7$

$x-2-2(y-x)=4(x-7)$

$3x-4x-2y=-28+2$

$x+2y=26$

$\frac{3x-y}{8}-\frac{3y-x}{6}=y-13$

$3(3x-y)-4(3y-x)=24(y-13)$

$13x-15y-24y=-312$

$13x-39y=-312$

$x+2y=26\quad(-13)$

$13x-39y=-312$

$-13x-26y=-338$

$13x-39y=-312$

$-65y=-650$

$y=10$

$x+2(10)=26$

$x+20=26$

$x=6$

$sol:\quad x=6\quad y=10$

21. $12-\frac{3x-2y}{6}=3y+2$

$72-3x+2y=6(3y+2)$

$-3x+2y-18y=12-72$

$3x+16y=60$

$\frac{5y-3x}{3}=x-y$

$5y-3x=3(x-y)$

$-3x-3x+5y+3y=0$

$6x-8y=0$

$3x+16y=60$

$6x-8y=0\quad(2)$

$3x+16y=60$

$12x-16y=0$

$15x=60$

$x=4$

$6(4)-8y=0$

$-8y=-24$

$y=3$

$sol:\quad x=4\quad y=3$

22. $y(x-4)=x(y-6)$

$xy-4y=xy-6x$

$6x-4y=0$

$\frac{5}{x-3}-\frac{11}{y-1}=0$

$5(y-1)-11(x-3)=0$

$5y-5-11x+33=0$

$11x-5y=28$

$6x-4y=0\quad(5)$

$11x-5y=28\quad(-4)$

$30x-20y=0$

$-44x+20y=-112$

$-14x=-112$

$x=8$

$11(8)-5y=28$

$-5y=28-88$

$-5y=-60$

$y=12$

$sol:\quad x=8\quad y=12$

23. $\frac{3(x+3y)}{5x+6y}=\frac{21}{17}$

$51(x+3y)=21(5x+6y)$

$51x+153y=105x+126y$

$-54x+27y=0$

$2x-y=0$

$\frac{4x-7y}{2y+1}=-2$

$4x-7y=-2(2y+1)$

$4x-7y=-4y-2$

$4x-3y=-2$

$2x-y=0\quad(-2)$

$4x-3y=2$

$-4x+2y=0$

$4x-3y=-2$

$y=2$

$2x-2=0$

$2x=2$

$x=1$

$sol:\quad x=1\quad y=2$

24. $\dfrac{7}{2x-3y+6}=-\dfrac{7}{3x-2y-1}$

$$7(3x-2y-1)=-7(2x-3y+6)$$
$$21x-14y-7=-14x+21y-42$$
$$35x-35y=-35$$
$$x-y=-1$$
$$\frac{6}{x-y+4}=\frac{10}{y+2}$$
$$6(y+2)=10(x-y+4)$$
$$6y+12=10x-10y+40$$
$$-28=10x-16y$$
$$-14=5x-8y$$
$$x-y=-1 \quad (-8)$$
$$\underline{5x-8y=-14}$$
$$-8x+8y=8$$
$$\underline{5x-8y=-14}$$
$$-3x=-6$$
$$x=2$$
$$2-y=-1$$
$$-y=-3$$
$$y=3$$
$$sol:\quad x=2 \quad y=3$$

25. $\dfrac{x+y}{x-y}=-7$

$$x+y=-7(x-y)$$
$$x+y=-7x+7y$$
$$8x-6y=0$$
$$\frac{x+y+1}{x+y-1}=\frac{3}{4}$$
$$4(x+y+1)=3(x+y-1)$$
$$4x+4y+4=3x+3y-3$$
$$x+y=-7$$
$$8x-6y=0$$
$$\underline{x+y=-7} \quad (6)$$
$$8x-6y=0$$
$$\underline{6x+6y=-42}$$
$$14x=-42$$
$$x=-3$$
$$-3+y=-7$$
$$y=-4$$
$$sol:\quad x=-3 \quad y=-4$$

26. $\dfrac{x}{4}-8=\dfrac{3y}{2}-\dfrac{33}{4}$

$$x-32=6y-33$$
$$x-6y=-1$$
$$\frac{y-x}{3}-\frac{2x+y}{2}=-\frac{17}{24}$$
$$8(y-x)-12(2x+y)=-17$$
$$8y-8x-24x-12y=-17$$
$$32x+4y=17$$
$$x-6y=-1 \quad (2)$$
$$\underline{32x+4y=17} \quad (3)$$
$$2x-12y=-2$$
$$\underline{96x+12y=51}$$
$$98x=49$$
$$x=\frac{1}{2}$$
$$\frac{1}{2}-6y=-1$$
$$1-12y=-2$$
$$-12y=-3$$
$$y=\frac{1}{4}$$
$$sol:\quad x=\frac{1}{2} \quad y=\frac{1}{4}$$

27. $\dfrac{x-2}{x+2}=\dfrac{y-7}{y-5}$

$$(y-5)(x-2)=(y-7)(x+2)$$
$$xy-2y-5x+10=xy+2y-7x-14$$
$$2x-4y=-24$$
$$x-2y=-12$$
$$x=-12+2y$$
$$\frac{x+1}{x-1}=\frac{y-3}{y-5}$$
$$(y-5)(x+1)=(x-1)(y-3)$$
$$xy+y-5x-5=xy-3x-y+3$$
$$-2x+2y=8$$
$$-x+y=4$$
$$x=y-4$$
$$-12+2y=y-4$$
$$y=8$$
$$x=8-4$$
$$x=4$$
$$sol:\quad x=4 \quad y=8$$

28. $\dfrac{x-y-1}{x+y+1}=-\dfrac{3}{17}$

$$17(x-y-1)=-3(x+y+1)$$
$$17x-17y-17=-3x-3y-3$$
$$20x-14y=14$$
$$10x-7y=7$$
$$\frac{x+y-1}{x-y+1}=-15$$
$$x+y-1=-15(x-y+1)$$
$$x+y-1=-15x+15y-15$$
$$16x-14y=-14$$
$$8x-7y=-7$$
$$10x-7y=7$$
$$\underline{8x-7y=-7} \quad (-1)$$
$$10x-7y=7$$
$$\underline{-8x+7y=7}$$
$$2x=14$$
$$x=7$$
$$10(7)-7y=7$$
$$-7y=-63$$
$$y=9$$
$$sol:\quad x=7 \quad y=9$$

29. $\dfrac{6x+9y-4}{4x-6y+5}=\dfrac{2}{5}$

$$5(6x+9y-4)=2(4x-6y+5)$$
$$30x+45y-20=8x-12y+10$$
$$22x+57y=30$$
$$\frac{2x+3y-3}{3x+2y-4}=\frac{6}{11}$$
$$11(2x+3y-3)=6(3x+2y-4)$$
$$22x+33y-33=18x+12y-24$$
$$4x+21y=9$$
$$22x+57y=30 \quad (2)$$
$$\underline{4x+21y=9} \quad (-11)$$
$$44x+114y=60$$
$$\underline{-44x-231y=-99}$$
$$-117y=-39$$
$$y=\frac{1}{3}$$
$$4x+21\left(\frac{1}{3}\right)=9$$
$$4x+7=9$$
$$4x=2$$
$$x=\frac{1}{2}$$
$$sol:\quad x=\frac{1}{2} \quad y=\frac{1}{3}$$

30. $\dfrac{3x+2y}{x+y-15}=-9$

$3x+2y=-9(x+y-15)$

$3x+2y=-9x-9y+135$

$12x+11y=135$

$\dfrac{4x}{3}-\dfrac{5(y-1)}{8}=-1$

$32x-15(y-1)=-24$

$32x-15y+15=-24$

$32x-15y=-39$

$12x+11y=135 \quad (8)$

$32x-15y=-39 \quad (-3)$

$96x+88y=1.080$

$-96x+45y=117$

$133y=1.197$

$y=9$

$12x+11(9)=135$

$12x=135-99$

$12x=36$

$x=3$

$sol: \; x=3 \quad y=9$

31. $\dfrac{2x+5}{17}-(5-y)=-60$

$2x+5-17(5-y)=-1.020$

$2x-85+17y=-1.025$

$2x+17y=-940$

$\dfrac{y+62}{2}-(1-x)=40$

$y+62-2(1-x)=80$

$y-2+2x=18$

$2x+y=20$

$2x+17y=-940$

$2x+y=20 \quad (-1)$

$2x+17y=-940$

$-2x-y=-20$

$16y=-960$

$y=-60$

$2x-60=20$

$2x=80$

$x=40$

$sol: \; x=40 \quad y=-60$

32. $\dfrac{3x+4y}{x-6y}=-\dfrac{30}{23}$

$23(3x+4y)=-30(x-6y)$

$69x+92y=-30x+180y$

$99x-88y=0$

$9x-8y=0$

$\dfrac{9x-y}{3+x-y}=-\dfrac{63}{37}$

$37(9x-y)=-63(3+x-y)$

$333x-37y=-189-63x+63y$

$396x-100y=-189$

$396x-100y=-189$

$9x-8y=0 \quad (-44)$

$396x-100y=-189$

$-396x+352y=0$

$252y=-189$

$y=\dfrac{-3}{4}$

$9x-8\left(\dfrac{-3}{4}\right)=0$

$9x+6=0$

$9x=-6$

$x=\dfrac{-2}{3}$

$sol: \; x=\dfrac{-2}{3} \quad y=\dfrac{-3}{4}$

33. $x-\dfrac{4x+1}{9}=\dfrac{2y-5}{3}$

$9x-(4x+1)=3(2y-5)$

$9x-4x-1=6y-15$

$5x-6y=-14$

$y-\dfrac{3y+2}{7}=\dfrac{x+18}{10}$

$70y-10(3y+2)=7(x+18)$

$70y-30y-20=7x+126$

$-146=7x-40y$

$7x-40y=-146 \quad (-5)$

$5x-6y=-14 \quad (7)$

$-35x+200y=730$

$35x-42y=-98$

$158y=632$

$y=4$

$5x-6(4)=-14$

$5x=-14+24$

$x=2$

$sol: \; x=2 \quad y=4$

EJERICIO 181

1. $x+y=a+b$; $x-y=a-b$

$x+y=a+b$

$x-y=a-b$

$x=a$

$a+y=a+b$

$y=b$

$sol: \; x=a \quad y=b$

2. $2x+y=b+2$

$2bx+by=b^2+2b$

$bx-y=0$

$-2bx+2y=0$

$-2bx+2y=0$

$2bx+by=b^2+2b$

$2y+by=b^2+2b$

$y(2+b)=b(b+2)$

$y=b$

$2x+b=b+2$

$2x=2$

$x=1$

$sol: \; x=1 \quad y=b$

3. $2x-y=3a$;

$x-2y=0$

$-2x+4y=0$

$2x-y=3a$

$-2x+4y=0$

$3y=3a$

$y=a$

$2x-a=3a$

$2x=4a$

$x=2a$

$sol: \; x=2a \quad y=a$

4. $x-y=1-a$; $x+y=1+a$

$x-y=1-a$

$x+y=1+a$

$2x=2$

$x=1$

$1+y=1+a$

$y=a$

$sol: \; x=1 \quad y=a$

5. $\frac{x}{a}+y=2b \quad ; \quad \frac{x}{b}-y=a-b$

$x+ay=2ab \quad ; \quad x-by=ab-b^2$

$x+ay = 2ab$

$-x+by=b^2-ab$

$ay+by=b^2+ab$

$y(a+b)=b(b+a)$

$y=b$

$x+ab=2ab$

$x=ab$

$sol: \quad x=ab \quad y=b$

6. $\frac{x}{b}+\frac{y}{a}=2$

$ax+by=2ab$

$abx+b^2y=2ab^2$

$\frac{x}{a}+\frac{y}{b}=\frac{a^2+b^2}{ab}$

$bx+ay=a^2+b^2$

$-abx-a^2y=-a^3-ab^2$

$abx+b^2y = 2ab^2$

$-abx-a^2y=-a^3-ab^2$

$b^2y-a^2y=-a^3+ab^2$

$y(b^2-a^2)=a(b^2-a^2)$

$y=a$

$ax+ba=2ab$

$a(x+b)=2ab$

$x+b=2b$

$x=b$

$sol: \quad x=b \quad y=a$

7. $x+y=a+b$

$-ax-ay=-a^2-ab$

$ax+by=a^2+b^2$

$-ax-ay=-a^2-ab$

$ax+by = a^2+b^2$

$by-ay= b^2-ab$

$y(b-a)=b(b-a)$

$y=b$

$x+b=a+b$

$x=a$

$sol: \quad x=a \quad y=b$

8. $ax-by=0 \quad ; \quad x+y=\frac{a+b}{ab}$

$abx-b^2y=0 \quad abx+aby=a+b$

$abx-b^2y = 0$

$-abx-aby=-a-b$

$-b^2y-aby=-(a+b)$

$-yb(b+a)=-(a+b)$

$-yb=-1 \Rightarrow y=\frac{1}{b}$

$ax-b\left(\frac{1}{b}\right)=0$

$ax=1 \Rightarrow x=\frac{1}{a}$

$sol: \quad x=\frac{1}{a} \quad y=\frac{1}{b}$

9. $mx-ny=m^2+n^2 \quad ; \quad nx+my=m^2+n^2$

$mnx-n^2y=m^2n+n^3 \quad mnx+m^2y=m^3+mn^2$

$-mnx+n^2y = -m^2n -n^3$

$mnx+m^2y=m^3+mn^2$

$n^2y+m^2y=m^3-m^2n+mn^2-n^3$

$y(m^2+n^2)=m^2(m-n)+n^2(m-n)$

$y(m^2+n^2)=(m^2+n^2)(m-n)$

$y=m-n$

$m^2x-mny=m^3+mn^2$

$n^2x+mny= m^2n+n^3$

$m^2x+n^2x=m^3+mn^2+m^2n+n^3$

$x(m^2+n^2)=m(m^2+n^2)+n(m^2+n^2)$

$x(m^2+n^2)=(m+n)(m^2+n^2)$

$x=m+n$

$sol: \quad x=m+n \quad y=m-n$

10. $\frac{x}{m}+\frac{y}{n}=2m \quad ; \quad mx-ny=m^3-mn^2$

$xn+ym=2m^2n \quad xmn-n^2y=m^3n-mn^3$

$xmn+ym^2 = 2m^3n$

$-xmn+n^2y= mn^3-m^3n$

$ym^2+yn^2=mn^3+m^3n$

$y(m^2+n^2)=mn(n^2+m^2)$

$y=mn$

$xn+m^2n=2m^2n$

$xn=m^2n \Rightarrow x=m^2$

$sol: \quad x=m^2 \quad y=mn$

11. $x+y=a$

$ax+ay=a^2$

$ax-by=a(a+b)+b^2$

$ax-by=a^2+ab+b^2$

$-ax-ay \quad =-a^2$

$ax \quad -by= \ a^2+ab+b^2$

$-ay-by= \quad ab+b^2$

$-y(a+b)=b(a+b)$

$-y=b$

$y=-b$

$x-b=a$

$x=a+b$

$sol: \quad x=a+b \quad y=-b$

12. $x-y=m-n$

$mx-my=m^2-mn$

$mx \quad -ny= \quad m^2 \quad -n^2$

$-mx+my \quad =-m^2+mn$

$my-ny= \quad mn-n^2$

$y(m-n)=n(m-n)$

$y=n$

$x-n=m-n$

$x=m$

$sol: \quad x=m \quad y=n$

13. $\frac{x}{a}+\frac{y}{b}=0 \quad ; \quad \frac{x}{b}+\frac{2y}{a}=\frac{2b^2-a^2}{ab}$

$bx+ay=0 \qquad ax+2by=2b^2-a^2$

$-abx \quad -a^2y=0$

$abx+2b^2y \quad =2b^3-a^2b$

$2b^2y-a^2y=2b^3-a^2b$

$y(2b^2-a^2)=b(2b^2-a^2)$

$y=b$

$\frac{x}{a}+\frac{b}{b}=0$

$bx+ab=0$

$bx=-ab$

$x=-a$

$sol: \ x=-a \quad y=b$

14. $x+y=2c \quad ; \quad a^2(x-y)=2a^3$

$-a^2x-a^2y=-2a^2c \quad ; \quad a^2x-a^2y=2a^3$

$-a^2x-a^2y= \quad -2a^2c$

$a^2x-a^2y=2a^3$

$-2a^2y=2a^3-2a^2c$

$-2a^2y=2a^2(a-c)$

$-y=a-c$

$y=c-a$

$x+c-a=2c$

$x-a=c \Rightarrow x=c+a$

$sol: \ x=a+c \quad y=c-a$

15. $ax-by=0 \quad ; \quad ay-bx=\frac{a^2-b^2}{ab}$

$b^2ax-b^3y=0 \qquad a^2by-b^2ax=a^2-b^2$

$b^2ax \quad -b^3y=0$

$-b^2ax+a^2by \quad =a^2-b^2$

$a^2by-b^3y=a^2-b^2$

$by(a^2-b^2)=a^2-b^2$

$by=1$

$y=\frac{1}{b}$

$ax-b\left(\frac{1}{b}\right)=0$

$ax=1$

$x=\frac{1}{a}$

$sol: \quad x=\frac{1}{a} \quad y=\frac{1}{b}$

16. $\frac{x}{b^2}+\frac{y}{a^2}=a+b \quad ; \quad x-y=ab(b-a)$

$a^2x+b^2y=a^3b^2+a^2b^3 \quad ; \quad x-y=ab^2-a^2b$

$a^2x-a^2y=a^3b^2-a^4b$

$-a^2x-b^2y \quad =-a^3b^2-a^2b^3$

$a^2x \quad -a^2y= \quad a^3b^2 \quad -a^4b$

$-b^2y-a^2y= \quad -a^2b^3-a^4b$

$-y(a^2+b^2)=-a^2b(b^2+a^2)$

$-y=-a^2b$

$y=a^2b$

$\frac{x}{b^2}+\frac{a^2b}{a^2}=a+b$

$x+b^3=ab^2+b^3$

$x=ab^2$

$sol: \ x=ab^2 \quad y=a^2b$

17. $nx+my=m+n \quad ; \quad mx-ny=\dfrac{m^3-n^3}{mn}$

$m^2nx+m^3y=m^3+m^2n \qquad m^2nx-mn^2y=m^3-n^3$

$-m^2nx-m^3y=-m^3-m^2n$

$m^2nx-mn^2y=m^3-n^3$

$-m^3y-mn^2y=-m^2n-n^3$

$-ym\left(m^2+n^2\right)=-n\left(m^2+n^2\right)$

$ym=n$

$y=\dfrac{n}{m}$

$nx+m\left(\dfrac{n}{m}\right)=m+n$

$nx+n=m+n$

$nx=m$

$x=\dfrac{m}{n}$

$sol: \quad x=\dfrac{m}{n} \quad y=\dfrac{n}{m}$

18. $(a-b)x-(a+b)y=b^2-3ab$

$ax-bx-ay-by=b^2-3ab$

$(a+b)x-(a-b)y=ab-b^2$

$ax+bx-ay+by=ab-b^2$

$-ax+bx+ay+by=-b^2+3ab$

$ax+bx-ay+by=-b^2+ab$

$2bx+2by=-2b^2+4ab$

$2b(x+y)=2b(2a-b)$

$x+y=2a-b$

$ax-bx-ay-by=b^2-3ab$

$ax+bx-ay+by=-b^2+ab$

$2ax-2ay=-2ab$

$2a(x-y)=-2ab$

$x-y=-b$

$x+y=2a-b$

$x-y=-b$

$2x=2(a-b)$

$x=a-b$

$a-b-y=-b$

$-y=-a$

$y=a$

$sol: \quad x=a-b \quad y=a$

19. $\dfrac{x+b}{a}+\dfrac{y-b}{b}=\dfrac{a+b}{b}$

$b(x+b)+a(y-b)=a(a+b)$

$bx+b^2+ay-ab=a^2+ab$

$abx+ab^2+a^2y-a^2b=a^3+a^2b$

$-abx-ab^2-a^2y=-a^3-2a^2b$

$\dfrac{x-a}{b}-\dfrac{y-a}{a}=-\dfrac{a+b}{a}$

$a(x-a)-b(y-a)=-b(a+b)$

$ax-a^2-by+ba=-ba-b^2$

$abx-a^2b-b^2y+b^2a=-b^2a-b^3$

$abx-a^2b-b^2y=-2b^2a-b^3$

$-abx-ab^2-a^2y=-a^3-2a^2b$

$abx-a^2b-b^2y=-2b^2a-b^3$

$-ab^2-a^2b-a^2y-b^2y=-a^3-2a^2b-2b^2a-b^3$

$ab^2+a^2b+a^3+b^3=a^2y+b^2y$

$b^2(a+b)+a^2(b+a)=y\left(a^2+b^2\right)$

$\left(b^2+a^2\right)(a+b)=y\left(a^2+b^2\right)$

$a+b=y$

$\dfrac{x+b}{a}+\dfrac{a+b-b}{b}=\dfrac{a+b}{b}$

$b(x+b)+a^2=a(a+b)$

$bx+b^2+a^2=a^2+ab$

$bx=ab-b^2 \Rightarrow bx=b(a-b) \Rightarrow x=a-b$

$sol: \quad x=a-b \quad y=a+b$

20. $\dfrac{x}{a+b}+\dfrac{y}{a+b}=\dfrac{1}{ab} \quad ; \quad \dfrac{x}{b}+\dfrac{y}{a}=\dfrac{a^2+b^2}{a^2b^2}$

$abx+aby=a+b \qquad a^2bx+ab^2y=a^2+b^2$

$-a^2bx-a^2by=-a^2-ab$

$-a^2bx-a^2by=-a^2-ab$

$a^2bx+ab^2y=a^2+b^2$

$ab^2y-a^2by=b^2-ab$

$ayb(b-a)=b(b-a)$

$ayb=b$

$ay=1$

$y=\dfrac{1}{a}$

$abx+ab\left(\dfrac{1}{a}\right)=a+b$

$abx+b=a+b$

$abx=a \Rightarrow bx=1 \Rightarrow x=\dfrac{1}{b}$

$sol: \quad x=\dfrac{1}{b} \quad y=\dfrac{1}{a}$

EJERCICIO 182

1. $\frac{1}{x}+\frac{2}{y}=\frac{7}{6}$; $\frac{2}{x}+\frac{1}{y}=\frac{4}{3}$

$-\frac{2}{x}-\frac{4}{y}=-\frac{7}{3}$

$\frac{2}{x}+\frac{1}{y}=\frac{4}{3}$

$\frac{-3}{y}=-1$

$3=y$

$\frac{2}{x}+\frac{1}{3}=\frac{4}{3}$

$6+x=4x$

$6=3x \Rightarrow 2=x$

sol: $x=2$ $y=3$

2. $\frac{3}{x}-\frac{2}{y}=\frac{1}{2}$; $\frac{2}{x}+\frac{5}{y}=\frac{23}{12}$

$-\frac{6}{x}+\frac{4}{y}=-1$

$\frac{6}{x}+\frac{15}{y}=\frac{23}{4}$

$\frac{19}{y}=\frac{19}{4}$

$4=y$

$\frac{3}{x}-\frac{2}{4}=\frac{1}{2}$

$6-x=x$

$6=2x$

$3=x$

sol: $x=3$ $y=4$

3. $\frac{5}{x}+\frac{4}{y}=7$; $\frac{7}{x}-\frac{6}{y}=4$

$\frac{15}{x}+\frac{12}{y}=21$

$\frac{14}{x}-\frac{12}{y}=8$

$\frac{29}{x}=29$

$1=x$

$\frac{5}{1}+\frac{4}{y}=7$

$5y+4=7y$

$4=2y$

$2=y$

sol: $x=1$ $y=2$

4. $\frac{12}{x}+\frac{5}{y}=-\frac{13}{2}$; $\frac{18}{x}+\frac{7}{y}=-\frac{19}{2}$

$-\frac{36}{x}-\frac{15}{y}=\frac{39}{2}$

$\frac{36}{x}+\frac{14}{y}=-\frac{38}{2}$

$-\frac{1}{y}=\frac{1}{2}$

$-2=y$

$36-7x=-19x$

$12x=-36$

$x=-3$

sol: $x=-3$ $y=-2$

5. $\frac{9}{x}+\frac{3}{y}=27$; $\frac{5}{x}+\frac{4}{y}=22$

$\frac{3}{x}+\frac{1}{y}=9$

$-\frac{12}{x}-\frac{4}{y}=-36$

$\frac{5}{x}+\frac{4}{y}=22$

$-\frac{7}{x}=-14$

$\frac{1}{2}=x$

$\frac{5}{\frac{1}{2}}+\frac{4}{y}=22 \Rightarrow 10+\frac{4}{y}=22$

$10y+4=22y$

$4=12y$

$\frac{1}{3}=y$

sol: $x=\frac{1}{2}$ $y=\frac{1}{3}$

6. $\frac{6}{x}-\frac{8}{y}=-23$; $\frac{4}{x}+\frac{11}{y}=50$

$-\frac{12}{x}+\frac{16}{y}=46$

$\frac{12}{x}+\frac{33}{y}=150$

$\frac{49}{y}=196$

$\frac{1}{4}=y$

Continúa

6. Continuación

$\frac{4}{x}+\frac{11}{\frac{1}{4}}=50$

$\frac{4}{x}+44=50$

$4+44x=50x$

$4=6x$

$\frac{2}{3}=x$

sol: $x=\frac{2}{3}$ $y=\frac{1}{4}$

7. $\frac{9}{x}+\frac{10}{y}=-11$; $\frac{7}{x}-\frac{15}{y}=-4$

$\frac{27}{x}+\frac{30}{y}=-33$

$\frac{14}{x}-\frac{30}{y}=-8$

$\frac{41}{x}=-41$

$-1=x$

$-\frac{7}{1}-\frac{15}{y}=-4$

$-7y-15=-4y$

$-15=3y$

$-5=y$

sol: $x=-1$ $y=-5$

8. $\frac{1}{2x}-\frac{3}{y}=\frac{3}{4}$; $\frac{1}{x}+\frac{5}{2y}=-\frac{4}{3}$

$\frac{1}{2x}-\frac{3}{y}=\frac{3}{4}$

$-\frac{1}{2x}-\frac{5}{4y}=\frac{2}{3}$

$-\frac{17}{4y}=\frac{17}{12}$

$-12=4y$

$-3=y$

$\frac{1}{x}+\frac{5}{2(-3)}=-\frac{4}{3}$

$6-5x=-8x$

$3x=-6$

$x=-2$

sol: $x=-2$ $y=-3$

9. $\frac{2}{5x}-\frac{1}{3y}=-\frac{11}{45}$; $\frac{1}{10x}-\frac{3}{5y}=\frac{4}{5}$

$$-\frac{2}{10x}+\frac{1}{6y}=\frac{11}{90}$$
$$\frac{2}{10x}-\frac{6}{5y}=\frac{8}{5}$$
$$-\frac{31}{30y}=\frac{31}{18}$$
$$-18=30y$$
$$-\frac{3}{5}=y$$
$$\frac{1}{10x}+\frac{3}{5\left(\frac{3}{5}\right)}=\frac{4}{5}$$
$$3+30x=24x$$
$$6x=-3\Rightarrow x=-\frac{1}{2}$$
$$sol:\quad x=-\frac{1}{2}\quad y=-\frac{3}{5}$$

10. $\frac{3}{x}-\frac{7}{3y}=\frac{2}{3}$; $\frac{1}{4x}+\frac{8}{y}=\frac{103}{84}$

$$\frac{3}{4x}-\frac{7}{12y}=\frac{1}{6}$$
$$-\frac{3}{4x}-\frac{24}{y}=-\frac{309}{84}$$
$$\frac{-295}{12y}=\frac{-295}{84}$$
$$y=7$$
$$\frac{3}{x}-\frac{7}{3(7)}=\frac{2}{3}$$
$$9-x=2x$$
$$9=3x$$
$$3=x$$
$$sol:\ x=3\quad y=7$$

11. $\frac{3}{10x}+\frac{1}{3y}=\frac{107}{60}$; $\frac{6}{5x}+\frac{1}{4y}=\frac{14}{5}$

$$\frac{3}{5x}+\frac{2}{3y}=\frac{107}{30}$$
$$-\frac{3}{5x}-\frac{1}{8y}=-\frac{7}{5}$$
$$\frac{13}{24y}=\frac{13}{6}$$
$$\frac{1}{4}=y$$
$$\frac{6}{5x}+\frac{1}{4\left(\frac{1}{4}\right)}=\frac{14}{5}$$
$$6+5x=14x$$
$$6=9x$$
$$\frac{2}{3}=x$$
$$sol:\ x=\frac{2}{3}\quad y=\frac{1}{4}$$

12. $\frac{1}{x}+\frac{1}{y}=a$; $\frac{1}{x}-\frac{1}{y}=b$

$$\frac{1}{x}+\frac{1}{y}=a$$
$$-\frac{1}{x}+\frac{1}{y}=-b$$
$$\frac{2}{y}=a-b\Rightarrow\ \frac{2}{a-b}=y$$
$$\frac{1}{x}-\frac{1}{\frac{2}{a-b}}=b$$
$$2-x(a-b)=2bx$$
$$2-ax+xb=2bx$$
$$-ax=bx-2$$
$$-ax-bx=-2$$
$$x(a+b)=2\Rightarrow x=\frac{2}{a+b}$$
$$sol:\ x=\frac{2}{a+b}\quad y=\frac{2}{a-b}$$

13. $\frac{2}{x}-\frac{3b}{y}=\frac{2-3a}{a}$; $\frac{a}{x}+\frac{b}{y}=2$

$$\frac{2a}{x}-\frac{3ab}{y}=2-3a$$
$$-\frac{2a}{x}-\frac{2b}{y}=-4$$
$$\frac{b(-3a-2)}{y}=-3a-2$$
$$b=y$$
$$\frac{a}{x}+\frac{b}{b}=2$$
$$a+x=2x$$
$$a=x$$
$$sol:\ x=a\quad y=b$$

14. $\frac{2}{x}+\frac{2}{y}=\frac{m+n}{mn}$; $\frac{m}{x}-\frac{n}{y}=0$

$$\frac{2n}{x}+\frac{2n}{y}=\frac{m+n}{m}$$
$$\frac{2m}{x}-\frac{2n}{y}=0$$
$$\frac{2(m+n)}{x}=\frac{m+n}{m}$$
$$2m=x$$
$$\frac{m}{2m}-\frac{n}{y}=0$$
$$y-2n=0$$
$$y=2n$$
$$sol:\ x=2m\quad y=2n$$

EJERCICIO 183

1. $\begin{vmatrix}4 & 5\\ 2 & 3\end{vmatrix}$ $4\cdot3-2\cdot5=12-10=2$

2. $\begin{vmatrix}2 & 7\\ 3 & 5\end{vmatrix}$ $2\cdot5-3\cdot7=10-21=-11$

3. $\begin{vmatrix}-2 & 5\\ 4 & 3\end{vmatrix}$ $-2\cdot3-4\cdot5=-6-20=-26$

4. $\begin{vmatrix}7 & 9\\ 5 & -2\end{vmatrix}$ $7(-2)-9\cdot5=-14-45=-59$

5. $\begin{vmatrix}5 & -3\\ -2 & -8\end{vmatrix}$ $5(-8)-(-3)(-2)=-40-6=-46$

6. $\begin{vmatrix}9 & -11\\ -3 & 7\end{vmatrix}$ $9\cdot7-(-11)(-3)=63-33=30$

7. $\begin{vmatrix}-15 & -1\\ 13 & 2\end{vmatrix}$ $-15\cdot2-13(-1)=-30+13=-17$

8. $\begin{vmatrix}12 & -1\\ 13 & -9\end{vmatrix}$ $12(-9)-(13)(-1)=-108+13=-95$

9. $\begin{vmatrix}10 & 3\\17 & 13\end{vmatrix}$ $10\cdot13-3\cdot17=130-51=79$

10. $\begin{vmatrix}-5 & -8\\-19 & -21\end{vmatrix}$ $(-5)(-21)-(-8)(-19)=105-152=-47$

11. $\begin{vmatrix}8 & 2\\-3 & 0\end{vmatrix}$ $8(0)-2(-3)=0+6=6$

12. $\begin{vmatrix}31 & -85\\-20 & 43\end{vmatrix}$ $31\cdot43-(-85)(-20)=1.333-1.700=-367$

EJERCICIO 184

1. $7x+8y=29$

$5x+11y=26$

$$x=\frac{\begin{vmatrix}29 & 8\\26 & 11\end{vmatrix}}{\begin{vmatrix}7 & 8\\5 & 11\end{vmatrix}}=\frac{319-208}{77-40}=\frac{111}{37}=3$$

$$y=\frac{\begin{vmatrix}7 & 29\\5 & 26\end{vmatrix}}{\begin{vmatrix}7 & 8\\5 & 11\end{vmatrix}}=\frac{182-145}{37}=\frac{37}{37}=1$$

sol: $x=3 \quad y=1$

2. $3x-4y=13$

$8x-5y=-5$

$$x=\frac{\begin{vmatrix}13 & -4\\-5 & -5\end{vmatrix}}{\begin{vmatrix}3 & -4\\8 & -5\end{vmatrix}}=\frac{-65-20}{-15+32}=\frac{-85}{17}=-5$$

$$y=\frac{\begin{vmatrix}3 & 13\\8 & -5\end{vmatrix}}{\begin{vmatrix}3 & -4\\8 & -5\end{vmatrix}}=\frac{-15-104}{17}=\frac{-119}{17}=-7$$

sol: $x=-5 \quad y=-7$

3. $13x-31y=-326$

$25x+37y=146$

$$x=\frac{\begin{vmatrix}-326 & -31\\146 & 37\end{vmatrix}}{\begin{vmatrix}13 & -31\\25 & 37\end{vmatrix}}$$

$$x=\frac{-12.062+4.526}{481+775}=\frac{-7.536}{1.256}=-6$$

$$y=\frac{\begin{vmatrix}13 & -326\\25 & 146\end{vmatrix}}{\begin{vmatrix}13 & -31\\25 & 37\end{vmatrix}}$$

$$y=\frac{1.898+8.150}{1.256}=\frac{10.048}{1.256}=8$$

sol: $x=-6 \quad y=8$

4. $15x-44y=-6$

$-27x+32y=-1$

$$x=\frac{\begin{vmatrix}-6 & -44\\-1 & 32\end{vmatrix}}{\begin{vmatrix}15 & -44\\-27 & 32\end{vmatrix}}$$

$$x=\frac{-192-44}{480-1.188}=\frac{-236}{-708}=\frac{1}{3}$$

$$y=\frac{\begin{vmatrix}15 & -6\\-27 & -1\end{vmatrix}}{\begin{vmatrix}15 & -44\\-27 & 32\end{vmatrix}}$$

$$y=\frac{-15-162}{-708}=\frac{-177}{-708}=\frac{1}{4}$$

sol: $x=\frac{1}{3} \quad y=\frac{1}{4}$

5. $8x=-9y$

$8x+9y=0$

$2x+5+3y=\frac{7}{2}$

$4x+10+6y=7$

$4x+6y=-3$

$$x=\frac{\begin{vmatrix}0 & 9\\-3 & 6\end{vmatrix}}{\begin{vmatrix}8 & 9\\4 & 6\end{vmatrix}}$$

$$x=\frac{0+27}{48-36}=\frac{27}{12}=\frac{9}{4}=2\tfrac{1}{4}$$

$$y=\frac{\begin{vmatrix}8 & 0\\4 & -3\end{vmatrix}}{\begin{vmatrix}8 & 9\\4 & 6\end{vmatrix}}$$

$$y=\frac{-24-0}{12}=\frac{-24}{12}=-2$$

sol: $x=2\tfrac{1}{4} \quad y=-2$

6. $ax-by=-1$

$ax+by=7$

$$x=\frac{\begin{vmatrix}-1 & -b\\7 & b\end{vmatrix}}{\begin{vmatrix}a & -b\\a & b\end{vmatrix}}$$

$$x=\frac{-b+7b}{ab+ab}=\frac{6b}{2ab}=\frac{3}{a}$$

$$y=\frac{\begin{vmatrix}a & -1\\a & 7\end{vmatrix}}{\begin{vmatrix}a & -b\\a & b\end{vmatrix}}$$

$$y=\frac{7a+a}{2ab}=\frac{8a}{2ab}=\frac{4}{b}$$

sol: $x=\frac{3}{a} \quad y=\frac{4}{b}$

7. $3x-(y+2)=2y+1$

$3x-y-2=2y+1$

$3x-3y=3$

$x-y=1$

$5y-(x+3)=3x+1$

$5y-x-3=3x+1$

$-4x+5y=4$

$$x=\frac{\begin{vmatrix}1 & -1\\4 & 5\end{vmatrix}}{\begin{vmatrix}1 & -1\\-4 & 5\end{vmatrix}} \quad x=\frac{5+4}{5-4}=9$$

$$y=\frac{\begin{vmatrix}1 & 1\\-4 & 4\end{vmatrix}}{\begin{vmatrix}1 & -1\\-4 & 5\end{vmatrix}} \quad y=\frac{4+4}{1}=8$$

sol: $x=9 \quad y=8$

8. $ax+2y=2$

$\frac{ax}{2}-3y=-1$

$ax-6y=-2$

$$x=\frac{\begin{vmatrix}2 & 2\\ -2 & -6\end{vmatrix}}{\begin{vmatrix}a & 2\\ a & -6\end{vmatrix}}=\frac{-12+4}{-6a-2a}=\frac{-8}{-8a}=\frac{1}{a}$$

$$y=\frac{\begin{vmatrix}a & 2\\ a & -2\end{vmatrix}}{\begin{vmatrix}a & 2\\ a & -6\end{vmatrix}}=\frac{-2a-2a}{-8a}=\frac{-4a}{-8a}=\frac{1}{2}$$

sol: $x=\frac{1}{a}\quad y=\frac{1}{2}$

9. $\frac{x}{4}+\frac{y}{6}=-4$

$3x+2y=-48$

$\frac{x}{8}-\frac{y}{12}=0$

$3x-2y=0$

$$x=\frac{\begin{vmatrix}-48 & 2\\ 0 & -2\end{vmatrix}}{\begin{vmatrix}3 & 2\\ 3 & -2\end{vmatrix}}=\frac{96-0}{-6-6}=\frac{96}{-12}=-8$$

$$y=\frac{\begin{vmatrix}3 & -48\\ 3 & 0\end{vmatrix}}{\begin{vmatrix}3 & 2\\ 3 & -2\end{vmatrix}}=\frac{0+144}{-12}=-12$$

sol: $x=-8\quad y=-12$

10. $3x+ay=3a+1$

$\frac{x}{a}+ay=2$

$x+a^2y=2a$

$$x=\frac{\begin{vmatrix}3a+1 & a\\ 2a & a^2\end{vmatrix}}{\begin{vmatrix}3 & a\\ 1 & a^2\end{vmatrix}}$$

$$x=\frac{3a^3+a^2-2a^2}{3a^2-a}=\frac{a^2(3a-1)}{a(3a-1)}=a$$

$$y=\frac{\begin{vmatrix}3 & 3a+1\\ 1 & 2a\end{vmatrix}}{\begin{vmatrix}3 & a\\ 1 & a^2\end{vmatrix}}$$

$$y=\frac{6a-3a-1}{a(3a-1)}=\frac{3a-1}{a(3a-1)}=\frac{1}{a}$$

sol: $x=a\quad y=\frac{1}{a}$

11. $\frac{x+2}{3}-\frac{y-3}{8}=\frac{5}{6}$

$8(x+2)-3(y-3)=20$

$8x+16-3y+9=20$

$8x-3y+25=20$

$8x-3y=-5$

$\frac{y-5}{6}-\frac{2x-3}{5}=0$

$5(y-5)-6(2x-3)=0$

$5y-25-12x+18=0$

$-12x+5y-7=0$

$-12x+5y=7$

$$x=\frac{\begin{vmatrix}-5 & -3\\ 7 & 5\end{vmatrix}}{\begin{vmatrix}8 & -3\\ -12 & 5\end{vmatrix}}$$

$$x=\frac{-25+21}{40-36}=\frac{-4}{4}=-1$$

$$y=\frac{\begin{vmatrix}8 & -5\\ -12 & 7\end{vmatrix}}{\begin{vmatrix}8 & -3\\ -12 & 5\end{vmatrix}}$$

$$y=\frac{56-60}{40-36}=\frac{-4}{4}=-1$$

sol: $x=-1\quad y=-1$

12. $3x-2y=5$

$mx+4y=2(m+1)$

$$x=\frac{\begin{vmatrix}5 & -2\\ 2(m+1) & 4\end{vmatrix}}{\begin{vmatrix}3 & -2\\ m & 4\end{vmatrix}}$$

$$x=\frac{20+4(m+1)}{12+2m}=\frac{4(6+m)}{2(6+m)}=2$$

$$y=\frac{\begin{vmatrix}3 & 5\\ m & 2(m+1)\end{vmatrix}}{\begin{vmatrix}3 & -2\\ m & 4\end{vmatrix}}$$

$$y=\frac{6(m+1)-5m}{2(6+m)}=\frac{m+6}{2(6+m)}=\frac{1}{2}$$

sol: $x=2\quad y=\frac{1}{2}$

13. $2x-\frac{2y+3}{17}=y+2$

$34x-(2y+3)=17(y+2)$

$34x-2y-3=17y+34$

$34x-19y=37$

$3y-\frac{4x+1}{21}=3x+5$

$63y-(4x+1)=21(3x+5)$

$63y-4x-1=63x+105$

$-67x+63y=106$

$$x=\frac{\begin{vmatrix}37 & -19\\ 106 & 63\end{vmatrix}}{\begin{vmatrix}34 & -19\\ -67 & 63\end{vmatrix}}$$

$$x=\frac{2.331+2.014}{2.142-1.273}=\frac{4.345}{869}=5$$

$$y=\frac{\begin{vmatrix}34 & 37\\ -67 & 106\end{vmatrix}}{\begin{vmatrix}34 & -19\\ -67 & 63\end{vmatrix}}$$

$$y=\frac{3.604+2.479}{869}=7$$

sol: $x=5\quad y=7$

14. $\frac{x+y}{x-y}=4$

$x+y=4(x-y)$

$x+y=4x-4y$

$-3x+5y=0$

$\frac{x-y-1}{x+y+1}=\frac{1}{9}$

$9(x-y-1)=x+y+1$

$9x-9y-9=x+y+1$

$8x-10y=10$

$4x-5y=5$

$$x=\frac{\begin{vmatrix}0 & 5\\ 5 & -5\end{vmatrix}}{\begin{vmatrix}-3 & 5\\ 4 & -5\end{vmatrix}}=\frac{0-25}{15-20}=\frac{-25}{-5}=5$$

$$y=\frac{\begin{vmatrix}-3 & 0\\ 4 & 5\end{vmatrix}}{\begin{vmatrix}-3 & 5\\ 4 & -5\end{vmatrix}}=\frac{-15-0}{-5}=3$$

sol: $x=5\quad y=3$

15. $x-y=2b$

$$\frac{x}{a+b}+\frac{y}{a-b}=2$$

$$x(a-b)+y(a+b)=2(a^2-b^2)$$

$$x=\frac{\begin{vmatrix}2b & -1\\ 2(a^2-b^2) & a+b\end{vmatrix}}{\begin{vmatrix}1 & -1\\ a-b & a+b\end{vmatrix}}$$

$$x=\frac{2b(a+b)+2(a^2-b^2)}{a+b+a-b}=\frac{2a(b+a)}{2a}=b+a$$

$$y=\frac{\begin{vmatrix}1 & 2b\\ a-b & 2(a^2-b^2)\end{vmatrix}}{\begin{vmatrix}1 & -1\\ a-b & a+b\end{vmatrix}}$$

$$y=\frac{2(a^2-b^2)-2b(a-b)}{2a}=\frac{2a(a-b)}{2a}=a-b$$

$sol:\quad x=a+b \quad y=a-b$

16. $\frac{x+9}{x-9}=\frac{y+21}{y+39}$

$$(x+9)(y+39)=(x-9)(y+21)$$

$$xy+39x+9y+351=xy+21x-9y-189$$

$$18x+18y=-540$$

$$x+y=-30$$

$$\frac{x+8}{x-8}=\frac{y+19}{y+11}$$

$$(y+11)(x+8)=(y+19)(x-8)$$

$$xy+8y+11x+88=xy-8y+19x-152$$

$$-8x+16y=-240$$

$$-x+2y=-30$$

$$x=\frac{\begin{vmatrix}-30 & 1\\ -30 & 2\end{vmatrix}}{\begin{vmatrix}1 & 1\\ -1 & 2\end{vmatrix}}=\frac{-60+30}{2+1}=\frac{-30}{3}=-10$$

$$y=\frac{\begin{vmatrix}1 & -30\\ -1 & -30\end{vmatrix}}{\begin{vmatrix}1 & 1\\ -1 & 2\end{vmatrix}}=\frac{-30-30}{3}=\frac{-60}{3}=-20$$

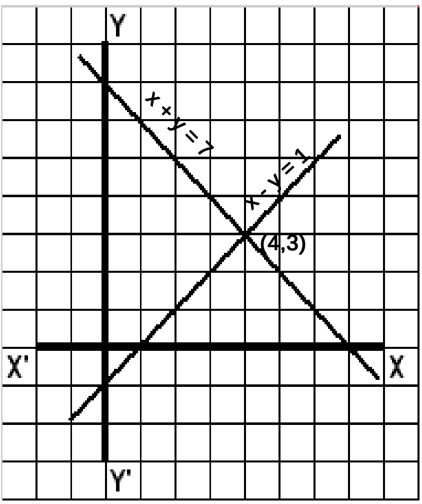
$sol:\quad x=-10 \quad y=-20$

EJERCICIO 185

1. $x-y=1 \qquad x+y=7$

$x=0 \quad y=-1 \qquad x=0 \quad y=7$

$y=0 \quad x=1 \qquad y=0 \quad x=7$

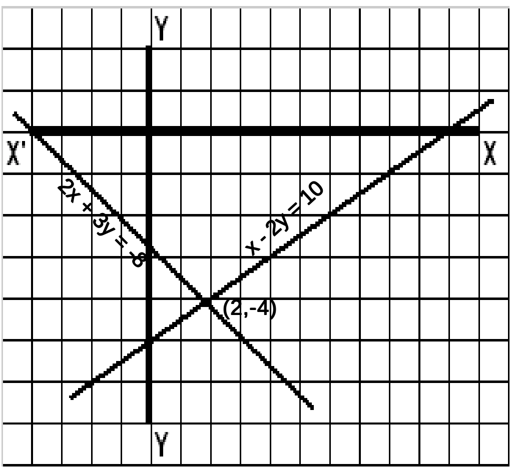


sol : x = 4 y = 3

2. $x-2y=10 \qquad 2x+3y=-8$

$x=0 \quad y=-5 \qquad x=0 \quad y=-2\frac{2}{3}$

$y=0 \quad x=10 \qquad y=0 \quad x=-4$



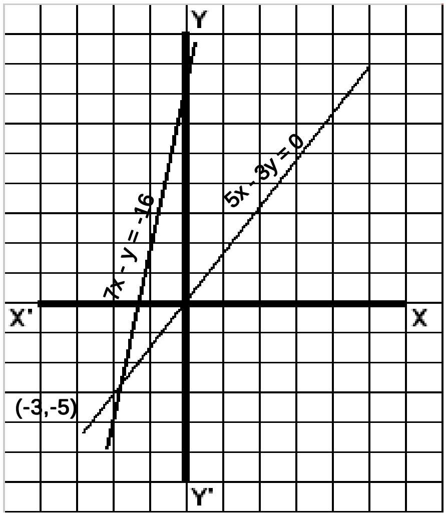
sol : x = 2 Y = - 4

3. $5x-3y=0$ $\quad$ $7x-y=-16$

$x=3 \quad y=5$ $\quad$ $x=0 \quad y=16$

$y=0 \quad x=-2\frac{2}{7}$

Escala 1:2

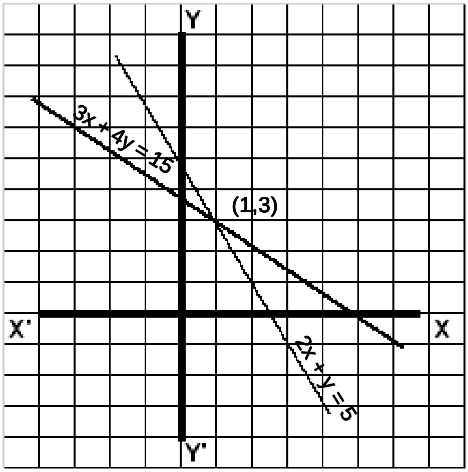


sol : x = - 3 y = - 5

5. $3x+4y=15$ $\quad$ $2x+y=5$

$x=5 \quad y=0$ $\quad$ $x=0 \quad y=5$

$x=1 \quad y=3$ $\quad$ $x=2 \quad y=1$

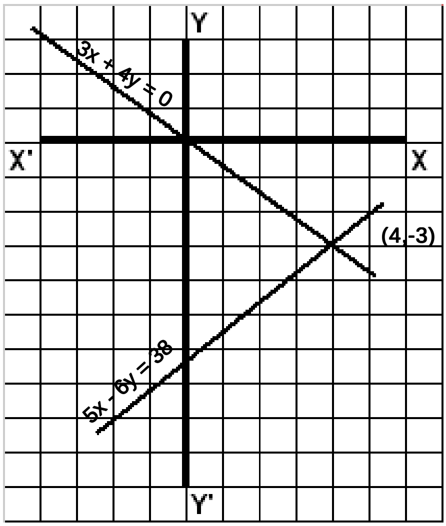


sol : x = 1 y = 3

4. $3x=-4y$ $\quad$ $5x-6y=38$

$3x+4y=0$ $\quad$ $x=-2 \quad y=-8$

$x=4 \quad y=-3$ $\quad$ $y=-3 \quad x=4$

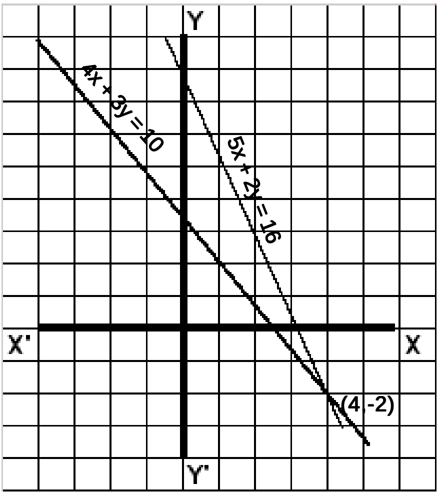


sol : x = 4 y = - 3

6. $5x+2y=16$ $\quad$ $4x+3y=10$

$x=2 \quad y=3$ $\quad$ $x=-2 \quad y=6$

$x=4 \quad y=-2$ $\quad$ $x=1 \quad y=2$



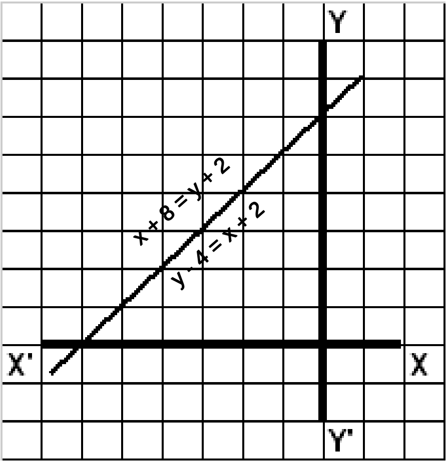
sol : x = 4 y = - 2

7. $x+8=y+2$ $\quad y-4=x+2$

$x-y=-6$ $\quad -x+y=6$

$x=0 \quad y=6$ $\quad x=0 \quad y=6$

$x=-6 \quad y=0$ $\quad x=-6 \quad y=0$



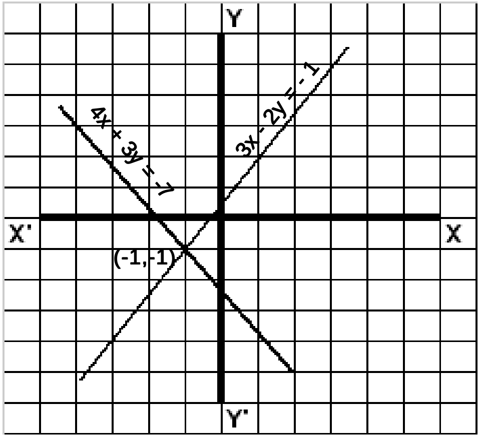
sol : Equivalentes

9. $\frac{x}{2}-\frac{y}{3}=-\frac{1}{6} \quad 3x-2y=-1$

$x=3 \quad y=5$; $x=-3 \quad y=-4$

$\frac{x}{3}+\frac{y}{4}=-\frac{7}{12} \quad 4x+3y=-7$

$x=-1 \quad y=-1$; $x=-4 \quad y=3$



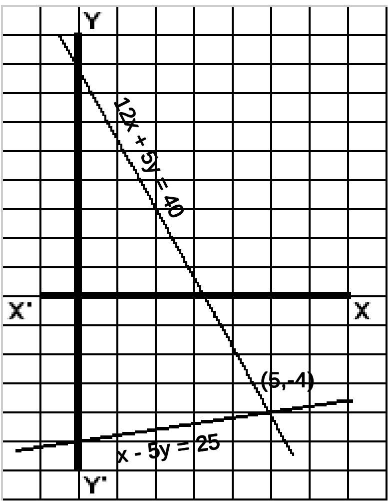
sol : x = - 1 y = - 1

8. $\frac{3x}{5}+\frac{y}{4}=2 \quad x-5y=25$

$12x+5y=40$ $\quad x=0 \quad y=-5$

$x=0 \quad y=8$ $\quad x=5 \quad y=-4$

$x=5 \quad y=-4$

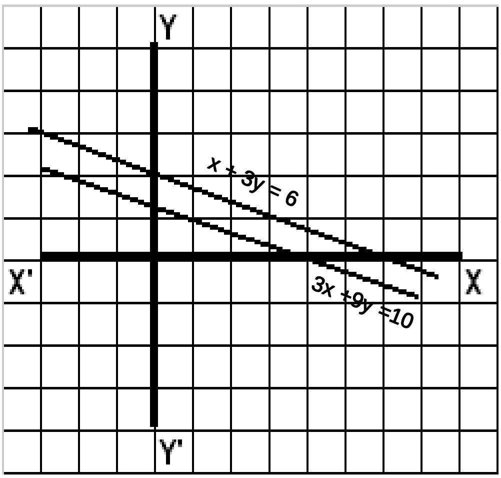


sol : x = 5 y = - 4

10. $x+3y=6$ $\quad 3x+9y=10$

$x=0 \quad y=2$ $\quad x=0 \quad y=1\frac{1}{9}$

$x=6 \quad y=0$ $\quad x=3\frac{1}{3} \quad y=0$



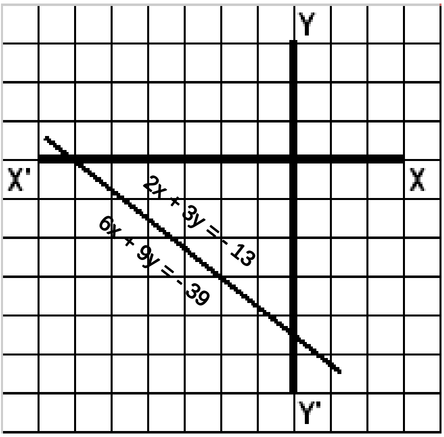
sol : Incompatibles

11. $2x+3y=-13$

$x=-2 \quad y=-3 \; ; x=-5 \quad y=-1$

$6x+9y=-39$

$x=-2 \quad y=-3 \; , \quad x=-5 \quad y=-1$



sol : Equivalentes

12. $\frac{x-2}{2}-\frac{y-3}{3}=4$ $\qquad$ $\frac{y-2}{2}+\frac{x-3}{3}=-\frac{11}{3}$

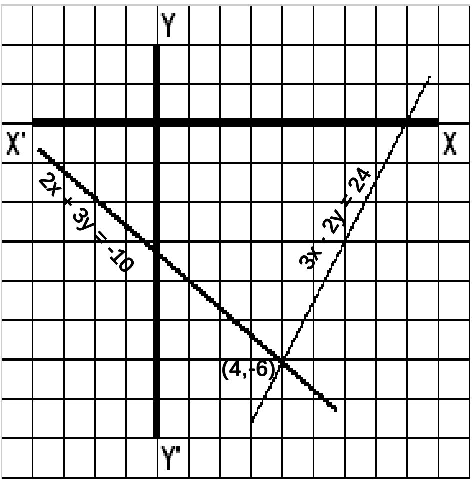
$3(x-2)-2(y-3)=24 \qquad 3(y-2)+2(x-3)=-22$

$3x-6-2y+6=24 \qquad 3y-6+2x-6=-22$

$3x-2y=24 \qquad 2x+3y=-10$

$x=6 \quad y=-3 \qquad x=4 \quad y=-6$

$x=4 \quad y=-6 \qquad x=-2 \quad y=-2$



sol : x = 4 y = - 6

13. $x+y=9$

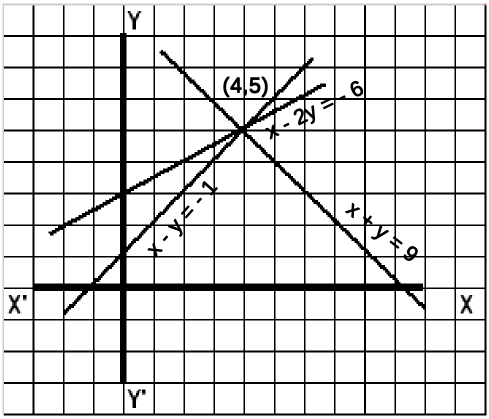
$x=6 \;\; y=3 \;\; ; x=4 \quad y=5$

$x-y=-1$

$x=0 \quad y=1 \;\; ; x=2 \;\; y=3$

$x-2y=-6$

$x=0 \quad y=3 \; ; x=2 \quad y=4$



sol : x = 4 y = 5

14. $x+y=5$

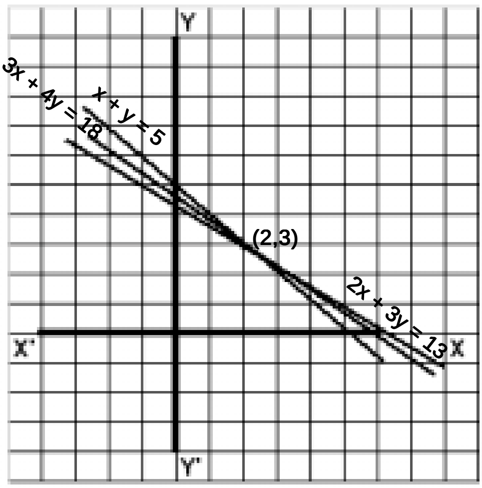
$x=0 \;\; y=5 \;\; ; \; x=2 \quad y=3$

$3x+4y=18$

$x=2 \quad y=3 \; ; x=-2 \;\; y=6$

$2x+3y=13$

$x=2 \quad y=3 \; ; \; x=5 \quad y=1$



sol : x = 2 y = 3

15. $2x+y=-1$

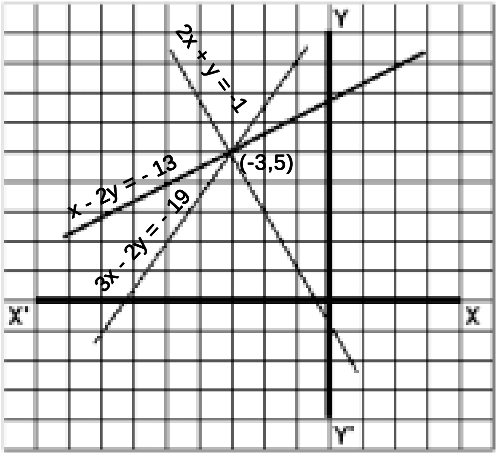
$x=0 \quad y=-1 \;;\; x=-1 \quad y=1$

$x-2y=-13$

$x=-5 \quad y=4 \;;\; x=-3 \quad y=5$

$3x-2y=-19$

$x=-5 \quad y=2 \;;\; x=-3 \quad y=5$

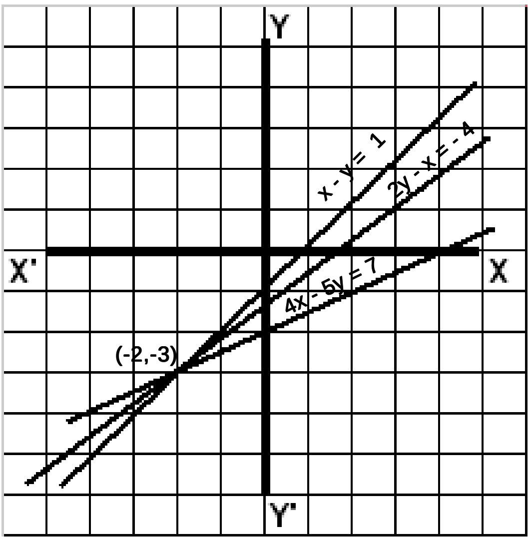


sol : x = - 3 y = 5

16. $x-y=1 \qquad 2y-x=-4 \qquad 4x-5y=7$

$x=0 \quad y=-1 \quad x=0 \quad y=-2 \quad x=-2 \quad y=-3$

$x=1 \quad y=0 \quad x=2 \quad y=-1 \quad x=3 \quad y=1$



sol : x = - 2 y = - 3

EJERCICIO 186

1. $x+y+z=6$

$x-y+2z=5$

$x-y-3z=-10$

$$\begin{array}{r} x+y+z=6 \\ \underline{x-y+2z=5} \\ 2x \quad +3z=11 \end{array}$$

$$\begin{array}{r} x+y+z=6 \\ \underline{x-y-3z=-10} \\ 2x \quad -2z=-4 \end{array}$$

$x-z=-2$

$$\begin{array}{rl} 2x+3z=11 & \\ \underline{x-z=-2} & (3) \\ 2x+3z=11 & \\ \underline{3x-3z=-6} & \\ 5x \qquad =5 & \\ x=1 & \end{array}$$

$2(1)+3z=11$

$2+3z=11$

$3z=9$

$z=3$

$1+y+3=6$

$y+4=6$

$y=2$

$sol: \quad x=1 \quad y=2 \quad z=3$

2. $x+y+z=12$

$2x-y+z=7$

$x+2y-z=6$

$$\begin{array}{r} x+y+z=12 \\ \underline{2x-y+z=7} \\ 3x \quad +2z=19 \end{array}$$

$$\begin{array}{rl} 2x-y+z=7 & (2) \\ \underline{x+2y-z=6} & \\ 4x-2y+2z=14 & \\ \underline{x+2y-z=6} & \\ 5x \qquad +z=20 & \end{array}$$

$$\begin{array}{rl} 3x+2z=19 & \\ \underline{5x+z=20} & (-2) \\ 3x+2z=19 & \\ \underline{-10x-2z=-40} & \\ -7x \qquad =-21 & \\ x=3 & \end{array}$$

Continúa

2. Continuación

$$\begin{array}{l}
5(3)+z=20\\
15+z=20\\
z=5\\
3+y+5=12\\
y+8=12\\
y=4\\
sol:\ x=3\ \ y=4\ \ z=5
\end{array}$$

3.

$$\begin{array}{l}
x-y+z=2\\
x+y+z=4\\
2x+2y-z=-4
\end{array}$$

$$\begin{array}{l}
x-y+z=2\\
\underline{x+y+z=4}\\
2x+2z=6\\
x+z=3
\end{array}$$

$$\begin{array}{l}
x-y+z=2\quad(2)\\
\underline{2x+2y-z=-4}\\
2x-2y+2z=4\\
\underline{2x+2y-z=-4}\\
4x+z=0
\end{array}$$

$$\begin{array}{l}
x+z=3\quad(-1)\\
\underline{4x+z=0}\\
-x-z=-3\\
\underline{4x+z=0}\\
3x=-3\\
x=-1
\end{array}$$

$$\begin{array}{l}
-1+z=3\\
z=4\\
-1-y+4=2\\
-y+3=2\\
-y=-1\\
y=1\\
sol:\ x=-1\ \ y=1\ \ z=4
\end{array}$$

4.

$$\begin{array}{l}
2x+y-3z=-1\\
x-3y-2z=-12\\
3x-2y-z=-5
\end{array}$$

$$\begin{array}{l}
2x+y-3z=-1\quad(2)\\
\underline{3x-2y-z=-5}\\
4x+2y-6z=-2\\
\underline{3x-2y-z=-5}\\
7x-7z=-7\\
x-z=-1
\end{array}$$

Continúa

4. Continuación

$$\begin{array}{l}
2x+y-3z=-1\quad(3)\\
\underline{x-3y-2z=-12}\\
6x+3y-9z=-3\\
\underline{x-3y-2z=-12}\\
7x-11z=-15
\end{array}$$

$$\begin{array}{l}
x-z=-1\quad(-7)\\
\underline{7x-11z=-15}\\
-7x+7z=7\\
\underline{7x-11z=-15}\\
-4z=-8\\
z=2
\end{array}$$

$$\begin{array}{l}
x-2=-1\\
x=1\\
2(1)+y-3(2)=-1\\
2+y-6=-1\\
y-4=-1\\
y=3\\
sol:\ x=1\ \ y=3\ \ z=2
\end{array}$$

5.

$$\begin{array}{l}
2x+3y+z=1\\
6x-2y-z=-14\\
3x+y-z=1
\end{array}$$

$$\begin{array}{l}
2x+3y+z=1\\
\underline{3x+y-z=1}\\
5x+4y=2
\end{array}$$

$$\begin{array}{l}
2x+3y+z=1\\
\underline{6x-2y-z=-14}\\
8x+y=-13
\end{array}$$

$$\begin{array}{l}
5x+4y=2\\
\underline{8x+y=-13}\quad(-4)\\
5x+4y=2\\
\underline{-32x-4y=52}\\
-27x=54\\
x=-2
\end{array}$$

$$\begin{array}{l}
8(-2)+y=-13\\
-16+y=-13\\
y=3\\
2(-2)+3(3)+z=1\\
-4+9+z=1\\
5+z=1\\
z=-4\\
sol:\ x=-2\ \ y=3\ \ z=-4
\end{array}$$

6.

$$\begin{array}{l}
5x-2y+z=24\\
2x+5y-2z=-14\\
x-4y+3z=26
\end{array}$$

$$\begin{array}{l}
5x-2y+z=24\quad(2)\\
\underline{2x+5y-2z=-14}\\
10x-4y+2z=48\\
\underline{2x+5y-2z=-14}\\
12x+y=34
\end{array}$$

$$\begin{array}{l}
5x-2y+z=24\quad(-3)\\
\underline{x-4y+3z=26}\\
-15x+6y-3z=-72\\
\underline{x-4y+3z=26}\\
-14x+2y=-46
\end{array}$$

$$\begin{array}{l}
12x+y=34\quad(-2)\\
\underline{-14x+2y=-46}\\
-24x-2y=-68\\
\underline{-14x+2y=-46}\\
-38x=-114\\
x=3
\end{array}$$

$$\begin{array}{l}
12(3)+y=34\\
36+y=34\\
y=-2\\
3-4(-2)+3z=26\\
3+8+3z=26\\
3z=15\\
z=5\\
sol:\ x=3\ \ y=-2\ \ z=5
\end{array}$$

7.

$$\begin{array}{l}
4x+2y+3z=8\\
3x+4y+2z=-1\\
2x-y+5z=3
\end{array}$$

$$\begin{array}{l}
4x+2y+3z=8\quad(-2)\\
\underline{3x+4y+2z=-1}\\
-8x-4y-6z=-16\\
\underline{3x+4y+2z=-1}\\
-5x-4z=-17\\
5x+4z=17
\end{array}$$

$$\begin{array}{l}
4x+2y+3z=8\\
\underline{2x-y+5z=3}\quad(2)\\
4x+2y+3z=8\\
\underline{4x-2y+10z=6}\\
8x+13z=14
\end{array}$$

Continúa

7. Continuación

$8x+13z=14 \quad (-5)$
$5x+4z=17 \quad (8)$
$-40x-65z=-70$
$40x+32z=136$
$-33z=66$
$z=-2$
$5x+4(-2)=17$
$5x-8=17$
$5x=25$
$x=5$
$2(5)-y+5(-2)=3$
$10-y-10=3$
$-y=3$
$y=-3$
$sol: x=5 \quad y=-3 \quad z=-2$

8. $6x+3y+2z=12$
$9x-y+4z=37$
$10x+5y+3z=21$
$6x+3y+2z=12$
$9x-y+4z=37 \quad (3)$
$6x+3y+2z=12$
$27x-3y+12z=111$
$33x+14z=123$
$9x-y+4z=37 \quad (5)$
$10x+5y+3z=21$
$45x-5y+20z=185$
$10x+5y+3z=21$
$55x+23z=206$
$33x+14z=123 \quad (5)$
$55x+23z=206 \quad (-3)$
$165x+70z=615$
$-165x-69z=-618$
$z=-3$
$33x+14(-3)=123$
$33x-42=123$
$33x=165$
$x=5$
$6(5)+3y+2(-3)=12$
$30+3y-6=12$
$3y+24=12$
$3y=-12$
$y=-4$
$sol: x=5 \quad y=-4 \quad z=-3$

9. $2x+4y+3z=3$
$10x-8y-9z=0$
$4x+4y-3z=2$
$2x+4y+3z=3 \quad (3)$
$10x-8y-9z=0$
$6x+12y+9z=9$
$10x-8y-9z=0$
$16x+4y=9$
$10x-8y-9z=0$
$4x+4y-3z=2 \quad (-3)$
$10x-8y-9z=0$
$-12x-12y+9z=-6$
$-2x-20y=-6$
$x+10y=3$
$16x+4y=9 \quad (5)$
$x+10y=3 \quad (-2)$
$80x+20y=45$
$-2x-20y=-6$
$78x=39$
$x=\frac{1}{2}$
$16\left(\frac{1}{2}\right)+4y=9$
$8+4y=9$
$4y=1$
$y=\frac{1}{4}$
$2\left(\frac{1}{2}\right)+4\left(\frac{1}{4}\right)+3z=3$
$1+1+3z=3$
$3z=1$
$z=\frac{1}{3}$
$sol: x=\frac{1}{2} \quad y=\frac{1}{4} \quad z=\frac{1}{3}$

10. $3x+y+z=1$
$x+2y-z=1$
$x+y+2z=-17$
$3x+y+z=1$
$x+2y-z=1$
$4x+3y=2$
$x+2y-z=1 \quad (2)$
$x+y+2z=-17$

Continúa

10. Continuación

$2x+4y-2z=2$
$x+y+2z=-17$
$3x+5y=-15$
$4x+3y=2 \quad (3)$
$3x+5y=-15 \quad (-4)$
$12x+9y=6$
$-12x-20y=60$
$-11y=66$
$y=-6$
$4x+3(-6)=2$
$4x-18=2$
$4x=20$
$x=5$
$5-6+2z=-17$
$2z=-16$
$z=-8$
$sol: x=5 \quad y=-6 \quad z=-8$

11. $7x+3y-4z=-35$
$3x-2y+5z=38$
$x+y-6z=-27$
$7x+3y-4z=-35$
$x+y-6z=-27 \quad (-3)$
$7x+3y-4z=-35$
$-3x-3y+18z=81$
$4x+14z=46$
$2x+7z=23$
$3x-2y+5z=38$
$x+y-6z=-27 \quad (2)$
$3x-2y+5z=38$
$2x+2y-12z=-54$
$5x-7z=-16$
$2x+7z=23$
$5x-7z=-16$
$7x=7$
$x=1$
$2(1)+7z=23$
$7z=21$
$z=3$
$1+y-6(3)=-27$
$y-17=-27$
$y=-10$
$sol: x=1 \quad y=-10 \quad z=3$

12. $4x-y+5z=-6$
$3x+3y-4z=30$
$6x+2y-3z=33$
$4x-y+5z=-6 \quad (3)$
$3x+3y-4z=30$
$12x-3y+15z=-18$
$3x+3y-4z=30$
$15x+11z=12$
$4x-y+5z=-6 \quad (2)$
$6x+2y-3z=33$
$8x-2y+10z=-12$
$6x+2y-3z=33$
$14x+7z=21$
$2x+z=3$
$15x+11z=12$
$2x+z=3 \quad (-11)$
$15x+11z=12$
$-22x-11z=-33$
$-7x=-21$
$x=3$
$2(3)+z=3$
$z=-3$
$4(3)-y+5(-3)=-6$
$12-y-15=-6$
$-y-3=-6$
$-y=-3$
$y=3$
$sol: x=3 \quad y=3 \quad z=-3$

13. $9x+4y-10z=6$
$6x-8y+5z=-1$
$12x+12y-15z=10$
$6x-8y+5z=-1 \quad (-2)$
$12x+12y-15z=10$
$-12x+16y-10z=2$
$12x+12y-15z=10$
$28y-25z=12$
$9x+4y-10z=6 \quad (2)$
$6x-8y+5z=-1 \quad (-3)$
$18x+8y-20z=12$
$-18x+24y-15z=3$
$32y-35z=15$

Continúa

13. Continuación

$28y-25z=12 \quad (-8)$
$32y-35z=15 \quad (7)$
$-224y+200z=-96$
$224y-245z=105$
$-45z=9$
$z=-\frac{1}{5}$
$28y-25\left(-\frac{1}{5}\right)=12$
$28y+5=12$
$28y=7$
$y=\frac{1}{4}$
$6x-8\left(\frac{1}{4}\right)+5\left(-\frac{1}{5}\right)=-1$
$6x-2-1=-1$
$6x=2$
$x=\frac{1}{3}$
$sol:\ x=\frac{1}{3} \quad y=\frac{1}{4} \quad z=-\frac{1}{5}$

14. $5x+3y-z=-11$
$10x-y+z=10$
$15x+2y-z=-7$

$5x+3y-z=-11$
$10x-y+z=10$
$15x+2y=-1$

$10x-y+z=10$
$15x+2y-z=-7$
$25x+y=3$

$15x+2y=-1$
$25x+y=3 \quad (-2)$
$15x+2y=-1$
$-50x-2y=-6$
$-35x=-7$
$x=\frac{1}{5}$
$15\left(\frac{1}{5}\right)+2y=-1$
$3+2y=-1$
$2y=-4$
$y=-2$

Continúa

14. Continuación

$10x-(-2)+z=10$
$10\left(\frac{1}{5}\right)-(-2)+z=10$
$2+2+z=10$
$z=6$
$sol:\ x=\frac{1}{5} \quad y=-2 \quad z=6$

15. $x+y=1$
$y+z=-1$
$z+x=-6$

$x+y=1$
$y+z=-1 \quad (-1)$
$x+y=1$
$-y-z=1$
$x-z=2$

$x-z=2$
$x+z=-6$
$2x=-4$
$x=-2$
$-2+y=1$
$y=3$
$3+z=-1$
$z=-4$
$sol:\ x=-2 \quad y=3 \quad z=-4$

16. $x+2y=-1$
$2y+z=0$
$x+2z=11$

$x+2y=-1$
$2y+z=0 \quad (-1)$
$x+2y=-1$
$-2y-z=0$
$x-z=-1$

$x-z=-1 \quad (-1)$
$x+2z=11$
$-x+z=1$
$x+2z=11$
$3z=12$
$z=4$
$x-4=-1 \Rightarrow x=3$
$3+2y=-1$
$2y=-4 \Rightarrow y=-2$
$sol:\ x=3 \quad y=-2 \quad z=4$

17. $y+z=-8$
$2x+z=9$
$3y+2x=-3$

$y+z=-8$
$2x+z=9 \quad (-1)$
$y+z=-8$
$-2x-z=-9$
$-2x+y=-17$

$-2x+y=-17$
$2x+3y=-3$
$4y=-20$
$y=-5$
$-5+z=-8$
$z=-3$
$2x-3=9$
$2x=12$
$x=6$
$sol: x=6 \quad y=-5 \quad z=-3$

18. $3x-2y=0$
$3y-4z=25$
$z-5x=-14$

$3y-4z=25$
$-5x+z=-14 \quad (4)$
$3y-4z=25$
$-20x+4z=-56$
$-20x+3y=-31$

$-20x+3y=-31 \quad (2)$
$3x-2y=0 \quad (3)$
$-40x+6y=-62$
$9x-6y=0$
$-31x=-62$
$x=2$
$3(2)-2y=0$
$6-2y=0$
$-2y=-6$
$y=3$
$3(3)-4z=25$
$-4z=16$
$z=-4$
$sol: x=2 \quad y=3 \quad z=-4$

19. $3z-5x=10$
$5x-3y=-7$
$3y-5z=-13$

$-5x+3z=10$
$5x-3y=-7$
$-3y+3z=3$
$-y+z=1$

$3y-5z=-13$
$-y+z=1 \quad (3)$
$3y-5z=-13$
$-3y+3z=3$
$-2z=-10$
$z=5$
$-y+5=1$
$-y=-4$
$y=4$
$5x-3(4)=-7$
$5x-12=-7$
$5x=5$
$x=1$
$sol:\ x=1 \quad y=4 \quad z=5$

20. $x-2y=0$
$y-2z=5$
$x+y+z=8$

$x+y+z=8$
$x-2y=0 \quad (-1)$
$x+y+z=8$
$-x+2y=0$
$3y+z=8$

$3y+z=8 \quad (2)$
$y-2z=5$
$6y+2z=16$
$y-2z=5$
$7y=21$
$y=3$
$x-2(3)=0$
$x-6=0$
$x=6$
$6+3+z=8$
$z=-1$
$sol: x=6 \quad y=3 \quad z=-1$

21. $5x-3z=2$

$2z-y=-5$

$x+2y-4z=8$

$$\begin{array}{l} x+2y-4z=8 \\ \underline{-y+2z=-5}\ (2) \\ x+2y-4z=8 \\ \underline{-2y+4z=-10} \\ x\qquad\qquad=-2 \end{array}$$

$5(-2)-3z=2$

$-10-3z=2$

$-3z=12$

$z=-4$

$2(-4)-y=-5$

$-8-y=-5$

$-y=3$

$y=-3$

$sol:\ x=-2\ y=-3\ z=-4$

22. $2x-z=14$

$4x+y-z=41$

$3x-y+5z=53$

$$\begin{array}{l} 4x+y-z=41 \\ \underline{3x-y+5z=53} \\ 7x\qquad+4z=94 \end{array}$$

$$\begin{array}{l} 7x+4z=94 \\ \underline{2x-z=14}\ (4) \\ 7x+4z=94 \\ \underline{8x-4z=56} \\ 15x\quad=150 \end{array}$$

$x=10$

$2(10)-z=14$

$20-z=14$

$-z=-6$

$z=6$

$4(10)+y-6=41$

$y+34=41$

$y=7$

$sol:\ x=10\ y=7\ z=6$

23. $x+y-z=1$

$z+x-y=3$

$z-x+y=7$

$$\begin{array}{l} z+x-y=3 \\ \underline{z-x+y=7} \\ 2z\qquad=10 \Rightarrow z=5 \end{array}$$

Continúa

23. Continuación

$$\begin{array}{l} x-y+z=3 \\ \underline{x+y-z=1} \\ 2x\qquad=4 \end{array}$$

$x=2$

$2-y+5=3$

$-y+7=3$

$-y=-4$

$y=4$

$sol:\ x=2\quad y=4\quad z=5$

24. $\frac{x}{2}+\frac{y}{2}-\frac{z}{3}=3$

$3x+3y-2z=18$

$\frac{x}{3}+\frac{y}{6}-\frac{z}{2}=-5$

$2x+y-3z=-30$

$\frac{x}{6}-\frac{y}{3}+\frac{z}{6}=0$

$x-2y+z=0$

$$\begin{array}{l} 3x+3y-2z=18\quad(2) \\ \underline{x-2y+z=0}\quad(3) \\ 6x+6y-4z=36 \\ \underline{3x-6y+3z=0} \\ 9x\qquad-z=36 \end{array}$$

$$\begin{array}{l} 2x+y-3z=-30\quad(2) \\ \underline{x-2y+z=0} \\ 4x+2y-6z=-60 \\ \underline{x-2y+z=0} \\ 5x\qquad-5z=-60 \end{array}$$

$x-z=-12$

$$\begin{array}{l} 9x-z=36 \\ \underline{x-z=-12}\quad(-1) \\ 9x-z=36 \\ \underline{-x+z=12} \\ 8x\qquad=48 \end{array}$$

$x=6$

$6-z=-12$

$-z=-18$

$z=18$

Continúa

24. Continuación

$6-2y+18=0$

$-2y+24=0$

$-2y=-24$

$y=12$

$sol:\ x=6\quad y=12\quad z=18$

25. $\frac{x}{3}+\frac{y}{4}+\frac{z}{3}=21$

$4x+3y+4z=252$

$\frac{x}{5}+\frac{y}{6}-\frac{z}{3}=0$

$6x+5y-10z=0$

$\frac{x}{10}+\frac{y}{3}-\frac{z}{6}=3$

$3x+10y-5z=90$

$$\begin{array}{l} 6x+5y-10z=0\quad(-2) \\ \underline{3x+10y-5z=90} \\ -12x-10y+20z=0 \\ \underline{3x+10y-5z=90} \\ -9x\qquad+15z=90 \end{array}$$

$-3x+5z=30$

$$\begin{array}{l} 4x+3y+4z=252\quad(10) \\ \underline{3x+10y-5z=90}\quad(-3) \\ 40x+30y+40z=2.520 \\ \underline{-9x-30y+15z=-270} \\ 31x\qquad+55z=2.250 \end{array}$$

$$\begin{array}{l} -3x+5z=30\quad(-11) \\ \underline{31x+55z=2.250} \\ 33x-55z=-330 \\ \underline{31x+55z=2.250} \\ 64x\qquad=1.920 \end{array}$$

$x=30$

$-3(30)+5z=30$

$-90+5z=30$

$5z=120$

$z=24$

$6(30)+5y-10(24)=0$

$180-240+5y=0$

$5y=60$

$y=12$

$sol:\ x=30\quad y=12\quad z=24$

26. $x-\frac{y+z}{3}=4$

$3x-y-z=12$

$y-\frac{x+z}{8}=10$

$8y-x-z=80$

$z-\frac{y-x}{2}=5$

$2z-y+x=10$

$3x-y-z=12\ (-1)$

$-x+8y-z=80$

$-3x+y+z=-12$

$-x+8y-z=80$

$-4x+9y=68$

$-x+8y-z=80\ (2)$

$x-y+2z=10$

$-2x+16y-2z=160$

$x-y+2z=10$

$-x+15y=170$

$-4x+9y=68$

$-x+15y=170\ (-4)$

$-4x+9y=68$

$4x-60y=-680$

$-51y=-612$

$y=12$

$-4x+9(12)=68$

$-4x+108=68$

$-4x=-40$

$x=10$

$3(10)-12-z=12$

$30-12-z=12$

$18-z=12$

$z=6$

$sol: x=10\ \ y=12\ \ z=6$

27. $\frac{x+y}{7}=\frac{y+4}{5}$

$5(x+y)=7(y+4)$

$5x+5y=7y+28$

$5x-2y=28$

$\frac{x-z}{5}=\frac{y-4}{2}$

$2(x-z)=5(y-4)$

$2x-2z=5y-20$

$2x-5y-2z=-20$

Continúa

27. Continuación

$\frac{y-z}{3}=\frac{x+2}{10}$

$10(y-z)=3(x+2)$

$10y-10z=3x+6$

$3x-10y+10z=-6$

$2x-5y-2z=-20\ (5)$

$3x-10y+10z=-6$

$10x-25y-10z=-100$

$3x-10y+10z=-6$

$13x-35y=-106$

$13x-35y=-106\ (2)$

$5x-2y=28\ (-35)$

$26x-70y=-212$

$-175x+70y=-980$

$-149x=-1.192$

$x=8$

$5(8)-2y=28$

$40-2y=28$

$-2y=-12$

$y=6$

$2(8)-5(6)-2z=-20$

$16-30-2z=-20$

$-2z=-6$

$z=3$

$sol: x=8\ \ y=6\ \ z=3$

28. $y-\frac{z+4}{2}=x-6$

$2y-z-4=2x-12$

$-2x+2y-z=-8$

$z-\frac{x-7}{3}=y-5$

$3z-x+7=3y-15$

$-x-3y+3z=-22$

$x-\frac{y+2}{5}=z+4$

$5x-y-2=5z+20$

$5x-y-5z=22$

$-2x+2y-z=-8$

$-x-3y+3z=-22\ (-2)$

$-2x+2y-z=-8$

$2x+6y-6z=44$

$8y-7z=36$

Continúa

28. Continuación

$5x-y-5z=22$

$-x-3y+3z=-22\ (5)$

$5x-y-5z=22$

$-5x-15y+15z=-110$

$-16y+10z=-88$

$-8y+5z=-44$

$-8y+5z=-44$

$8y-7z=36$

$-2z=-8$

$z=4$

$8y-7(4)=36$

$8y-28=36$

$8y=64$

$y=8$

$-x-3(8)+3(4)=-22$

$-x-12=-22$

$x=10$

$sol: x=10\ \ y=8\ \ z=4$

29. $x-y+\frac{y-z}{2}=3$

$2x-2y+y-z=6$

$2x-y-z=6$

$\frac{x-y}{2}-\frac{x-z}{4}=0$

$2x-2y-x+z=0$

$x-2y+z=0$

$\frac{y-z}{2}-x=-5$

$y-z-2x=-10$

$2x-y-z=6$

$-2x+y-z=-10$

$-2z=-4$

$z=2$

$x-2y+z=0$

$2x-y-z=6\ (-2)$

$x-2y+z=0$

$-4x+2y+2z=-12$

$-3x+3z=-12$

$-3x+3(2)=-12$

$-3x+6=-12$

$-3x=-18$

$x=6$

Continúa

29. Continuación

$6-2y+2=0$

$-2y+8=0$

$-2y=-8$

$y=4$

sol: $x=6\ \ y=4\ \ z=2$

30. $\frac{1}{x}+\frac{1}{y}=5$

$\frac{1}{x}+\frac{1}{z}=6\ (-1)$

$\frac{1}{x}+\frac{1}{y}=5$

$-\frac{1}{x}-\frac{1}{z}=-6$

$\frac{1}{y}-\frac{1}{z}=-1$

$\frac{1}{y}-\frac{1}{z}=-1$

$\frac{1}{y}+\frac{1}{z}=7$

$\frac{2}{y}=6$

$\frac{1}{3}=y$

$\frac{1}{x}+\frac{1}{\frac{1}{3}}=5$

$\frac{1}{x}+3=5$

$\frac{1}{x}=2$

$\frac{1}{2}=x$

$\frac{1}{\frac{1}{2}}+\frac{1}{z}=6$

$2+\frac{1}{z}=6$

$\frac{1}{z}=4$

$\frac{1}{4}=z$

sol: $x=\frac{1}{2}\ \ y=\frac{1}{3}\ \ z=\frac{1}{4}$

31. $\frac{3}{x}+\frac{2}{y}=2$

$\frac{2}{y}+\frac{2}{z}=\frac{3}{2}\ (-1)$

$\frac{3}{x}+\frac{2}{y}=2$

$-\frac{2}{y}-\frac{2}{z}=-\frac{3}{2}$

$\frac{3}{x}-\frac{2}{z}=\frac{1}{2}$

$\frac{3}{x}-\frac{2}{z}=\frac{1}{2}\ (2)$

$\frac{1}{x}+\frac{4}{z}=\frac{4}{3}$

$\frac{6}{x}-\frac{4}{z}=1$

$\frac{1}{x}+\frac{4}{z}=\frac{4}{3}$

$\frac{7}{x}=\frac{7}{3}$

$x=3$

$\frac{3}{3}+\frac{2}{y}=2$

$1+\frac{2}{y}=2$

$\frac{2}{y}=1$

$2=y$

$\frac{3}{3}-\frac{2}{z}=\frac{1}{2}$

$1-\frac{2}{z}=\frac{1}{2}$

$-\frac{2}{z}=-\frac{1}{2}$

$4=z$

sol: $x=3\ \ y=2\ \ z=4$

32. $\frac{1}{x}+\frac{4}{y}+\frac{2}{z}=-6$

$\frac{3}{x}+\frac{2}{y}+\frac{4}{z}=3\ (-2)$

$\frac{1}{x}+\frac{4}{y}+\frac{2}{z}=-6$

$-\frac{6}{x}-\frac{4}{y}-\frac{8}{z}=-6$

$-\frac{5}{x}-\frac{6}{z}=-12$

32. Continuación

$\frac{1}{x}+\frac{4}{y}+\frac{2}{z}=-6\ (5)$

$\frac{6}{x}-\frac{5}{y}-\frac{6}{z}=31\ (4)$

$\frac{5}{x}+\frac{20}{y}+\frac{10}{z}=-30$

$\frac{24}{x}-\frac{20}{y}-\frac{24}{z}=124$

$\frac{29}{x}-\frac{14}{z}=94$

$-\frac{5}{x}-\frac{6}{z}=12\ (-7)$

$\frac{29}{x}-\frac{14}{z}=94\ (3)$

$\frac{35}{x}+\frac{42}{z}=84$

$\frac{87}{x}-\frac{42}{z}=282$

$\frac{122}{x}=366$

$122=366x$

$\frac{1}{3}=x$

$-\frac{5}{\frac{1}{3}}-\frac{6}{z}=-12$

$-15-\frac{6}{z}=-12$

$-\frac{6}{z}=3$

$-2=z$

$\frac{1}{\frac{1}{3}}+\frac{4}{y}+\frac{2}{-2}=-6$

$3+\frac{4}{y}-1=-6$

$\frac{4}{y}=-8$

$-\frac{1}{2}=y$

sol: $x=\frac{1}{3}\ \ y=-\frac{1}{2}\ \ z=-2$

Continúa

EJERCICIO 187

1. $$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 4 \end{vmatrix}$$
$\Rightarrow 6+0+8-3-0-4=7$

2. $$\begin{vmatrix} 1 & 2 & -2 \\ 1 & -3 & 3 \\ -1 & 4 & 5 \\ 1 & 2 & -2 \\ 1 & -3 & 3 \end{vmatrix}$$
$\Rightarrow -15-8-6+6-12-10=-45$

3. $$\begin{vmatrix} -3 & 4 & 1 \\ 2 & -3 & 0 \\ 1 & 2 & 7 \\ -3 & 4 & 1 \\ 2 & -3 & 0 \end{vmatrix}$$
$\Rightarrow 63+4+0+3+0-56=14$

4. $$\begin{vmatrix} 2 & 5 & -1 \\ 3 & -4 & 3 \\ 6 & 2 & 4 \\ 2 & 5 & -1 \\ 3 & -4 & 3 \end{vmatrix}$$
$\Rightarrow -32-6+90-24-12-60=-44$

5. $$\begin{vmatrix} 5 & -1 & -6 \\ -2 & 5 & 3 \\ 3 & 4 & 2 \\ 5 & -1 & -6 \\ -2 & 5 & 3 \end{vmatrix}$$
$\Rightarrow 50+48-9+90-60-4=115$

6. $$\begin{vmatrix} 4 & 1 & 5 \\ 3 & 2 & -6 \\ 12 & 3 & 2 \\ 4 & 1 & 5 \\ 3 & 2 & -6 \end{vmatrix}$$
$\Rightarrow 16+45-72-120+72-6=-65$

7. $$\begin{vmatrix} 5 & 2 & -8 \\ -3 & -7 & 3 \\ 4 & 0 & -1 \\ 5 & 2 & -8 \\ -3 & -7 & 3 \end{vmatrix}$$
$\Rightarrow 35+0+24-224+0-6=-171$

8. $$\begin{vmatrix} 3 & 2 & 5 \\ -1 & -3 & 4 \\ 3 & 2 & 5 \\ 3 & 2 & 5 \\ -1 & -3 & 4 \end{vmatrix}$$
$\Rightarrow -45-10+24+45-24+10=0$

9. $$\begin{vmatrix} 5 & 2 & 3 \\ 6 & 1 & 2 \\ 3 & 4 & 5 \\ 5 & 2 & 3 \\ 6 & 1 & 2 \end{vmatrix}$$
$\Rightarrow 25+72+12-9-40-60=0$

10. $$\begin{vmatrix} 12 & 5 & 10 \\ 8 & -6 & 9 \\ 7 & 4 & -2 \\ 12 & 5 & 10 \\ 8 & -6 & 9 \end{vmatrix}$$
$\Rightarrow 144+320+315+420-432+80=847$

11. $$\begin{vmatrix} -9 & 3 & -4 \\ 7 & -5 & -3 \\ 4 & 6 & 1 \\ -9 & 3 & -4 \\ 7 & -5 & -3 \end{vmatrix}$$
$\Rightarrow 45-168-36-80-162-21=-422$

12. $$\begin{vmatrix} 11 & -5 & 7 \\ -12 & 3 & 8 \\ -13 & 1 & 9 \\ 11 & -5 & 7 \\ -12 & 3 & 8 \end{vmatrix}$$
$\Rightarrow 297-84+520+273-88-540=378$

EJERCICIO 188

1. $x+y+z=11$

$x-y+3z=13$

$2x+2y-z=7$

$$x=\frac{\begin{vmatrix}11&1&1\\13&-1&3\\7&2&-1\\11&1&1\\13&-1&3\end{vmatrix}}{\begin{vmatrix}1&1&1\\1&-1&3\\2&2&-1\\1&1&1\\1&-1&3\end{vmatrix}}$$

$$x=\frac{11+26+21+7-66+13}{1+2+6+2-6+1}=\frac{12}{6}=2$$

$$y=\begin{vmatrix}1&11&1\\1&13&3\\2&7&-1\\1&11&1\\1&13&3\end{vmatrix}$$

$$y=\frac{-13+7+66-26-21+11}{6}=\frac{24}{6}=4$$

$$z=\begin{vmatrix}1&1&11\\1&-1&13\\2&2&7\\1&1&11\\1&-1&13\end{vmatrix}$$

$$z=\frac{-7+22+26+22-26-7}{6}=\frac{30}{6}=5$$

sol: $x=2 \quad y=4 \quad z=5$

2. $x+y+z=-6$

$2x+y-z=-1$

$x-2y+3z=-6$

$$x=\frac{\begin{vmatrix}-6&1&1\\-1&1&-1\\-6&-2&3\\-6&1&1\\-1&1&-1\end{vmatrix}}{\begin{vmatrix}1&1&1\\2&1&-1\\1&-2&3\\1&1&1\\2&1&-1\end{vmatrix}}$$

2. Continuación

$$x=\frac{-18+2+6+6+12+3}{3-4-1-1-2-6}=\frac{11}{-11}=-1$$

$$y=\begin{vmatrix}1&-6&1\\2&-1&-1\\1&-6&3\\1&-6&1\\2&-1&-1\end{vmatrix}$$

$$y=\frac{-3-12+6+1-6+36}{-11}=\frac{22}{-11}=-2$$

$$z=\begin{vmatrix}1&1&-6\\2&1&-1\\1&-2&-6\\1&1&-6\\2&1&-1\end{vmatrix}$$

$$z=\frac{-6+24-1+6-2+12}{-11}=\frac{33}{-11}=-3$$

sol: $x=-1 \quad y=-2 \quad z=-3$

3. $2x+3y+4z=3$

$2x+6y+8z=5$

$4x+9y-4z=4$

$$x=\frac{\begin{vmatrix}3&3&4\\5&6&8\\4&9&-4\\3&3&4\\5&6&8\end{vmatrix}}{\begin{vmatrix}2&3&4\\2&6&8\\4&9&-4\\2&3&4\\2&6&8\end{vmatrix}}$$

$$x=\frac{-72+180+96-96-216+60}{-48+72+96-96-144+24}=\frac{-48}{-96}=\frac{1}{2}$$

$$y=\begin{vmatrix}2&3&4\\2&5&8\\4&4&-4\\2&3&4\\2&5&8\end{vmatrix}$$

$$y=\frac{-40+32+96-80-64+24}{-96}=\frac{-32}{-96}=\frac{1}{3}$$

$$z=\begin{vmatrix}2&3&3\\2&6&5\\4&9&4\\2&3&3\\2&6&5\end{vmatrix}$$

$$z=\frac{48+54+60-72-90-24}{-96}=\frac{-24}{-96}=\frac{1}{4}$$

sol: $x=\frac{1}{2} \quad y=\frac{1}{3} \quad z=\frac{1}{4}$

4. $4x-y+z=4$

$2x+2y-z=2$

$6x-2y+3z=12$

$$x=\frac{\begin{vmatrix}4&-1&1\\2&2&-1\\12&-2&3\\4&-1&1\\2&2&-1\end{vmatrix}}{\begin{vmatrix}4&-1&1\\2&2&-1\\6&-2&3\\4&-1&1\\2&2&-1\end{vmatrix}}$$

$$x=\frac{24-4+12-24-8+6}{24-4+6-12-8+6}=\frac{6}{12}=\frac{1}{2}$$

$$y=\begin{vmatrix}4&4&1\\2&2&-1\\6&12&3\\4&4&1\\2&2&-1\end{vmatrix}$$

$$y=\frac{24+24-24-12+48-24}{12}=\frac{36}{12}=3$$

$$z=\begin{vmatrix}4&-1&4\\2&2&2\\6&-2&12\\4&-1&4\\2&2&2\end{vmatrix}$$

$$z=\frac{96-16-12-48+16+24}{12}=\frac{60}{12}=5$$

sol: $x=\frac{1}{2}$ $y=3$ $z=5$

5. $x+4y+5z=11$

$3x-2y+z=5$

$4x+y-3z=-26$

$$x=\frac{\begin{vmatrix}11&4&5\\5&-2&1\\-26&1&-3\\11&4&5\\5&-2&1\end{vmatrix}}{\begin{vmatrix}1&4&5\\3&-2&1\\4&1&-3\\1&4&5\\3&-2&1\end{vmatrix}}$$

Continúa

5. Continuación

$$x=\frac{66+25-104-260-11+60}{6+15+16+40-1+36}=\frac{-224}{112}=-2$$

$$y=\begin{vmatrix}1&11&5\\3&5&1\\4&-26&-3\\1&11&5\\3&5&1\end{vmatrix}$$

$$y=\frac{-15-390+44-100+26+99}{112}=\frac{-336}{112}=-3$$

$$z=\begin{vmatrix}1&4&11\\3&-2&5\\4&1&-26\\1&4&11\\3&-2&5\end{vmatrix}$$

$$z=\frac{52+33+80+88-5+312}{112}=\frac{560}{112}=5$$

sol: $x=-2$ $y=-3$ $z=5$

6. $7x+10y+4z=-2$

$5x-2y+6z=38$

$3x+y-z=21$

$$x=\frac{\begin{vmatrix}-2&10&4\\38&-2&6\\21&1&-1\\-2&10&4\\38&-2&6\end{vmatrix}}{\begin{vmatrix}7&10&4\\5&-2&6\\3&1&-1\\7&10&4\\5&-2&6\end{vmatrix}}$$

$$x=\frac{-4+152+1{.}260+168+12+380}{14+20+180+24-42+50}=\frac{1{.}968}{246}=8$$

$$y=\begin{vmatrix}7&-2&4\\5&38&6\\3&21&-1\\7&-2&4\\5&38&6\end{vmatrix}$$

$$y=\frac{-266+420-36-456-882-10}{246}=\frac{-1{.}230}{246}=-5$$

$$z=\begin{vmatrix}7&10&-2\\5&-2&38\\3&1&21\\7&10&-2\\5&-2&38\end{vmatrix}$$

$$z=\frac{-294-10+1{.}140-12-266-1{.}050}{246}=\frac{-492}{246}=-2$$

sol: $x=8$ $y=-5$ $z=-2$

7. $4x+7y+5z=-2$

$6x+3y+7z=6$

$x-y+9z=-21$

$$x=\frac{\begin{vmatrix}-2 & 7 & 5\\ 6 & 3 & 7\\ -21 & -1 & 9\\ -2 & 7 & 5\\ 6 & 3 & 7\end{vmatrix}}{\begin{vmatrix}4 & 7 & 5\\ 6 & 3 & 7\\ 1 & -1 & 9\\ 4 & 7 & 5\\ 6 & 3 & 7\end{vmatrix}}$$

$$x=\frac{-54-30-1.029+315-14-378}{108-30+49-15+28-378}=\frac{-1.190}{-238}=5$$

$$y=\begin{vmatrix}4 & -2 & 5\\ 6 & 6 & 7\\ 1 & -21 & 9\\ 4 & -2 & 5\\ 6 & 6 & 7\end{vmatrix}$$

$$y=\frac{216-630-14-30+588+108}{-238}=\frac{238}{-238}=-1$$

$$z=\begin{vmatrix}4 & 7 & -2\\ 6 & 3 & 6\\ 1 & -1 & -21\\ 4 & 7 & -2\\ 6 & 3 & 6\end{vmatrix}$$

$$z=\frac{-252+12+42+6+24+882}{-238}=\frac{714}{-238}=-3$$

sol: $x=5 \qquad y=-1 \qquad z=-3$

8. $3x-5y+2z=-22$

$2x-y+6z=32$

$8x+3y-5z=-33$

$$x=\frac{\begin{vmatrix}-22 & -5 & 2\\ 32 & -1 & 6\\ -33 & 3 & -5\\ -22 & -5 & 2\\ 32 & -1 & 6\end{vmatrix}}{\begin{vmatrix}3 & -5 & 2\\ 2 & -1 & 6\\ 8 & 3 & -5\\ 3 & -5 & 2\\ 2 & -1 & 6\end{vmatrix}}$$

Continúa

8. Continuación

$$x=\frac{-110+192+990-66+396-800}{15+12-240+16-54-50}=\frac{602}{-301}=-2$$

$$y=\begin{vmatrix}3 & -22 & 2\\ 2 & 32 & 6\\ 8 & -33 & -5\\ 3 & -22 & 2\\ 2 & 32 & 6\end{vmatrix}$$

$$y=\frac{-480-132-1.056-512+594-220}{-301}=\frac{-1.806}{-301}=6$$

$$z=\begin{vmatrix}3 & -5 & -22\\ 2 & -1 & 32\\ 8 & 3 & -33\\ 3 & -5 & -22\\ 2 & -1 & 32\end{vmatrix}$$

$$z=\frac{99-132-1.280-176-288-330}{-301}=\frac{-2.107}{-301}=7$$

sol: $x=-2 \qquad y=6 \qquad z=7$

9. $x+y+z=3$

$x+2y=6$

$2x+3y=6$

$$x=\frac{\begin{vmatrix}3 & 1 & 1\\ 6 & 2 & 0\\ 6 & 3 & 0\\ 3 & 1 & 1\\ 6 & 2 & 0\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 0\\ 2 & 3 & 0\\ 1 & 1 & 1\\ 1 & 2 & 0\end{vmatrix}}$$

$$x=\frac{0+18+0-12+0+0}{0+3+0-4+0+0}=\frac{6}{-1}=-6$$

$$y=\begin{vmatrix}1 & 3 & 1\\ 1 & 6 & 0\\ 2 & 6 & 0\\ 1 & 3 & 1\\ 1 & 6 & 0\end{vmatrix}$$

$$y=\frac{0+6+0-12+0+0}{-1}=\frac{-6}{-1}=6$$

$$z=\begin{vmatrix}1 & 1 & 3\\ 1 & 2 & 6\\ 2 & 3 & 6\\ 1 & 1 & 3\\ 1 & 2 & 6\end{vmatrix}$$

$$z=\frac{12+9+12-12-18-6}{-1}=\frac{-3}{-1}=3$$

sol: $x=-6 \qquad y=6 \qquad z=3$

10. $3x-2y=-1$

$4x+z=-28$

$x+2y+3z=-43$

$$x=\frac{\begin{vmatrix}-1 & -2 & 0\\ -28 & 0 & 1\\ -43 & 2 & 3\\ -1 & -2 & 0\\ -28 & 0 & 1\end{vmatrix}}{\begin{vmatrix}3 & -2 & 0\\ 4 & 0 & 1\\ 1 & 2 & 3\\ 3 & -2 & 0\\ 4 & 0 & 1\end{vmatrix}}$$

$$x=\frac{0+0+86+0+2-168}{0+0-2+0-6+24}=\frac{-80}{16}=-5$$

$$y=\begin{vmatrix}3 & -1 & 0\\ 4 & -28 & 1\\ 1 & -43 & 3\\ 3 & -1 & 0\\ 4 & -28 & 1\end{vmatrix}$$

$$y=\frac{-252+0-1+0+129+12}{16}=\frac{-112}{16}=-7$$

$$z=\begin{vmatrix}3 & -2 & -1\\ 4 & 0 & -28\\ 1 & 2 & -43\\ 3 & -2 & -1\\ 4 & 0 & -28\end{vmatrix}$$

$$z=\frac{0-8+56+0+168-344}{16}=\frac{-128}{16}=-8$$

sol: $x=-5 \quad y=-7 \quad z=-8$

11. $\frac{x}{3}-\frac{y}{4}+\frac{z}{4}=1$

$4x-3y+3z=12$

$\frac{x}{6}+\frac{y}{2}-z=1$

$2x+6y-12z=12$

$x+3y-6z=6$

$\frac{x}{2}-\frac{y}{8}-\frac{z}{2}=0$

$4x-y-4z=0$

$$x=\frac{\begin{vmatrix}12 & -3 & 3\\ 6 & 3 & -6\\ 0 & -1 & -4\\ 12 & -3 & 3\\ 6 & 3 & -6\end{vmatrix}}{\begin{vmatrix}4 & -3 & 3\\ 1 & 3 & -6\\ 4 & -1 & -4\\ 4 & -3 & 3\\ 1 & 3 & -6\end{vmatrix}}$$

Continúa

11. Continuación

$$x=\frac{-144-18+0+0-72-72}{-48-3+72-36-24-12}=\frac{-306}{-51}=6$$

$$y=\begin{vmatrix}4 & 12 & 3\\ 1 & 6 & -6\\ 4 & 0 & -4\\ 4 & 12 & 3\\ 1 & 6 & -6\end{vmatrix}$$

$$y=\frac{-96+0-288-72+0+48}{-51}=\frac{-408}{-51}=8$$

$$z=\begin{vmatrix}4 & -3 & 12\\ 1 & 3 & 6\\ 4 & -1 & 0\\ 4 & -3 & 12\\ 1 & 3 & 6\end{vmatrix}$$

$$z=\frac{0-12-72-144+24+0}{-51}=\frac{-204}{-51}=4$$

sol: $x=6 \quad y=8 \quad z=4$

12. $\frac{x}{3}+y=2z+3$; $x-y=1$; $x+z=\frac{y}{4}+11$

$x+3y=6z+y$; ; $4x+4z=y+44$

$x+3y-6z=9$ $\qquad$ $4x-y+4z=44$

$$x=\frac{\begin{vmatrix}9 & 3 & -6\\ 1 & -1 & 0\\ 44 & -1 & 4\\ 9 & 3 & -6\\ 1 & -1 & 0\end{vmatrix}}{\begin{vmatrix}1 & 3 & -6\\ 1 & -1 & 0\\ 4 & -1 & 4\\ 1 & 3 & -6\\ 1 & -1 & 0\end{vmatrix}}$$

$$x=\frac{-36+6+0-264+0-12}{-4+6+0-24+0-12}=\frac{-306}{-34}=9$$

$$y=\begin{vmatrix}1 & 9 & -6\\ 1 & 1 & 0\\ 4 & 44 & 4\\ 1 & 9 & -6\\ 1 & 1 & 0\end{vmatrix}$$

$$y=\frac{4-264+0+24+0-36}{-34}=\frac{-272}{-34}=8$$

$$z=\begin{vmatrix}1 & 3 & 9\\ 1 & -1 & 1\\ 4 & -1 & 44\\ 1 & 3 & 9\\ 1 & -1 & 1\end{vmatrix}$$

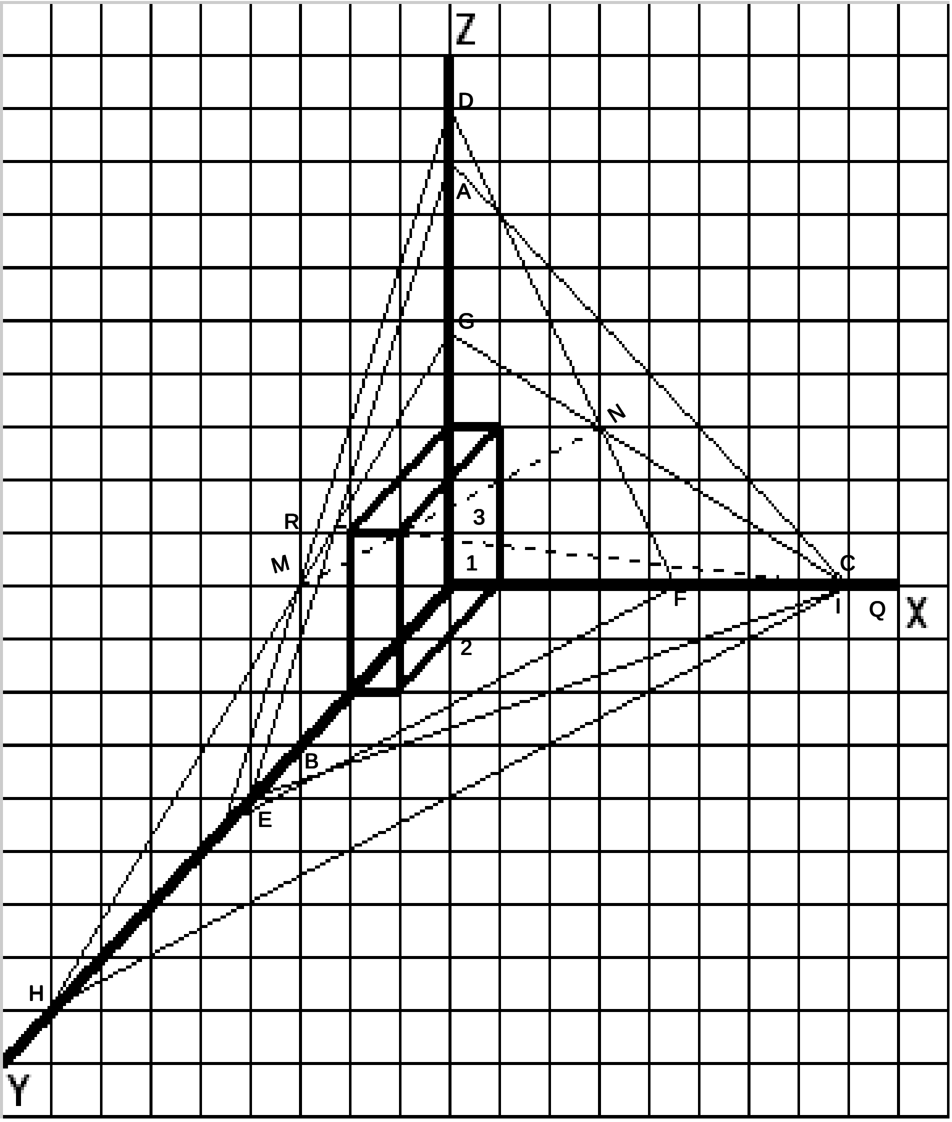
$$z=\frac{-44-9+12+36+1-132}{-34}=\frac{-136}{-34}=4$$

sol: $x=9 \quad y=8 \quad z=4$

EJERCICIO 191

1. $x+2y+z=8$ $\quad$ $2x+2y+z=9$ $\quad$ $3x+3y+5z=24$

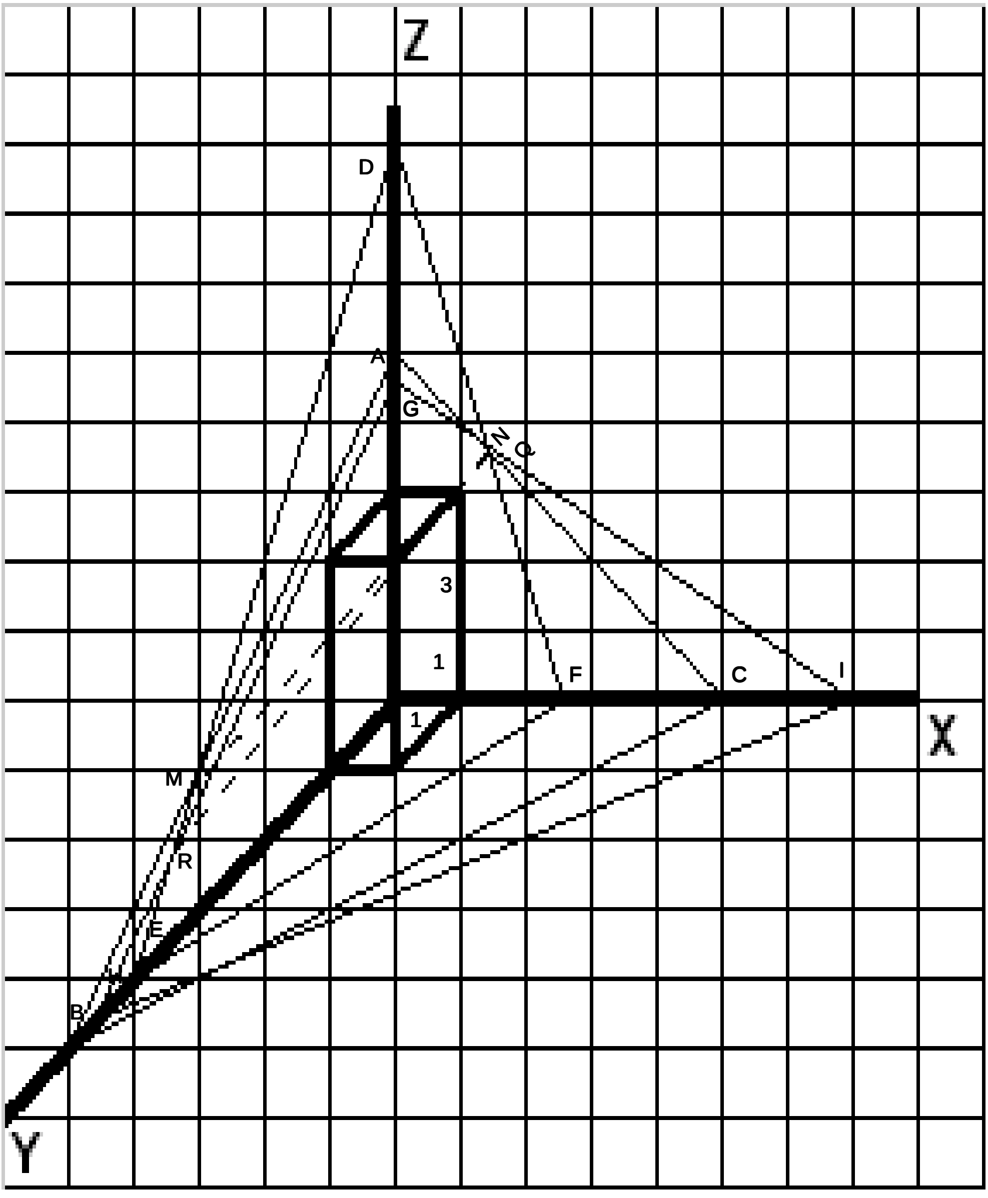
Plano ABC $x+2y+z=8$	*Plano DEF* $2x+2y+z=9$	*Plano GHI* $3x+3y+5z=24$
Para $y=0$ $z=0$ $x=8$	*Para* $y=0$ $z=0$ $x=4\frac{1}{2}$	*Para* $y=0$ $z=0$ $x=8$
Para $x=0$ $z=0$ $y=4$	*Para* $x=0$ $z=0$ $y=4\frac{1}{2}$	*Para* $x=0$ $z=0$ $y=8$
Para $x=0$ $y=0$ $z=8$	*Para* $x=0$ $y=0$ $z=9$	*Para* $x=0$ $y=0$ $z=4\frac{4}{5}$



Sol : x = 1 y= 2 z = 3

2. $x+y+z=5$ $\quad$ $3x+2y+z=8$ $\quad$ $2x+3y+3z=14$

Plano ABC $x+y+z=5$	*Plano DEF* $3x+2y+z=8$	*Plano GHI* $2x+3y+3z=14$
Para $y=0$ $z=0$ $x=5$	*Para* $y=0$ $z=0$ $x=2\frac{2}{3}$	*Para* $y=0$ $z=0$ $x=7$
Para $x=0$ $z=0$ $y=5$	*Para* $x=0$ $z=0$ $y=4$	*Para* $x=0$ $z=0$ $y=4\frac{2}{3}$
Para $x=0$ $y=0$ $z=5$	*Para* $x=0$ $y=0$ $z=8$	*Para* $x=0$ $y=0$ $z=4\frac{2}{3}$



sol : x = 1 y = 1 z = 3

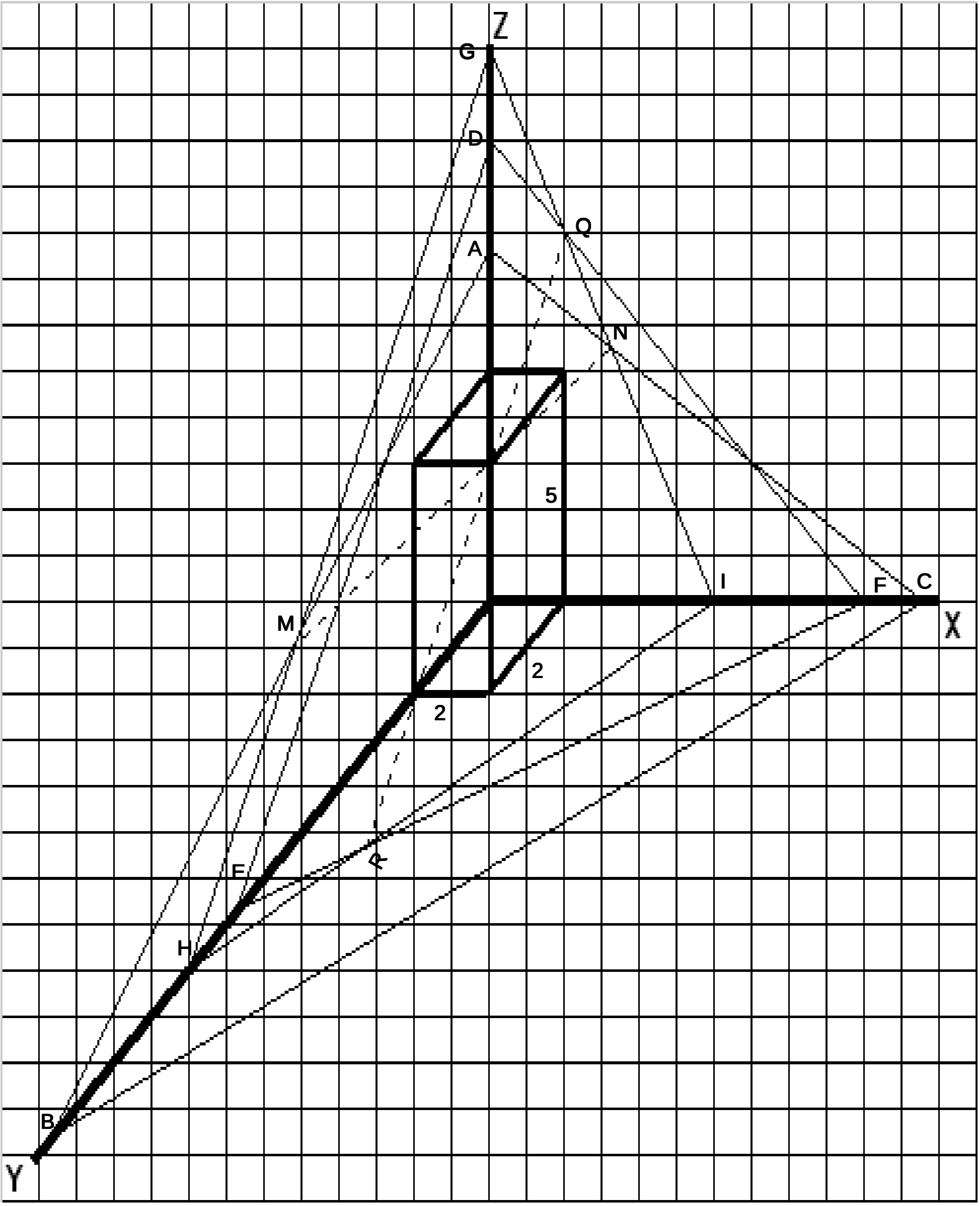
3. $2x+2y+3z=23$ $\qquad 2x+3y+2z=20$ $\qquad 4x+3y+2z=24$

$Plano\ \ ABC\ \ 2x+2y+3z=23$ $\qquad Plano\ DEF\ \ 2x+3y+2z=20$ $\qquad Plano\ GHI\ \ 4x+3y+2z=24$

$Para\ \ y=0\ \ z=0\ \ x=11\frac{1}{2}$ $\qquad Para\ \ y=0\ \ z=0\ \ x=10$ $\qquad Para\ \ y=0\ \ z=0\ \ x=6$

$Para\ \ x=0\ \ z=0\ \ y=11\frac{1}{2}$ $\qquad Para\ \ x=0\ \ z=0\ \ y=6\frac{2}{3}$ $\qquad Para\ \ x=0\ \ z=0\ \ y=8$

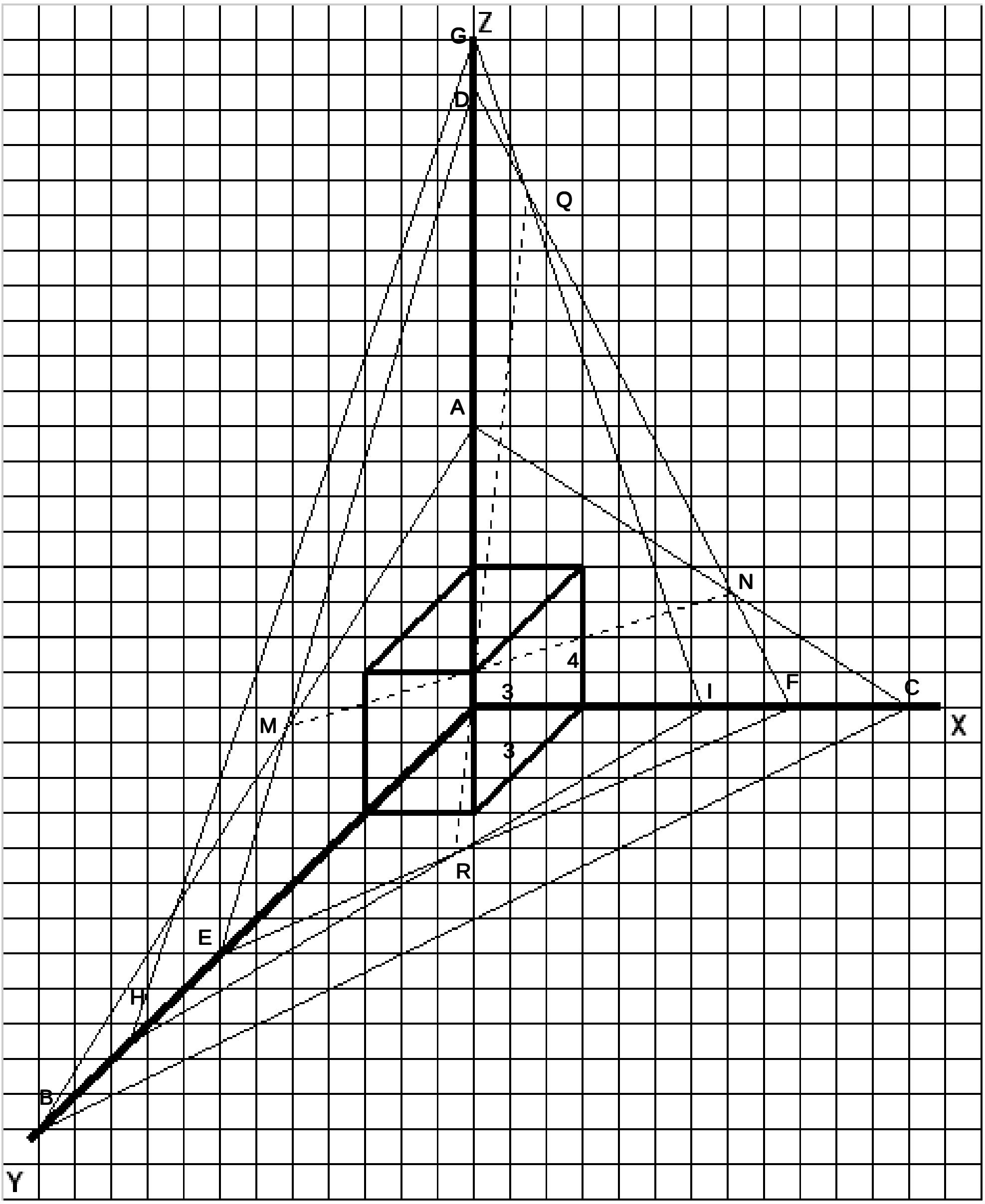
$Para\ \ x=0\ \ y=0\ \ z=7\frac{2}{3}$ $\qquad Para\ \ x=0\ \ y=0\ \ z=10$ $\qquad Para\ \ x=0\ \ y=0\ \ z=12$



sol : x = 2 y = 2 z = 5

4. $2x+2y+3z=24$ $\quad$ $4x+5y+2z=35$ $\quad$ $3x+2y+z=19$

Plano ABC $2x+2y+3z=24$	*Plano DEF* $4x+5y+2z=35$	*Plano GHI* $3x+2y+z=19$
Para $y=0$ $z=0$ $x=12$	*Para* $y=0$ $z=0$ $x=8\frac{3}{4}$	*Para* $y=0$ $z=0$ $x=6\frac{1}{3}$
Para $x=0$ $z=0$ $y=12$	*Para* $x=0$ $z=0$ $y=7$	*Para* $x=0$ $z=0$ $y=9\frac{1}{2}$
Para $x=0$ $y=0$ $z=8$	*Para* $x=0$ $y=0$ $z=17\frac{1}{2}$	*Para* $x=0$ $y=0$ $z=19$



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sol : x = 3 y = 3 z = 4

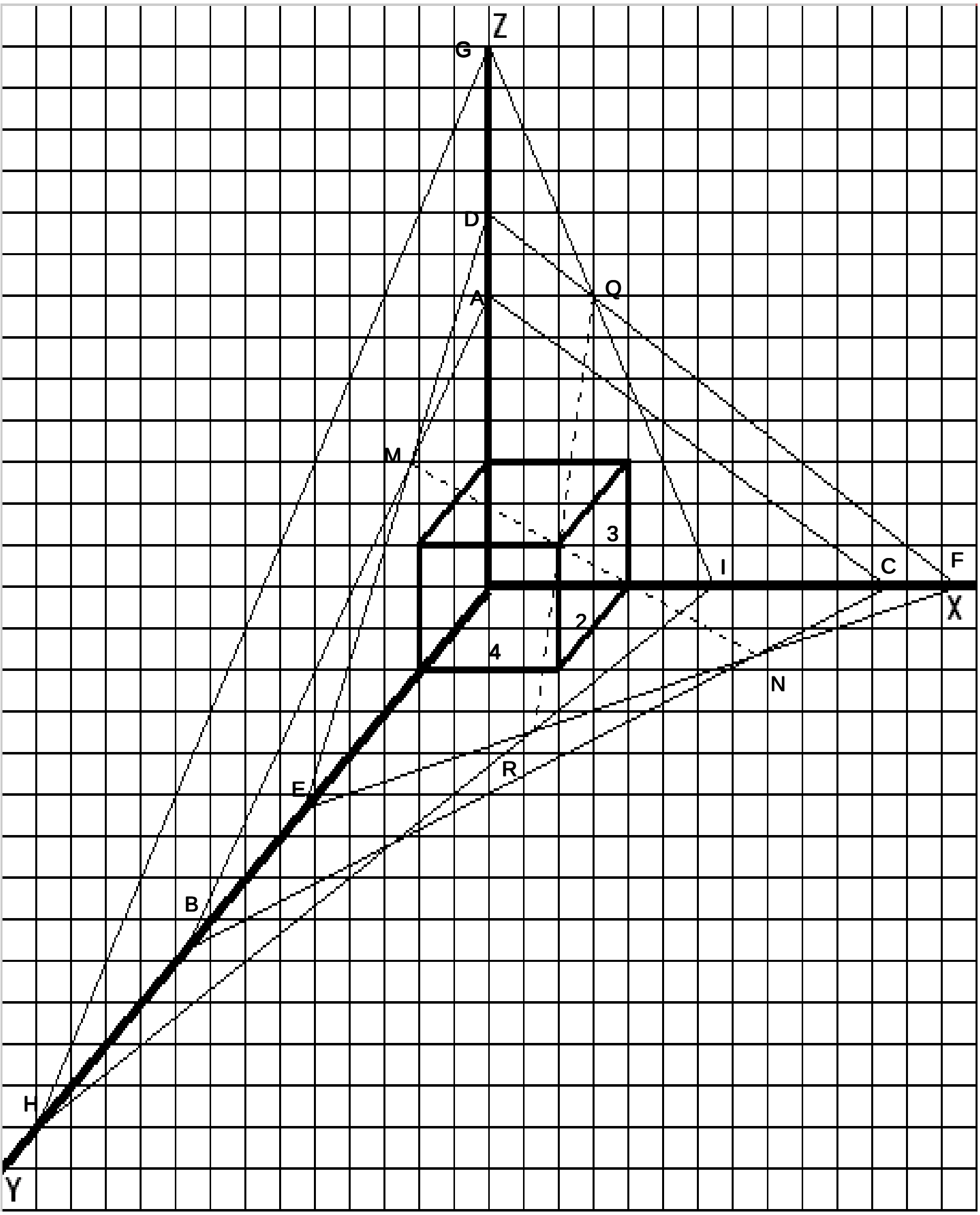
5. $3x+4y+5z=35$ $2x+5y+3z=27$ $2x+y+z=13$

Plano ABC $3x+4y+5z=35$ *Plano DEF* $2x+5y+3z=27$ *Plano GHI* $2x+y+z=13$

Para $y=0$ $z=0$ $x=11\frac{2}{3}$ *Para* $y=0$ $z=0$ $x=13\frac{1}{2}$ *Para* $y=0$ $z=0$ $x=6\frac{1}{2}$

Para $x=0$ $z=0$ $y=8\frac{3}{4}$ *Para* $x=0$ $z=0$ $y=5\frac{2}{5}$ *Para* $x=0$ $z=0$ $y=13$

Para $x=0$ $y=0$ $z=7$ *Para* $x=0$ $y=0$ $z=9$ *Para* $x=0$ $y=0$ $z=13$



sol : **x = 4** **y = 2** **z = 3**

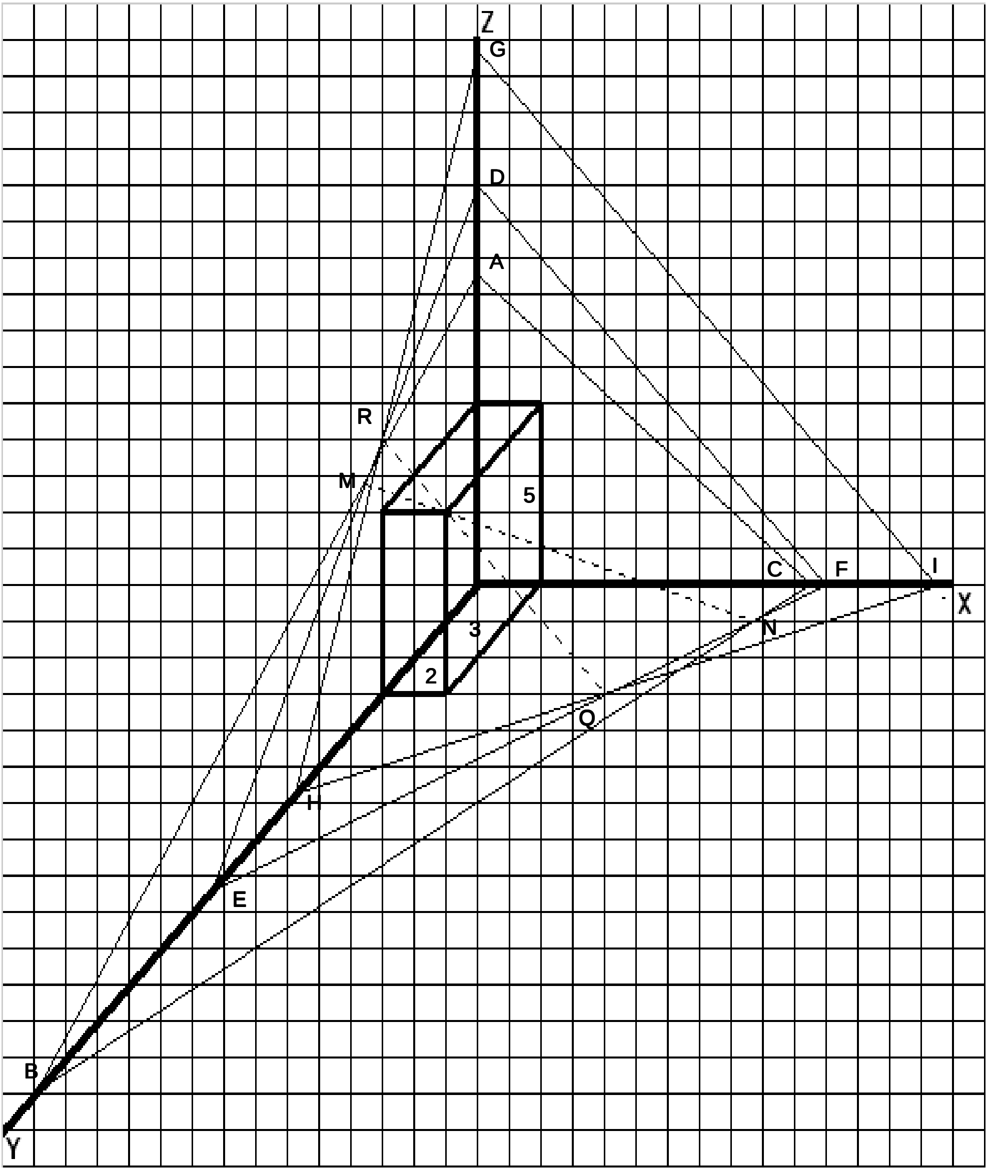
6. $4x+3y+5z=42$ $\quad$ $3x+4y+3z=33$ $\quad$ $2x+5y+2z=29$

Plano ABC $\quad 4x+3y+5z=42$ $\quad$ *Plano* DEF $\quad 3x+4y+3z=33$ $\quad$ *Plano* GHI $\quad 2x+5y+2z=29$

Para $y=0 \quad z=0 \quad x=10\frac{1}{2}$ $\quad$ *Para* $y=0 \quad z=0 \quad x=11$ $\quad$ *Para* $y=0 \quad z=0 \quad x=14\frac{1}{2}$

Para $x=0 \quad z=0 \quad y=14$ $\quad$ *Para* $x=0 \quad z=0 \quad y=8\frac{1}{4}$ $\quad$ *Para* $x=0 \quad z=0 \quad y=5\frac{4}{5}$

Para $x=0 \quad y=0 \quad z=8\frac{2}{5}$ $\quad$ *Para* $x=0 \quad y=0 \quad z=11$ $\quad$ *Para* $x=0 \quad y=0 \quad z=14\frac{1}{2}$



sol : x = 2 y = 3 z = 5

5. $x \to Cifra\ decenas$

$y \to Cifra\ unidades$

$$\frac{10x+y-17}{x+y}=5$$

$$10x+y-17=5(x+y)$$

$$5x-4y=17$$

$$\frac{10x+y-2}{y-2}=19$$

$$10x+y-2=19(y-2)$$

$$10x-18y=-36$$

$$5x-9y=-18$$

$$5x-4y=17$$

$$5x-9y=-18\quad(-1)$$

$$5x-4y=17$$

$$-5x+9y=18$$

$$5y=35 \Rightarrow y=7$$

$$5x-4(7)=17$$

$$5x-28=17$$

$$5x=45 \Rightarrow x=9 \quad sol: 97$$

6. $x \to Cifra\ decenas$

$y \to Cifra\ unidades$

$$10x+y+9=10y+x$$

$$9x-9y=-9$$

$$x-y=-1$$

$$\frac{10y+x-1}{7}=6$$

$$x+10y=43$$

$$x-y=-1\quad(-1)$$

$$x+10y=43$$

$$-x+y=1$$

$$x+10y=43$$

$$11y=44 \Rightarrow y=4$$

$$x-4=-1 \Rightarrow x=3 \quad sol:34$$

7. $x \to Cifra\ decenas$

$y \to Cifra\ unidades$

$$x+y=9$$

$$10(x+1)+y-1=10y+x$$

$$10x+10+y-1=10y+x$$

$$9x-9y=-9$$

$$x-y=-1$$

$$x+y=9$$

$$x-y=-1$$

$$2x=8 \Rightarrow x=4$$

$$4+y=9 \Rightarrow y=5 \quad sol: 45$$

EJERCICIO 199

1. $x \to Monedas\ de\ 20\ cts.$

$y \to Monedas\ de\ 10\ cts.$

$$x+y=78\quad(-10)$$

$$20x+10y=1.130$$

$$-10x-10y=-780$$

$$20x+10y=1.130$$

$$10x=350$$

$$x=35$$

$$35+y=78$$

$$y=43$$

$sol: 35\ Mon.\ de\ 20\ cts.$

$43\ Mon.\ de\ 10\ cts.$

2. $x \to Monedas\ de\ \$5$

$y \to Monedas\ de\ \$4$

$$x+y=91\quad(-4)$$

$$5x+4y=404$$

$$-4x-4y=-364$$

$$5x+4y=404$$

$$x=40$$

$$40+y=91$$

$$y=51$$

$sol: 40\ Mon.\ de \$5$

$51\ Mon.\ de \$4$

3. $x \to N^\circ\ de\ adultos$

$y \to N^\circ\ de\ niños$

$$x+y=700\quad(-15)$$

$$40x+15y=18.000$$

$$-15x-15y=-10.500$$

$$40x+15y=18.000$$

$$25x=7.500$$

$$x=300$$

$$300+y=700$$

$$y=400$$

$sol: 300\ adultos\ y\ 400\ niños$

4. $x \to Monedas\ de\ 20\ cts.$

$y \to Monedas\ de\ 25\ cts.$

$$x+y=44\quad(-20)$$

$$20x+25y=995$$

$$-20x-20y=-880$$

$$20x+25y=995$$

Continúa

4. Continuación

$$5y=115$$

$$y=23$$

$$x+23=44$$

$$x=21$$

$sol: 21\ Monedas\ de\ 20\ cts.$

$23\ Monedas\ de\ 25\ cts.$

5. $x \to Billetes\ de\ \$1$

$y \to Billetes\ de\ \$2$

$$x+y=287\quad(-1)$$

$$x+2y=419$$

$$-x-y=-287$$

$$x+2y=419$$

$$y=132$$

$$x+132=287$$

$$x=155$$

$sol: 155\ Billetes\ de\ \$1$

$132\ Billetes\ de\ \$2$

6. $x \to Libros\ de\ 3\ col.$

$y \to Libros\ de\ 7\ col.$

$$x+y=34\quad(-3)$$

$$3x+7y=174$$

$$-3x-3y=-102$$

$$3x+7y=174$$

$$4y=72$$

$$y=18$$

$$x+18=34$$

$$x=16$$

$sol: 16\ libros\ de\ 3\ col.$

$18\ libros\ de\ 7\ col.$

7. $x \to N^\circ\ de\ trajes$

$y \to N^\circ\ de\ sombreros$

$$x+y=54\quad(-45)$$

$$375x+45y=6.720$$

$$-45x-45y=-2.430$$

$$375x+45y=6.720$$

$$330x=4.290$$

$$x=13$$

$$13+y=54 \Rightarrow y=41$$

$sol: 13\ trajes$

$41\ sombreros$

EJERCICIO 200

1. $A-1=B+1$

$A-B=2$

$A+1=3(B-1)$

$A-3B=-4$

$A-B=2 \quad (-1)$

$\underline{A-3B=-4}$

$-A+B=-2$

$\underline{A-3B=-4}$

$-2B=-6$

$B=3$

$A-3=2$

$A=5$

$sol: A\to \$5 \quad B\to \$3$

2. $B-2=A+2$

$A-B=-4$

$2(A-2)=B+2$

$2A-B=6$

$A-B=-4 \quad (-1)$

$\underline{2A-B=6}$

$-A+B=4$

$\underline{2A-B=6}$

$A \quad =10$

$10-B=-4$

$-B=-14$

$B=14$

$sol: A\to 10\ soles$

$B\to 14\ soles$

3. $P\to\ Pedro$

$J\to\ Juan$

$P-3=J+3$

$P-J=6$

$4(J-3)=P+3$

$P-4J=-15$

$P-J=6 \quad (-1)$

$\underline{P-4J=-15}$

$-P+J=-6$

$\underline{P-4J=-15}$

$-3J=-21$

$J=7$

$P-7=6 \Rightarrow P=13$

$sol: \ Pedro\to \$13$

$Juan\to \$7$

4. $A-10=2(B-10)$

$A-2B=-10$

$\frac{3}{4}(A+10)=B+10$

$3(A+10)=4(B+10)$

$3A-4B=10$

$A-2B=-10 \quad (-2)$

$\underline{3A-4B=10}$

$-2A+4B=20$

$\underline{3A-4B=10}$

$A \quad =30$

$30-2B=-10$

$-2B=-40$

$B=20$

$sol: \ A\to 30\ Años$

$B\to 20\ Años$

5. $A-6=2(B-6)$

$A-2B=-6$

$A+6=\frac{8}{5}(B+6)$

$5(A+6)=8(B+6)$

$5A-8B=18$

$A-2B=-6 \quad (-4)$

$\underline{5A-8B=18}$

$-4A+8B=24$

$\underline{5A-8B=18}$

$A \quad =42$

$42-2B=-6$

$-2B=-48$

$B=24$

$sol: \ A\to 42\ Años$

$B\to 24\ Años$

6. $A-5=\frac{3}{2}(B-5)$

$2(A-5)=3(B-5)$

$2A-3B=-5$

$\frac{7}{9}(A+10)=B+10$

$7(A+10)=9(B+10)$

$7A-9B=20$

$2A-3B=-5 \quad (-3)$

$\underline{7A-9B=20}$

Continúa

6. Continuación

$-6A+9B=15$

$\underline{7A-9B=20}$

$A \quad =35$

$2(35)-3B=-5$

$70-3B=-5$

$B=25$

$sol: \ A\to\ 35\ Años$

$B\to\ 25\ Años$

7. $x\to\ Edad\ Hombre$

$y\to\ Edad\ Esposa$

$x=\frac{9y}{5}$

$\frac{3}{5}(x+4)=y+4$

$3x+12=5y+20$

$3x-5y=8$

$5x-9y=0 \quad (3)$

$\underline{3x-5y=8} \quad (-5)$

$15x-27y=0$

$\underline{-15x+25y=-40}$

$y=20$

$x=\frac{9}{5}(20)$

$x=9(4)$

$x=36$

$sol: 36\ Años\to\ tiene\ el\ hombre$

$20\ Años\to\ tiene\ la\ esposa$

8. $A-25=B+25$

$A-B=50$

$\frac{5}{17}(A+35)=B-35$

$5A+175=17B-595$

$5A-17B=-770$

$A-B=50 \quad (-5)$

$\underline{5A-17B=-770}$

$-5A+5B=-250$

$\underline{5A-17B=-770}$

$-12B=-1.020$

$B=85$

$A-85=50$

$A=135$

$sol: A\to\ 135\ Lempiras$

$B\to\ 85\ Lempiras$

9. $x \rightarrow$ *Edad padre*
$y \rightarrow$ *Edad hijo*

$$\frac{1}{5}(x-6)=y-6$$
$$x-6=5y-30$$
$$x-5y=-24$$
$$\frac{2}{5}(x+9)=y+9$$
$$2x+18=5y+45$$
$$2x-5y=27$$
$$x-5y=-24 \quad (-1)$$
$$2x-5y=27$$
$$-x+5y=24$$
$$2x-5y=27$$
$$x=51$$
$$51-5y=-24$$
$$-5y=-75$$
$$y=15$$

sol: 51 *Años* $\rightarrow$ *Edad padre*
15 *Años* $\rightarrow$ *Edad hijo*

10.
$$P+15=5(J-15)$$
$$P-5J=-90$$
$$3(P-20)=J+20$$
$$3P-J=80$$
$$P-5J=-90 \quad (-3)$$
$$3P-J=80$$
$$-3P+15J=270$$
$$3P-J=80$$
$$14J=350$$
$$J=25$$
$$3P-25=80$$
$$3P=105$$
$$P=35$$

sol: 35 *cts.* $\rightarrow$ *tenia Pedro*
25 *cts.* $\rightarrow$ *tenia Juan*

11.
$$A+\frac{B}{2}+60=4\left(B-\frac{B}{2}-60\right)$$
$$2A+B+120=8B-4B-480$$
$$2A+B+120=4B-480$$
$$2A-3B=-600$$
$$B+80-310=A-80$$
$$B-230=A-80$$
$$A-B=-150$$
$$2A-3B=-600$$
$$A-B=-150 \quad (-2)$$
$$2A-3B=-600$$
$$-2A+2B=300$$
$$B=300$$
$$A-300=-150$$
$$A=150 \text{ cts.}$$

sol: \$1,50 $\rightarrow$ *tiene A*
\$3 $\rightarrow$ *tiene B*

12. $x \rightarrow$ *Edad de Enrique*
$y \rightarrow$ *Edad hermana*

$$x-6=\frac{3}{2}(y-6)$$
$$2x-12=3y-18$$
$$2x-3y=-6$$
$$4(x+6)=5(y+6)$$
$$4x+24=5y+30$$
$$4x-5y=6$$
$$2x-3y=-6 \quad (-2)$$
$$4x-5y=6$$
$$-4x+6y=12$$
$$4x-5y=6$$
$$y=18$$
$$2x-3(18)=-6$$
$$2x-54=-6$$
$$2x=48$$
$$x=24$$

sol: 24 *Añ.* $\rightarrow$ *Ed. Enrique*
18 *Añ.* $\rightarrow$ *Ed. herm.*

EJERCICIO 201

1. $x \rightarrow$ *Veloc. bote agua tranq.*
$y \rightarrow$ *Veloc. del rio*
$x+y \rightarrow$ *Veloc. bote a favor del agua*
$x-y \rightarrow$ *Veloc. bote contra la corriente*

$$\frac{10}{x+y}=1$$
$$x+y=10$$
$$\frac{4}{x-y}=1$$
$$x-y=4$$
$$x+y=10$$
$$x-y=4$$
$$2x=14$$
$$x=7$$
$$7-y=4$$
$$y=3$$

sol: $7\,{}^{Km}\!/_{h}$ $\rightarrow$ *Veloc. bote agua tranq.*
$3\,{}^{Km}\!/_{h}$ $\rightarrow$ *Veloc. del rio*

2. $x \rightarrow$ *Veloc. bote agua tranq.*
$y \rightarrow$ *Veloc. del rio*
$x+y \rightarrow$ *Veloc. bote a favor del agua*
$x-y \rightarrow$ *Veloc. bote contra agua*

$$\frac{28}{x+y}=\frac{7}{4}$$
$$7x+7y=112$$
$$x+y=16$$
$$\frac{24}{x-y}=3$$
$$3x-3y=24$$
$$x-y=8$$
$$x+y=16$$
$$x-y=8$$
$$2x=24$$
$$x=12$$
$$12-y=8$$
$$y=4$$

sol: $12\,{}^{Km}\!/_{h}$ $\rightarrow$ *Bote*
$4\,{}^{Km}\!/_{h}$ $\rightarrow$ *Rio*

3. $x \rightarrow$ *Tiempo ida*
$y \rightarrow$ *Tiempo vuelta*

$$x+y=5$$
$$\frac{8}{x}=\frac{12}{y}$$
$$12x-8y=0$$
$$x+y=5 \quad (8)$$
$$12x-8y=0$$
$$8x+8y=40$$
$$20x=40$$
$$x=2$$
$$12(2)-8y=0$$
$$-8y=-24$$
$$y=3$$

sol: $2h \rightarrow$ *Tiempo ida*
$3h \rightarrow$ *Tiempo vuelta*

4. $x \to$ *Veloc. bote agua tranq.*

$y \to$ *Veloc. rio*

$x+y \to$ *Veloc. bote a favor de la cte*

$x-y \to$ *Veloc. bote en contra de la cte*

$$\frac{40}{x+y}=\frac{5}{2}$$

$5x+5y=80$

$x+y=16$

$$\frac{40}{x-y}=5$$

$5x-5y=40$

$x-y=8$

$x+y=16$

$\underline{x-y=\ 8}$

$2x \quad =24 \ \Rightarrow x=12$

$12-y=8 \ \Rightarrow y=4$

sol: $12h \to$ *Veloc. bote*

$4h \to$ *Veloc. rio*

5. $x \to$ *tiempo ida*

$y \to$ *tiempo vuelta*

$x+y=6$

$$\frac{20}{x}=\frac{40}{y}$$

$40x-20y=0$

$2x-y=0$

$x+y=6$

$\underline{2x-y=0}$

$3x \quad =6 \ \Rightarrow \ x=2$

$2(2)-y=0$

$-y=-4 \Rightarrow y=4$

sol: $2h \to$ *tiempo ida*

$4h \to$ *tiempo vuelta*

6. $x \to$ *tiempo ida*

$y \to$ *tiempo vuelta*

$x+y=5$

$$\frac{8}{x}=\frac{12}{y}$$

$12x-8y=0$

$3x-2y=0$

$x+\ y=5 \ (2)$

$\underline{3x-2y=0}$

$2x+2y=10$

$\underline{3x-2y=\ 0}$

$5x \quad =10$

$x=2$

$2+y=5$

$y=3$

sol: $2h \to$ *tiempo ida*

$3h \to$ *tiempo vuelta*

Entonces

$$\frac{32Km}{2h}=16\, Km/h \to \textit{Veloc.rio abajo}$$

$$\frac{12Km}{3h}=4\, Km/h \to \textit{Veloc.rio arriba}$$

Luego

$x \to$ *Veloc. bote*

$y \to$ *Veloc. rio*

$x+y=16$

$\underline{x-y=\ 4}$

$2x \quad =20$

$x=10$

$10+y=16$

$y=6$

sol: $10\, Km/h \to$ *Veloc. bote*

$6\, Km/h \to$ *Veloc. rio*

EJERCICIO 202

1. $x \to$ *N° mayor*

$y \to$ *N° medio*

$z \to$ *N° menor*

$$z-1=\frac{x+y}{3}$$

$3z-3=x+y$

$x+y-3z=-3$

$x+y+z=37$

$y-z=x-13$

$x-y+z=13$

$x+y+\ z=37 \ (-1)$

$\underline{x+y-3z=-3}$

$-x-y-z\ =-37$

$\underline{x+y-3z=-\ 3}$

$-4z=-40$

$z=10$

$x+y+z\ =\ 37$

$\underline{x-y+z\ =\ 13}$

$2x \quad +2z=50$

$2x+2(10)=50$

$2x=30$

$x=15$

$15+y+10=37$

$y+25=37$

$y=12$

sol: 15 , 12 , 10

2. $x \to$ *Precio kilo azúcar*

$y \to$ *Precio kilo café*

$z \to$ *Precio kilo fríjoles*

$5x+3y+4z=1{,}18$

$4x+5y+3z=1{,}45$

$2x+y+2z=0{,}46$

$5x+3y+4z=1{,}18 \ (-3)$

$\underline{4x+5y+3z=1{,}45 \ (4)}$

Continúa

2. Continuación

$-15x-\ 9y\ -12z=-3{,}54$

$\underline{16x+20y+12z=\ 5{,}80}$

$x\ +11y \qquad =\ 2{,}26$

$4x+5y+3z=1{,}45 \quad (2)$

$\underline{2x+\ y\ +2z=0{,}46} \ (-3)$

$8x+10y+6z=\ 2{,}90$

$\underline{-6x-\ 3y\ -6z=-1{,}38}$

Continúa

2. Continuación

$2x+\ 7y \qquad =1{,}52$

$x+11y=2{,}26 \quad (-2)$

$\underline{2x+\ 7y=1{,}52}$

$-2x-22y=-4{,}52$

$\underline{2x+\ 7y=\ \ 1{,}52}$

$-15y=-3$

$$y=\frac{1}{5}$$

$y=0{,}2$

Continúa

2. Continuación

$2x+7(0{,}2)=1{,}52$

$2x+1{,}4=1{,}52$

$2x=0{,}12$

$x=0{,}06$

$2(0{,}06)+0{,}2+2z=0{,}46$

$0{,}12+0{,}2+2z=0{,}46$

$2z=0{,}14$

$z=0{,}07$

sol: 6 *cts.* $\to$ *Kilo Az.*

20 *cts.* $\to$ *Kilo café*

7 *cts.* $\to$ *Kilo fríjoles*

3. $x \rightarrow$ *Cifra centenas*

$y \rightarrow$ *Cifra decenas*

$z \rightarrow$ *Cifra unidades*

$$x+y+z=15$$
$$x+y=\frac{3z}{2}$$
$$2x+2y=3z$$
$$2x+2y-3z=0$$
$$100x+10y+z-99=100z+10y+x$$
$$99x-99z=99$$
$$x-z=1$$
$$x+y+z=15 \quad (-2)$$
$$2x+2y-3z=0$$
$$-2x-2y-2z=-30$$
$$2x+2y-3z=0$$
$$-5z=-30$$
$$z=6$$
$$x-6=1$$
$$x=7$$
$$7+y+6=15$$
$$y+13=15$$
$$y=2 \qquad sol: 726$$

4. $x \rightarrow$ *N° mayor*

$y \rightarrow$ *N° medio*

$z \rightarrow$ *N° menor*

$$x+y+z=127$$
$$\frac{z}{2}+\frac{y}{3}+\frac{x}{9}=39$$
$$2x+6y+9z=702$$
$$x-4=\frac{y+z}{2}$$
$$2x-8=y+z$$
$$2x-y-z=8$$
$$x+y+z=127$$
$$2x-y-z=8$$
$$3x=135$$
$$x=45$$
$$2x+6y+9z=702$$
$$2x-y-z=8 \quad (9)$$
$$2x+6y+9z=702$$
$$18x-9y-9z=72$$
$$20x-3y=774$$

Continúa

4. Continuación

$$20(45)-3y=774$$
$$900-3y=774$$
$$-3y=-126$$
$$y=42$$
$$45+42+z=127$$
$$z=40$$

$sol:\ 45,\ 42,\ 40$

5. $x \rightarrow$ *Cifra centenas*

$y \rightarrow$ *Cifra decenas*

$z \rightarrow$ *Cifra unidades*

$$x+y+z=6$$
$$\frac{100x+10y+z}{x+y}=41$$
$$100x+10y+z=41x+41y$$
$$59x-31y+z=0$$
$$100x+10y+z+198=100z+10y+x$$
$$99x-99z=-198$$
$$x-z=-2$$
$$x+y+z=6 \quad (31)$$
$$59x-31y+z=0$$
$$31x+31y+31z=186$$
$$59x-31y+z=0$$
$$90x+32z=186$$
$$45x+16z=93$$
$$x-z=-2 \quad (-45)$$
$$45x+16z=93$$
$$-45x+45z=90$$
$$45x+16z=93$$
$$61z=183$$
$$z=3$$
$$x-3=-2$$
$$x=1$$
$$1+y+3=6$$
$$y+4=6$$
$$y=2 \qquad sol:\ 123$$

6. $x \rightarrow$ *Angulo mayor*

$y \rightarrow$ *Angulo medio*

$z \rightarrow$ *Angulo menor*

$$x+y+z=180$$
$$x-35=z$$
$$x-z=35$$

Continúa

6. Continuación

$$z-20=x-y$$
$$x-y-z=-20$$
$$x+y+z=180$$
$$x-y-z=-20$$
$$2x=160$$
$$x=80$$
$$80-z=35$$
$$z=45$$
$$80+y+45=180$$
$$y+125=180$$
$$y=55$$

$sol:\ 80^\circ, 55^\circ, 45^\circ$

7. $x \rightarrow$ *N° de vacas*

$y \rightarrow$ *N° de caballos*

$z \rightarrow$ *N° de terneros*

$$x+y+z=110$$
$$\frac{x}{8}+\frac{y}{9}+\frac{z}{5}=15$$
$$45x+40y+72z=5.400$$
$$z+x=65$$
$$x+y+z=110$$
$$x+z=65 \quad (-1)$$
$$x+y+z=110$$
$$-x-z=-65$$
$$y=45$$
$$45x+40y+72z=5.400$$
$$x+z=65 \quad (-45)$$
$$45x+40y+72z=5.400$$
$$-45x-45z=-2.925$$
$$40y+27z=2.475$$
$$40(45)+27z=2.475$$
$$1.800+27z=2.475$$
$$27z=675$$
$$z=25$$
$$x+45+25=110$$
$$x+70=110$$
$$x=40$$

sol: 40 *vacas*;

45 *caballos*;

25 *terneros*

8. $x \rightarrow$ *Cifra centenas*

$y \rightarrow$ *Cifra decenas*

$z \rightarrow$ *Cifra unidades*

$$x+y+z=10$$
$$x+y-4=z$$
$$x+y-z=4$$
$$x+z-6=y$$
$$x-y+z=6$$
$$\begin{array}{r} x+y+z=10 \\ \underline{x+y-z=4} \;(-1) \end{array}$$
$$\begin{array}{r} x+y+z=10 \\ \underline{-x-y+z=-4} \\ 2z=6 \\ z=3 \end{array}$$
$$\begin{array}{r} x+y+z=10 \\ \underline{x-y+z=6} \\ 2x+2z=16 \\ x+z=8 \end{array}$$
$$x+3=8$$
$$x=5$$
$$5+y+3=10$$
$$y=2 \quad sol: 523$$

9. $x \rightarrow$ *Angulo mayor*

$y \rightarrow$ *Angulo medio*

$z \rightarrow$ *Angulo menor*

$$x+y+z=180$$
$$x+y=135$$
$$y+z=110$$
$$\begin{array}{r} x+y+z=180 \\ \underline{y+z=110} \;(-1) \end{array}$$
$$\begin{array}{r} x+y+z=180 \\ \underline{-y-z=-110} \\ x=70 \end{array}$$
$$70+y=135$$
$$y=65$$
$$70+65+z=180$$
$$135+z=180$$
$$z=45$$
$$sol: 70^\circ,\ 65^\circ,\ 45^\circ$$

10. $A+B+C=140$

$$C=\frac{A}{2}$$
$$A-2C=0$$
$$A-10=B$$
$$A-B=10$$
$$\begin{array}{r} A+B+C=140 \\ \underline{A-B=10} \\ 2A+C=150 \end{array}$$
$$\begin{array}{r} 2A+C=150 \;(2) \\ \underline{A-2C=0} \\ 4A+2C=300 \\ \underline{A-2C=0} \\ 5A=300 \\ A=60 \end{array}$$
$$2(60)+C=150$$
$$C=30$$
$$60+B+30=140$$
$$B=50$$
$$sol: A \rightarrow 60bs.\ B \rightarrow 50bs.\ C \rightarrow 30bs.$$

11. $A-1=C+1$

$$A-C=2$$
$$B-1=C$$
$$B-C=1$$
$$A+5=2C$$
$$A-2C=-5$$
$$\begin{array}{r} A-C=2 \\ \underline{B-C=1} \\ A+B-2C=3 \end{array}$$
$$\begin{array}{r} A+B-2C=3 \\ \underline{A-2C=-5} \;(-1) \end{array}$$
$$\begin{array}{r} A+B-2C=3 \\ \underline{-A+2C=5} \\ B=8 \end{array}$$
$$8-C=1$$
$$C=7$$
$$A-7=2$$
$$A=9$$
$$sol: A \rightarrow \$9\ B \rightarrow \$8\ C \rightarrow \$7$$

12. $x+y+z=6$

$$100z+10y+x=\frac{41(100x+10y+z)}{107}$$
$$10.700z+1.070y+107x=4.100x+410y+41z$$
$$3.993x-660y-10.659z=0$$
$$1.331x-220y-3.553z=0$$
$$100x=300$$
$$x=3$$
$$\begin{array}{r} 1.331x-220y-3.553z=0 \\ \underline{x+y+z=6} \;(220) \end{array}$$
$$\begin{array}{r} 1.331x-220y-3.553z=0 \\ \underline{220x+220y+220z=1.320} \\ 1.551x-3.333z=1.320 \end{array}$$
$$1.551(3)-3.333z=1.320$$
$$4.653-3.333z=1.320$$
$$z=1$$
$$3+y+1=6$$
$$y+4=6$$
$$y=2 \quad sol: 321$$

13. $A-2=B+2$

$$A-B=4$$
$$B-1=C+1$$
$$B-C=2$$
$$A=\frac{8}{5}C$$
$$5A-8C=0$$
$$\begin{array}{r} A-B=4 \\ \underline{B-C=2} \\ A-C=6 \end{array}$$
$$\begin{array}{r} A-C=6 \;(-8) \\ \underline{5A-8C=0} \end{array}$$
$$\begin{array}{r} -8A+8C=-48 \\ \underline{5A-8C=0} \\ -3A=-48 \\ A=16 \end{array}$$
$$16-B=4$$
$$B=12$$
$$12-C=2$$
$$C=10$$
$$sol: A \rightarrow 16Q\ B \rightarrow 12Q\ C \rightarrow 10Q$$

14. $x \rightarrow Cifra\ centena$
$y \rightarrow Cifra\ decena$
$z \rightarrow Cifra\ unidades$
$100x = 400$
$x = 4$
$x + y + z = 9$
$$100z + 10y + x = \frac{16(100x + 10y + z)}{49}$$
$4.900z + 490y + 49x = 1.600x + 160y + 16z$
$1.551x - 330y - 4.884z = 0$
$517x - 110y - 1.628z = 0$
$517x - 110y - 1.628z = 0$
$x + y + z = 9 \quad (110)$
$517x - 110y - 1.628z = 0$
$110x + 110y + 110z = 990$
$627x - 1.518z = 990$
$627(4) - 1.518z = 990$
$2.508 - 1.518z = 990$
$-1.518z = -1.518$
$z = 1$
$4 + y + 1 = 9$
$y + 5 = 9$
$y = 4 \quad sol:\ 441$

15. $2A + B = C + 32$
$2A + B - C = 32$
$\frac{B}{3} + 2C = A + 9$
$B + 6C = 3A + 27$
$3A - B - 6C = -27$
$\frac{A + B}{3} = C - 1$
$A + B = 3C - 3$
$A + B - 3C = -3$
$2A + B - C = 32$
$3A - B - 6C = -27$
$5A - 7C = 5$
$3A - B - 6C = -27$
$A + B - 3C = -3$
$4A - 9C = -30$
$5A - 7C = 5 \quad (-4)$
$4A - 9C = -30 \quad (5)$
$-20A + 28C = -20$
$20A - 45C = -150$
$-17C = -170$

15. Continuación

$C = 10$
$5A - 7(10) = 5$
$5A = 75$
$A = 15$
$2(15) + B - 10 = 32$
$B + 20 = 32$
$B = 12$
$sol:\ 15,\ 12\ 10$

EJERCICIO 203

1. $x \rightarrow Ancho$
$y \rightarrow Largo$
$2(x + y) = 18$
$4y = 5x$
$5x - 4y = 0$
$x + y = 9 \quad (4)$
$5x - 4y = 0$
$4x + 4y = 36$
$5x - 4y = 0$
$9x = 36$
$x = 4$
$4 + y = 9$
$y = 5$
$sol:\ 5m \cdot 4m$

2. $A = 2B$
$A - 2B = 0$
$A - 12 = B + 12$
$A - B = 24$
Continúa

2. Continuación
$A - 2B = 0$
$A - B = 24 \quad (-1)$
$A - 2B = 0$
$-A + B = -24$
$B = 24$
$A - 24 = 24$
$A = 48$
$sol:\ A \rightarrow 48\ balboas$
$B \rightarrow 24\ balboas$

3. $x \rightarrow Ancho$
$y \rightarrow Largo$
$(x + 1)(y + 1) = 26 + xy$
$xy + x + y + 1 = 26 + xy$
$x + y = 25$
$(x + 2)(y - 3) = 19 + xy$
$xy - 3x + 2y - 6 = 19 + xy$
$-3x + 2y = 25$
Continúa

3. Continuación
$x + y = 25 \quad (3)$
$-3x + 2y = 25$
$3x + 3y = 75$
$-3x + 2y = 25$
$5y = 100$
$y = 20$
$x + 20 = 25$
$x = 5$
$sol:\ 20m \cdot 5m$

4. $x \rightarrow Precio\ carro$
$y \rightarrow Precio\ caballo$
$z \rightarrow Precio\ arreos$
$x + y + z = 200$
$x + z = y + 20$
$x - y + z = 20$
$y + z = x + 40$
$x - y - z = -40$
Continúa

4. Continuación
$x + y + z = 200$
$x - y - z = -40$
$2x = 160$
$x = 80$
$x + y + z = 200$
$x - y + z = 20$
$2x + 2z = 220$
$x + z = 110$
$80 + z = 110$
$z = 30$
$80 + y + 30 = 200$
$y + 110 = 200$
$y = 90$
$sol: \$80 \rightarrow Costo\ carro$
$\$90 \rightarrow Costo\ caballo$
$\$30 \rightarrow Costo\ arreos$

5. $x \rightarrow 1^{er}$ *Número*

$y \rightarrow 2^{o}$ *Número*

$z \rightarrow 3^{er}$ *Número*

$x+y=z+18$

$x+y-z=18$

$x+z-78=y$

$x-y+z=78$

$y+z=x+102$

$x-y-z=-102$

$$\begin{array}{l} x+y-z=18 \\ \underline{x-y+z=78} \\ 2x \qquad =96 \\ \qquad x=48 \end{array}$$

$$\begin{array}{l} x-y+z= \quad 78 \\ \underline{x-y-z=-102} \\ 2x-2y=-24 \end{array}$$

$x-y=-12$

$48-y=-12$

$y=60$

$48+60-z=18$

$108-18=z$

$90=z$

sol: 48 , 60 , 90

6. $x \rightarrow$ *Cifra decenas*

$y \rightarrow$ *Cifra unidades*

$x+y=6$

$10x+y-36=10y+x$

$9x-9y=36$

$x-y=4$

$$\begin{array}{l} x+y=6 \\ \underline{x-y=4} \\ 2x \quad =10 \end{array}$$

$x=5$

$5+y=6$

$y=1$ *sol*: 51

7. $x \rightarrow$ *Veloc. pájaro*

$y \rightarrow$ *Veloc. viento*

$x+y \rightarrow$ *Veloc. pájaro a favor*

$x-y \rightarrow$ *Veloc. pájaro contra*

$\frac{55}{x+y}=1$

$x+y=55$

$\frac{25}{x-y}=1$

Continúa

7. Continuación

$$\begin{array}{l} x-y=25 \\ \underline{x+y=55} \\ 2x \quad =80 \end{array}$$

$x=40$

$40-y=25$

$y=15$

sol: $40 \, {}^{Km}\!/\!_{h} \rightarrow$ *Veloc. del pájaro tranq.*

$15 \, {}^{Km}\!/\!_{h} \rightarrow$ *Veloc. del viento*

8. $x \rightarrow N^{o}$ *de libros*

$y \rightarrow$ *Precio de c / u*

$xy=(x+5)(y-2)$

$xy=xy-2x+5y-10$

$2x-5y=-10$

$xy=(x-5)(y+4)$

$xy=xy+4x-5y-20$

$4x-5y=20$

$$\begin{array}{l} 2x-5y=-10 \\ \underline{4x-5y= \ \ 20} \quad (-1) \end{array}$$

$$\begin{array}{l} 2x-5y=-10 \\ \underline{-4x+5y=-20} \\ -2x \qquad =-30 \end{array}$$

$x=15$

$2(15)-5y=-10$

$30-5y=-10$

$-5y=-40$

$y=8$

sol: 15 → *Libros compró*

$ 8 → *costó c / u*

9. $x \rightarrow$ *Precio kilo café*

$y \rightarrow$ *Precio kilo té*

$7x+6y=4,8 \quad (-9)$

$\underline{8x+9y=6,45} \quad (6)$

$$\begin{array}{l} -63x-54y=-43,2 \\ \underline{\ 48x+54y= \ \ 38,7} \\ -15x \qquad =-4,5 \end{array}$$

$x=0,3$

Continúa

9. Continuación

$7(0,3)+6y=4,8$

$2,1+6y=4,8$

$6y=2,7$

$y=0,45$

sol: 30 *cts.* → *Costo kilo café*

45 *cts.* → *Costo kilo té*

10. $x \rightarrow$ *trajes de* $40

$y \rightarrow$ *trajes de* $35

$x + y = 50 \quad (-40)$

$\underline{40x+35y=1.910}$

$$\begin{array}{l} -40x-40y=-2.000 \\ \underline{\ 40x+35y= \ \ 1.910} \\ \qquad -5y=-90 \end{array}$$

$y=18$

$x+18=50$

$x=32$

sol: 32 → *trajes de* $40

18 → *trajes de* $35

11. $x \rightarrow$ *Numerador*

$y \rightarrow$ *Denominador*

$\frac{x-1}{y}=\frac{1}{3}$

$3x-3=y$

$3x-y=3$

$\frac{x}{y-2}=\frac{1}{2}$

$2x=y-2$

$-2x+y=2$

$$\begin{array}{l} 3x-y=3 \\ \underline{-2x+y=2} \\ x \qquad =5 \end{array}$$

$3(5)-y=3$

$-y=-12$

$y=12$ *sol*: $\frac{5}{12}$

12. $x \rightarrow 1^{a}$ *bolsa*

$y \rightarrow 2^{a}$ *bolsa*

$x+y=200$

$x-15=y+15$

$x-y=30$

Continúa

12. Continuación

$x+y=200$
$x-y=\ 30$
$2x\quad =230$
$x=115$
$115+y=200$
$y=85$
sol: 115 *soles* $\rightarrow 1^a$ *bolsa*
85 *soles* $\rightarrow 2^a$ *bolsa*

13. $x \rightarrow$ *Precio caballo*
$y \rightarrow$ *Precio coche*
$x+20=3y$
$x-3y=-20$
$20+y=\frac{3x}{5}$
$3x=100+5y$
$3x-5y=100$
$x-3y=-20\ (-3)$
$3x-5y=100$
$-3x+9y=\ 60$
$3x-5y=100$
$4y=160$
$y=40$
$x-3(40)=-20$
$x=100$
sol: 100 *soles* $\rightarrow$ *Costó el caballo*
40 *soles* $\rightarrow$ *Costó el coche*

14. $x \rightarrow$ *Cifra decenas*
$y \rightarrow$ *Cifra unidades*
$10x+y=6(x+y)$
$10x+y=6x+6y$
$4x-5y=0$
$10x+y-9=10y+x$
$9x-9y=9$
$x-y=1$
$4x-5y=0$
$x-\ y=1\ \ (-5)$
$4x-5y=\ 0$
$-5x+5y=-5$
$-x\quad =-5$
$x=5$
$5-y=1$
$y=4$ *sol*: 54

15. $x \rightarrow$ *N° de personas*
$y \rightarrow$ *Precio c/u*
$xy=(x+10)(y-5)$
$xy=xy-5x+10y-50$
$5x-10y=-50$
$x-2y=-10$
$xy=(x-6)(y+5)$
$xy=xy+5x-6y-30$
$5x-6y=30$
$5x-6y=\ 30$
$x-2y=-10\ \ (-3)$
$5x-6y=\ 30$
$-3x+6y=30$
$2x\quad =60$
$x=30$
$30-2y=-10$
$-2y=-40$
$y=20$
sol: 30 $\rightarrow$ *Personas*
20*bs.* $\rightarrow$ *c/u*

16. $A+B=1.080$
$A-\frac{2A}{5}=B-\frac{B}{4}$
$20A-8A=20B-5B$
$12A-15B=0$
$4A-5B=0$
$A+\ \ B=1.080\ (5)$
$4A-5B=\ \ 0$
$5A+5B=5.400$
$4A-5B=\ \ 0$
$9A\quad =5.400$
$A=600$
$600+B=1.080$
$B=480$
sol: $A \rightarrow$ 600 *sucres*
$B \rightarrow$ 480 *sucres*

17. $x \rightarrow$ *gané ayer*
$y \rightarrow$ *gané hoy*
$y+10=x$
$y=\frac{5x}{6}$
$5x-6y=0$
$5x-6y=\ 0$
$x-\ y=10\ \ (-6)$

Continúa

17. Continuación

$5x-6y=\ 0$
$-6x+6y=-60$
$-x\quad =-60$
$x=60$
$60-y=10$
$y=50$
sol: \$ 60 $\rightarrow$ *gané ayer*
\$ 50 $\rightarrow$ *gané hoy*

18. $x \rightarrow 1^{er}$ *nùmero*
$y \rightarrow 2^{o}$ *nùmero*
$\frac{x}{y}=\frac{3}{5}$
$\frac{x-10}{y-10}=\frac{1}{2}$
$5x-3y=\ 0$
$2x-\ y=10\ (-3)$
$5x-3y=\ \ 0$
$-6x+3y=-30$
$-x\quad =-30$
$x=30$
$2(30)-y=10$
$60-y=10$
$y=50$
sol: 30 , 50

19. $A+4=B-4$
$A-B=-8$
$B+4=\frac{9}{5}(A-4)$
$5B+20=9A-36$
$9A-5B=56$
$A-\ B=-8\ \ (-5)$
$9A-5B=56$
$-5A+5B=40$
$9A-5B=56$
$4A\quad =96$
$A=24$
$24-B=-8$
$B=32$
sol: $A \rightarrow$ 24 *lempiras*
$B \rightarrow$ 32 *lempiras*

20. $A-20=2(B-20)$

$A-2B=-20$

$A+30=\frac{9}{7}(B+30)$

$7A+210=9B+270$

$7A-9B=60$

$A-2B=-20 \quad (-7)$

$7A-9B=60$

$-7A+14B=140$

$7A-9B=60$

$5B=200$

$B=40$

$A-2(40)=-20$

$A-80=-20$

$A=60$

$sol:\ A \to 60\ Años$

$B \to 40\ Años$

21. $x \to$ *tiempo ida*

$y \to$ *tiempo vuelta*

$x+y=3$

$\frac{4}{x}=\frac{8}{y}$

$8x-4y=0$

$2x-y=0$

$x+y=3$

$2x-y=0$

$3x=3$

$x=1$

$1+y=3$

$y=2$

Si $1h \to$ *tiempo de ida*

$2h \to$ *tiempo de vuelta*

Entonces

$\frac{16Km}{1h}=16\,{}^{Km}/_{h} \to$ *Veloc. a favor*

$\frac{16Km}{2h}=8\,{}^{Km}/_{h} \to$ *Veloc. en contra*

Luego

$x \to$ *Veloc. del bote*

$y \to$ *Veloc. delrio*

$x+y=16$

$x-y=8$

$2x=24$

$x=12$

Continúa

21. Continuación

$12+y=16$

$y=4$

$sol: 12\,{}^{Km}/_{h} \to$ *Veloc. bote*

$4\,{}^{Km}/_{h} \to$ *Veloc. rio*

22. $\frac{A}{9}-2=\frac{B}{5}$

$5A-90=9B$

$5A-9B=90$

$2B=A-15$

$A-2B=15$

$5A-9B=90$

$A-2B=15 \quad (-5)$

$5A-9B=90$

$-5A+10B=-75$

$B=15$

$A-2(15)=15$

$A-30=15$

$A=45$

$sol:\ A \to 45\ Años$

$B \to 15\ Años$

23. $5A-4=4B$

$5A-4B=4$

$7A-2=6B$

$7A-6B=2$

$5A-4B=4 \quad (6)$

$7A-6B=2 \quad (-4)$

$30A-24B=24$

$-28A+24B=-8$

$2A=16$

$A=8$

$5(8)-4B=4$

$40-4B=4$

$-4B=-36$

$B=9$

$sol:\ A \to 8\,Km\ \ B \to 9\,Km$

24. $x \to$ *Cifra decenas*

$y \to$ *Cifra unidades*

$y-x=4$

$10x+y+10y+x=66$

$11x+11y=66$

$x+y=6$

Continúa

24. Continuación

$-x+y=4$

$x+y=6$

$2y=10$

$y=5$

$x+5=6$

$x=1$

sol: 15

25. $x \to Largo$

$y \to Ancho$

$2(x+y)=58$

$2x+2y=58$

$x+y=29$

$(x+2)(y-2)=xy-46$

$xy-2x+2y-4=xy-46$

$-2x+2y=-42$

$x-y=21$

$x+y=29$

$x-y=21$

$2x=50$

$x=25$

$25+y=29$

$y=4$

$sol:\ 25m \cdot 4m$

26. $x \to Largo$

$y \to Ancho$

$2x+2y=56$

$x+y=28$

$(y-2)(x+2)=xy$

$xy+2y-2x-4=xy$

$2x-2y=-4$

$x-y=-4$

$x+y=28$

$x-y=-4$

$2x=24$

$x=12$

$12+y=28$

$y=16$

$sol:\ 16m \cdot 12m$

EJERCICIO 204

1. $m=6 \quad n=3$

$^{3}A_6=6\cdot 5....(6-3+1)$

$^{3}A_6=6\cdot 5\cdot 4$

$^{3}A_6=120$

2. $P_5=5!$

$P_5=5\cdot 4\cdot 3\cdot 2$

$P_5=120$

3. $m=7 \quad n=5$

$^{5}C_7=\frac{^{5}A_7}{P_5}=\frac{7\cdot 6....(7-5+1)}{5!}$

$^{5}C_7=\frac{7\cdot 6\cdot 5\cdot 4\cdot 3}{5\cdot 4\cdot 3\cdot 2\cdot 1}$

$^{5}C_7=\frac{2.520}{120}$

$^{5}C_7=21$

4. $m=6 \quad n=2$

$^{2}A_6=6\cdot 5....(6-2+1)$

$^{2}A_6=6\cdot 5$

$^{2}A_6=30$

5. $m=5 \quad n=3$

$^{3}A_5=5\cdot 4....(5-3+1)$

$^{3}A_5=5\cdot 4\cdot 3$

$^{3}A_5=60$

6. $m=12 \quad n=5$

$^{5}C_{12}=\frac{^{5}A_{12}}{P_5}=\frac{12\cdot 11....(12-5+1)}{5!}$

$^{5}C_{12}=\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1}$

$^{5}C_{12}=3\cdot 11\cdot 2\cdot 3\cdot 4$

$^{5}C_{12}=792$

7. $P_7=7!$

$P_7=7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2$

$P_7=5.040$

8. $m=7 \quad n=4$

$^{4}C_7=\frac{^{4}A_7}{P_4}=\frac{7\cdot 6....(7-4+1)}{4!}$

$^{4}C_7=\frac{7\cdot 6\cdot 5\cdot 4}{4\cdot 3\cdot 2\cdot 1}$

$^{4}C_7=7\cdot 5$

$^{4}C_7=35$

9. $P_{5-1}=P_4$

$P_4=4!$

$P_4=4\cdot 3\cdot 2$

$P_4=24$

10. $P_6=6!$

$P_6=6\cdot 5\cdot 4\cdot 3\cdot 2$

$P_6=720$

11. *Siendo el 1º*

$P_6=6!$

$P_6=6\cdot 5\cdot 4\cdot 3\cdot 2$

$P_6=720$

sin *lugar fijo*

$P_7=7!$

$P_7=7P_6$

$P_7=7\cdot 720$

$P_7=5.040$

12. *En un banco*

$P_6=720$

En una mesa redonda

$P_{6-1}=P_5$

$P_5=5!$

$P_5=120$

13. $m=9 \quad n=3$

$^{3}A_9=9\cdot 8....(9-3+1)$

$^{3}A_9=9\cdot 8\cdot 7$

$^{3}A_9=504$

14. $P_{4-1}=P_3$

$P_3=3!$

$P_3=6$

15. $m=5 \quad n=3$

$^{3}C_5=\frac{^{3}A_5}{P_3}=\frac{5\cdot 4....(5-3+1)}{3!}$

$^{3}C_5=\frac{5\cdot 4\cdot 3}{3\cdot 2}$

$^{3}C_5=5\cdot 2$

$^{3}C_5=10$

16. $P_{5-2}=P_3$

$P_3=3!$

$P_3=3\cdot 2$

$P_3=6$

17. $m=5 \quad n=3$

$^{3}A_5=5\cdot 4....(5-3+1)$

$^{3}A_5=5\cdot 4\cdot 3$

$^{3}A_5=60$

18. $P_{11-1}=P_{10}$

$P_{10}=10!$

$P_{10}=10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2$

$P_{10}=3.628.800$

19. $m=8 \quad n=3$

$^{3}C_8=\frac{^{3}A_8}{P_3}=\frac{8\cdot 7....(8-3+1)}{3!}$

$^{3}C_8=\frac{8\cdot 7\cdot 6}{3\cdot 2}$

$^{3}C_8=4\cdot 7\cdot 2$

$^{3}C_8=56$

20. $m=6 \quad n=3$

$^{5}A_6=6\cdot 5....(6-3+1)$

$^{5}A_6=6\cdot 5\cdot 4$

$^{5}A_6=120$

21. *Palabras distintas*

$P_8=8!$

$P_8=8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2$

$P_8=40.320$

Vocales fijas

$P_5=5!$

$P_5=120$

22. $P_{5-1}=P_4$

$P_4=4!$

$P_4=4\cdot 3\cdot 2$

$P_4=24$

EJERCICIO 205

1. $(4a^2)^2 = 16a^4$

2. $(-5a)^3 = -125a^3$

3. $(3xy)^3 = 27x^3y^3$

4. $(-6a^2b)^2 = 36a^4b^2$

5. $(-2x^2y^3)^3 = -8x^6y^9$

6. $(4a^2b^3c^4)^3 = 64a^6b^9c^{12}$

7. $(-6x^4y^5)^2 = 36x^8y^{10}$

8. $(-7ab^3c^4)^3 = -343a^3b^9c^{12}$

9. $(a^mb^n)^x = a^{mx}b^{nx}$

10. $(-2x^3y^5z^6)^4 = 16x^{12}y^{20}z^{24}$

11. $(-3m^3n)^3 = -27m^9n^3$

12. $(a^2b^3c)^m = a^{2m}b^{3m}c^m$

13. $(-m^2nx^3)^4 = m^8n^4x^{12}$

14. $(-3a^2b)^5 = -243a^{10}b^5$

15. $(7x^5y^6z^8)^2 = 49x^{10}y^{12}z^{16}$

16. $\left(-\frac{x}{2y}\right)^2 = \frac{x^2}{4y^2}$

17. $\left(-\frac{2m}{n^2}\right)^3 = -\frac{8m^3}{n^6}$

18. $\left(\frac{ab^2}{5}\right)^3 = \frac{a^3b^6}{125}$

19. $\left(-\frac{3x^2}{4y}\right)^2 = \frac{9x^4}{16y^2}$

20. $\left(-\frac{2ab^2}{3m^3}\right)^4 = \frac{16a^4b^8}{81m^{12}}$

21. $\left(\frac{2m^3n}{3x^4}\right)^5 = \frac{32m^{15}n^5}{243x^{20}}$

22. $\left(-\frac{3a^3b^2}{4}\right)^2 = \frac{9a^6b^4}{16}$

23. $\left(-\frac{mn^2}{3}\right)^4 = \frac{m^4n^8}{81}$

24. $\left(-\frac{a^2b^4}{2}\right)^5 = -\frac{a^{10}b^{20}}{32}$

EJERCICIO 206

1. $(a^5+7b^4)^2$
$= a^{10} + 2a^5(7b^4) + 49b^8$
$= a^{10} + 14a^5b^4 + 49b^8$

2. $(3x^4-5xy^3)^2$
$= 9x^8 - 2(3x^4)(5xy^3) + 25x^2y^6$
$= 9x^8 - 30x^5y^3 + 25x^2y^6$

3. $(a^2b^3-a^5)^2$
$= a^4b^6 - 2a^2b^3(a^5) + a^{10}$
$= a^4b^6 - 2a^7b^3 + a^{10}$

4. $(7x^5-8x^3y^4)^2$
$= 49x^{10} - 2(7x^5)(8x^3y^4) + 64x^6y^8$
$= 49x^{10} - 112x^8y^4 + 64x^6y^8$

5. $(9ab^2+5a^2b^3)^2$
$= 81a^2b^4 + 2(9ab^2)(5a^2b^3) + 25a^4b^6$
$= 81a^2b^4 + 90a^3b^5 + 25a^4b^6$

6. $(3x^2y^3-7x^3y^2)^2$
$= 9x^4y^6 - 2(3x^2y^3)(7x^3y^2) + 49x^6y^4$
$= 9x^4y^6 - 42x^5y^5 + 49x^6y^4$

7. $(xy-a^2b^2)^2$
$= x^2y^2 - 2(xy)(a^2b^2) + a^4b^4$
$= x^2y^2 - 2a^2b^2xy + a^4b^4$

8. $\left(\frac{x}{2}+\frac{2y}{3}\right)^2$
$= \frac{x^2}{4} + 2\left(\frac{x}{2}\right)\left(\frac{2y}{3}\right) + \frac{4y^2}{9}$
$= \frac{x^2}{4} + \frac{2xy}{3} + \frac{4y^2}{9}$

9. $\left(\frac{3a^2}{4}-\frac{2b^2}{5}\right)^2$
$= \frac{9a^4}{16} - 2\left(\frac{3a^2}{4}\right)\left(\frac{2b^2}{5}\right) + \frac{4b^4}{25}$
$= \frac{9a^4}{16} - \frac{3a^2b^2}{5} + \frac{4b^4}{25}$

10. $\left(\frac{5x^3}{6}+\frac{3xy^2}{5}\right)^2$
$= \frac{25x^6}{36} + 2\left(\frac{5x^3}{6}\right)\left(\frac{3xy^2}{5}\right) + \frac{9x^2y^4}{25}$
$= \frac{25x^6}{36} + x^4y^2 + \frac{9x^2y^4}{25}$

11. $\left(\frac{a^5}{9}-\frac{3a^3b^7}{7}\right)^2$
$= \frac{a^{10}}{81} - 2\left(\frac{a^5}{9}\right)\left(\frac{3a^3b^7}{7}\right) + \frac{9a^6b^{14}}{49}$
$= \frac{a^{10}}{81} - \frac{2a^8b^7}{21} + \frac{9a^6b^{14}}{49}$

12. $\left(\frac{2m^4}{5}-\frac{5n^3}{4}\right)^2$
$= \frac{4m^8}{25} - 2\left(\frac{2m^4}{5}\right)\left(\frac{5n^3}{4}\right) + \frac{25n^6}{16}$
$= \frac{4m^8}{25} - m^4n^3 + \frac{25n^6}{16}$

13. $\left(\frac{x}{3}+\frac{y^2}{4}\right)^2$
$= \frac{x^2}{9} + 2\left(\frac{x}{3}\right)\left(\frac{y^2}{4}\right) + \frac{y^4}{16}$
$= \frac{x^2}{9} + \frac{xy^2}{6} + \frac{y^4}{16}$

14. $\left(\frac{2x}{3}-\frac{3y}{5}\right)^2$
$= \frac{4x^2}{9} - 2\left(\frac{2x}{3}\right)\left(\frac{3y}{5}\right) + \frac{9y^2}{25}$
$= \frac{4x^2}{9} - \frac{4xy}{5} + \frac{9y^2}{25}$

15. $\left(\frac{a^3}{8}+\frac{4a^2}{7b}\right)^2$

$=\frac{a^6}{64}+2\left(\frac{a^3}{8}\right)\left(\frac{4a^2}{7b}\right)+\frac{16a^4}{49b^2}=\frac{a^6}{64}+\frac{a^5}{7b}+\frac{16a^4}{49b^2}$

16. $\left(\frac{3}{2x}-\frac{2x^4}{3}\right)^2$

$=\frac{9}{4x^2}-2\left(\frac{3}{2x}\right)\left(\frac{2x^4}{3}\right)+\frac{4x^8}{9}=\frac{9}{4x^2}-2x^3+\frac{4x^8}{9}$

17. $\left(\frac{5x^7}{6y^4}-\frac{3y^6}{10x^2}\right)^2=\frac{25x^{14}}{36y^8}-2\left(\frac{5x^7}{6y^4}\right)\left(\frac{3y^6}{10x^2}\right)+\frac{9y^{12}}{100x^4}$

$=\frac{25x^{14}}{36y^8}-\frac{x^5y^2}{2}+\frac{9y^{12}}{100x^4}$

18. $\left(\frac{3a^6}{8}-\frac{4a^2}{9b^5}\right)^2=\frac{9a^{12}}{64}-2\left(\frac{3a^6}{8}\right)\left(\frac{4a^2}{9b^5}\right)+\frac{16a^4}{81b^{10}}$

$=\frac{9a^{12}}{64}-\frac{a^8}{3b^5}+\frac{16a^4}{81b^{10}}$

EJERCICIO 207

1. $(2a+3b)^3=8a^3+3(4a^2)(3b)+3(2a)(9b^2)+27b^3=8a^3+36a^2b+54ab^2+27b^3$

2. $(4a-3b^2)^3=64a^3-3(16a^2)(3b^2)+3(4a)(9b^4)-27b^6=64a^3-144a^2b^2+108ab^4-27b^6$

3. $(5x^2+6y^3)^3=125x^6+3(25x^4)(6y^3)+3(5x^2)(36y^6)+216y^9=125x^6+450x^4y^3+540x^2y^6+216y^9$

4. $(4x^3-3xy^2)^3=64x^9-3(16x^6)(3xy^2)+3(4x^3)(9x^2y^4)-27x^3y^6=64x^9-144x^7y^2+108x^5y^4-27x^3y^6$

5. $(7a^4-5a^2b^3)^3$

$=343a^{12}-3(49a^8)(5a^2b^3)+3(7a^4)(25a^4b^6)-125a^6b^9=343a^{12}-735a^{10}b^3+525a^8b^6-125a^6b^9$

6. $(a^8+9a^5x^4)^3=a^{24}+3(a^{16})(9a^5x^4)+3(a^8)(81a^{10}x^8)+729a^{15}x^{12}=a^{24}+27a^{21}x^4+243a^{18}x^8+729a^{15}x^{12}$

7. $(8x^4-7x^2y^4)^3$

$=512x^{12}-3(64x^8)(7x^2y^4)+3(8x^4)(49x^4y^8)-343x^6y^{12}=512x^{12}-1.344x^{10}y^4+1.176x^8y^8-343x^6y^{12}$

8. $(3a^2b-5a^3b^2)^3$

$=27a^6b^3-3(9a^4b^2)(5a^3b^2)+3(3a^2b)(25a^6b^4)-125a^9b^6=27a^6b^3-135a^7b^4+225a^8b^5-125a^9b^6$

9. $\left(\frac{1}{2}a+\frac{2b^2}{3}\right)^3=\frac{a^3}{8}+3\left(\frac{a^2}{4}\right)\left(\frac{2b^2}{3}\right)+3\left(\frac{a}{2}\right)\left(\frac{4b^4}{9}\right)+\frac{8b^6}{27}=\frac{a^3}{8}+\frac{a^2b^2}{2}+\frac{2ab^4}{3}+\frac{8b^6}{27}$

10. $\left(\frac{3a^2}{4}-\frac{4b^2}{5}\right)^3=\frac{27a^6}{64}-3\left(\frac{9a^4}{16}\right)\left(\frac{4b^2}{5}\right)+3\left(\frac{3a^2}{4}\right)\left(\frac{16b^4}{25}\right)-\frac{64b^6}{125}=\frac{27a^6}{64}-\frac{27a^4b^2}{20}+\frac{36a^2b^4}{25}-\frac{64b^6}{125}$

11. $\left(\frac{5a^2b}{6}-\frac{3b^4}{10}\right)^3$

$=\frac{125a^6b^3}{216}-3\left(\frac{25a^4b^2}{36}\right)\left(\frac{3b^4}{10}\right)+3\left(\frac{5a^2b}{6}\right)\left(\frac{9b^8}{100}\right)-\frac{27b^{12}}{1.000}=\frac{125a^6b^3}{216}-\frac{5a^4b^6}{8}+\frac{9a^2b^9}{40}-\frac{27b^{12}}{1.000}$

12. $\left(\frac{7x^5}{8}-\frac{4y^6}{7}\right)^3$

$=\frac{343x^{15}}{512}-3\left(\frac{49x^{10}}{64}\right)\left(\frac{4y^6}{7}\right)+3\left(\frac{7x^5}{8}\right)\left(\frac{16y^{12}}{49}\right)+\frac{64y^{18}}{343}=\frac{343x^{15}}{512}-\frac{21x^{10}y^6}{16}+\frac{6x^5y^{12}}{7}+\frac{64y^{18}}{343}$

13. $\left(\dfrac{x}{2y}+\dfrac{3y}{x^2}\right)^3=\dfrac{x^3}{8y^3}+3\left(\dfrac{x^2}{4y^2}\right)\left(\dfrac{3y}{x^2}\right)+3\left(\dfrac{x}{2y}\right)\left(\dfrac{9y^2}{x^4}\right)+\dfrac{27y^3}{x^6}=\dfrac{x^3}{8y^3}+\dfrac{9}{4y}+\dfrac{27y}{2x^3}+\dfrac{27y^3}{x^6}$

14. $\left(\dfrac{2a^2}{5}-\dfrac{5}{2b^3}\right)^3=\dfrac{8a^6}{125}-3\left(\dfrac{4a^4}{25}\right)\left(\dfrac{5}{2b^3}\right)+3\left(\dfrac{2a^2}{5}\right)\left(\dfrac{25}{4b^6}\right)-\dfrac{125}{8b^9}=\dfrac{8a^6}{125}-\dfrac{6a^4}{5b^3}+\dfrac{15a^2}{2b^6}-\dfrac{125}{8b^9}$

15. $\left(4x^4-\dfrac{3x}{y^3}\right)=64x^{12}-3\left(16x^8\right)\left(\dfrac{3x}{y^3}\right)+3\left(4x^4\right)\left(\dfrac{9x^2}{y^6}\right)-\dfrac{27x^3}{y^9}=64x^{12}-\dfrac{144x^9}{y^3}+\dfrac{108x^6}{y^6}-\dfrac{27x^3}{y^9}$

16. $\left(\dfrac{3a}{2b}+\dfrac{4b^2}{5}\right)^3=\dfrac{27a^3}{8b^3}+3\left(\dfrac{9a^2}{4b^2}\right)\left(\dfrac{4b^2}{5}\right)+3\left(\dfrac{3a}{2b}\right)\left(\dfrac{16b^4}{25}\right)+\dfrac{64b^6}{125}=\dfrac{27a^3}{8b^3}+\dfrac{27a^2}{5}+\dfrac{72ab^3}{25}+\dfrac{64b^6}{125}$

17. $\left(\dfrac{7}{8}-x^4y^5\right)^3=\dfrac{343}{512}-3\left(\dfrac{49}{64}\right)x^4y^5+3\left(\dfrac{7}{8}\right)x^8y^{10}-x^{12}y^{15}=\dfrac{343}{512}-\dfrac{147x^4y^5}{64}+\dfrac{21x^8y^{10}}{8}-x^{12}y^{15}$

18. $\left(\dfrac{m^3}{6}-\dfrac{6n^2}{m^2}\right)^3=\dfrac{m^9}{216}-3\left(\dfrac{m^6}{36}\right)\left(\dfrac{6n^2}{m^2}\right)+3\left(\dfrac{m^3}{6}\right)\left(\dfrac{36n^4}{m^4}\right)-\dfrac{216n^6}{m^6}=\dfrac{m^9}{216}-\dfrac{m^4n^2}{2}+\dfrac{18n^4}{m}-\dfrac{216n^6}{m^6}$

EJERCICIO 208

1. x^2-2x+1

$=x^4+4x^2+1+2x^2\left(-2x\right)+2x^2\left(1\right)+2\left(-2x\right)\left(1\right)=x^4+4x^2+1-4x^3+2x^2-4x=x^4-4x^3+6x^2-4x+1$

2. $2x^2+x+1$

$=4x^4+x^2+1+2\left(2x^2\right)\left(x\right)+2\left(2x^2\right)\left(1\right)+2\left(x\right)\left(1\right)=4x^4+x^2+1+4x^3+4x^2+2x=4x^4+4x^3+5x^2+2x+1$

3. $x^2-5x+2=x^4+25x^2+4+2\left(x^2\right)\left(-5x\right)+2\left(x^2\right)\left(2\right)+2\left(-5x\right)\left(2\right)=x^4+25x^2+4-10x^3+4x^2-20x$

$=x^4-10x^3+29x^2-20x+4$

4. $x^3-5x^2+6=x^6+25x^4+36+2\left(x^3\right)\left(-5x^2\right)+2\left(x^3\right)\left(6\right)+2\left(-5x^2\right)\left(6\right)=x^6+25x^4+36-10x^5+12x^3-60x^2$

$=x^6-10x^5+25x^4+12x^3-60x^2+36$

5. $4a^4-3a^2+5=16a^8+9a^4+25+2\left(4a^4\right)\left(-3a^2\right)+2\left(4a^4\right)\left(5\right)+2\left(-3a^2\right)\left(5\right)$

$=16a^8+9a^4+25-24a^6+40a^4-30a^2=16a^8-24a^6+49a^4-30a^2+25$

6. $x+2y-z=x^2+4y^2+z^2+2\left(x\right)\left(2y\right)+2\left(x\right)\left(-z\right)+2\left(2y\right)\left(-z\right)=x^2+4y^2+z^2+4xy-2xz-4yz$

7. $3-x^3-x^6$

$=9+x^6+x^{12}+2\left(3\right)\left(-x^3\right)+2\left(3\right)\left(-x^6\right)+2\left(-x^3\right)\left(-x^6\right)=9+x^6+x^{12}-6x^3-6x^6+2x^9=x^{12}+2x^9-5x^6-6x^3+9$

8. $5x^4-7x^2+3x=25x^8+49x^4+9x^2+2\left(5x^4\right)\left(-7x^2\right)+2\left(5x^4\right)\left(3x\right)+2\left(-7x^2\right)\left(3x\right)$

$=25x^8+49x^4+9x^2-70x^6+30x^5-42x^3=25x^8-70x^6+30x^5+49x^4-42x^3+9x^2$

9. $2a^2+2ab-3b^2=4a^4+4a^2b^2+9b^4+2\left(2a^2\right)\left(2ab\right)+2\left(2a^2\right)\left(-3b^2\right)+2\left(2ab\right)\left(-3b^2\right)$

$=4a^4+4a^2b^2+9b^4+8a^3b-12a^2b^2-12ab^3=4a^4+8a^3b-8a^2b^2-12ab^3+9b^4$

10. $m^3-2m^2n+2n^4=m^6+4m^4n^2+4n^8+2\left(m^3\right)\left(-2m^2n\right)+2\left(m^3\right)\left(2n^4\right)+2\left(-2m^2n\right)\left(2n^4\right)$

$=m^6+4m^4n^2+4n^8-4m^5n+4m^3n^4-8m^2n^5=m^6-4m^5n+4m^4n^2+4m^3n^4-8m^2n^5+4n^8$

11. $\dfrac{a}{2}-b+\dfrac{c}{4}=\dfrac{a^2}{4}+b^2+\dfrac{c^2}{16}+2\left(\dfrac{a}{2}\right)\left(-b\right)+2\left(\dfrac{a}{2}\right)\left(\dfrac{c}{4}\right)+2\left(-b\right)\left(\dfrac{c}{4}\right)=\dfrac{a^2}{4}+b^2+\dfrac{c^2}{16}-ab+\dfrac{ac}{4}-\dfrac{bc}{2}$

12. $\frac{x}{5}-5y+\frac{5}{3}=\frac{x^2}{25}+25y^2+\frac{25}{9}+2\left(\frac{x}{5}\right)(-5y)+2\left(\frac{x}{5}\right)\left(\frac{5}{3}\right)+2(-5y)\left(\frac{5}{3}\right)=\frac{x^2}{25}+25y^2+\frac{25}{9}-2xy+\frac{2x}{3}-\frac{50y}{3}$

$=\frac{x^2}{25}-2xy+\frac{2x}{3}+25y^2-\frac{50y}{3}+\frac{25}{9}$

13. $\frac{x^2}{2}-x+\frac{2}{3}=\frac{x^4}{4}+x^2+\frac{4}{9}+2\left(\frac{x^2}{2}\right)(-x)+2\left(\frac{x^2}{2}\right)\left(\frac{2}{3}\right)+2\left(\frac{2}{3}\right)(-x)=\frac{x^4}{4}+x^2+\frac{4}{9}-x^3+\frac{2x^2}{3}-\frac{4x}{3}$

$=\frac{x^4}{4}-x^3+\frac{5x^2}{3}-\frac{4x}{3}+\frac{4}{9}$

14. $\frac{a}{x}-\frac{1}{3}+\frac{x}{a}=\frac{a^2}{x^2}+\frac{1}{9}+\frac{x^2}{a^2}+2\left(\frac{a}{x}\right)\left(-\frac{1}{3}\right)+2\left(\frac{a}{x}\right)\left(\frac{x}{a}\right)+2\left(-\frac{1}{3}\right)\left(\frac{x}{a}\right)=\frac{a^2}{x^2}+\frac{1}{9}+\frac{x^2}{a^2}-\frac{2a}{3x}+2-\frac{2x}{3a}$

$=\frac{a^2}{x^2}-\frac{2a}{3x}-\frac{2x}{3a}+\frac{x^2}{a^2}+\frac{19}{9}$

15. $\frac{3a^2}{4}-\frac{a}{2}+\frac{4}{5}=\frac{9a^4}{16}+\frac{a^2}{4}+\frac{16}{25}+2\left(\frac{3a^2}{4}\right)\left(-\frac{a}{2}\right)+2\left(\frac{3a^2}{4}\right)\left(\frac{4}{5}\right)+2\left(-\frac{a}{2}\right)\left(\frac{4}{5}\right)$

$=\frac{9a^4}{16}+\frac{a^2}{4}+\frac{16}{25}-\frac{3a^3}{4}+\frac{6a^2}{5}-\frac{4a}{5}=\frac{9a^4}{16}-\frac{3a^3}{4}+\frac{29a^2}{20}-\frac{4a}{5}+\frac{16}{25}$

16. $\frac{a^2}{4}-\frac{3}{5}+\frac{b^2}{9}=\frac{a^4}{16}+\frac{9}{25}+\frac{b^4}{81}+2\left(\frac{a^2}{4}\right)\left(-\frac{3}{5}\right)+2\left(\frac{a^2}{4}\right)\left(\frac{b^2}{9}\right)+2\left(-\frac{3}{5}\right)\left(\frac{b^2}{9}\right)$

$=\frac{a^4}{16}+\frac{9}{25}+\frac{b^4}{81}-\frac{3a^2}{10}+\frac{a^2b^2}{18}-\frac{2b^2}{15}=\frac{a^4}{16}-\frac{3a^2}{10}+\frac{a^2b^2}{18}+\frac{9}{25}-\frac{2b^2}{15}+\frac{b^4}{81}$

17. $x^3-x^2+x+1=x^6+x^4+x^2+1+2(x^3)(-x^2)+2(x^3)(x)+2x^3+2(-x^2)(x)+2(-x^2)+2x$

$=x^6+x^4+x^2+1-2x^5+2x^4+2x^3-2x^3-2x^2+2x=x^6-2x^5+3x^4-x^2+2x+1$

18. x^3-3x^2-2x+2

$=x^6+9x^4+4x^2+4+2(x^3)(-3x^2)+2(x^3)(-2x)+2(x^3)(2)+2(-3x^2)(-2x)+2(-3x^2)(2)+2(-2x)(2)$

$=x^6+9x^4+4x^2+4-6x^5-4x^4+4x^3+12x^3-12x^2-8x=x^6-6x^5+5x^4+16x^3-8x^2-8x+4$

19. x^4+3x^2-4x+5

$=x^8+9x^4+16x^2+25+2(x^4)(3x^2)+2(x^4)(-4x)+2(x^4)(5)+2(3x^2)(-4x)+2(3x^2)(5)+2(-4x)(5)$

$=x^8+9x^4+16x^2+25+6x^6-8x^5+10x^4-24x^3+30x^2-40x=x^8+6x^6-8x^5+19x^4-24x^3+46x^2-40x+25$

20. x^4-4x^3+2x-3

$=x^8+16x^6+4x^2+9+2(x^4)(-4x^3)+2(x^4)(2x)+2(x^4)(-3)+2(-4x^3)(2x)+2(-4x^3)(-3)+2(2x)(-3)$

$=x^8+16x^6+4x^2+9-8x^7+4x^5-6x^4-16x^4+24x^3-12x=x^8-8x^7+16x^6+4x^5-22x^4+24x^3+4x^2-12x+9$

21. $3-6a+a^2-a^3$

$=9+36a^2+a^4+a^6+2(3)(-6a)+2(3)(a^2)+2(3)(-a^3)+2(-6a)(a^2)+2(-6a)(-a^3)+2(a^2)(-a^3)$

$=9+36a^2+a^4+a^6-36a+6a^2-6a^3-12a^3+12a^4-2a^5=a^6-2a^5+13a^4-18a^3+42a^2-36a+9$

22. $\frac{x^3}{2}-x^2+\frac{2x}{3}+2$

$=\frac{x^6}{4}+x^4+\frac{4x^2}{9}+4+2\left(\frac{x^3}{2}\right)(-x^2)+2\left(\frac{x^3}{2}\right)\left(\frac{2x}{3}\right)+2\left(\frac{x^3}{2}\right)(2)+2(-x^2)\left(\frac{2x}{3}\right)+2(-x^2)(2)+2\left(\frac{2x}{3}\right)(2)$

$=\frac{x^6}{4}+x^4+\frac{4x^2}{9}+4-x^5+\frac{2x^4}{3}+2x^3-\frac{4x^3}{3}-4x^2+\frac{8x}{3}=\frac{x^6}{4}-x^5+\frac{5x^4}{3}+\frac{2x^3}{3}-\frac{32x^2}{9}+\frac{8x}{3}+4$

23. $\frac{a^3}{2}-\frac{2a^2}{3}+\frac{3a}{4}-\frac{1}{2}$

$=\frac{a^6}{4}+\frac{4a^4}{9}+\frac{9a^2}{16}+\frac{1}{4}+2\left(\frac{a^3}{2}\right)\left(-\frac{2a^2}{3}\right)+2\left(\frac{a^3}{2}\right)\left(\frac{3a}{4}\right)+2\left(\frac{a^3}{2}\right)\left(-\frac{1}{2}\right)+2\left(-\frac{2a^2}{3}\right)\left(\frac{3a}{4}\right)+2\left(-\frac{2a^2}{3}\right)\left(-\frac{1}{2}\right)+2\left(\frac{3a}{4}\right)\left(-\frac{1}{2}\right)$

$=\frac{a^6}{4}+\frac{4a^4}{9}+\frac{9a^2}{16}+\frac{1}{4}-\frac{2a^5}{3}+\frac{3a^4}{4}-\frac{a^3}{2}-a^3+\frac{2a^2}{3}-\frac{3a}{4}=\frac{a^6}{4}-\frac{2a^5}{3}+\frac{43a^4}{36}-\frac{3a^3}{2}+\frac{59a^2}{48}-\frac{3a}{4}+\frac{1}{4}$

24. $x^5-x^4+x^3-x^2+x-2$

$=x^{10}+x^8+x^6+x^4+x^2+4+2(x^5)(-x^4)+2(x^5)(x^3)+2(x^5)(-x^2)+2(x^5)(x)+2(x^5)(-2)+2(-x^4)(x^3)+2(-x^4)(-x^2)$

$+2(-x^4)(x)+2(-x^4)(-2)+2(x^3)(-x^2)+2(x^3)(x)+2(x^3)(-2)+2(-x^2)(x)+2(-x^2)(-2)+2(x)(-2)$

$=x^{10}+x^8+x^6+x^4+x^2+4-2x^9+2x^8-2x^7+2x^6-4x^5-2x^7+2x^6-2x^5+4x^4-2x^5+2x^4-4x^3-2x^3+4x^2-4x$

$=x^{10}-2x^9+3x^8-4x^7+5x^6-8x^5+7x^4-6x^3+5x^2-4x+4$

EJERCICIO 209

1. $x^2+x+1=x^6+x^3+1+3(x^2)^2(x)+3(x^2)^2(1)+3(x)^2(x^2)+3(x)^2(1)+3(x^2)+3(x)+6(x^2)(x)(1)$

$=x^6+x^3+1+3x^5+3x^4+3x^4+3x^2+3x^2+3x+6x^3=x^6+3x^5+6x^4+7x^3+6x^2+3x+1$

2. $2x^2-x-1=8x^6-x^3-1+3(2x^2)^2(-x)+3(2x^2)^2(-1)+3(-x)^2(-1)+3(-x)^2(2x^2)+3(-1)^2(2x^2)+3(-1)^2(-x)$

$+6(2x^2)(-x)(-1)$

$=8x^6-x^3-1-12x^5-12x^4-3x^2+6x^4+6x^2-3x+12x^3=8x^6-12x^5-6x^4+11x^3+3x^2-3x-1$

3. $1-3x+2x^2$

$=1-27x^3+8x^6+3(1)^2(-3x)+3(1^2)(2x^2)+3(-3x)^2(1)+3(-3x)^2(2x^2)+3(2x^2)^2(1)+3(2x^2)^2(-3x)$

$+6(1)(-3x)(2x^2)$

$=1-27x^3+8x^6-9x+6x^2+27x^2+54x^4+12x^4-36x^5-36x^3=1-9x+33x^2-63x^3+66x^4-36x^5+8x^6$

4. $2-3x+x^2$

$=8-27x^3+x^6+3(2)^2(-3x)+3(2)^2(x^2)+3(-3x)^2(2)+3(-3x)^2(x^2)+3(x^2)^2(2)+3(x^2)^2(-3x)+6(2)(-3x)(x^2)$

$=8-27x^3+x^6-36x+12x^2+54x^2+27x^4+6x^4-9x^5-36x^3=x^6-9x^5+33x^4-63x^3+66x^2-36x+8$

5. x^3-2x^2-4

$=x^9-8x^6-64+3(x^3)^2(-2x^2)+3(x^3)^2(-4)+3(-2x^2)^2(x^3)+3(-2x^2)^2(-4)+3(-4)^2(x^3)+3(-4)^2(-2x^2)$

$+6(x^3)(-2x^2)(-4)$

$=x^9-8x^6-64-6x^8-12x^6+12x^7-48x^4+48x^3-96x^2+48x^5$

$=x^9-6x^8+12x^7-20x^6+48x^5-48x^4+48x^3-96x^2-64$

6. x^4-x^2-2

$=x^{12}-x^6-8+3(x^4)^2(-x^2)+3(x^4)^2(-2)+3(-x^2)^2(x^4)+3(-x^2)^2(-2)+3(-2)^2(x^4)+3(-2)^2(-x^2)$

$+6(x^4)(-x^2)(-2)$

$=x^{12}-x^6-8-3x^{10}-6x^8+3x^8-6x^4+12x^4-12x^2+12x^6=x^{12}-3x^{10}-3x^8+11x^6+6x^4-12x^2-8$

7. $a^3+\frac{a^2}{2}-\frac{a}{3}=a^9+\frac{a^6}{8}-\frac{a^3}{27}+3(a^3)^2\left(\frac{a^2}{2}\right)+3(a^3)^2\left(-\frac{a}{3}\right)+3\left(\frac{a^2}{2}\right)^2(a^3)+3\left(\frac{a^2}{2}\right)^2\left(-\frac{a}{3}\right)+3\left(-\frac{a}{3}\right)^2(a^3)$

$+3\left(-\frac{a}{3}\right)^2\left(\frac{a^2}{2}\right)+6a^3\left(\frac{a^2}{2}\right)\left(-\frac{a}{3}\right)$

$=a^9+\frac{a^6}{8}-\frac{a^3}{27}+\frac{3a^8}{2}-a^7+\frac{3a^7}{4}-\frac{a^5}{4}+\frac{a^5}{3}+\frac{a^4}{6}-a^6=a^9+\frac{3a^8}{2}-\frac{a^7}{4}-\frac{7a^6}{8}+\frac{a^5}{12}+\frac{a^4}{6}-\frac{a^3}{27}$

8. $\frac{x^2}{2}-\frac{x}{3}+2=\frac{x^6}{8}-\frac{x^3}{27}+8+3\left(\frac{x^2}{2}\right)^2\left(-\frac{x}{3}\right)+3(x^2)2^2(2)+3\left(-\frac{x}{3}\right)^2\left(\frac{x^2}{2}\right)+3\left(-\frac{x}{3}\right)^2(2)+3(2)^2\left(\frac{x^2}{2}\right)$

$+3(2)^2\left(-\frac{x}{3}\right)+6\left(\frac{x^2}{2}\right)\left(-\frac{x}{3}\right)(2)$

$=\frac{x^6}{8}-\frac{x^3}{27}+8-\frac{x^5}{4}+\frac{3x^4}{2}+\frac{x^4}{6}+\frac{2x^2}{3}+6x^2-4x-2x^3=\frac{x^6}{8}-\frac{x^5}{4}+\frac{5x^4}{3}-\frac{55x^3}{27}+\frac{20x^2}{3}-4x+8$

9. a^3-a^2+a-1

$=a^9-a^6+a^3-1+3(a^3)^2(-a^2)+3(a^3)^2(a)+3(a^3)^2(-1)+3(-a^2)^2(a^3)+3(-a^2)^2(a)$

$+3(-a^2)^2(-1)+3a^2(a^3)+3a^2(-a^2)+3a^2(-1)+3(-1)^2(a^3)+3(-1)^2(-a^2)+3(-1)^2(a)$

$+6(a^3)(-a^2)(a)+6(a^3)(-a^2)(-1)+6(a^3)(a)(-1)+6(-a^2)(a)(-1)$

$=a^9-a^6+a^3-1-3a^8+3a^7-3a^6+3a^7+3a^5-3a^4+3a^5-3a^4-3a^2+3a^3-3a^2+3a-6a^6+6a^5-6a^4+6a^3$

$=a^9-3a^8+6a^7-10a^6+12a^5-12a^4+10a^3-6a^2+3a-1$

10. $x^3-2x^2+x-3=x^9-8x^6+x^3-27+3(x^3)^2(-2x^2)+3(x^3)^2(x)+3(x^3)^2(-3)+3(-2x^2)^2(x^3)+3(-2x^2)^2(x)$

$+3(-2x^2)^2(-3)+3x^2(x^3)+3x^2(-2x^2)+3x^2(-3)+6(x^3)(-2x^2)(x)+6(x^3)(-2x^2)(-3)$

$+6(x^3)(x)(-3)+6(-2x^2)(x)(-3)+3(-3)^2(x^3)+3(-3)^2(-2x^2)+3(-3)^2x$

$=x^9-8x^6+x^3-27-6x^8+3x^7-9x^6+12x^7+12x^5-36x^4+3x^5-6x^4-9x^2-12x^6+36x^5$

$-18x^4+36x^3+27x^3-54x^2+27x$

$=x^9-6x^8+15x^7-29x^6+51x^5-60x^4+64x^3-63x^2+27x-27$

11. x^3-4x^2+2x-3

$=x^9-64x^6+8x^3-27+3(x^3)^2(-4x^2)+3(x^3)^2(2x)+3(x^3)^2(-3)+3(-4x^2)^2(x^3)+3(-4x^2)^2(2x)$

$+3(-4x^2)^2(-3)+3(2x)^2(x^3)+3(2x)^2(-4x^2)+3(2x)^2(-3)+3(-3)^2(x^3)+3(-3)^2(-4x^2)$

$+3(-3)^2(2x)+6(x^3)(-4x^2)(2x)+6(x^3)(-4x^2)(-3)+6(x^3)(2x)(-3)+6(-4x^2)(2x)(-3)$

$=x^9-64x^6+8x^3-27-12x^8+6x^7-9x^6+48x^7+96x^5-144x^4+12x^5-48x^4-36x^2+27x^3-108x^2$

$+54x-48x^6+72x^5-36x^4+144x^3$

$=x^9-12x^8+54x^7-121x^6+180x^5-228x^4+179x^3-144x^2+54x-27$

12. $1-x^2+2x^4-x^6=1-x^6+8x^{12}-x^{18}+3(-x^2)+3(2x^4)+3(-x^6)+3(-x^2)^2+3(-x^2)^2(2x^4)+3(-x^2)^2(-x^6)$

$+3(2x^4)^2+3(2x^4)^2(-x^2)+3(2x^4)^2(-x^6)+3(-x^6)^2+3(-x^6)^2(-x^2)+3(-x^6)^2(2x^4)$

$+6(-x^2)(2x^4)+6(2x^4)(-x^6)+6(-x^2)(-x^6)+6(-x^2)(2x^4)(-x^6)$

$=1-x^6+8x^{12}-x^{18}-3x^2+6x^4-3x^6+3x^4+6x^8-3x^{10}+12x^8-12x^{10}-12x^{14}+3x^{12}$

$-3x^{14}+6x^{16}-12x^6-12x^{10}+6x^8+12x^{12}$

$=1-3x^2+9x^4-16x^6+24x^8-27x^{10}+23x^{12}-15x^{14}+6x^{16}-x^{18}$

EJERCICIO 210

1. $(x-2)^4 = x^4 - 4x^3(2) + \frac{4\cdot 3}{2}x^2(2)^2 - \frac{6\cdot 2}{3}x(2)^3 + (2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$

2. $(a+3)^4 = a^4 + 4a^3(3) + \frac{4\cdot 3}{2}a^2(3)^2 + \frac{6\cdot 2}{3}a(3)^3 + (3)^4 = a^4 + 12a^3 + 54a^2 + 108a + 81$

3. $(2-x)^5 = (2)^5 - 5(2)^4x + \frac{5\cdot 4}{2}(2)^3x^2 - \frac{10\cdot 3}{3}(2)^2x^3 + \frac{10\cdot 2}{4}(2)x^4 - x^5 = 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

4. $(2x+5y)^4$

$= (2x)^4 + 4(2x)^3(5y) + \frac{4\cdot 3}{2}(2x)^2(5y)^2 + \frac{6\cdot 2}{3}(2x)(5y)^3 + (5y)^4 = 16x^4 + 160x^3y + 600x^2y^2 + 1.000xy^3 + 625y^4$

5. $(a-3)^6 = a^6 - 6a^5(3) + \frac{6\cdot 5}{2}a^4(3)^2 - \frac{15\cdot 4}{3}a^3(3)^3 + \frac{20\cdot 3}{4}a^2(3)^4 - \frac{15\cdot 2}{5}a(3)^5 + (3)^6$

$= a^6 - 18a^5 + 135a^4 - 540a^3 + 1.215a^2 - 1.458a + 729$

6. $(2a-b)^6 = (2a)^6 - 6(2a)^5b + \frac{6\cdot 5}{2}(2a)^4b^2 - \frac{15\cdot 4}{3}(2a)^3b^3 + \frac{20\cdot 3}{4}(2a)^2b^4 - \frac{15\cdot 2}{5}(2a)b^5 + b^6$

$= 64a^6 - 192a^5b + 240a^4b^2 - 160a^3b^3 + 60a^2b^4 - 12ab^5 + b^6$

7. $(x^2+2y^3)^5 = (x^2)^5 + 5(x^2)^4(2y^3) + \frac{5\cdot 4}{2}(x^2)^3(2y^3)^2 + \frac{10\cdot 3}{3}(x^2)^2(2y^3)^3 + \frac{10\cdot 2}{4}(x^2)(2y^3)^4 + (2y^3)^5$

$= x^{10} + 10x^8y^3 + 40x^6y^6 + 80x^4y^9 + 80x^2y^{12} + 32y^{15}$

8. $(x^3+1)^6 = (x^3)^6 + 6(x^3)^5(1) + \frac{6\cdot 5}{2}(x^3)^4(1)^2 + \frac{15\cdot 4}{3}(x^3)^3(1)^3 + \frac{20\cdot 3}{4}(x^3)^2(1)^4 + \frac{15\cdot 2}{5}(x^3)(1)^5 + (1)^6$

$= x^{18} + 6x^{15} + 15x^{12} + 20x^9 + 15x^6 + 6x^3 + 1$

9. $(2a-3b)^5 = (2a)^5 - 5(2a)^4(3b) + \frac{5\cdot 4}{2}(2a)^3(3b)^2 - \frac{10\cdot 3}{3}(2a)^2(3b)^3 + \frac{10\cdot 2}{4}(2a)(3b)^4 - (3b)^5$

$= 32a^5 - 240a^4b + 720a^3b^2 - 1.080a^2b^3 + 810ab^4 - 243b^5$

10. $(x^4-5y^3)^6$

$= (x^4)^6 - 6(x^4)^5(5y^3) + \frac{6\cdot 5}{2}(x^4)^4(5y^3)^2 - \frac{15\cdot 4}{3}(x^4)^3(5y^3)^3 + \frac{20\cdot 3}{4}(x^4)^2(5y^3)^4 - \frac{15\cdot 2}{5}(x^4)(5y^3)^5 + (5y^3)^6$

$= x^{24} - 30x^{20}y^3 + 375x^{16}y^6 - 2.500x^{12}y^9 + 9.375x^8y^{12} - 18.750x^4y^{15} + 15.625y^{18}$

11. $\left(2x - \frac{y}{2}\right)^6$

$= (2x)^6 - 6(2x)^5\left(\frac{y}{2}\right) + \frac{6\cdot 5}{2}(2x)^4\left(\frac{y}{2}\right)^2 - \frac{15\cdot 4}{3}(2x)^3\left(\frac{y}{2}\right)^3 + \frac{20\cdot 3}{4}(2x)^2\left(\frac{y}{2}\right)^4 - \frac{15\cdot 2}{5}(2x)\left(\frac{y}{2}\right)^5 + \left(\frac{y}{2}\right)^6$

$= 64x^6 - 96x^5y + 60x^4y^2 - 20x^3y^3 + \frac{15}{4}x^2y^4 - \frac{3}{8}xy^5 + \frac{y^6}{64}$

12. $\left(3 - \frac{x^2}{3}\right)^5 = (3)^5 - 5(3)^4\left(\frac{x^2}{3}\right) + \frac{5\cdot 4}{2}(3)^3\left(\frac{x^2}{3}\right)^2 - \frac{10\cdot 3}{3}(3)^2\left(\frac{x^2}{3}\right)^3 + \frac{10\cdot 2}{4}(3)\left(\frac{x^2}{3}\right)^4 - \left(\frac{x^2}{3}\right)^5$

$= 243 - 135x^2 + 30x^4 - \frac{10}{3}x^6 + \frac{5}{27}x^8 - \frac{x^{10}}{243}$

13. $(2m^3-3n^4)^6 = (2m^3)^6 - 6(2m^3)^5(3n^4) + \frac{6\cdot 5}{2}(2m^3)^4(3n^4)^2 - \frac{15\cdot 4}{3}(2m^3)^3(3n^4)^3 + \frac{20\cdot 3}{4}(2m^3)^2(3n^4)^4$

$- \frac{15\cdot 2}{5}(2m^3)(3n^4)^5 + (3n^4)^6$

$= 64m^{18} - 576m^{15}n^4 + 2.160m^{12}n^8 - 4.320m^9n^{12} + 4.860m^6n^{16} - 2.916m^3n^{20} + 729n^{24}$

14. $\left(x^2-3\right)^7$

$=\left(x^2\right)^7-7\left(x^2\right)^6(3)+\frac{7\cdot 6}{2}\left(x^2\right)^5(3)^2-\frac{21\cdot 5}{3}\left(x^2\right)^4(3)^3+\frac{35\cdot 4}{4}\left(x^2\right)^3(3)^4-\frac{35\cdot 3}{5}\left(x^2\right)^2(3)^5+\frac{21\cdot 2}{6}\left(x^2\right)(3)^6-(3)^7$

$=x^{14}-21x^{12}+189x^{10}-945x^8+2.835x^6-5.103x^4+5.103x^2-2.187$

15. $\left(3a-\frac{b^2}{3}\right)^5=(3a)^5-5(3a)^4\left(\frac{b^2}{3}\right)+\frac{5\cdot 4}{2}(3a)^3\left(\frac{b^2}{3}\right)^2-\frac{10\cdot 3}{3}(3a)^2\left(\frac{b^2}{3}\right)^3+\frac{10\cdot 2}{4}(3a)\left(\frac{b^2}{3}\right)^4-\left(\frac{b^2}{3}\right)^5$

$=243a^5-135a^4b^2+30a^3b^4-\frac{10}{3}a^2b^6+\frac{5}{27}ab^8-\frac{b^{10}}{243}$

16. $\left(x^2+2y^2\right)^7=\left(x^2\right)^7+7\left(x^2\right)^6\left(2y^2\right)+\frac{7\cdot 6}{2}\left(x^2\right)^5\left(2y^2\right)^2+\frac{21\cdot 5}{3}\left(x^2\right)^4\left(2y^2\right)^3+\frac{35\cdot 4}{4}\left(x^2\right)^3\left(2y^2\right)^4$

$+\frac{35\cdot 3}{5}\left(x^2\right)^2\left(2y^2\right)^5+\frac{21\cdot 2}{6}\left(x^2\right)\left(2y^2\right)^6+\left(2y^2\right)^7$

$=x^{14}+14x^{12}y^2+84x^{10}y^4+280x^8y^6+560x^6y^8+672x^4y^{10}+448x^2y^{12}+128y^{14}$

17. $\left(x^3-1\right)^8=\left(x^3\right)^8-8\left(x^3\right)^7(1)+\frac{8\cdot 7}{2}\left(x^3\right)^6(1)^2-\frac{28\cdot 6}{3}\left(x^3\right)^5(1)^3+\frac{56\cdot 5}{4}\left(x^3\right)^4(1)^4-\frac{70\cdot 4}{5}\left(x^3\right)^3(1)^5$

$+\frac{56\cdot 3}{6}\left(x^3\right)^2(1)^6-\frac{28\cdot 2}{7}\left(x^3\right)(1)^7+(1)^8$

$=x^{24}-8x^{21}+28x^{18}-56x^{15}+70x^{12}-56x^9+28x^6-8x^3+1$

18. $\left(x^2-\frac{y}{2}\right)^9=\left(x^2\right)^9-9\left(x^2\right)^8\left(\frac{y}{2}\right)+\frac{9\cdot 8}{2}\left(x^2\right)^7\left(\frac{y}{2}\right)^2-\frac{36\cdot 7}{3}\left(x^2\right)^6\left(\frac{y}{2}\right)^3+\frac{84\cdot 6}{4}\left(x^2\right)^5\left(\frac{y}{2}\right)^4-\frac{126\cdot 5}{5}\left(x^2\right)^4\left(\frac{y}{2}\right)^5$

$+\frac{126\cdot 4}{6}\left(x^2\right)^3\left(\frac{y}{2}\right)^6-\frac{84\cdot 3}{7}\left(x^2\right)^2\left(\frac{y}{2}\right)^7+\frac{36\cdot 2}{8}\left(x^2\right)\left(\frac{y}{2}\right)^8-\left(\frac{y}{2}\right)^9$

$=x^{18}-\frac{9}{2}x^{16}y+9x^{14}y^2-\frac{21}{2}x^{12}y^3+\frac{63}{8}x^{10}y^4-\frac{63}{16}x^8y^5+\frac{21}{16}x^6y^6-\frac{9}{32}x^4y^7+\frac{9}{256}x^2y^8-\frac{y^9}{512}$

19. $\left(2m^3-n^4\right)^7=\left(2m^3\right)^7-7\left(2m^3\right)^6\left(n^4\right)+\frac{7\cdot 6}{2}\left(2m^3\right)^5\left(n^4\right)^2-\frac{21\cdot 5}{3}\left(2m^3\right)^4\left(n^4\right)^3+\frac{35\cdot 4}{4}\left(2m^3\right)^3\left(n^4\right)^4$

$-\frac{35\cdot 3}{5}\left(2m^3\right)^2\left(n^4\right)^5+\frac{21\cdot 2}{6}\left(2m^3\right)\left(n^4\right)^6-\left(n^4\right)^7$

$=128m^{21}-448m^{18}n^4+672m^{15}n^8-560m^{12}n^{12}+280m^9n^{16}-84m^6n^{20}+14m^3n^{24}-n^{28}$

20. $\left(\frac{x^2}{2}+\frac{2y^2}{3}\right)^5$

$=\left(\frac{x^2}{2}\right)^5+5\left(\frac{x^2}{2}\right)^4\left(\frac{2y^2}{3}\right)+\frac{5\cdot 4}{2}\left(\frac{x^2}{2}\right)^3\left(\frac{2y^2}{3}\right)^2+\frac{10\cdot 3}{3}\left(\frac{x^2}{2}\right)^2\left(\frac{2y^2}{3}\right)^3+\frac{10\cdot 2}{4}\left(\frac{x^2}{2}\right)\left(\frac{2y^2}{3}\right)^4+\left(\frac{2y^2}{3}\right)^5$

$=\frac{x^{10}}{32}+\frac{5x^8y^2}{24}+\frac{5x^6y^4}{9}+\frac{20x^4y^6}{27}+\frac{40x^2y^8}{81}+\frac{32y^{10}}{243}$

21. $\left(\frac{1}{5}-\frac{5a}{2}\right)^6=\left(\frac{1}{5}\right)^6-6\left(\frac{1}{5}\right)^5\left(\frac{5a}{2}\right)+\frac{6\cdot 5}{2}\left(\frac{1}{5}\right)^4\left(\frac{5a}{2}\right)^2-\frac{15\cdot 4}{3}\left(\frac{1}{5}\right)^3\left(\frac{5a}{2}\right)^3+\frac{20\cdot 3}{4}\left(\frac{1}{5}\right)^2\left(\frac{5a}{2}\right)^4$

$-\frac{15\cdot 2}{5}\left(\frac{1}{5}\right)\left(\frac{5a}{2}\right)^5+\left(\frac{5a}{2}\right)^6$

$=\frac{1}{15.625}-\frac{3a}{625}+\frac{3a^2}{20}-\frac{5a^3}{2}+\frac{375a^4}{16}-\frac{1.875a^5}{16}+\frac{15.625a^6}{64}$

EJERCICIO 211

NOTA: Triángulo necesario para el desarrollo de los problemas del presente ejercicio

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

1. $(a+2b)^6 = a^6+6a^5(2b)+15a^4(2b)^2+20a^3(2b)^3+15a^2(2b)^4+6a(2b)^5+(2b)^6$

$= a^6+12a^5b+60a^4b^2+160a^3b^3+240a^2b^4+192ab^5+64b^6$

2. $(2m^2-3n^3)^5=(2m^2)^5-5(2m^2)^4(3n^3)+10(2m^2)^3(3n^3)^2-10(2m^2)^2(3n^3)^3+5(2m^2)(3n^3)^4-(3n^3)^5$

$=32m^{10}-240m^8n^3+720m^6n^6-1.080m^4n^9+810m^2n^{12}-243n^{15}$

3. $(x^2+y^3)^6=(x^2)^6+6(x^2)^5(y^3)+15(x^2)^4(y^3)^2+20(x^2)^3(y^3)^3+15(x^2)^2(y^3)^4+6(x^2)(y^3)^5+(y^3)^6$

$=x^{12}+6x^{10}y^3+15x^8y^6+20x^6y^9+15x^4y^{12}+6x^2y^{15}+y^{18}$

4. $(3-y^7)^7=(3)^7-7(3)^6(y^7)+21(3)^5(y^7)^2-35(3)^4(y^7)^3+35(3)^3(y^7)^4-21(3)^2(y^7)^5+7(3)(y^7)^6-(y^7)^7$

$=2.187-5.103y^7+5.103y^{14}-2.835y^{21}+945y^{28}-189y^{35}+21y^{42}-y^{49}$

5. $(2x^3-3y^4)^6$

$=(2x^3)^6-6(2x^3)^5(3y^4)+15(2x^3)^4(3y^4)^2-20(2x^3)^3(3y^4)^3+15(2x^3)^2(3y^4)^4-6(2x^3)(3y^4)^5+(3y^4)^6$

$=64x^{18}-576x^{15}y^4+2.160x^{12}y^8-4.320x^9y^{12}+4.860x^6y^{16}-2.916x^3y^{20}+729y^{24}$

6. $\left(\frac{x^2}{2}+y^3\right)^5=\left(\frac{x^2}{2}\right)^5+5\left(\frac{x^2}{2}\right)^4y^3+10\left(\frac{x^2}{2}\right)^3(y^3)^2+10\left(\frac{x^2}{2}\right)^2(y^3)^3+5\left(\frac{x^2}{2}\right)(y^3)^4+(y^3)^5$

$=\frac{x^{10}}{32}+\frac{5x^8y^3}{16}+\frac{5x^6y^6}{4}+\frac{5x^4y^9}{2}+\frac{5x^2y^{12}}{2}+y^{15}$

7. $\left(\frac{a}{3}-\frac{3}{b}\right)^6=\left(\frac{a}{3}\right)^6-6\left(\frac{a}{3}\right)^5\left(\frac{3}{b}\right)+15\left(\frac{a}{3}\right)^4\left(\frac{3}{b}\right)^2-20\left(\frac{a}{3}\right)^3\left(\frac{3}{b}\right)^3+15\left(\frac{a}{3}\right)^2\left(\frac{3}{b}\right)^4-6\left(\frac{a}{3}\right)\left(\frac{3}{b}\right)^5+\left(\frac{3}{b}\right)^6$

$=\frac{a^6}{729}-\frac{2a^5}{27b}+\frac{5a^4}{3b^2}-\frac{20a^3}{b^3}+\frac{135a^2}{b^4}-\frac{486a}{b^5}+\frac{729}{b^6}$

8. $(1-x^4)^8$

$=(1)^8-8(1)^7(x^4)+28(1)^6(x^4)^2-56(1)^5(x^4)^3+70(1)^4(x^4)^4-56(1)^3(x^4)^5+28(1)^2(x^4)^6-8(1)(x^4)^7+(x^4)^8$

$=1-8x^4+28x^8-56x^{12}+70x^{16}-56x^{20}+28x^{24}-8x^{28}+x^{32}$

9. $\left(\frac{2}{3x}-\frac{3}{2y}\right)^7=\left(\frac{2}{3x}\right)^7-7\left(\frac{2}{3x}\right)^6\left(\frac{3}{2y}\right)+21\left(\frac{2}{3x}\right)^5\left(\frac{3}{2y}\right)^2-35\left(\frac{2}{3x}\right)^4\left(\frac{3}{2y}\right)^3+35\left(\frac{2}{3x}\right)^3\left(\frac{3}{2y}\right)^4$

$-21\left(\frac{2}{3x}\right)^2\left(\frac{3}{2y}\right)^5+7\left(\frac{2}{3x}\right)\left(\frac{3}{2y}\right)^6-\left(\frac{3}{2y}\right)^7$

$=\frac{128}{2.187x^7}-\frac{224}{243x^6y}+\frac{56}{9x^5y^2}-\frac{70}{3x^4y^3}+\frac{105}{2x^3y^4}-\frac{567}{8x^2y^5}+\frac{1.701}{32xy^6}-\frac{2.187}{128y^7}$

10. $\left(\frac{2}{m}-\frac{m^2}{2}\right)^7=\left(\frac{2}{m}\right)^7-7\left(\frac{2}{m}\right)^6\left(\frac{m^2}{2}\right)+21\left(\frac{2}{m}\right)^5\left(\frac{m^2}{2}\right)^2-35\left(\frac{2}{m}\right)^4\left(\frac{m^2}{2}\right)^3+35\left(\frac{2}{m}\right)^3\left(\frac{m^2}{2}\right)^4$

$-21\left(\frac{2}{m}\right)^2\left(\frac{m^2}{2}\right)^5+7\left(\frac{2}{m}\right)\left(\frac{m^2}{2}\right)^6-\left(\frac{m^2}{2}\right)^7$

$=\frac{128}{m^7}-\frac{224}{m^4}+\frac{168}{m}-70m^2+\frac{35m^5}{2}-\frac{21m^8}{8}+\frac{7m^{11}}{32}-\frac{m^{14}}{128}$

11. $\left(x^3+mn\right)^8=\left(x^3\right)^8+8\left(x^3\right)^7mn+28\left(x^3\right)^6(mn)^2+56\left(x^3\right)^5(mn)^3+70\left(x^3\right)^4(mn)^4+56\left(x^3\right)^3(mn)^5$

$+28\left(x^3\right)^2(mn)^6+8\left(x^3\right)(mn)^7+(mn)^8$

$=x^{24}+8x^{21}mn+28x^{18}m^2n^2+56x^{15}m^3n^3+70x^{12}m^4n^4+56x^9m^5n^5+28x^6m^6n^6+8x^3m^7n^7+m^8n^8$

12. $\left(3-\frac{b^2}{3}\right)^9=(3)^9-9(3)^8\left(\frac{b^2}{3}\right)+36(3)^7\left(\frac{b^2}{3}\right)^2-84(3)^6\left(\frac{b^2}{3}\right)^3+126(3)^5\left(\frac{b^2}{3}\right)^4-126(3)^4\left(\frac{b^2}{3}\right)^5$

$+84(3)^3\left(\frac{b^2}{3}\right)^6-36(3)^2\left(\frac{b^2}{3}\right)^7+9(3)\left(\frac{b^2}{3}\right)^8-\left(\frac{b^2}{3}\right)^9$

$=19.683-19.683b^2+8.748b^4-2.268b^6+378b^8-42b^{10}+\frac{28b^{12}}{9}-\frac{4b^{14}}{27}+\frac{b^{16}}{243}-\frac{b^{18}}{19.683}$

13. $\left(1-\frac{1}{x}\right)^{10}=(1)^{10}-10(1)^9\left(\frac{1}{x}\right)+45(1)^8\left(\frac{1}{x}\right)^2-120(1)^7\left(\frac{1}{x}\right)^3+210(1)^6\left(\frac{1}{x}\right)^4-252(1)^5\left(\frac{1}{x}\right)^5+210(1)^4\left(\frac{1}{x}\right)^6$

$-120(1)^3\left(\frac{1}{x}\right)^7+45(1)^2\left(\frac{1}{x}\right)^8-10(1)\left(\frac{1}{x}\right)^9+\left(\frac{1}{x}\right)^{10}$

$=1-\frac{10}{x}+\frac{45}{x^2}-\frac{120}{x^3}+\frac{210}{x^4}-\frac{252}{x^5}+\frac{210}{x^6}-\frac{120}{x^7}+\frac{45}{x^8}-\frac{10}{x^9}+\frac{1}{x^{10}}$

14. $\left(2m^2-5n^5\right)^6=\left(2m^2\right)^6-6\left(2m^2\right)^5\left(5n^5\right)+15\left(2m^2\right)^4\left(5n^5\right)^2-20\left(2m^2\right)^3\left(5n^5\right)^3+15\left(2m^2\right)^2\left(5n^5\right)^4$

$-6\left(2m^2\right)\left(5n^5\right)^5+\left(5n^5\right)^6$

$=64m^{12}-960m^{10}n^5+6.000m^8n^{10}-20.000m^6n^{15}+37.500m^4n^{20}-37.500m^2n^{25}+15.625n^{30}$

15. $\left(4-\frac{x^5}{4}\right)^7$

$=(4)^7-7(4)^6\left(\frac{x^5}{4}\right)+21(4)^5\left(\frac{x^5}{4}\right)^2-35(4)^4\left(\frac{x^5}{4}\right)^3+35(4)^3\left(\frac{x^5}{4}\right)^4-21(4)^2\left(\frac{x^5}{4}\right)^5+7(4)\left(\frac{x^5}{4}\right)^6-\left(\frac{x^5}{4}\right)^7$

$=16.384-7.168x^5+1.344x^{10}-140x^{15}+\frac{35x^{20}}{4}-\frac{21x^{25}}{64}+\frac{7x^{30}}{1.024}-\frac{x^{35}}{16.384}$

EJERCICIO 212

1. $(x-y)^5$ *donde* $r=3$

$=\frac{5\cdot4}{1\cdot2}(x)^{5-2}(-y)^2=10x^3y^2$

2. $(a-4b)^7$ *donde* $r=4$

$=\frac{7\cdot6\cdot5}{1\cdot2\cdot3}(a)^{7-3}(-4b)^{4-1}$

$=35a^4(-64b^3)$

$=-2.240a^4b^3$

3. $(1+x)^{11}$ *donde* $r=5$

$=\frac{11\cdot10\cdot9\cdot8}{1\cdot2\cdot3\cdot4}(1)^{11-4}(x)^{5-1}$

$=330x^4$

4. $(3x-2y)^6$ *donde* $r=4$

$=\frac{6\cdot5\cdot4}{1\cdot2\cdot3}(3x)^{6-3}(-2y)^{4-1}$

$=20(27x^3)(-8y^3)$

$=-4.320x^3y^3$

5. $(a^2-2b)^9$ *donde* $r=5$

$=\frac{9\cdot8\cdot7\cdot6}{1\cdot2\cdot3\cdot4}(a^2)^{9-4}(-2b)^{5-1}$

$=126(a^{10})(16b^4)$

$=2.016a^{10}b^4$

6. $\left(2a-\frac{b}{2}\right)^8$ *donde* $r=6$

$=\frac{8\cdot7\cdot6\cdot5\cdot4}{1\cdot2\cdot3\cdot4\cdot5}(2a)^{8-5}\left(-\frac{b}{2}\right)^{6-1}$

$=56(8a^3)\left(-\frac{b^5}{32}\right)$

$=-14a^3b^5$

7. $(x^2-2y)^{10}$ *donde* $r=7$

$=\frac{10\cdot9\cdot8\cdot7\cdot6\cdot5}{1\cdot2\cdot3\cdot4\cdot5\cdot6}(x^2)^{10-6}(-2y)^{7-1}$

$=210(x^8)(64y^6)$

$=13.440x^8y^6$

8. $(x-y^2)^{11}$ *donde* $r=8$

$=\frac{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7}(x)^{11-7}(-y^2)^{8-1}$

$=330x^4(-y^{14})$

$=-330x^4y^{14}$

9. $(a^2+b)^{15}$ *donde* $r=10$

$=\frac{15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}(a^2)^{15-9}(b)^{10-1}$

$=5.005a^{12}b^9$

10. $(1-x^2)^{12}$ *donde* $r=9$

$=\frac{12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8}(1)^{12-8}(-x^2)^{9-1}$

$=495x^{16}$

11. $(2a-b^2)^6$ *donde* $r=6$

$=\frac{6\cdot5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4\cdot5}(2a)^{6-5}(-b^2)^{6-1}$

$=6(2a)(-b^{10})$

$=-12ab^{10}$

12. $(3x^2-y^2)^8$ *donde* $r=5$

$=\frac{8\cdot7\cdot6\cdot5}{1\cdot2\cdot3\cdot4}(3x^2)^{8-4}(-y^2)^{5-1}$

$=70(81x^8)(y^8)$

$=5.670x^8y^8$

EJERCICIO 213

1. $\sqrt{4a^2b^4}=\pm2ab^2$

2. $\sqrt{25x^6y^8}=\pm5x^3y^4$

3. $\sqrt[3]{27a^3b^9}=3ab^3$

4. $\sqrt[3]{-8a^3b^6x^{12}}=-2ab^2x^4$

5. $\sqrt{64x^8y^{10}}=\pm8x^4y^5$

6. $\sqrt[4]{16a^8b^{16}}=\pm2a^2b^4$

7. $\sqrt[5]{x^{15}y^{20}z^{25}}=x^3y^4z^5$

8. $\sqrt[3]{-64a^3x^6y^{18}}=-4ax^2y^6$

9. $\sqrt[5]{-243m^5n^{15}}=-3mn^3$

10. $\sqrt{81x^6y^8z^{20}}=\pm9x^3y^4z^{10}$

11. $\sqrt[3]{1.000x^9y^{18}}=10x^3y^6$

12. $\sqrt[4]{81a^{12}b^{24}}=\pm3a^3b^6$

13. $\sqrt[6]{64a^{12}b^{18}c^{30}}=\pm2a^2b^3c^5$

14. $\sqrt{49a^{2n}b^{4n}}=\pm7a^nb^{2n}$

15. $\sqrt[5]{-x^{5n}y^{10x}}=-x^ny^{2x}$

16. $\sqrt{\frac{9a^2}{25x^4}}=\pm\frac{3a}{5x^2}$

17. $\sqrt[3]{-\frac{27a^3}{64x^9}}=-\frac{3a}{4x^3}$

18. $\sqrt[5]{-\frac{a^5b^{10}}{32x^{15}}}=-\frac{ab^2}{2x^3}$

19. $\sqrt[4]{\frac{a^8}{81b^4c^{12}}}=\pm\frac{a^2}{3bc^3}$

20. $\sqrt[7]{\frac{128}{x^{14}}}=\frac{2}{x^2}$

21. $\sqrt{\frac{x^{2m}}{121y^{4n}}}=\pm\frac{x^m}{11y^{2n}}$

22. $\sqrt[3]{-\frac{125x^9}{216m^{12}}}=-\frac{5x^3}{6m^4}$

23. $\sqrt[9]{\frac{a^{18}}{b^9c^{27}}}=\frac{a^2}{bc^3}$

24. $\sqrt[10]{\frac{x^{20}}{1.024y^{30}}}=\pm\frac{x^2}{2y^3}$

EJERCICIO 214

1.
$$
\begin{array}{l|l}
\sqrt{\;16x^2-24xy^2+9y^4} & 4x-3y^2 \\ \hline
-16x^2 & \left(8x-3y^2\right)\left(-3y^2\right) \\ \hline
\qquad -24xy^2+9y^4 & \qquad =-24xy^2+9y^4 \\
\qquad 24xy^2-9y^4 & \\ \hline
\qquad\qquad 0 &
\end{array}
$$

2.
$$
\begin{array}{l|l}
\sqrt{\;25a^4-70a^3x+49a^2x^2} & 5a^2-7ax \\ \hline
-25a^4 & \left(10a^2-7ax\right)\left(-7ax\right) \\ \hline
\qquad -70a^3x+49a^2x^2 & \qquad =-70a^3x+49a^2x^2 \\
\qquad 70a^3x-49a^2x^2 & \\ \hline
\qquad\qquad 0 &
\end{array}
$$

3.
$$
\begin{array}{l|l}
\sqrt{\;x^4-4x^3+6x^2-4x+1} & x^2-2x+1 \\ \hline
-x^4 & \left(2x^2-2x\right)\left(-2x\right) \\ \hline
\qquad -4x^3+6x^2 & \qquad =-4x^3+4x^2 \\
\qquad 4x^3-4x^2 & \\ \hline
\qquad\qquad 2x^2-4x+1 & \left(2x^2-4x+1\right)(1) \\
\qquad\qquad -2x^2+4x-1 & \qquad =2x^2-4x+1 \\ \hline
\qquad\qquad\qquad 0 &
\end{array}
$$

4.
$$
\begin{array}{l|l}
\sqrt{\;4a^4+4a^3+5a^2+2a+1} & 2a^2+a+1 \\ \hline
-4a^4 & \left(4a^2+a\right)(a) \\ \hline
\qquad 4a^3+5a^2 & \qquad =4a^3+a^2 \\
\qquad -4a^3-a^2 & \\ \hline
\qquad\qquad 4a^2+2a+1 & \left(4a^2+2a+1\right)(1) \\
\qquad\qquad -4a^2-2a-1 & \qquad =4a^2+2a+1 \\ \hline
\qquad\qquad\qquad 0 &
\end{array}
$$

5.
$$
\begin{array}{l|l}
\sqrt{\;n^4-10n^3+29n^2-20n+4} & n^2-5n+2 \\ \hline
-n^4 & \left(2n^2-5n\right)\left(-5n\right) \\ \hline
\qquad -10n^3+29n^2 & \qquad =-10n^3+25n^2 \\
\qquad 10n^3-25n^2 & \\ \hline
\qquad\qquad 4n^2-20n+4 & \left(2n^2-10n+2\right)(2) \\
\qquad\qquad -4n^2+20n-4 & \qquad =4n^2-20n+4 \\ \hline
\qquad\qquad\qquad 0 &
\end{array}
$$

6. $\sqrt{x^6 - 10x^5 + 25x^4 + 12x^3 - 60x^2 + 36} = x^3 - 5x^2 + 6$

$$-x^6$$

$$-10x^5 + 25x^4 \qquad (2x^3 - 5x^2)(-5x^2) = -10x^5 + 25x^4$$

$$10x^5 - 25x^4$$

$$12x^3 - 60x^2 + 36 \qquad (2x^3 - 10x^2 + 6)(6) = 12x^3 - 60x^2 + 36$$

$$-12x^3 + 60x^2 - 36$$

$$0$$

7. $\sqrt{16a^8 - 24a^6 + 49a^4 - 30a^2 + 25} = 4a^4 - 3a^2 + 5$

$$-16a^8$$

$$-24a^6 + 49a^4 \qquad (8a^4 - 3a^2)(-3a^2) = -24a^6 + 9a^4$$

$$24a^6 - 9a^4$$

$$40a^4 - 30a^2 + 25 \qquad (8a^4 - 6a^2 + 5)(5) = 40a^4 - 30a^2 + 25$$

$$-40a^4 + 30a^2 - 25$$

$$0$$

8. $\sqrt{x^2 + 4xy - 2xz - 4yz + 4y^2 + z^2} = x + 2y - z$

$$-x^2$$

$$4xy - 2xz + 4y^2 \qquad (2x + 2y)(2y) = 4xy + 4y^2$$

$$-4xy - 4y^2$$

$$-2xz - 4yz + z^2 \qquad (2x + 4y - z)(-z) = -2xz - 4yz + z^2$$

$$2xz + 4yz - z^2$$

$$0$$

9. $\sqrt{x^{12} + 2x^9 - 5x^6 - 6x^3 + 9} = x^6 + x^3 - 3$

$$-x^{12}$$

$$2x^9 - 5x^6 \qquad (2x^6 + x^3)(x^3) = 2x^9 + x^6$$

$$-2x^9 - x^6$$

$$-6x^6 - 6x^3 + 9 \qquad (2x^6 + 2x^3 - 3)(-3) = -6x^6 - 6x^3 + 9$$

$$6x^6 + 6x^3 - 9$$

$$0$$

10. $\sqrt{25x^8 - 70x^6 + 30x^5 + 49x^4 - 42x^3 + 9x^2} = 5x^4 - 7x^2 + 3x$

$$-25x^8$$

$$-70x^6 + 30x^5 + 49x^4 \qquad (10x^4 - 7x^2)(-7x^2) = -70x^6 + 49x^4$$

$$70x^6 - 49x^4$$

$$30x^5 - 42x^3 + 9x^2 \qquad (10x^4 - 14x^2 + 3x)(3x) = 30x^5 - 42x^3 + 9x^2$$

$$-30x^5 + 42x^3 - 9x^2$$

$$0$$

11. $\sqrt{4a^4+8a^3b-8a^2b^2-12ab^3+9b^4}$ $\quad 2a^2+2ab-3b^2$

$-4a^4$

$8a^3b-8a^2b^2$ $\qquad (4a^2+2ab)\ (2ab) = 8a^3b+4a^2b^2$

$-8a^3b-4a^2b^2$

$-12a^2b^2-12ab^3+9b^4$ $\qquad (4a^2+4ab-3b^2)\ (-3b^2) = -12a^2b^2-12ab^3+9b^4$

$12a^2b^2+12ab^3-9b^4$

0

12. $\sqrt{x^6-2x^5+3x^4-x^2+2x+1}$ $\quad x^3-x^2+x+1$

$-x^6$ $\qquad (2x^3-x^2)\ (-x^2) = -2x^5+x^4$

$-2x^5+3x^4$

$2x^5-x^4$

$2x^4-x^2+2x$ $\qquad (2x^3-2x^2+x)\ (x) = 2x^4-2x^3+x^2$

$2x^3-2x^4-x^2$

$2x^3 \quad -2x^2+2x+1$

$-2x^3 \quad +2x^2-2x-1$ $\qquad (2x^3-2x^2+2x+1)\ (1) = 2x^3-2x^2+2x+1$

0

13. $\sqrt{x^6-6x^5+5x^4+16x^3-8x^2-8x+4}$ $\quad x^3-3x^2-2x+2$

$-x^6$ $\qquad (2x^3-3x^2)\ (-3x^2) = -6x^5+9x^4$

$-6x^5+5x^4$

$6x^5-9x^4$

$-4x^4+16x^3-8x^2$ $\qquad (2x^3-6x^2-2x)\ (-2x) = -4x^4+12x^3+4x^2$

$4x^4-12x^3-4x^2$

$4x^3-12x^2-8x+4$

$-4x^3+12x^2+8x-4$ $\qquad (2x^3-6x^2-4x+2)\ (2) = 4x^3-12x^2-8x+4$

0

14. $\sqrt{x^8+6x^6-8x^5+19x^4-24x^3+46x^2-40x+25}$ $\quad x^4+3x^2-4x+5$

$-x^8$ $\qquad (2x^4+3x^2)\ (3x^2) = 6x^6+9x^4$

$6x^6-8x^5+19x^4$

$-6x^6 \quad -9x^4$ $\qquad (2x^4+6x^2-4x)\ (-4x) = -8x^5-24x^3+16x^2$

$-8x^5+10x^4-24x^3+46x^2$

$8x^5 \quad +24x^3-16x^2$

$10x^4 \quad +30x^2-40x+25$ $\qquad (2x^4+6x^2-8x+5)\ (5) = 10x^4+30x^2-40x+25$

$-10x^4 \quad -30x^2+40x-25$

0

15. $$\sqrt{x^8-8x^7+16x^6+4x^5-22x^4+24x^3+4x^2-12x+9} \;\Big|\; x^4-4x^3+2x-3$$

$$-x^8$$
$$-8x^7+16x^6$$
$$8x^7-16x^6$$
$$4x^5-22x^4+24x^3+4x^2$$
$$-4x^5+16x^4 \qquad -4x^2$$
$$-\;6x^4+24x^3 \qquad -12x+9$$
$$6x^4-24x^3 \qquad +12x-9$$
$$0$$

$$\left(2x^4-4x^3\right)\left(-4x^3\right)=-8x^7+16x^6$$
$$\left(2x^4-8x^3+2x\right)(2x)=4x^5-16x^4+4x^2$$
$$\left(2x^4-8x^3+4x-3\right)(-3)=-6x^4+24x^3-12x+9$$

16. $$\sqrt{9-36a+42a^2-18a^3+13a^4-2a^5+a^6} \;\Big|\; 3-6a+a^2-a^3$$

$$-9$$
$$-36a+42a^2$$
$$36a-36a^2$$
$$6a^2-18a^3+13a^4$$
$$-6a^2+12a^3-a^4$$
$$-\;6a^3+12a^4-2a^5+a^6$$
$$6a^3-12a^4+2a^5-a^6$$
$$0$$

$$\left(6-6a\right)\left(-6a\right)=-36a+36a^2$$
$$\left(6-12a+a^2\right)\left(a^2\right)=6a^2-12a^3+a^4$$
$$\left(6-12a+2a^2-a^3\right)\left(-a^3\right)=-6a^3+12a^4-2a^5+a^6$$

17. $$\sqrt{9x^6-24x^5+28x^4-22x^3+12x^2-4x+1} \;\Big|\; 3x^3-4x^2+2x-1$$

$$-9x^6$$
$$-24x^5+28x^4$$
$$24x^5-16x^4$$
$$12x^4-22x^3+12x^2$$
$$-12x^4+16x^3-4x^2$$
$$-\;6x^3+8x^2-4x+1$$
$$6x^3-8x^2+4x-1$$
$$0$$

$$\left(6x^3-4x^2\right)\left(-4x^2\right)=-24x^5+16x^4$$
$$\left(6x^3-8x^2+2x\right)(2x)=12x^4-16x^3+4x^2$$
$$\left(6x^3-8x^2+4x-1\right)(-1)=-6x^3+8x^2-4x+1$$

18. $$\sqrt{16x^6-40x^5+73x^4-84x^3+66x^2-36x+9} \;\Big|\; 4x^3-5x^2+6x-3$$

$$-16x^6$$
$$-40x^5+73x^4$$
$$40x^5-25x^4$$
$$48x^4-84x^3+66x^2$$
$$-48x^4+60x^3-36x^2$$
$$-24x^3+30x^2-36x+9$$
$$24x^3-30x^2+36x-9$$
$$0$$

$$\left(8x^3-5x^2\right)\left(-5x^2\right)=-40x^5+25x^4$$
$$\left(8x^3-10x^2+6x\right)(6x)=48x^4-60x^3+36x^2$$
$$\left(8x^3-10x^2+12x-3\right)(-3)=-24x^3+30x^2-36x+9$$

19. $\sqrt{m^6-4m^5n+4m^4n^2+4m^3n^4-8m^2n^5+4n^8}$ | $m^3-2m^2n+2n^4$

$$-m^6$$
$$-4m^5n+4m^4n^2$$
$$4m^5n-4m^4n^2$$
$$4m^3n^4-8m^2n^5+4n^8$$
$$-4m^3n^4+8m^2n^5-4n^8$$
$$0$$

$$\left(2m^3-2m^2n\right)\left(-2m^2n\right)=-4m^5n+4m^4n^2$$

$$\left(2m^3-4m^2n+2n^4\right)\left(2n^4\right)=4m^3n^4-8m^2n^5+4n^8$$

20. $\sqrt{9x^6-6x^5y+13x^4y^2-16x^3y^3+8x^2y^4-8xy^5+4y^6}$ | $3x^3-x^2y+2xy^2-2y^3$

$$-9x^6$$
$$-6x^5y+13x^4y^2$$
$$6x^5y-\ x^4y^2$$
$$12x^4y^2-16x^3y^3+8x^2y^4$$
$$-12x^4y^2+\ 4x^3y^3-4x^2y^4$$
$$-12x^3y^3+4x^2y^4-8xy^5+4y^6$$
$$12x^3y^3-4x^2y^4+8xy^5-4y^6$$
$$0$$

$$\left(6x^3-x^2y\right)\left(-x^2y\right)=-6x^5y+x^4y^2$$

$$\left(6x^3-2x^2y+2xy^2\right)\left(2xy^2\right)=12x^4y^2-4x^3y^3+4x^2y^4$$

$$\left(6x^3-2x^2y+4xy^2-2y^3\right)\left(-2y^3\right)=-12x^3y^3+4x^2y^4-8xy^5+4y^6$$

21. $\sqrt{16a^6-24a^5b+25a^4b^2-20a^3b^3+10a^2b^4-4ab^5+b^6}$ | $4a^3-3a^2b+2ab^2-b^3$

$$-16a^6$$
$$-24a^5b+25a^4b^2$$
$$24a^5b-\ 9a^4b^2$$
$$16a^4b^2-20a^3b^3+10a^2b^4$$
$$-16a^4b^2+12a^3b^3-\ 4a^2b^4$$
$$-\ 8a^3b^3+\ 6a^2b^4-4ab^5+b^6$$
$$8a^3b^3-\ 6a^2b^4+4ab^5-b^6$$
$$0$$

$$\left(8a^3-3a^2b\right)\left(-3a^2b\right)=-24a^5b+9a^4b^2$$

$$\left(8a^3-6a^2b+2ab^2\right)\left(2ab^2\right)=16a^4b^2-12a^3b^3+4a^2b^4$$

$$\left(8a^3-6a^2b+4ab^2-b^3\right)\left(-b^3\right)=-8a^3b^3+6a^2b^4-4ab^5+b^6$$

22. $\sqrt{36x^8-36x^6y^2+48x^5y^3-15x^4y^4-24x^3y^5+28x^2y^6-16xy^7+4y^8}$ | $6x^4-3x^2y^2+4xy^3-2y^4$

$$-36x^8$$
$$-36x^6y^2+48x^5y^3-15x^4y^4$$
$$36x^6y^2\qquad-\ 9x^4y^4$$
$$48x^5y^3-24x^4y^4-24x^3y^5+28x^2y^6$$
$$-48x^5y^3\qquad+24x^3y^5-16x^2y^6$$
$$-24x^4y^4\qquad+12x^2y^6-16xy^7+4y^8$$
$$24x^4y^4\qquad-\ 12x^2y^6+16xy^7-4y^8$$
$$0$$

$$\left(12x^4-3x^2y^2\right)\left(-3x^2y^2\right)=-36x^6y^2+9x^4y^4$$

$$\left(12x^4-6x^2y^2+4xy^3\right)\left(4xy^3\right)=48x^5y^3-24x^3y^5+16x^2y^6$$

$$\left(12x^4-6x^2y^2+8xy^3-2y^4\right)\left(-2y^4\right)=-24x^4y^4+12x^2y^6-16xy^7+4y^8$$

23. $\sqrt{25a^6-40a^5x+26a^4x^2-28a^3x^3+17a^2x^4-4ax^5+4x^6}$ $\quad 5a^3-4a^2x+ax^2-2x^3$

$-25a^6$

$-40a^5x+26a^4x^2$

$40a^5x-16a^4x^2$

$10a^4x^2-28a^3x^3+17a^2x^4$

$-10a^4x^2+8a^3x^3-a^2x^4$

$-20a^3x^3+16a^2x^4-4ax^5+4x^6$

$20a^3x^3-16a^2x^4+4ax^5-4x^6$

0

$(10a^3-4a^2x)(-4a^2x)=-40a^5x+16a^4x^2$

$(10a^3-8a^2x+ax^2)(ax^2)=10a^4x^2-8a^3x^3+a^2x^4$

$(10a^3-8a^2x+2ax^2-2x^3)(-2x^3)$

$=-20a^3x^3+16a^2x^4-4ax^5+4x^6$

24. $\sqrt{4a^8-12a^7+17a^6-16a^5+14a^4-10a^3+5a^2-2a+1}$ $\quad 2a^4-3a^3+2a^2-a+1$

$-4a^8$

$-12a^7+17a^6$

$12a^7-9a^6$

$8a^6-16a^5+14a^4$

$-8a^6+12a^5-4a^4$

$-4a^5+10a^4-10a^3+5a^2$

$4a^5-6a^4+4a^3-a^2$

$4a^4-6a^3+4a^2-2a+1$

$-4a^4+6a^3-4a^2+2a-1$

0

$(4a^4-3a^3)(-3a^3)=-12a^7+9a^6$

$(4a^4-6a^3+2a^2)(2a^2)$

$=8a^6-12a^5+4a^4$

$(4a^4-6a^3+4a^2-a)(-a)$

$=-4a^5+6a^4-4a^3+a^2$

$(4a^4-6a^3+4a^2-2a+1)(1)$

$=4a^4-6a^3+4a^2-2a+1$

25. $\sqrt{x^{10}-2x^9+3x^8-4x^7+5x^6-8x^5+7x^4-6x^3+5x^2-4x+4}$ $\quad x^5-x^4+x^3-x^2+x-2$

$-x^{10}$

$-2x^9+3x^8$

$2x^9-x^8$

$2x^8-4x^7+5x^6$

$-2x^8+2x^7-x^6$

$-2x^7+4x^6-8x^5+7x^4$

$2x^7-2x^6+2x^5-x^4$

$2x^6-6x^5+6x^4-6x^3+5x^2$

$-2x^6+2x^5-2x^4+2x^3-x^2$

$-4x^5+4x^4-4x^3+4x^2-4x+4$

$4x^5-4x^4+4x^3-4x^2+4x-4$

0

$(2x^5-x^4)(-x^4)=-2x^9+x^8$

$(2x^5-2x^4+x^3)(x^3)=2x^8-2x^7+x^6$

$(2x^5-2x^4+2x^3-x^2)(-x^2)$

$=-2x^7+2x^6-2x^5+x^4$

$(2x^5-2x^4+2x^3-2x^2+x)(x)$

$=2x^6-2x^5+2x^4-2x^3+x^2$

$(2x^5-2x^4+2x^3-2x^2+2x-2)(-2)$

$=-4x^5+4x^4-4x^3+4x^2-4x+4$

EJERCICIO 215

1. $\sqrt{\dfrac{x^4}{4}-x^3+\dfrac{5x^2}{3}-\dfrac{4x}{3}+\dfrac{4}{9}}$ $\quad\Big|\quad \dfrac{x^2}{2}-x+\dfrac{2}{3}$

$-\dfrac{x^4}{4}$

$-x^3+\dfrac{5x^2}{3}$

x^3-x^2

$\dfrac{2x^2}{3}-\dfrac{4x}{3}+\dfrac{4}{9}$

$-\dfrac{2x^2}{3}+\dfrac{4x}{3}-\dfrac{4}{9}$

0

$(x^2-x)(-x)=-x^3+x^2$

$\left(x^2-2x+\dfrac{2}{3}\right)\left(\dfrac{2}{3}\right)=\dfrac{2x^2}{3}-\dfrac{4x}{3}+\dfrac{4}{9}$

2. $\sqrt{\dfrac{a^2}{x^2}-\dfrac{2a}{3x}+\dfrac{19}{9}-\dfrac{2x}{3a}+\dfrac{x^2}{a^2}}$ $\quad\Big|\quad \dfrac{a}{x}-\dfrac{1}{3}+\dfrac{x}{a}$

$-\dfrac{a^2}{x^2}$

$-\dfrac{2a}{3x}+\dfrac{19}{9}$

$\dfrac{2a}{3x}-\dfrac{1}{9}$

$2-\dfrac{2x}{3a}+\dfrac{x^2}{a^2}$

$-2+\dfrac{2x}{3a}-\dfrac{x^2}{a^2}$

0

$\left(\dfrac{2a}{x}-\dfrac{1}{3}\right)\left(-\dfrac{1}{3}\right)=-\dfrac{2a}{3x}+\dfrac{1}{9}$

$\left(\dfrac{2a}{x}-\dfrac{2}{3}+\dfrac{x}{a}\right)\left(\dfrac{x}{a}\right)=2-\dfrac{2x}{3a}+\dfrac{x^2}{a^2}$

3. $\sqrt{\dfrac{a^2}{4}-ab+b^2+\dfrac{ac}{4}-\dfrac{bc}{2}+\dfrac{c^2}{16}}$ $\quad\Big|\quad \dfrac{a}{2}-b+\dfrac{c}{4}$

$-\dfrac{a^2}{4}$

$-ab+b^2$

$ab-b^2$

$\dfrac{ac}{4}-\dfrac{bc}{2}+\dfrac{c^2}{16}$

$-\dfrac{ac}{4}+\dfrac{bc}{2}-\dfrac{c^2}{16}$

0

$(a-b)(-b)=-ab+b^2$

$\left(a-2b+\dfrac{c}{4}\right)\left(\dfrac{c}{4}\right)=\dfrac{ac}{4}-\dfrac{bc}{2}+\dfrac{c^2}{16}$

4. $$\sqrt{\frac{9a^4}{16}-\frac{3a^3}{4}+\frac{29a^2}{20}-\frac{4a}{5}+\frac{16}{25}} \quad \Big| \quad \frac{3a^2}{4}-\frac{a}{2}+\frac{4}{5}$$

$$-\frac{9a^4}{16}$$

$$-\frac{3a^3}{4}+\frac{29a^2}{20}$$

$$\frac{3a^3}{4}-\frac{a^2}{4}$$

$$\frac{6a^2}{5}-\frac{4a}{5}+\frac{16}{25}$$

$$-\frac{6a^2}{5}+\frac{4a}{5}-\frac{16}{25}$$

$$0$$

$$\left(\frac{3a^2}{2}-\frac{a}{2}\right)\left(-\frac{a}{2}\right)=-\frac{3a^3}{4}+\frac{a^2}{4}$$

$$\left(\frac{3a^2}{2}-a+\frac{4}{5}\right)\left(\frac{4}{5}\right)=\frac{6a^2}{5}-\frac{4a}{5}+\frac{16}{25}$$

5. $$\sqrt{\frac{a^4}{16}+\frac{a^3b}{2}+\frac{3a^2b^2}{4}-ab^3+\frac{b^4}{4}} \quad \Big| \quad \frac{a^2}{4}+ab-\frac{b^2}{2}$$

$$-\frac{a^4}{16}$$

$$\frac{a^3b}{2}+\frac{3a^2b^2}{4}$$

$$-\frac{a^3b}{2}-a^2b^2$$

$$-\frac{a^2b^2}{4}-ab^3+\frac{b^4}{4}$$

$$\frac{a^2b^2}{4}+ab^3-\frac{b^4}{4}$$

$$0$$

$$\left(\frac{a^2}{2}+ab\right)(ab)=\frac{a^3b}{2}+a^2b^2$$

$$\left(\frac{a^2}{2}+2ab-\frac{b^2}{2}\right)\left(-\frac{b^2}{2}\right)=-\frac{a^2b^2}{4}-ab^3+\frac{b^4}{4}$$

6. $$\sqrt{\frac{x^2}{25}+\frac{2x}{3}-2xy+\frac{25}{9}-\frac{50y}{3}+25y^2} \quad \Big| \quad \frac{x}{5}+\frac{5}{3}-5y$$

$$-\frac{x^2}{25}$$

$$\frac{2x}{3}-2xy+\frac{25}{9}$$

$$-\frac{2x}{3} \qquad -\frac{25}{9}$$

$$-2xy \qquad -\frac{50y}{3}+25y^2$$

$$2xy \qquad +\frac{50y}{3}-25y^2$$

$$0$$

$$\left(\frac{2x}{5}+\frac{5}{3}\right)\left(\frac{5}{3}\right)=\frac{2x}{3}+\frac{25}{9}$$

$$\left(\frac{2x}{5}+\frac{10}{3}-5y\right)(-5y)=-2xy-\frac{50y}{3}+25y^2$$

7. $$\sqrt{\frac{x^4}{9}-\frac{4x^3y}{3}+\frac{62x^2y^2}{15}-\frac{4xy^3}{5}+\frac{y^4}{25}} = \frac{x^2}{3}-2xy+\frac{y^2}{5}$$

$$-\frac{x^4}{9}$$

$$-\frac{4x^3y}{3}+\frac{62x^2y^2}{15}-\frac{4xy^3}{5}$$

$$\frac{4x^3y}{3}-4x^2y^2$$

$$\frac{2x^2y^2}{15}-\frac{4xy^3}{5}+\frac{y^4}{25}$$

$$-\frac{2x^2y^2}{15}+\frac{4xy^3}{5}-\frac{y^4}{25}$$

$$0$$

$$\left(\frac{2x^2}{3}-2xy\right)\left(-2xy\right)=-\frac{4x^3y}{3}+4x^2y^2$$

$$\left(\frac{2x^2}{3}-4xy+\frac{y^2}{5}\right)\left(\frac{y^2}{5}\right)=\frac{2x^2y^2}{15}-\frac{4xy^3}{5}+\frac{y^4}{25}$$

8. $$\sqrt{\frac{a^4}{16}-\frac{3a^2}{10}+\frac{9}{25}+\frac{a^2b^2}{18}-\frac{2b^2}{15}+\frac{b^4}{81}} = \frac{a^2}{4}-\frac{3}{5}+\frac{b^2}{9}$$

$$-\frac{a^4}{16}$$

$$-\frac{3a^2}{10}+\frac{9}{25}$$

$$\frac{3a^2}{10}-\frac{9}{25}$$

$$\frac{a^2b^2}{18}-\frac{2b^2}{15}+\frac{b^4}{81}$$

$$-\frac{a^2b^2}{18}+\frac{2b^2}{15}-\frac{b^4}{81}$$

$$0$$

$$\left(\frac{a^2}{2}-\frac{3}{5}\right)\left(-\frac{3}{5}\right)=-\frac{3a^2}{10}+\frac{9}{25}$$

$$\left(\frac{a^2}{2}-\frac{6}{5}+\frac{b^2}{9}\right)\left(\frac{b^2}{9}\right)=\frac{a^2b^2}{18}-\frac{2b^2}{15}+\frac{b^4}{81}$$

9. $$\sqrt{x^2+4x+2-\frac{4}{y}+\frac{1}{x^2}} = x+2-\frac{1}{x}$$

$$-x^2$$

$$4x+2$$

$$-4x-4$$

$$-2-\frac{4}{y}+\frac{1}{x^2}$$

$$2+\frac{4}{y}-\frac{1}{x^2}$$

$$0$$

$$\left(2x+2\right)\left(2\right)=4x+4$$

$$\left(2x+4-\frac{1}{x}\right)\left(-\frac{1}{x}\right)=-2-\frac{4}{x}+\frac{1}{x^2}$$

10. $$\sqrt{\frac{x^2}{9}-\frac{10x}{3}+\frac{79}{3}-\frac{20}{x}+\frac{4}{x^2}} \;\Big|\; \frac{x}{3}-5+\frac{2}{x}$$

$$-\frac{x^2}{9}$$

$$-\frac{10x}{3}+\frac{79}{3}$$

$$\frac{10x}{3}-25$$

$$\frac{4}{3}-\frac{20}{x}+\frac{4}{x^2}$$

$$-\frac{4}{3}+\frac{20}{x}-\frac{4}{x^2}$$

$$0$$

$$\left(\frac{2x}{3}-5\right)(-5)=-\frac{10x}{3}+25$$

$$\left(\frac{2x}{3}-10+\frac{2}{x}\right)\left(\frac{2}{x}\right)=\frac{4}{3}-\frac{20}{x}+\frac{4}{x^2}$$

11. $$\sqrt{\frac{a^4}{4}-5a^2+28-\frac{30}{a^2}+\frac{9}{a^4}} \;\Big|\; \frac{a^2}{2}-5+\frac{3}{a^2}$$

$$-\frac{a^4}{4}$$

$$-5a^2+28$$

$$5a^2-25$$

$$3-\frac{30}{a^2}+\frac{9}{a^4}$$

$$-3+\frac{30}{a^2}-\frac{9}{a^4}$$

$$0$$

$$\left(a^2-5\right)(-5)=-5a^2+25$$

$$\left(a^2-10+\frac{3}{a^2}\right)\left(\frac{3}{a^2}\right)=3-\frac{30}{a^2}+\frac{9}{a^4}$$

12. $$\sqrt{\frac{a^4}{9}+\frac{2a^3}{3x}+\frac{a^2}{x^2}-\frac{2ax}{3}-2+\frac{x^2}{a^2}} \;\Big|\; \frac{a^2}{3}+\frac{a}{x}-\frac{x}{a}$$

$$-\frac{a^4}{9}$$

$$\frac{2a^3}{3x}+\frac{a^2}{x^2}$$

$$-\frac{2a^3}{3x}-\frac{a^2}{x^2}$$

$$-\frac{2ax}{3}-2+\frac{x^2}{a^2}$$

$$\frac{2ax}{3}+2-\frac{x^2}{a^2}$$

$$0$$

$$\left(\frac{2a^2}{3}+\frac{a}{x}\right)\left(\frac{a}{x}\right)=\frac{2a^3}{3x}+\frac{a^2}{x^2}$$

$$\left(\frac{2a^2}{3}+\frac{2a}{x}-\frac{x}{a}\right)\left(-\frac{x}{a}\right)=-\frac{2ax}{3}-2+\frac{x^2}{a^2}$$

13. $\sqrt{\dfrac{9a^2}{x^2}-\dfrac{3a}{2x}+\dfrac{65}{16}-\dfrac{x}{3a}+\dfrac{4x^2}{9a^2}}$ $\quad\Big|\quad \dfrac{3a}{x}-\dfrac{1}{4}+\dfrac{2x}{3a}$

$$-\frac{9a^2}{x^2}$$

$$-\frac{3a}{2x}+\frac{65}{16}$$

$$\frac{3a}{2x}-\frac{1}{16}$$

$$4-\frac{x}{3a}+\frac{4x^2}{9a^2}$$

$$-4+\frac{x}{3a}-\frac{4x^2}{9a^2}$$

$$0$$

$$\left(\frac{6a}{x}-\frac{1}{4}\right)\left(-\frac{1}{4}\right)=-\frac{3a}{2x}+\frac{1}{16}$$

$$\left(\frac{6a}{x}-\frac{1}{2}+\frac{2x}{3a}\right)\left(\frac{2x}{3a}\right)=4-\frac{x}{3a}+\frac{4x^2}{9a^2}$$

14. $\sqrt{9x^4+30x^2+55+\dfrac{50}{x^2}+\dfrac{25}{x^4}}$ $\quad\Big|\quad 3x^2+5+\dfrac{5}{x^2}$

$$-9x^4$$

$$30x^2+55$$

$$-30x^2-25$$

$$30+\frac{50}{x^2}+\frac{25}{x^4}$$

$$-30-\frac{50}{x^2}-\frac{25}{x^4}$$

$$0$$

$$\left(6x^2+5\right)(5)=30x^2+25$$

$$\left(6x^2+10+\frac{5}{x^2}\right)\left(\frac{5}{x^2}\right)=30+\frac{50}{x^2}+\frac{25}{x^4}$$

15. $\sqrt{\dfrac{4a^2}{25x^2}-\dfrac{2a}{5x}+\dfrac{19}{12}-\dfrac{5x}{3a}+\dfrac{25x^2}{9a^2}}$ $\quad\Big|\quad \dfrac{2a}{5x}-\dfrac{1}{2}+\dfrac{5x}{3a}$

$$-\frac{4a^2}{25x^2}$$

$$-\frac{2a}{5x}+\frac{19}{12}$$

$$\frac{2a}{5x}-\frac{1}{4}$$

$$\frac{4}{3}-\frac{5x}{3a}+\frac{25x^2}{9a^2}$$

$$-\frac{4}{3}+\frac{5x}{3a}-\frac{25x^2}{9a^2}$$

$$0$$

$$\left(\frac{4a}{5x}-\frac{1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{2a}{5x}+\frac{1}{4}$$

$$\left(\frac{4a}{5x}-1+\frac{5x}{3a}\right)\left(\frac{5x}{3a}\right)=\frac{4}{3}-\frac{5x}{3a}+\frac{25x^2}{9a^2}$$

16. $$\sqrt{\frac{x^4}{16}-\frac{x^3y}{4}+\frac{3x^2y^2}{20}+\frac{xy^3}{5}+\frac{y^4}{25}} \quad \Big| \quad \frac{x^2}{4}-\frac{xy}{2}-\frac{y^2}{5}$$

$$-\frac{x^4}{16}$$

$$-\frac{x^3y}{4}+\frac{3x^2y^2}{20}$$

$$\frac{x^3y}{4}-\frac{x^2y^2}{4}$$

$$-\frac{x^2y^2}{10}+\frac{xy^3}{5}+\frac{y^4}{25}$$

$$\frac{x^2y^2}{10}-\frac{xy^3}{5}-\frac{y^4}{25}$$

$$0$$

$$\left(\frac{x^2}{2}-\frac{xy}{2}\right)\left(-\frac{xy}{2}\right)=-\frac{x^3y}{4}+\frac{x^2y^2}{4}$$

$$\left(\frac{x^2}{2}-xy-\frac{y^2}{5}\right)\left(-\frac{y^2}{5}\right)=-\frac{x^2y^2}{10}+\frac{xy^3}{5}+\frac{y^4}{25}$$

17. $$\sqrt{\frac{4a^2b^2}{49x^2y^2}-\frac{2ab}{7xy}+\frac{21}{20}-\frac{7xy}{5ab}+\frac{49x^2y^2}{25a^2b^2}} \quad \Big| \quad \frac{2ab}{7xy}-\frac{1}{2}+\frac{7xy}{5ab}$$

$$-\frac{4a^2b^2}{49x^2y^2}$$

$$-\frac{2ab}{7xy}+\frac{21}{20}$$

$$\frac{2ab}{7xy}-\frac{1}{4}$$

$$\frac{4}{5}-\frac{7xy}{5ab}+\frac{49x^2y^2}{25a^2b^2}$$

$$-\frac{4}{5}+\frac{7xy}{5ab}-\frac{49x^2y^2}{25a^2b^2}$$

$$0$$

$$\left(\frac{4ab}{7xy}-\frac{1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{2ab}{7xy}+\frac{1}{4}$$

$$\left(\frac{4ab}{7xy}-1+\frac{7xy}{5ab}\right)\left(\frac{7xy}{5ab}\right)$$

$$=\frac{4}{5}-\frac{7xy}{5ab}+\frac{49x^2y^2}{25a^2b^2}$$

18. $$\sqrt{\frac{9a^2x^2}{25m^2n^2}-\frac{6ax}{25mn}+\frac{23}{75}-\frac{4mn}{45ax}+\frac{4m^2n^2}{81a^2x^2}} \quad \Big| \quad \frac{3ax}{5mn}-\frac{1}{5}+\frac{10mn}{45ax}$$

$$-\frac{9a^2x^2}{25m^2n^2}$$

$$-\frac{6ax}{25mn}+\frac{23}{75}$$

$$\frac{6ax}{25mn}-\frac{1}{25}$$

$$\frac{20}{75}-\frac{4mn}{45ax}+\frac{4m^2n^2}{81a^2x^2}$$

$$-\frac{20}{75}+\frac{4mn}{45ax}-\frac{4m^2n^2}{81a^2x^2}$$

$$0$$

$$\left(\frac{6ax}{5mn}-\frac{1}{5}\right)\left(-\frac{1}{5}\right)=-\frac{6ax}{25mn}+\frac{1}{25}$$

$$\left(\frac{6ax}{5mn}-\frac{2}{5}+\frac{10mn}{45ax}\right)\left(\frac{10mn}{45ax}\right)$$

$$=\frac{20}{75}-\frac{4mn}{45ax}+\frac{4m^2n^2}{81a^2x^2}$$

19. $\sqrt{\frac{x^6}{4}-x^5+\frac{5x^4}{3}+\frac{2x^3}{3}-\frac{32x^2}{9}+\frac{8x}{3}+4}$ | $\frac{x^3}{2}-x^2+\frac{2x}{3}+2$

$$-\frac{x^6}{4}$$

$$-x^5+\frac{5x^4}{3}$$

$$x^5-x^4$$

$$\frac{2x^4}{3}+\frac{2x^3}{3}-\frac{32x^2}{9}$$

$$-\frac{2x^4}{3}+\frac{4x^3}{3}-\frac{4x^2}{9}$$

$$2x^3-4x^2+\frac{8x}{3}+4$$

$$-2x^3+4x^2-\frac{8x}{3}-4$$

$$0$$

$$\left(x^3-x^2\right)\left(-x^2\right)=-x^5+x^4$$

$$\left(x^3-2x^2+\frac{2x}{3}\right)\left(\frac{2x}{3}\right)=\frac{2x^4}{3}-\frac{4x^3}{3}+\frac{4x^2}{9}$$

$$\left(x^3-2x^2+\frac{4x}{3}+2\right)(2)=2x^3-4x^2+\frac{8x}{3}+4$$

20. $\sqrt{\frac{1}{4}-\frac{3a}{4}+\frac{59a^2}{48}-\frac{3a^3}{2}+\frac{43a^4}{36}-\frac{2a^5}{3}+\frac{1a^6}{4}}$ | $\frac{1}{2}-\frac{3a}{4}+\frac{2a^2}{3}-\frac{a^3}{2}$

$$-\frac{1}{4}$$

$$-\frac{3a}{4}+\frac{59a^2}{48}$$

$$\frac{3a}{4}-\frac{9a^2}{16}$$

$$\frac{2a^2}{3}-\frac{3a^3}{2}+\frac{43a^4}{36}$$

$$-\frac{2a^2}{3}+a^3-\frac{4a^4}{9}$$

$$-\frac{a^3}{2}+\frac{3a^4}{4}-\frac{2a^5}{3}+\frac{a^6}{4}$$

$$\frac{a^3}{2}-\frac{3a^4}{4}+\frac{2a^5}{3}-\frac{a^6}{4}$$

$$0$$

$$\left(1-\frac{3a}{4}\right)\left(-\frac{3a}{4}\right)=-\frac{3a}{4}+\frac{9a^2}{16}$$

$$\left(1-\frac{3a}{2}+\frac{2a^2}{3}\right)\left(\frac{2a^2}{3}\right)=\frac{2a^2}{3}-a^3+\frac{4a^4}{9}$$

$$\left(1-\frac{3a}{2}+\frac{4a^2}{3}-\frac{a^3}{2}\right)\left(-\frac{a^3}{2}\right)$$

$$=-\frac{a^3}{2}+\frac{3a^4}{4}-\frac{2a^5}{3}+\frac{a^6}{4}$$

EJERCICIO 216

1. $\sqrt[3]{8-36y+54y^2-27y^3}$ | $2-3y$

$$-8$$

$$-36y+54y^2-27y^3$$

$$36y-54y^2+27y^3$$

$$0$$

$$3(2)^2=12$$

$$3(2)^2\left(-3y\right)=-36y$$

$$3(2)\left(-3y\right)^2=54y^2$$

$$\left(-3y\right)^3=-27y^3$$

2. $\sqrt[3]{64a^6+240a^4b^2+300a^2b^4+125b^6}$ | $4a^2+5b^2$

$$-64a^6$$

$$240a^4b^2+300a^2b^4+125b^6$$

$$-240a^4b^2-300a^2b^4-125b^6$$

$$0$$

$$3\left(4a^2\right)^2=48a^4$$

$$3\left(4a^2\right)^2\left(5b^2\right)=240a^4b^2$$

$$3\left(4a^2\right)\left(5b^2\right)^2=300a^2b^4$$

$$\left(5b^2\right)^3=125b^6$$

3. $\sqrt[3]{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$ | x^2+x+1

$$-x^6$$

$$3x^5+6x^4+7x^3$$

$$-3x^5-3x^4-\ x^3$$

$$3x^4+6x^3+6x^2+3x+1$$

$$-3x^4-6x^3-6x^2-3x-1$$

$$0$$

$$3\left(x^2\right)^2=3x^4$$

$$3\left(x^2\right)^2(x)=3x^5$$

$$3\left(x^2\right)\left(x^2\right)=3x^4$$

$$x^3=x^3$$

$$3\left(x^2+x\right)^2=3x^4+6x^3+3x^2$$

$$3\left(x^2+x\right)^2(1)=3x^4+6x^3+3x^2$$

$$3\left(x^2+x\right)(1)^2=3x^2+3x$$

$$(1)^3=1$$

4. $\sqrt[3]{8x^6-12x^5-6x^4+11x^3+3x^2-3x-1}$ | $2x^2-x-1$

$$-8x^6$$

$$-12x^5-6x^4\ +11x^3$$

$$12x^5-6x^4\ +\ \ x^3$$

$$-12x^4+12x^3+3x^2-3x-1$$

$$12x^4-12x^3-3x^2+3x+1$$

$$0$$

$$3\left(2x^2\right)^2=12x^4$$

$$3\left(2x^2\right)^2(-x)=-12x^5$$

$$3\left(2x^2\right)(-x)^2=6x^4$$

$$(-x)^3=-x^3$$

$$3\left(2x^2-x\right)^2=12x^4-12x^3+3x^2$$

$$3\left(2x^2-x\right)^2(-1)=-12x^4+12x^3-3x^2$$

$$3\left(2x^2-x\right)(-1)^2=6x^2-3x$$

$$(-1)^3=-1$$

5. $\sqrt[3]{1-9x+33x^2-63x^3+66x^4-36x^5+8x^6}$ | $1-3x+2x^2$

$$-1$$

$$-9x+33x^2-63x^3$$

$$9x-27x^2+27x^3$$

$$6x^2\ -36x^3+66x^4-36x^5+8x^6$$

$$-6x^2\ +36x^3-66x^4+36x^5-8x^6$$

$$0$$

$$3(1)^2=3$$

$$3(1)^2(-3x)=-9x$$

$$3(1)(-3x)^2=27x^2$$

$$(-3x)^3=-27x^3$$

$$3(1-3x)^2=3-18x+27x^2$$

$$3(1-3x)^2\left(2x^2\right)=6x^2-36x^3+54x^4$$

$$3(1-3x)\left(2x^2\right)^2=12x^4-36x^5$$

$$\left(2x^2\right)^3=8x^6$$

6. $\sqrt[3]{x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8} \;\Big|\; x^2 - 3x + 2$

$$-x^6$$
$$-9x^5 + 33x^4 - 63x^3$$
$$9x^5 - 27x^4 + 27x^3$$
$$6x^4 - 36x^3 + 66x^2 - 36x + 8$$
$$-6x^4 + 36x^3 - 66x^2 + 36x - 8$$
$$0$$

$$3\left(x^2\right)^2 = 3x^4$$
$$3\left(x^2\right)^2\left(-3x\right) = -9x^5$$
$$3\left(x^2\right)\left(-3x\right)^2 = 27x^4$$
$$\left(-3x\right)^3 = -27x^3$$

$$3\left(x^2 - 3x\right)^2 = 3x^4 - 18x^3 + 27x^2$$
$$3\left(x^2 - 3x\right)^2(2) = 6x^4 - 36x^3 + 54x^2$$
$$3\left(x^2 - 3x\right)(2)^2 = 12x^2 - 36x$$
$$(2)^3 = 8$$

7. $\sqrt[3]{x^9 - 6x^8 + 12x^7 - 20x^6 + 48x^5 - 48x^4 + 48x^3 - 96x^2 - 64} \;\Big|\; x^3 - 2x^2 - 4$

$$-x^9$$
$$-6x^8 + 12x^7 - 20x^6$$
$$6x^8 - 12x^7 + 8x^6$$
$$-12x^6 + 48x^5 - 48x^4 + 48x^3 - 96x^2 - 64$$
$$12x^6 - 48x^5 + 48x^4 - 48x^3 + 96x^2 + 64$$
$$0$$

$$3\left(x^3\right)^2 = 3x^6$$
$$3\left(x^3\right)^2\left(-2x^2\right) = -6x^8$$
$$3(x)^3\left(-2x^2\right)^2 = 12x^7$$
$$\left(-2x^2\right)^3 = -8x^6$$

$$3\left(x^3 - 2x^2\right)^2 = 3x^6 - 12x^5 + 12x^4$$
$$3\left(x^3 - 2x^2\right)^2\left(-4\right) = -12x^6 + 48x^5 - 48x^4$$
$$3\left(x^3 - 2x^2\right)\left(-4\right)^2 = 48x^3 - 96x^2$$
$$\left(-4\right)^3 = -64$$

8. $\sqrt[3]{x^{12} - 3x^{10} - 3x^8 + 11x^6 + 6x^4 - 12x^2 - 8} \;\Big|\; x^4 - x^2 - 2$

$$-x^{12}$$
$$-3x^{10} - 3x^8 + 11x^6$$
$$3x^{10} - 3x^8 + x^6$$
$$-6x^8 + 12x^6 + 6x^4 - 12x^2 - 8$$
$$6x^8 - 12x^6 - 6x^4 + 12x^2 + 8$$
$$0$$

$$3\left(x^4\right)^2 = 3x^8$$
$$3\left(x^4\right)^2\left(-x^2\right) = -3x^{10}$$
$$3\left(x^4\right)\left(-x^2\right)^2 = 3x^8$$
$$\left(-x^2\right)^3 = -x^6$$

$$3\left(x^4 - x^2\right)^2 = 3x^8 - 6x^6 + 3x^4$$
$$3\left(x^4 - x^2\right)^2\left(-2\right) = -6x^8 + 12x^6 - 6x^4$$
$$3\left(x^4 - x^2\right)\left(-2\right)^2 = 12x^4 - 12x^2$$
$$\left(-2\right)^3 = -8$$

9. $\sqrt[3]{8x^6-36x^5+66x^4-63x^3+33x^2-9x+1}$ $\quad\Big|\quad 2x^2-3x+1$

$$-8x^6$$

$$-36x^5+66x^4-63x^3$$

$$36x^5-54x^4+27x^3$$

$$12x^4-36x^3+33x^2-9x+1$$

$$-12x^4+36x^3-33x^2+9x-1$$

$$0$$

$$3\left(2x^2\right)^2=12x^4$$

$$3\left(2x^2\right)^2\left(-3x\right)=-36x^5$$

$$3\left(2x^2\right)\left(-3x\right)^2=54x^4$$

$$\left(-3x\right)^3=-27x^3$$

$$3\left(2x^2-3x\right)^2=12x^4-36x^3+27x^2$$

$$3\left(2x^2-3x\right)^2(1)=12x^4-36x^3+27x^2$$

$$3\left(2x^2-3x\right)(1)^2=6x^2-9x$$

$$(1)^3=1$$

10. $\sqrt[3]{27a^6-135a^5+117a^4+235a^3-156a^2-240a-64}$ $\quad\Big|\quad 3a^2-5a-4$

$$-27a^6$$

$$-135a^5+117a^4+235a^3$$

$$135a^5-225a^4+125a^3$$

$$-108a^4+360a^3-156a^2-240a-64$$

$$108a^4-360a^3+156a^2+240a+64$$

$$0$$

$$3\left(3a^2\right)^2=27a^4$$

$$3\left(3a^2\right)^2\left(-5a\right)=-135a^5$$

$$3\left(3a^2\right)\left(-5a\right)^2=225a^4$$

$$\left(-5a\right)^3=-125a^3$$

$$3\left(3a^2-5a\right)^2=27a^4-90a^3+75a^2$$

$$3\left(3a^2-5a\right)^2\left(-4\right)=-108a^4+360a^3-300a^2$$

$$3\left(3a^2-5a\right)\left(-4\right)^2=144a^2-240a$$

$$\left(-4\right)^3=-64$$

11. $\sqrt[3]{a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6}$ $\quad\Big|\quad a^2-2ab+b^2$

$$-a^6$$

$$-6a^5b+15a^4b^2-20a^3b^3$$

$$6a^5b-12a^4b^2+8a^3b^3$$

$$3a^4b^2-12a^3b^3+15a^2b^4-6ab^5+b^6$$

$$-3a^4b^2+12a^3b^3-15a^2b^4+6ab^5-b^6$$

$$0$$

$$3\left(a^2\right)^2=3a^4$$

$$3\left(a^2\right)^2\left(-2ab\right)=-6a^5b$$

$$3\left(a^2\right)\left(-2ab\right)^2=12a^4b^2$$

$$\left(-2ab\right)^3=-8a^3b^3$$

$$3\left(a^2-2ab\right)^2=3a^4-12a^3b+12a^2b^2$$

$$3\left(a^2-2ab\right)^2(b)^2=3a^4b^2-12a^3b^3+12a^2b^4$$

$$3\left(a^2-2ab\right)\left(b^2\right)^2=3a^2b^4-6ab^5$$

$$\left(b^2\right)^3=b^6$$

12. $$\sqrt[3]{x^6-9x^5y+42x^4y^2-117x^3y^3+210x^2y^4-225xy^5+125y^6} \quad \Big| \quad x^2-3xy+5y^2$$

$$-x^6$$

$$-9x^5y+42x^4y^2-117x^3y^3$$

$$9x^5y-27x^4y^2+27x^3y^3$$

$$15x^4y^2-80x^3y^3+210x^2y^4-225xy^5+125y^6$$

$$-15x^4y^2+80x^3y^3-210x^2y^4+225xy^5-125y^6$$

$$0$$

$$3\left(x^2\right)^2=3x^4$$

$$3\left(x^2\right)^2\left(-3xy\right)=-9x^5y$$

$$3\left(x^2\right)\left(-3xy\right)^2=27x^4y^2$$

$$\left(-3xy\right)^3=-27x^3y^3$$

$$3\left(x^2-3xy\right)^2=3x^4-18x^3y+27x^2y^2$$

$$3\left(x^2-3xy\right)^2\left(5y^2\right)=15x^4y^2-80x^3y^3+135x^2y^4$$

$$3\left(x^2-3xy\right)\left(5y^2\right)^2=75x^2y^4-225xy^5$$

$$\left(5y^2\right)^3=125y^6$$

13. $$\sqrt[3]{a^{12}-3a^{10}+15a^8-25a^6+60a^4-48a^2+64} \quad \Big| \quad a^4-a^2+4$$

$$-a^{12}$$

$$-3a^{10}+15a^8-25a^6$$

$$3a^{10}-3a^8+a^6$$

$$12a^8-24a^6+60a^4-48a^2+64$$

$$-12a^8+24a^6-60a^4+48a^2-64$$

$$0$$

$$3\left(a^4\right)^2=3a^8$$

$$3\left(a^4\right)^2\left(-a^2\right)=-3a^{10}$$

$$3\left(a^4\right)\left(-a^2\right)^2=3a^8$$

$$\left(-a^2\right)^3=-a^6$$

$$3\left(a^4-a^2\right)^2=3a^8-6a^6+3a^4$$

$$3\left(a^4-a^2\right)^2(4)=12a^8-24a^6+12a^4$$

$$3\left(a^4-a^2\right)(4)^2=48a^4-48a^2$$

$$(4)^3=64$$

14. $$\sqrt[3]{a^9-9a^8x+27a^7x^2-21a^6x^3-36a^5x^4+54a^4x^5+12a^3x^6-36a^2x^7+8x^9} \quad \Big| \quad a^3-3a^2x+2x^3$$

$$-a^9$$

$$-9a^8x+27a^7x^2-21a^6x^3$$

$$9a^8x-27a^7x^2+27a^6x^3$$

$$6a^6x^3-36a^5x^4+54a^4x^5+12a^3x^6-36a^2x^7+8x^9$$

$$-6a^6x^3+36a^5x^4-54a^4x^5-12a^3x^6+36a^2x^7-8x^9$$

$$0$$

$$3\left(a^3\right)^2=3a^6$$

$$3\left(a^3\right)^2\left(-3a^2x\right)=-9a^8x$$

$$3\left(a^3\right)\left(-3a^2x\right)^2=27a^7x^2$$

$$\left(-3a^2x\right)^3=-27a^6x^3$$

$$3\left(a^3-3a^2x\right)^2=3a^6-18a^5x+27a^4x^2$$

$$3\left(a^3-3a^2x\right)^2\left(2x^3\right)=6a^6x^3-36a^5x^4+54a^4x^5$$

$$3\left(a^3-3a^2x\right)\left(2x^3\right)^2=12a^3x^6-36a^2x^7$$

$$\left(2x^3\right)^3=8x^9$$

15. $\sqrt[3]{a^9-3a^8+6a^7-10a^6+12a^5-12a^4+10a^3-6a^2+3a-1} = a^3-a^2+a-1$

$$-a^9$$

$$-3a^8+6a^7-10a^6$$

$$3a^8-3a^7+a^6$$

$$3a^7-9a^6+12a^5-12a^4+10a^3$$

$$-3a^7+6a^6-6a^5+3a^4-a^3$$

$$-3a^6+6a^5-9a^4+9a^3-6a^2+3a-1$$

$$3a^6-6a^5+9a^4-9a^3+6a^2-3a+1$$

$$0$$

$$3\left(a^3\right)^2=3a^6$$

$$3\left(a^3\right)^2\left(-a^2\right)=-3a^8$$

$$3\left(a^3\right)\left(-a^2\right)^2=3a^7$$

$$\left(-a^2\right)^3=-a^6$$

$$3\left(a^3-a^2\right)^2=3a^6-6a^5+3a^4$$

$$3\left(a^3-a^2\right)^2(a)=3a^7-6a^6+3a^5$$

$$3\left(a^3-a^2\right)\left(a^2\right)=3a^5-3a^4$$

$$\left(a^3\right)=a^3$$

$$3\left(a^3-a^2+a\right)^2=3a^6+3a^4+3a^2-6a^5+6a^4-6a^3$$

$$3\left(a^3-a^2+a\right)^2(-1)=-3a^6+6a^5-9a^4+6a^3-3a^2$$

$$3\left(a^3-a^2+a\right)(-1)^2=3a^3-3a^2+3a$$

$$(-1)^3=-1$$

16. $\sqrt[3]{x^9-12x^8+54x^7-121x^6+180x^5-228x^4+179x^3-144x^2+54x-27} = x^3-4x^2+2x-3$

$$-x^9$$

$$-12x^8+54x^7-121x^6$$

$$12x^8-48x^7+64x^6$$

$$6x^7-57x^6+180x^5-228x^4+179x^3$$

$$-6x^7+48x^6-108x^5+48x^4-8x^3$$

$$-9x^6+72x^5-180x^4+171x^3-144x^2+54x-27$$

$$9x^6-72x^5+180x^4-171x^3+144x^2-54x+27$$

$$0$$

$$3\left(x^3\right)^2=3x^6 \qquad 3\left(x^3\right)^2\left(-4x^2\right)=-12x^8$$

$$3\left(x^3\right)\left(-4x^2\right)^2=48x^7 \qquad \left(-4x^2\right)^3=-64x^6$$

$$3\left(x^3-4x^2\right)^2=3x^6-24x^5+48x^4$$

$$3\left(x^3-4x^2\right)^2(2x)=6x^7-48x^6+96x^5$$

$$3\left(x^3-4x^2\right)(2x)^2=12x^5-48x^4$$

$$(2x)^3=8x^3$$

$$3\left(x^3-4x^2+2x\right)^2=3x^6+48x^4+12x^2-24x^5+12x^4-48x^3$$

$$3\left(x^3-4x^2+2x\right)^2(-3)=-9x^6-144x^4-36x^2+72x^5-36x^4+144x^3$$

$$3\left(x^3-4x^2+2x\right)(-3)^2=27x^3-108x^2+54x$$

$$(-3)^3=-27$$

EJERCICIO 217

1. $\sqrt[3]{\frac{x^6}{8}-\frac{x^5}{4}+\frac{5x^4}{3}-\frac{55x^3}{27}+\frac{20x^2}{3}-4x+8}$ | $\frac{x^2}{2}-\frac{x}{3}+2$

$-\frac{x^6}{8}$

$-\frac{x^5}{4}+\frac{5x^4}{3}-\frac{55x^3}{27}$

$\frac{x^5}{4}-\frac{x^4}{6}+\frac{x^3}{27}$

$\frac{3x^4}{2}-2x^3+\frac{20x^2}{3}-4x+8$

$-\frac{3x^4}{2}+2x^3-\frac{20x^2}{3}+4x-8$

0

$3\left(\frac{x^2}{2}\right)^2=\frac{3x^4}{4}$

$3\left(\frac{x^2}{2}\right)^2\left(-\frac{x}{3}\right)=-\frac{x^5}{4}$

$3\left(\frac{x^2}{2}\right)\left(-\frac{x}{3}\right)^2=\frac{x^4}{6}$

$\left(-\frac{x}{3}\right)^3=-\frac{x^3}{27}$

$3\left(\frac{x^2}{2}-\frac{x}{3}\right)^2=\frac{3x^4}{4}-x^3+\frac{x^2}{3}$

$3\left(\frac{x^2}{2}-\frac{x}{3}\right)^2(2)=\frac{3x^4}{2}-2x^3+\frac{2x^2}{3}$

$3\left(\frac{x^2}{2}-\frac{x}{3}\right)(2)^2=6x^2-4x$

$(2)^3=8$

2. $\sqrt[3]{a^9+\frac{3a^8}{2}-\frac{a^7}{4}-\frac{7a^6}{8}+\frac{a^5}{12}+\frac{a^4}{6}-\frac{a^3}{27}}$ | $a^3+\frac{a^2}{2}-\frac{a}{3}$

$-a^9$

$\frac{3a^8}{2}-\frac{a^7}{4}-\frac{7a^6}{8}$

$-\frac{3a^8}{2}-\frac{3a^7}{4}-\frac{a^6}{8}$

$-a^7-a^6+\frac{a^5}{12}+\frac{a^4}{6}-\frac{a^3}{27}$

$a^7+a^6-\frac{a^5}{12}-\frac{a^4}{6}+\frac{a^3}{27}$

0

$3\left(a^3\right)^2=3a^6$

$3\left(a^3\right)^2\left(\frac{a^2}{2}\right)=\frac{3a^8}{2}$

$3\left(a^3\right)\left(\frac{a^2}{2}\right)^2=\frac{3a^7}{4}$

$\left(\frac{a^2}{2}\right)^3=\frac{a^6}{8}$

$3\left(a^3+\frac{a^2}{2}\right)^2=3a^6+3a^5+\frac{3a^4}{4}$

$3\left(a^3+\frac{a^2}{2}\right)^2\left(-\frac{a}{3}\right)=-a^7-a^6-\frac{a^5}{4}$

$3\left(a^3+\frac{a^2}{2}\right)\left(-\frac{a}{3}\right)^2=\frac{a^5}{3}+\frac{a^4}{6}$

$\left(-\frac{a}{3}\right)^3=-\frac{a^3}{27}$

3. $$\sqrt[3]{\frac{x^3}{8}-\frac{9x^2}{4}+15x-45+\frac{60}{x}-\frac{36}{x^2}+\frac{8}{x^3}} \quad \Bigg| \quad \frac{x}{2}-3+\frac{2}{x}$$

$$-\frac{x^3}{8}$$

$$-\frac{9x^2}{4}+15x-45$$

$$\frac{9x^2}{4}-\frac{27x}{2}+27$$

$$\frac{3x}{2}-18+\frac{60}{x}-\frac{36}{x^2}+\frac{8}{x^3}$$

$$-\frac{3x}{2}+18-\frac{60}{x}+\frac{36}{x^2}-\frac{8}{x^3}$$

$$0$$

$$3\left(\frac{x}{2}\right)^2=\frac{3x^2}{4}$$

$$3\left(\frac{x}{2}\right)^2(-3)=-\frac{9x^2}{4}$$

$$3\left(\frac{x}{2}\right)(-3)^2=\frac{27x}{2}$$

$$(-3)^3=-27$$

$$3\left(\frac{x}{2}-3\right)^2=\frac{3x^2}{4}-9x+27$$

$$3\left(\frac{x}{2}-3\right)^2\left(\frac{2}{x}\right)=\frac{3x}{2}-18+\frac{54}{x}$$

$$3\left(\frac{x}{2}-3\right)\left(\frac{2}{x}\right)^2=\frac{6}{x}-\frac{36}{x^2}$$

$$\left(\frac{2}{x}\right)^3=\frac{8}{x^3}$$

4. $$\sqrt[3]{\frac{a^3}{8b^3}-\frac{3a^2}{4b^2}+\frac{15a}{8b}-\frac{5}{2}+\frac{15b}{8a}-\frac{3b^2}{4a^2}+\frac{b^3}{8a^3}} \quad \Bigg| \quad \frac{a}{2b}-1+\frac{b}{2a}$$

$$-\frac{a^3}{8b^3}$$

$$-\frac{3a^2}{4b^2}+\frac{15a}{8b}-\frac{5}{2}$$

$$\frac{3a^2}{4b^2}-\frac{3a}{2b}+1$$

$$\frac{3a}{8b}-\frac{3}{2}+\frac{15b}{8a}-\frac{3b^2}{4a^2}+\frac{b^3}{8a^3}$$

$$-\frac{3a}{8b}+\frac{3}{2}-\frac{15b}{8a}+\frac{3b^2}{4a^2}-\frac{b^3}{8a^3}$$

$$0$$

$$3\left(\frac{a}{2b}\right)^2=\frac{3a^2}{4b^2}$$

$$3\left(\frac{a}{2b}\right)^2(-1)=-\frac{3a^2}{4b^2}$$

$$3\left(\frac{a}{2b}\right)(-1)^2=\frac{3a}{2b}$$

$$(-1)^3=-1$$

$$3\left(\frac{a}{2b}-1\right)^2=\frac{3a^2}{4b^2}-\frac{3a}{b}+3$$

$$3\left(\frac{a}{2b}-1\right)^2\left(\frac{b}{2a}\right)=\frac{3a}{8b}-\frac{3}{2}+\frac{3b}{2a}$$

$$3\left(\frac{a}{2b}-1\right)\left(\frac{b}{2a}\right)^2=\frac{3b}{8a}-\frac{3b^2}{4a^2}$$

$$\left(\frac{b}{2a}\right)^3=\frac{b^3}{8a^3}$$

5. $$\sqrt[3]{\frac{8a^3}{27x^3}-\frac{2a^2}{3x^2}+\frac{a}{18x}+\frac{13}{24}-\frac{x}{36a}-\frac{x^2}{6a^2}-\frac{x^3}{27a^3}} \quad \Bigg| \quad \frac{2a}{3x}-\frac{1}{2}-\frac{x}{3a}$$

$$-\frac{8a^3}{27x^3}$$

$$-\frac{2a^2}{3x^2}+\frac{a}{18x}+\frac{13}{24}$$

$$\frac{2a^2}{3x^2}-\frac{a}{2x}+\frac{1}{8}$$

$$-\frac{4a}{9x}+\frac{2}{3}-\frac{x}{36a}-\frac{x^2}{6a^2}-\frac{x^3}{27a^3}$$

$$\frac{4a}{9x}-\frac{2}{3}+\frac{x}{36a}+\frac{x^2}{6a^2}+\frac{x^3}{27a^3}$$

$$0$$

$$3\left(\frac{2a}{3x}\right)^2=\frac{4a^2}{3x^2}$$

$$3\left(\frac{2a}{3x}\right)^2\left(-\frac{1}{2}\right)=-\frac{2a^2}{3x^2}$$

$$3\left(\frac{2a}{3x}\right)\left(-\frac{1}{2}\right)^2=\frac{a}{2x}$$

$$\left(-\frac{1}{2}\right)^3=-\frac{1}{8}$$

$$3\left(\frac{2a}{3x}-\frac{1}{2}\right)^2=\frac{4a^2}{3x^2}-\frac{2a}{x}+\frac{3}{4}$$

$$3\left(\frac{2a}{3x}-\frac{1}{2}\right)^2\left(-\frac{x}{3a}\right)=-\frac{4a}{9x}+\frac{2}{3}-\frac{x}{4a}$$

$$3\left(\frac{2a}{3x}-\frac{1}{2}\right)\left(-\frac{x}{3a}\right)^2=\frac{2x}{9a}-\frac{x^2}{6a^2}$$

$$\left(-\frac{x}{3a}\right)^3=-\frac{x^3}{27a^3}$$

6. $$\sqrt[3]{\frac{8a^3}{27b^3}+\frac{4a^2}{3b^2}+\frac{3a}{b}+4+\frac{27b}{8a}+\frac{27b^2}{16a^2}+\frac{27b^3}{64a^3}} \quad \Bigg| \quad \frac{2a}{3b}+1+\frac{3b}{4a}$$

$$-\frac{8a^3}{27b^3}$$

$$\frac{4a^2}{3b^2}+\frac{3a}{b}+4$$

$$-\frac{4a^2}{3b^2}-\frac{2a}{b}-1$$

$$\frac{a}{b}+3+\frac{27b}{8a}+\frac{27b^2}{16a^2}+\frac{27b^3}{64a^3}$$

$$-\frac{a}{b}-3-\frac{27b}{8a}-\frac{27b^2}{16a^2}-\frac{27b^3}{64a^3}$$

$$0$$

$$3\left(\frac{2a}{3b}\right)^2=\frac{4a^2}{3b^2}$$

$$3\left(\frac{2a}{3b}\right)^2(1)=\frac{4a^2}{3b^2}$$

$$3\left(\frac{2a}{3b}\right)(1)^2=\frac{2a}{b}$$

$$(1)^3=1$$

$$3\left(\frac{2a}{3b}+1\right)^2=\frac{4a^2}{3b^2}+\frac{4a}{b}+3$$

$$3\left(\frac{2a}{3b}+1\right)^2\left(\frac{3b}{4a}\right)=\frac{a}{b}+3+\frac{9b}{4a}$$

$$3\left(\frac{2a}{3b}+1\right)\left(\frac{3b}{4a}\right)^2=\frac{9b}{8a}+\frac{27b^2}{16a^2}$$

$$\left(\frac{3b}{4a}\right)^3=\frac{27b^3}{64a^3}$$

EJERCICIO 218

1. $x^{\frac{1}{3}}=\sqrt[3]{x}$

2. $m^{\frac{3}{5}}=\sqrt[5]{m^3}$

3. $4a^{\frac{3}{4}}=4\sqrt[4]{a^3}$

4. $xy^{\frac{1}{2}}=x\sqrt{y}$

5. $a^{\frac{4}{5}}b^{\frac{3}{2}}=\sqrt[5]{a^4}\sqrt{b^3}$

$=\sqrt[5]{a^4}\sqrt{b^2b}$

$=b\sqrt[5]{a^4}\sqrt{b}$

6. $x^{\frac{3}{2}}y^{\frac{1}{4}}z^{\frac{1}{5}}=\sqrt{x^3}\sqrt[4]{y}\sqrt[5]{z}$

$=\sqrt{x^2x}\sqrt[4]{y}\sqrt[5]{z}$

$=x\sqrt{x}\sqrt[4]{y}\sqrt[5]{z}$

7. $2a^{\frac{4}{5}}b^{\frac{5}{2}}=2\sqrt[5]{a^4}\sqrt{b^5}$

$=2\sqrt[5]{a^4}\sqrt{b^2b^2b}$

$=2b^2\sqrt[5]{a^4}\sqrt{b}$

8. $3x^{\frac{2}{7}}y^{\frac{4}{5}}z^{\frac{2}{7}}=3\sqrt[7]{x^2}\sqrt[5]{y^4}\sqrt[7]{z^2}$

9. $a^{\frac{1}{4}}b^{\frac{5}{4}}c^{\frac{7}{4}}=\sqrt[4]{ab^5c^7}$

$=\sqrt[4]{ab^4bc^4c^3}$

$=bc\sqrt[4]{abc^3}$

10. $8mn^{\frac{8}{3}}=8m\sqrt[3]{n^8}$

$=8m\sqrt[3]{n^3n^3n^2}$

$=8mn^2\sqrt[3]{n^2}$

11. $4a^2b^{\frac{7}{3}}c^{\frac{5}{6}}=4a^2\sqrt[3]{b^7}\sqrt[6]{c^5}$

$=4a^2\sqrt[3]{b^3b^3b}\sqrt[6]{c^5}$

$=4a^2b^2\sqrt[3]{b}\sqrt[6]{c^5}$

12. $5m^{\frac{2}{5}}n^{\frac{3}{5}}x^{\frac{4}{5}}=5\sqrt[5]{m^2n^3x^4}$

13. $\sqrt{a^5}=a^{\frac{5}{2}}$

14. $\sqrt[3]{x^7}=x^{\frac{7}{3}}$

15. $\sqrt{x}=x^{\frac{1}{2}}$

16. $\sqrt[3]{m}=m^{\frac{1}{3}}$

17. $2\sqrt[4]{x^5}=2x^{\frac{5}{4}}$

18. $\sqrt{a^3}\sqrt[3]{b^5}=a^{\frac{3}{2}}b^{\frac{5}{3}}$

19. $3\sqrt{x^7}\sqrt[5]{y^6}=3x^{\frac{7}{2}}y^{\frac{6}{5}}$

20. $2\sqrt[4]{ab^3c^5}=2a^{\frac{1}{4}}b^{\frac{3}{4}}c^{\frac{5}{4}}$

21. $5a\sqrt[5]{x^2y^3z^9}=5ax^{\frac{2}{5}}y^{\frac{3}{5}}z^{\frac{9}{5}}$

22. $3\sqrt[6]{m^7}\sqrt[5]{n^8}=3m^{\frac{7}{6}}n^{\frac{8}{5}}$

23. $3\sqrt{a^m}\sqrt[3]{b^n}=3a^{\frac{m}{2}}b^{\frac{n}{3}}$

24. $\sqrt[m]{a}\sqrt[n]{b^3}\sqrt[r]{c^x}=a^{\frac{1}{m}}b^{\frac{3}{n}}c^{\frac{x}{r}}$

EJERCICIO 219

1. $a^2b^{-3}=\dfrac{a^2}{b^3}$

2. $3x^{-5}=\dfrac{3}{x^5}$

3. $a^{-4}b^{-\frac{1}{2}}=\dfrac{1}{a^4b^{\frac{1}{2}}}=\dfrac{1}{a^4\sqrt{b}}$

4. $3x^{-2}y^{-\frac{1}{3}}=\dfrac{3}{x^2y^{\frac{1}{3}}}=\dfrac{3}{x^2\sqrt[3]{y}}$

5. $m^{-\frac{1}{2}}n^{-5}=\dfrac{1}{m^{\frac{1}{2}}n^5}$

6. $a^2b^{-1}c=\dfrac{a^2c}{b}$

7. $4x^2y^{-\frac{3}{5}}=\dfrac{4x^2}{y^{\frac{3}{5}}}$

8. $5a^{-\frac{1}{3}}b^{-\frac{3}{4}}c^{-1}=\dfrac{5}{a^{\frac{1}{3}}b^{\frac{3}{4}}c}$

9. $\dfrac{1}{2x^{-2}}=\dfrac{x^2}{2}$

10. $\dfrac{3}{x^{-1}y^{-5}}=3xy^5$

11. $\dfrac{2a^{-2}b^{-3}}{a^{-4}c^{-1}}=\dfrac{2a^4c}{a^2b^3}=\dfrac{2a^2c}{b^3}$

12. $\dfrac{x^{-1}y^{-2}z^{-3}}{a^{-2}b^{-5}c^{-8}}=\dfrac{a^2b^5c^8}{xy^2z^3}$

13. $\dfrac{3m^{-4}n^{-\frac{1}{2}}}{8m^{-3}n^{-4}}=\dfrac{3m^3n^{-\frac{1}{2}}n^4}{8m^4}=\dfrac{3n^{\frac{7}{2}}}{8m}$

14. $\dfrac{4a^{\frac{1}{2}}}{7a^{-4}b^2c^{-\frac{2}{3}}}=\dfrac{4a^{\frac{1}{2}}a^4c^{\frac{2}{3}}}{7b^2}$

$=\dfrac{4a^{\frac{9}{2}}c^{\frac{2}{3}}}{7b^2}$

15. $\dfrac{2m^{-5}n^{-7}}{a^2m^3n^{-4}}=\dfrac{2}{a^2m^3m^5n^{-4}n^7}$

$=\dfrac{2}{a^2m^8n^3}$

16. $\dfrac{a^{-\frac{1}{2}}x^{-2}}{3a^3x^2y^{-1}}=\dfrac{y}{3a^3a^{\frac{1}{2}}x^2x^2}=\dfrac{y}{3a^{\frac{7}{2}}x^4}$

17. $\dfrac{c^2}{4b^{-\frac{1}{2}}x^3}=\dfrac{b^{\frac{1}{2}}c^2}{4x^3}$

18. $\dfrac{1}{3a^{-\frac{3}{4}}b^{-\frac{2}{5}}c^4}=\dfrac{a^{\frac{3}{4}}b^{\frac{2}{5}}}{3c^4}$

19. $\dfrac{3a^2mn}{a^{-3}m^{-\frac{1}{2}}n^{-\frac{3}{4}}}=3a^2a^3mm^{\frac{1}{2}}nn^{\frac{3}{4}}$

$=3a^5m^{\frac{3}{2}}n^{\frac{7}{4}}$

20. $\dfrac{x^{-\frac{2}{3}}y^{-\frac{1}{4}}}{x^2yz^{-\frac{1}{2}}}=\dfrac{z^{\frac{1}{2}}}{x^2x^{\frac{2}{3}}yy^{\frac{1}{4}}}=\dfrac{z^{\frac{1}{2}}}{x^{\frac{8}{3}}y^{\frac{5}{4}}}$

EJERCICIO 220

1. $\frac{a^2}{b^2}=\frac{1}{a^{-2}b^2}$

2. $\frac{3x^{-1}}{y^2}=\frac{3}{xy^2}$

3. $\frac{4mn^2}{x^3}=\frac{4}{m^{-1}n^{-2}x^3}$

4. $\frac{a^{-1}b^{-3}}{3}=\frac{1}{3ab^3}$

5. $\frac{3c^{-\frac{2}{3}}}{7}=\frac{3}{7c^{\frac{2}{3}}}$

6. $\frac{2x^{\frac{1}{4}}}{5y^2}=\frac{2}{5x^{-\frac{1}{4}}y^2}$

7. $\frac{m^{-3}}{5}=\frac{1}{5m^3}$

8. $\frac{3a^{-2}b^3}{c^4}=\frac{3}{a^2c^4b^{-3}}$

9. $x^{-\frac{1}{2}}y^2=\frac{1}{x^{\frac{1}{2}}y^{-2}}$

10. $a^{-\frac{2}{3}}b^3c^{-2}=\frac{1}{a^{\frac{2}{3}}c^2b^{-3}}$

11. $\frac{3x^{-1}y^{-\frac{1}{2}}}{y^3}=\frac{3}{xy^3y^{\frac{1}{2}}}=\frac{3}{xy^{\frac{7}{2}}}$

12. $\frac{2m^{-2}n^{\frac{1}{2}}}{9}=\frac{2}{9m^2n^{-\frac{1}{2}}}$

13. $\frac{2}{a}=2a^{-1}$

14. $\frac{3a}{b^2}=3ab^{-2}$

15. $\frac{x^2y}{y^{-2}}=x^2yy^2=x^2y^3$

16. $\frac{4}{x^{-\frac{1}{2}}y^2}=4x^{\frac{1}{2}}y^{-2}$

17. $\frac{3a^5}{7x^{-5}y^{-\frac{3}{4}}}=\frac{3a^5x^5y^{\frac{3}{4}}}{7}$

18. $\frac{1}{a^{-4}b^{-\frac{1}{3}}}=a^4b^{\frac{1}{3}}$

19. $\frac{2m^2}{3m^{-3}n^{-\frac{1}{4}}}=\frac{2m^2m^3n^{\frac{1}{4}}}{3}=\frac{2m^5n^{\frac{1}{4}}}{3}$

20. $\frac{a^3}{x^2y^{-\frac{1}{2}}}=a^3x^{-2}y^{\frac{1}{2}}$

21. $\frac{3a^2b^3}{a^{-1}x}=3a^2ab^3x^{-1}=3a^3b^3x^{-1}$

22. $\frac{3xy^2z^3}{x^{-1}y^{-2}z^{-3}}=3xxy^2y^2z^3z^3=3x^2y^4z^6$

23. $\frac{m^{-2}n^{-1}x^{-\frac{1}{2}}}{m^{-4}n^{-5}x^{-2}}=m^{-2}m^4n^{-1}n^5x^{-\frac{1}{2}}x^2$
$=m^2n^4x^{\frac{3}{2}}$

EJERCICIO 221

1. $x^{-\frac{1}{2}}=\frac{1}{x^{\frac{1}{2}}}=\frac{1}{\sqrt{x}}$

2. $\frac{1}{a^{-\frac{1}{2}}b^{\frac{2}{3}}}=\frac{a^{\frac{1}{2}}}{\sqrt[3]{b^2}}=\frac{\sqrt{a}}{\sqrt[3]{b^2}}$

3. $5a^{\frac{5}{7}}b^{-\frac{1}{3}}=\frac{5\sqrt[7]{a^5}}{b^{\frac{1}{3}}}=\frac{5\sqrt[7]{a^5}}{\sqrt[3]{b}}$

4. $\frac{3x^{-1}}{x^{-\frac{1}{2}}}=\frac{3}{x^{-\frac{1}{2}}x}=\frac{3}{x^{\frac{1}{2}}}=\frac{3}{\sqrt{x}}$

5. $2m^{-\frac{2}{5}}n^{\frac{3}{4}}=\frac{2\sqrt[4]{n^3}}{m^{\frac{2}{5}}}=\frac{2\sqrt[4]{n^3}}{\sqrt[5]{m^2}}$

6. $\frac{1}{4x^{\frac{1}{3}}}=\frac{1}{4\sqrt[3]{x}}$

7. $\frac{x^{\frac{3}{5}}}{y^{-\frac{2}{3}}}=\sqrt[5]{x^3}\,y^{\frac{2}{3}}=\sqrt[5]{x^3}\sqrt[3]{y^2}$

8. $\frac{3a^{-\frac{3}{2}}}{x^{\frac{1}{4}}}=\frac{3}{a^{\frac{3}{2}}\sqrt[4]{x}}$
$=\frac{3}{\sqrt[2]{a^2a}\sqrt[4]{x}}=\frac{3}{a\sqrt{a}\sqrt[4]{x}}$

9. $\frac{a^{-\frac{1}{2}}}{4a^2}=\frac{1}{4a^{\frac{1}{2}}a^2}$
$=\frac{1}{4a^{\frac{5}{2}}}=\frac{1}{4\sqrt[2]{a^2a^2a}}=\frac{1}{4a^2\sqrt{a}}$

10. $x^{-\frac{2}{3}}y^{\frac{3}{5}}z^{-\frac{4}{7}}=\frac{\sqrt[5]{y^3}}{x^{\frac{2}{3}}z^{\frac{4}{7}}}=\frac{\sqrt[5]{y^3}}{\sqrt[3]{x^2}\sqrt[7]{z^4}}$

11. $x^{-2}m^{-3}n^{-\frac{2}{5}}=\frac{1}{x^2m^3n^{\frac{2}{5}}}=\frac{1}{x^2m^3\sqrt[5]{n^2}}$

12. $\left(a^{-\frac{1}{2}}\right)^3=\frac{1}{a^{\frac{3}{2}}}=\frac{1}{\sqrt{a^3}}=\frac{1}{a\sqrt{a}}$

13. $\left(x^{\frac{2}{3}}\right)^{-2}=x^{-\frac{4}{3}}$
$=\frac{1}{x^{\frac{4}{3}}}=\frac{1}{\sqrt[3]{x^3x}}=\frac{1}{x\sqrt[3]{x}}$

14. $\left(\frac{a}{b}\right)^{-\frac{3}{2}}=\frac{a^{-\frac{3}{2}}}{b^{-\frac{3}{2}}}=\frac{b^{\frac{3}{2}}}{a^{\frac{3}{2}}}$
$=\frac{\sqrt{b^2b}}{\sqrt{a^2a}}=\frac{b\sqrt{b}}{a\sqrt{a}}$

15. $\left(x^{-\frac{1}{2}}\right)^{\frac{1}{3}}=x^{-\frac{1}{6}}=\frac{1}{x^{\frac{1}{6}}}=\frac{1}{\sqrt[6]{x}}$

16. $\sqrt{a^{-3}}=a^{-\frac{3}{2}}=\frac{1}{a^{\frac{3}{2}}}$

17. $2\sqrt{x^{-3}y^{-4}}=2x^{-\frac{3}{2}}y^{-\frac{4}{2}}=\frac{2}{x^{\frac{3}{2}}y^2}$

18. $\dfrac{a^{\frac{2}{3}}}{\sqrt{x^{-5}}}=\dfrac{a^{\frac{2}{3}}}{x^{-\frac{5}{2}}}=a^{\frac{2}{3}}x^{\frac{5}{2}}$

19. $\dfrac{3\sqrt[3]{m^2}}{5\sqrt[4]{n^{-3}}}=\dfrac{3m^{\frac{2}{3}}}{5n^{-\frac{3}{4}}}=\dfrac{3m^{\frac{2}{3}}n^{\frac{3}{4}}}{5}$

20. $a^{-\frac{3}{5}}\sqrt[4]{b^{-3}}=\dfrac{b^{-\frac{3}{4}}}{a^{\frac{3}{5}}}=\dfrac{1}{a^{\frac{3}{5}}b^{\frac{3}{4}}}$

21. $x^2\sqrt{x^{-1}}=x^2x^{-\frac{1}{2}}=x^{\frac{3}{2}}$

22. $\dfrac{1}{\sqrt{a^{-7}b^{-6}}}=\dfrac{1}{a^{-\frac{7}{2}}b^{-\frac{6}{2}}}=a^{\frac{7}{2}}b^3$

23. $\dfrac{3x^{-\frac{2}{3}}}{\sqrt{y^{-4}}}=\dfrac{3y^{\frac{4}{2}}}{x^{\frac{2}{3}}}=\dfrac{3y^2}{x^{\frac{2}{3}}}$

24. $\sqrt{m^{-1}}\,\sqrt[3]{n^{-3}}=m^{-\frac{1}{2}}n^{-\frac{3}{3}}=\dfrac{1}{m^{\frac{1}{2}}n}$

25. $16^{\frac{3}{2}}=\sqrt{(16)^3}$

$=\sqrt{(16)^2(16)}=16\sqrt{16}=64$

26. $8^{\frac{2}{3}}=\sqrt[3]{8^2}=\sqrt[3]{64}=4$

27. $81^{\frac{3}{4}}=81^{\frac{1}{2}}\,81^{\frac{1}{4}}$

$=\sqrt{81}\,\sqrt[4]{81}=9\cdot3=27$

28. $9^{-\frac{5}{2}}=\dfrac{1}{9^{\frac{5}{2}}}$

$=\dfrac{1}{\sqrt{9^5}}$

$=\dfrac{1}{\sqrt{9^29^29}}=\dfrac{1}{81\sqrt{9}}=\dfrac{1}{243}$

29. $(-27)^{\frac{2}{3}}=\sqrt[3]{(-27)^2}$

$=\sqrt[3]{-27}\,\sqrt[3]{-27}$

$=-3\cdot-3=9$

30. $(-32)^{\frac{2}{5}}=\sqrt[5]{(-32)^2}$

$=\sqrt[5]{-32}\,\sqrt[5]{-32}$

$=-2\cdot-2=4$

31. $49^{-\frac{3}{2}}=\dfrac{1}{\sqrt{49^3}}=\dfrac{1}{\sqrt{49^2}\,\sqrt{49}}$

$=\dfrac{1}{49\cdot7}=\dfrac{1}{343}$

32. $\left(\dfrac{4}{9}\right)^{\frac{5}{2}}=\dfrac{\sqrt{4^5}}{\sqrt{9^5}}$

$=\dfrac{\sqrt{4^24^24}}{\sqrt{9^29^29}}$

$=\dfrac{16\sqrt{4}}{81\sqrt{9}}$

$=\dfrac{16\cdot2}{81\cdot3}=\dfrac{32}{243}$

33. $\left(\dfrac{8}{27}\right)^{-\frac{1}{3}}=\dfrac{27^{\frac{1}{3}}}{8^{\frac{1}{3}}}$

$=\dfrac{\sqrt[3]{27}}{\sqrt[3]{8}}=\dfrac{3}{2}=1\frac{1}{2}$

34. $\left(\dfrac{25}{36}\right)^{-\frac{1}{2}}=\dfrac{36^{\frac{1}{2}}}{25^{\frac{1}{2}}}$

$=\dfrac{\sqrt{36}}{\sqrt{25}}=\dfrac{6}{5}=1\frac{1}{5}$

35. $\left(\dfrac{32}{243}\right)^{-\frac{1}{5}}=\dfrac{243^{\frac{1}{5}}}{32^{\frac{1}{5}}}$

$=\dfrac{\sqrt[5]{243}}{\sqrt[5]{32}}=\dfrac{3}{2}=1\frac{1}{2}$

36. $\left(-\dfrac{27}{64}\right)^{-\frac{2}{3}}=\dfrac{64^{\frac{2}{3}}}{-27^{\frac{2}{3}}}$

$=\dfrac{\sqrt[3]{64^2}}{\sqrt[3]{(-27)^2}}$

$=\dfrac{\sqrt[3]{64}\,\sqrt[3]{64}}{\sqrt[3]{-27}\,\sqrt[3]{-27}}$

$=\dfrac{4\cdot4}{-3\cdot-3}=\dfrac{16}{9}=1\frac{7}{9}$

37. $\dfrac{1}{9^{-3}}=9^3=729$

38. $\left(\dfrac{16}{81}\right)^{-\frac{5}{4}}=\dfrac{81^{\frac{5}{4}}}{16^{\frac{5}{4}}}$

$=\dfrac{\sqrt[4]{81^4}\,\sqrt[4]{81}}{\sqrt[4]{16^4}\,\sqrt[4]{16}}$

$=\dfrac{81\cdot3}{16\cdot2}=\dfrac{243}{32}=7\frac{19}{32}$

39. $\left(-\dfrac{32}{243}\right)^{-\frac{2}{5}}=\dfrac{243^{\frac{2}{5}}}{-32^{\frac{2}{5}}}$

$=\dfrac{\sqrt[5]{243}\,\sqrt[5]{243}}{\sqrt[5]{-32}\,\sqrt[5]{-32}}$

$=\dfrac{3\cdot3}{-2\cdot-2}=\dfrac{9}{4}=2\frac{1}{4}$

40. $\left(2\frac{7}{9}\right)^{-\frac{3}{2}}=\left(\dfrac{25}{9}\right)^{-\frac{3}{2}}$

$=\dfrac{9^{\frac{3}{2}}}{25^{\frac{3}{2}}}$

$=\dfrac{\sqrt{9^2}\,\sqrt{9}}{\sqrt{25^2}\,\sqrt{25}}$

$=\dfrac{9\cdot3}{25\cdot5}=\dfrac{27}{125}$

41. $\left(5\frac{1}{16}\right)^{-\frac{1}{4}}=\left(\dfrac{81}{16}\right)^{-\frac{1}{4}}$

$=\dfrac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}}=\dfrac{\sqrt[4]{16}}{\sqrt[4]{81}}=\dfrac{2}{3}$

42. $8^{\frac{2}{3}}\cdot4^{\frac{3}{2}}=\sqrt[3]{8}\,\sqrt[3]{8}\,\sqrt{4^2}\,\sqrt{4}$

$=2\cdot2\cdot4\cdot2=32$

43. $9^{\frac{5}{2}}\cdot27^{-\frac{1}{3}}=\dfrac{\sqrt{9^5}}{\sqrt[3]{27}}$

$=\dfrac{\sqrt{9^2}\,\sqrt{9^2}\,\sqrt{9}}{3}=81$

44. $243^{-\frac{1}{5}}\cdot128^{\frac{3}{7}}$

$=\dfrac{\sqrt[7]{128^3}}{\sqrt[5]{243}}=\dfrac{\sqrt[7]{128}\,\sqrt[7]{128}\,\sqrt[7]{128}}{3}$

$=\dfrac{2\cdot2\cdot2}{3}=\dfrac{8}{3}=2\frac{2}{3}$

EJERCICIO 222

1. $a^{-2}+a^{-1}b^{\frac{1}{2}}+x^{0}$ *para* $a=3$ $b=4$

$=3^{-2}+3^{-1}\cdot 4^{\frac{1}{2}}+x^{0}$

$=\dfrac{1}{3^{2}}+\dfrac{1\sqrt{4}}{3}+1$

$=\dfrac{1}{9}+\dfrac{2}{3}+1=\dfrac{1+6+9}{9}=\dfrac{16}{9}=1\tfrac{7}{9}$

2. $3x^{-\frac{1}{2}}+x^{2}y^{-3}+x^{0}y^{\frac{1}{3}}$ *para* $x=4$ $y=1$

$=3(4)^{\frac{1}{2}}+4^{2}(1)^{-3}+1(1)^{\frac{1}{3}}$

$=\dfrac{3}{\sqrt{4}}+16+1=\dfrac{3}{2}+17=\dfrac{3+34}{2}=\dfrac{37}{2}=18\tfrac{1}{2}$

3. $2a^{-3}b+\dfrac{a^{-4}}{b^{-1}}+a^{\frac{1}{2}}b^{-\frac{3}{4}}$ *para* $a=4$ $b=16$

$=2(4)^{-3}\cdot 16+\dfrac{4^{-4}}{16^{-1}}+4^{\frac{1}{2}}\cdot 16^{-\frac{3}{4}}$

$=\dfrac{2}{4^{3}}\cdot 16+\dfrac{16}{4^{4}}+\dfrac{\sqrt{4}}{\sqrt[4]{(16)^{2}\cdot 16}}$

$=\dfrac{1}{2}+\dfrac{1}{16}+\dfrac{2}{16^{\frac{1}{2}}\sqrt[4]{16}}=\dfrac{1}{2}+\dfrac{1}{16}+\dfrac{1}{4}=\dfrac{8+1+4}{16}=\dfrac{13}{16}$

4. $\dfrac{x^{\frac{3}{4}}}{y^{-2}}+x^{-\frac{1}{2}}y^{-\frac{1}{3}}-x^{0}y^{0}+\dfrac{x}{y^{\frac{4}{3}}}$ *para* $x=16$ $y=8$

$=\dfrac{16^{\frac{3}{4}}}{8^{-2}}+16^{-\frac{1}{2}}\cdot 8^{-\frac{1}{3}}-16^{0}\cdot 8^{0}+\dfrac{16}{8^{\frac{4}{3}}}$

$=\sqrt[4]{16^{2}}\ \sqrt[4]{16}\cdot 8^{2}+\dfrac{1}{\sqrt{16}\ \sqrt[3]{8}}-1+\dfrac{16}{\sqrt[3]{(8^{3})8}}$

$=4\cdot 2\cdot 64+\dfrac{1}{4\cdot 2}-1+\dfrac{16}{8\sqrt[3]{8}}=512+\dfrac{1}{8}-1+1$

$=512\tfrac{1}{8}$

5. $\dfrac{x^{0}}{x^{-1}}+\dfrac{y^{-3}}{y^{0}}+2x^{0}+x^{\frac{3}{4}}y^{-2}$ *para* $x=81$ $y=3$

$=\dfrac{1}{81^{-1}}+\dfrac{3^{-3}}{1}+2(1)+81^{\frac{3}{4}}\cdot 3^{-2}$

$=81+\dfrac{1}{3^{3}}+2+\dfrac{\sqrt[4]{81^{2}}\ \sqrt[4]{81}}{3^{2}}$

$=83+\dfrac{1}{27}+\dfrac{9\cdot 3}{9}=83+\dfrac{1}{27}+3=86+\dfrac{1}{27}=86\tfrac{1}{27}$

6. $a^{\frac{1}{2}}x^{\frac{1}{3}}+a^{-\frac{1}{2}}x^{-\frac{1}{3}}+\dfrac{1}{a^{-\frac{1}{4}}x^{-1}}+3x^{0}$ *para* $a=16$ $x=8$

$=16^{\frac{1}{2}}\cdot 8^{\frac{1}{3}}+16^{-\frac{1}{2}}\cdot 8^{-\frac{1}{3}}+\dfrac{1}{16^{-\frac{1}{4}}\cdot 8^{-1}}+3(1)$

$=\sqrt{16}\ \sqrt[3]{8}+\dfrac{1}{\sqrt{16}\ \sqrt[3]{8}}+\sqrt[4]{16}\cdot 8+3$

$=8+\dfrac{1}{4\cdot 2}+16+3=27+\dfrac{1}{8}=27\tfrac{1}{8}$

7. $\dfrac{a^{-2}}{b^{-1}}+3a^{-1}b^{2}c^{-3}-\dfrac{a^{-2}}{b^{\frac{1}{2}}c^{-1}}+b^{\frac{1}{4}}+c^{0}$

para $b=16$ $c=2$ $a=3$

$=\dfrac{3^{-2}}{16^{-1}}+3(3)^{-1}\cdot 16^{2}(2)^{-3}-\dfrac{3^{-2}}{16^{\frac{1}{2}}\cdot 2^{-1}}+16^{\frac{1}{4}}+1$

$=\dfrac{16}{3^{2}}+\dfrac{3}{3}\cdot\dfrac{256}{2^{3}}-\dfrac{2}{3^{2}\sqrt{16}}+\sqrt[4]{16}+1$

$=\dfrac{16}{9}+32-\dfrac{1}{18}+2+1$

$=\dfrac{16}{9}+35-\dfrac{1}{18}=\dfrac{32+630-1}{18}=\dfrac{661}{18}=36\tfrac{13}{18}$

8. $\dfrac{x^{0}}{3y^{0}}+x^{\frac{2}{3}}-y^{\frac{1}{5}}+\dfrac{x^{-2}}{y^{-1}}+y^{0}$ *para* $x=8$ $y=32$

$=\dfrac{1}{3(1)}+8^{\frac{2}{3}}-32^{\frac{1}{5}}+\dfrac{8^{-2}}{32^{-1}}+1$

$=\dfrac{1}{3}+\sqrt[3]{(2^{3})^{2}}-\sqrt[5]{32}+\dfrac{32}{8^{2}}+1$

$=\dfrac{1}{3}+2^{2}-2+\dfrac{1}{2}+1=\dfrac{1}{3}+3+\dfrac{1}{2}=\dfrac{2+18+3}{6}=\dfrac{23}{6}=3\tfrac{5}{6}$

9. $a^{-\frac{1}{3}}-\dfrac{1}{b^{-\frac{4}{5}}}+a^{0}b-\sqrt[3]{a}\,b^{\frac{2}{5}}-\dfrac{1}{a^{-\frac{2}{3}}}$ *para* $a=27$ $b=243$

$=27^{-\frac{1}{3}}-\dfrac{1}{243^{-\frac{4}{5}}}+(1)243-\sqrt[3]{27}\cdot 243^{\frac{2}{5}}-\dfrac{1}{27^{-\frac{2}{3}}}$

$=\dfrac{1}{\sqrt[3]{27}}-\sqrt[5]{(3^{5})^{4}}+243-3\sqrt[5]{(3^{5})^{2}}-\sqrt[3]{(3^{3})^{2}}$

$=\dfrac{1}{3}-3^{4}+243-3(3)^{2}-3^{2}$

$=\dfrac{1}{3}+162-27-9=\dfrac{1}{3}+126=126\tfrac{1}{3}$

EJERCICIO 223

1. $x^2x^{-3}=x^{2+(-3)}=x^{-1}$
2. $a^{-2}a^{-3}=a^{-2-3}=a^{-5}$
3. $x^3x^{-3}=x^{3-3}=x^0=1$
4. $a^{\frac{1}{2}}a=a^{\frac{1}{2}+1}=a^{\frac{3}{2}}$
5. $x^{\frac{1}{2}}x^{\frac{1}{4}}=x^{\frac{1}{2}+\frac{1}{4}}=x^{\frac{3}{4}}$
6. $a^{\frac{3}{4}}a^{\frac{1}{4}}=a^{\frac{3}{4}+\frac{1}{4}}=a^{\frac{4}{4}}=a$
7. $3m^{\frac{2}{5}}m^{-\frac{3}{5}}=3m^{\frac{2}{5}-\frac{3}{5}}=3m^{-\frac{1}{5}}$
8. $2a^{\frac{3}{4}}a^{-\frac{1}{2}}=2a^{\frac{3}{4}-\frac{1}{2}}=2a^{\frac{1}{4}}$
9. $x^{-2}x^{-\frac{1}{3}}=x^{-2-\frac{1}{3}}=x^{-\frac{7}{3}}$
10. $3n^2n^{-\frac{2}{3}}=3n^{2-\frac{2}{3}}=3n^{\frac{4}{3}}$
11. $4a^{-2}a^{-\frac{1}{2}}=4a^{-2-\frac{1}{2}}=4a^{-\frac{5}{2}}$
12. $a^{-1}b^{-2}ab^2$
    $=a^{-1+1}b^{-2+2}=a^0b^0=1$
13. $x^{-3}y^{\frac{1}{2}}x^{-2}y^{-\frac{1}{2}}=x^{-3-2}y^{\frac{1}{2}-\frac{1}{2}}=x^{-5}y^0=x^{-5}$
14. $3a^2b^{\frac{1}{2}}2a^{-2}b^{-\frac{1}{2}}=3\cdot 2a^{2-2}b^{\frac{1}{2}-\frac{1}{2}}=6a^0b^0=6$
15. $a^3b^{-1}a^{-2}b^{-2}=a^{3-2}b^{-1-2}=ab^{-3}$
16. $a^{-\frac{1}{2}}b^{\frac{3}{4}}a^{\frac{1}{2}}b^{\frac{1}{4}}=a^{-\frac{1}{2}+\frac{1}{2}}b^{\frac{3}{4}+\frac{1}{4}}=a^0b^{\frac{4}{4}}=b$
17. $m^{-\frac{2}{3}}n^{\frac{1}{3}}m^{-\frac{1}{3}}n^{\frac{2}{3}}=m^{-\frac{2}{3}-\frac{1}{3}}n^{\frac{1}{3}+\frac{2}{3}}=m^{-\frac{3}{3}}n^{\frac{3}{3}}=m^{-1}n$
18. $2a^{-1}b^{\frac{3}{4}}ab^{-2}=2a^{-1+1}b^{\frac{3}{4}-2}=2a^0b^{-\frac{5}{4}}=2b^{-\frac{5}{4}}$

EJERCICIO 224

1.
$$\begin{array}{l} a^{-4}+3a^{-2}+2 \\ \underline{a^{-4}-a^{-2}+1} \\ a^{-8}+3a^{-6}+2a^{-4} \\ \quad -a^{-6}-3a^{-4}-2a^{-2} \\ \qquad\qquad \underline{a^{-4}+3a^{-2}+2} \\ a^{-8}+2a^{-6} \qquad\quad +a^{-2}+2 \end{array}$$

2.
$$\begin{array}{l} x^2-1+x^{-2} \\ \underline{x^2+2-x^{-2}} \\ x^4-x^2+1 \\ \quad 2x^2-2+2x^{-2} \\ \qquad \underline{-1+x^{-2}-x^{-4}} \\ x^4+x^2-2+3x^{-2}-x^{-4} \end{array}$$

3.
$$\begin{array}{l} x+2x^{\frac{2}{3}}+x^{\frac{1}{3}} \\ \underline{x^{\frac{1}{3}}-2+x^{-\frac{1}{3}}} \\ x^{\frac{4}{3}}+2x+x^{\frac{2}{3}} \\ \quad -2x-4x^{\frac{2}{3}}-2x^{\frac{1}{3}} \\ \qquad \underline{x^{\frac{2}{3}}+2x^{\frac{1}{3}}+1} \\ x^{\frac{4}{3}} \qquad -2x^{\frac{2}{3}} \qquad +1 \end{array}$$

4.
$$\begin{array}{l} 2a^{\frac{3}{4}}-a^{\frac{1}{2}}+2a^{\frac{1}{4}} \\ \underline{a^{\frac{1}{4}}+1-a^{-\frac{1}{4}}} \\ 2a-a^{\frac{3}{4}}+2a^{\frac{1}{2}} \\ \quad 2a^{\frac{3}{4}}-a^{\frac{1}{2}}+2a^{\frac{1}{4}} \\ \qquad \underline{-2a^{\frac{1}{2}}+a^{\frac{1}{4}}-2} \\ 2a+a^{\frac{3}{4}}-a^{\frac{1}{2}}+3a^{\frac{1}{4}}-2 \end{array}$$

5.
$$\begin{array}{l} a^{\frac{2}{3}}-2+2a^{-\frac{2}{3}} \\ \underline{3+a^{-\frac{2}{3}}-4a^{-\frac{4}{3}}} \\ 3a^{\frac{2}{3}}-6+6a^{-\frac{2}{3}} \\ \quad +1-2a^{-\frac{2}{3}}+2a^{-\frac{4}{3}} \\ \qquad \underline{-4a^{-\frac{2}{3}}+8a^{-\frac{4}{3}}-8a^{-2}} \\ 3a^{\frac{2}{3}}-5 \qquad +10a^{-\frac{4}{3}}-8a^{-2} \end{array}$$

6.
$$\begin{array}{l} x^{\frac{3}{4}}+2x^{\frac{1}{4}}-x^{-\frac{1}{4}} \\ \underline{x^{\frac{1}{2}}-2+x^{-\frac{1}{2}}} \\ x^{\frac{5}{4}}+2x^{\frac{3}{4}}-x^{\frac{1}{4}} \\ \quad -2x^{\frac{3}{4}}-4x^{\frac{1}{4}}+2x^{-\frac{1}{4}} \\ \qquad \underline{x^{\frac{1}{4}}+2x^{-\frac{1}{4}}-x^{-\frac{3}{4}}} \\ x^{\frac{5}{4}} \qquad -4x^{\frac{1}{4}}+4x^{-\frac{1}{4}}-x^{-\frac{3}{4}} \end{array}$$

7.
$$\begin{array}{l} a^2b^{-1}+a+b \\ \underline{a^{-2}b^{-2}-a^{-3}b^{-1}+a^{-4}} \\ b^{-3}+a^{-1}b^{-2}+a^{-2}b^{-1} \\ \quad -a^{-1}b^{-2}-a^{-2}b^{-1}-a^{-3} \\ \qquad \underline{a^{-2}b^{-1}+a^{-3}+a^{-4}b} \\ b^{-3} \qquad +a^{-2}b^{-1} \qquad +a^{-4}b \end{array}$$

8.
$$\begin{array}{l} x^{-1}y^{-1}+x^{-3}y^{-3}+x^{-5}y^{-5} \\ \underline{x^{-3}y^{-2}-x^{-5}y^{-4}+x^{-7}y^{-6}} \\ x^{-4}y^{-3}+x^{-6}y^{-5}+x^{-8}y^{-7} \\ \quad -x^{-6}y^{-5}-x^{-8}y^{-7}-x^{-10}y^{-9} \\ \qquad \underline{+x^{-8}y^{-7}+x^{-10}y^{-9}+x^{-12}y^{-11}} \\ x^{-4}y^{-3} \qquad +x^{-8}y^{-7} \qquad +x^{-12}y^{-11} \end{array}$$

9. $$a^{\frac34}b^{-3}+a^{\frac14}b^{-2}-a^{-\frac14}b^{-1}$$
$$\underline{a^{\frac12}b^{-1}-2+3a^{-\frac12}b}$$
$$a^{\frac54}b^{-4}+a^{\frac34}b^{-3}-a^{\frac14}b^{-2}$$
$$-2a^{\frac34}b^{-3}-2a^{\frac14}b^{-2}+2a^{-\frac14}b^{-1}$$
$$\underline{3a^{\frac14}b^{-2}+3a^{-\frac14}b^{-1}-3a^{-\frac34}}$$
$$a^{\frac54}b^{-4}-a^{\frac34}b^{-3}+5a^{-\frac14}b^{-1}-3a^{-\frac34}$$

10. $$a^{-1}+2a^{-\frac12}b^{-\frac12}+2b^{-1}$$
$$\underline{a^{-1}-a^{-\frac12}b^{-\frac12}+b^{-1}}$$
$$a^{-2}+2a^{-\frac32}b^{-\frac12}+2a^{-1}b^{-1}$$
$$-a^{-\frac32}b^{-\frac12}-2a^{-1}b^{-1}-2a^{-\frac12}b^{-\frac32}$$
$$\underline{+a^{-1}b^{-1}+2a^{-\frac12}b^{-\frac32}+2b^{-2}}$$
$$a^{-2}+a^{-\frac32}b^{-\frac12}+a^{-1}b^{-1}+2b^{-2}$$

11. $$4x^2-x^{\frac32}y^{\frac12}+xy-x^{\frac12}y^{\frac32}$$
$$\underline{x^{\frac12}+y^{\frac12}}$$
$$4x^{\frac52}-x^2y^{\frac12}+x^{\frac32}y-xy^{\frac32}$$
$$\underline{4x^2y^{\frac12}-x^{\frac32}y+xy^{\frac32}-x^{\frac12}y^2}$$
$$4x^{\frac52}+3x^2y^{\frac12}-x^{\frac12}y^2$$

12. $$x-2a^{\frac13}x^{\frac23}+a^{\frac23}x^{\frac13}-3a$$
$$\underline{x^{\frac43}+3a^{\frac23}x^{\frac23}+2a^{\frac13}x}$$
$$x^{\frac73}-2a^{\frac13}x^2+a^{\frac23}x^{\frac53}-3ax^{\frac43}$$
$$+3a^{\frac23}x^{\frac53}-6ax^{\frac43}+3a^{\frac43}x-9a^{\frac53}x^{\frac23}$$
$$\underline{+2a^{\frac13}x^2-4a^{\frac23}x^{\frac53}+2ax^{\frac43}-6a^{\frac43}x}$$
$$x^{\frac73}-7ax^{\frac43}-3a^{\frac43}x-9a^{\frac53}x^{\frac23}$$

13. $$5a^2-3a+4-2a^{-1}$$
$$\underline{3a+2-5a^{-1}}$$
$$15a^3-9a^2+12a-6$$
$$10a^2-6a+8-4a^{-1}$$
$$\underline{-25a+15-20a^{-1}+10a^{-2}}$$
$$15a^3+a^2-19a+17-24a^{-1}+10a^{-2}$$

14. $$2x-3+x^{-1}+4x^{-2}$$
$$\underline{x^{-1}-2x^{-2}+x^{-3}}$$
$$2-3x^{-1}+x^{-2}+4x^{-3}$$
$$-4x^{-1}+6x^{-2}-2x^{-3}-8x^{-4}$$
$$\underline{+2x^{-2}-3x^{-3}+x^{-4}+4x^{-5}}$$
$$2-7x^{-1}+9x^{-2}-x^{-3}-7x^{-4}+4x^{-5}$$

15. $$m-m^{\frac12}n^{\frac12}+n-m^{-\frac12}n^{\frac32}$$
$$\underline{m^{\frac12}+n^{\frac12}+m^{-\frac12}n}$$
$$m^{\frac32}-mn^{\frac12}+m^{\frac12}n-n^{\frac32}$$
$$+mn^{\frac12}-m^{\frac12}n+n^{\frac32}-m^{-\frac12}n^2$$
$$\underline{+m^{\frac12}n-n^{\frac32}+m^{-\frac12}n^2-m^{-1}n^{\frac52}}$$
$$m^{\frac32}+m^{\frac12}n-n^{\frac32}-m^{-1}n^{\frac52}$$

16. $$a^{\frac35}+2a^{\frac15}-a^{-\frac15}$$
$$\underline{-2+a^{\frac25}-a^{-\frac25}}$$
$$-2a^{\frac35}-4a^{\frac15}+2a^{-\frac15}$$
$$+2a^{\frac35}-a^{\frac15}+a$$
$$\underline{-a^{\frac15}-2a^{-\frac15}+a^{-\frac35}}$$
$$-6a^{\frac15}+a+a^{-\frac35}$$

17. $$m+3m^{\frac23}+2m^{\frac13}$$
$$\underline{2-2m^{-\frac13}+2m^{-\frac23}}$$
$$2m+6m^{\frac23}+4m^{\frac13}$$
$$-2m^{\frac23}-6m^{\frac13}-4$$
$$\underline{+2m^{\frac13}+6+4m^{-\frac13}}$$
$$2m+4m^{\frac23}+2+4m^{-\frac13}$$

18. $$x^{-\frac34}y^{\frac32}+3x^{-\frac14}y-x^{\frac14}y^{\frac12}$$
$$\underline{x^{-\frac54}y^{\frac12}-3x^{-\frac34}-x^{-\frac14}y^{-\frac12}}$$
$$x^{-2}y^2+3x^{-\frac32}y^{\frac32}-x^{-1}y$$
$$-3x^{-\frac32}y^{\frac32}-9x^{-1}y+3x^{-\frac12}y^{\frac12}$$
$$\underline{-x^{-1}y-3x^{-\frac12}y^{\frac12}+1}$$
$$x^{-2}y^2-11x^{-1}y+1$$

19.

$$\begin{array}{l} x^2y^{-1}+5x^3y^{-3}+2x^4y^{-5} \\ \underline{x^{-3}y^3-x^{-2}y+3x^{-1}y^{-1}} \\ x^{-1}y^2+5+2xy^{-2} \\ \qquad -1-5xy^{-2}-2x^2y^{-4} \\ \qquad\qquad \underline{+3xy^{-2}+15x^2y^{-4}+6x^3y^{-6}} \\ x^{-1}y^2+4 \qquad +13x^2y^{-4}+6x^3y^{-6} \end{array}$$

20.

$$\begin{array}{l} a^{-\frac{2}{3}}b^{\frac{1}{2}}+2a^{-\frac{4}{3}}b-a^{-2}b^{\frac{3}{2}} \\ \underline{3a^{\frac{2}{3}}b^{-\frac{1}{2}}+1+a^{-\frac{2}{3}}b^{\frac{1}{2}}} \\ 3+6a^{-\frac{2}{3}}b^{\frac{1}{2}}-3a^{-\frac{4}{3}}b \\ \quad +a^{-\frac{2}{3}}b^{\frac{1}{2}}+2a^{-\frac{4}{3}}b-a^{-2}b^{\frac{3}{2}} \\ \qquad\qquad \underline{a^{-\frac{4}{3}}b+2a^{-2}b^{\frac{3}{2}}-a^{-\frac{8}{3}}b^2} \\ 3+7a^{-\frac{2}{3}}b^{\frac{1}{2}} \qquad +a^{-2}b^{\frac{3}{2}}-a^{-\frac{8}{3}}b^2 \end{array}$$

EJERCICIO 225

1. $a^2 \div a^{-2} = a^{2-(-2)} = a^{2+2} = a^4$

2. $x^{-3} \div x^2 = x^{-3-2} = x^{-5}$

3. $m^{\frac{1}{2}} \div m^{-\frac{1}{4}} = m^{\frac{1}{2}-\left(-\frac{1}{4}\right)} = m^{\frac{1}{2}+\frac{1}{4}} = m^{\frac{3}{4}}$

4. $a^2 \div a^5 = a^{2-5} = a^{-3}$

5. $x^{-3} \div x^{-7} = x^{-3-(-7)} = x^{-3+7} = x^4$

6. $a^{\frac{1}{2}} \div a = a^{\frac{1}{2}-1} = a^{-\frac{1}{2}}$

7. $x^{-\frac{2}{3}} \div x^{-\frac{1}{3}} = x^{-\frac{2}{3}-\left(-\frac{1}{3}\right)} = x^{-\frac{2}{3}+\frac{1}{3}} = x^{-\frac{1}{3}}$

8. $a^{\frac{2}{5}} \div a^{-\frac{1}{5}} = a^{\frac{2}{5}-\left(-\frac{1}{5}\right)} = a^{\frac{2}{5}+\frac{1}{5}} = a^{\frac{3}{5}}$

9. $m^{-\frac{3}{4}} \div m^{\frac{1}{2}} = m^{-\frac{3}{4}-\frac{1}{2}} = m^{-\frac{5}{4}}$

10. $a^{\frac{1}{3}} \div a = a^{\frac{1}{3}-1} = a^{-\frac{2}{3}}$

11. $4x^{\frac{2}{5}} \div 2x^{-\frac{1}{5}} = \frac{4}{2}x^{\frac{2}{5}-\left(-\frac{1}{5}\right)} = 2x^{\frac{2}{5}+\frac{1}{5}} = 2x^{\frac{3}{5}}$

12. $a^{-3} \div a^{-\frac{7}{4}} = a^{-3-\left(-\frac{7}{4}\right)} = a^{-3+\frac{7}{4}} = a^{-\frac{5}{4}}$

13. $x^{-2}y^{-1} \div x^{-3}y^{-2} = x^{-2+3}y^{-1+2} = xy$

14. $a^{\frac{1}{2}}b^{\frac{1}{3}} \div ab = a^{\frac{1}{2}-1}b^{\frac{1}{3}-1} = a^{-\frac{1}{2}}b^{-\frac{2}{3}}$

15. $a^2b^{-3} \div a^{-1}b = a^{2+1}b^{-3-1} = a^3b^{-4}$

16. $x^{-\frac{1}{2}}y^{-\frac{2}{3}} \div x^{-\frac{1}{2}}y^{-1} = x^{-\frac{1}{2}+\frac{1}{2}}y^{-\frac{2}{3}+1} = x^0y^{\frac{1}{3}} = y^{\frac{1}{3}}$

17. $m^{\frac{3}{4}}n^{-\frac{3}{4}} \div m^{-\frac{1}{2}}n^{\frac{3}{4}} = m^{\frac{3}{4}+\frac{1}{2}}n^{-\frac{3}{4}-\frac{3}{4}} = m^{\frac{5}{4}}n^{-\frac{3}{2}}$

18. $8x^{-2}y^{\frac{2}{5}} \div 4xy^{-\frac{1}{5}} = \frac{8}{4}x^{-2-1}y^{\frac{2}{5}+\frac{1}{5}} = 2x^{-3}y^{\frac{3}{5}}$

19. $a^{\frac{1}{3}}b \div a^{-\frac{1}{4}}b^{-3} = a^{\frac{1}{3}+\frac{1}{4}}b^{1+3} = a^{\frac{7}{12}}b^4$

20. $x^{-4}y^{-5} \div x^2y^{-1} = x^{-4-2}y^{-5+1} = x^{-6}y^{-4}$

EJERCICIO 226

1.

$$\begin{array}{l|l} x^{-8}+2x^{-6} \qquad + \; x^{-2} \; +2 & x^{-4}-x^{-2}+1 \\ \cline{2-2} \underline{-x^{-8}+x^{-6}-x^{-4}} & x^{-4}+3x^{-2}+2 \\ \quad 3x^{-6}-x^{-4}+x^{-2} & \\ \quad \underline{-3x^{-6}+3x^{-4}-3x^{-2}} & \\ \qquad 2x^{-4}-2x^{-2}+2 & \\ \qquad \underline{-2x^{-4}+2x^{-2}-2} & \end{array}$$

2.

$$\begin{array}{l|l} a^{\frac{4}{3}} \qquad -2a^{\frac{2}{3}} \qquad +1 & a+2a^{\frac{2}{3}}+a^{\frac{1}{3}} \\ \cline{2-2} \underline{-a^{\frac{4}{3}}-2a-a^{\frac{2}{3}}} & a^{\frac{1}{3}}-2+a^{-\frac{1}{3}} \\ \quad -2a-3a^{\frac{2}{3}} & \\ \quad \underline{+2a+4a^{\frac{2}{3}}+2a^{\frac{1}{3}}} & \\ \qquad a^{\frac{2}{3}}+2a^{\frac{1}{3}}+1 & \\ \qquad \underline{-a^{\frac{2}{3}}-2a^{\frac{1}{3}}-1} & \end{array}$$

3. $m^4+m^2-2+3m^{-2}-m^{-4} \;\Big|\; m^2-1+m^{-2}$

Quotient: m^2+2-m^{-2}

$-m^4+m^2-1$

$2m^2-3+3m^{-2}$

$-2m^2+2-2m^{-2}$

$-1+m^{-2}-m^{-4}$

$+1-m^{-2}+m^{-4}$

4. $2x+x^{\frac{3}{4}}-x^{\frac{1}{2}}+3x^{\frac{1}{4}}-2 \;\Big|\; x^{\frac{1}{4}}+1-x^{-\frac{1}{4}}$

Quotient: $2x^{\frac{3}{4}}-x^{\frac{1}{2}}+2x^{\frac{1}{4}}$

$-2x-2x^{\frac{3}{4}}+2x^{\frac{1}{2}}$

$-x^{\frac{3}{4}}+x^{\frac{1}{2}}+3x^{\frac{1}{4}}$

$+x^{\frac{3}{4}}+x^{\frac{1}{2}}-x^{\frac{1}{4}}$

$2x^{\frac{1}{2}}+2x^{\frac{1}{4}}-2$

$-2x^{\frac{1}{2}}-2x^{\frac{1}{4}}+2$

5. $3m^{\frac{2}{3}}-5+10m^{-\frac{4}{3}}-8m^{-2} \;\Big|\; 3+m^{-\frac{2}{3}}-4m^{-\frac{4}{3}}$

Quotient: $m^{\frac{2}{3}}-2+2m^{-\frac{2}{3}}$

$-3m^{\frac{2}{3}}-1+4m^{-\frac{2}{3}}$

$-6+4m^{-\frac{2}{3}}+10m^{-\frac{4}{3}}$

$+6+2m^{-\frac{2}{3}}-8m^{-\frac{4}{3}}$

$6m^{-\frac{2}{3}}+2m^{-\frac{4}{3}}-8m^{-2}$

$-6m^{-\frac{2}{3}}-2m^{-\frac{4}{3}}+8m^{-2}$

6. $a^{\frac{5}{4}}-4a^{\frac{1}{4}}+4a^{-\frac{1}{4}}-a^{-\frac{3}{4}} \;\Big|\; a^{\frac{1}{2}}-2+a^{-\frac{1}{2}}$

Quotient: $a^{\frac{3}{4}}+2a^{\frac{1}{4}}-a^{-\frac{1}{4}}$

$-a^{\frac{5}{4}}+2a^{\frac{3}{4}}-a^{\frac{1}{4}}$

$2a^{\frac{3}{4}}-5a^{\frac{1}{4}}+4a^{-\frac{1}{4}}$

$-2a^{\frac{3}{4}}+4a^{\frac{1}{4}}-2a^{-\frac{1}{4}}$

$-a^{\frac{1}{4}}+2a^{-\frac{1}{4}}-a^{-\frac{3}{4}}$

$+a^{\frac{1}{4}}-2a^{-\frac{1}{4}}+a^{-\frac{3}{4}}$

7. $4x^{-5}-7x^{-4}-x^{-3}+9x^{-2}-7x^{-1}+2 \;\Big|\; 4x^{-2}+x^{-1}-3+2x$

Quotient: $x^{-3}-2x^{-2}+x^{-1}$

$-4x^{-5}-x^{-4}+3x^{-3}-2x^{-2}$

$-8x^{-4}+2x^{-3}+7x^{-2}-7x^{-1}$

$8x^{-4}+2x^{-3}-6x^{-2}+4x^{-1}$

$4x^{-3}+x^{-2}-3x^{-1}+2$

$-4x^{-3}-x^{-2}+3x^{-1}-2$

8. $a^{-12}b^{-11}+a^{-8}b^{-7}+a^{-4}b^{-3} \;\Big|\; a^{-7}b^{-6}-a^{-5}b^{-4}+a^{-3}b^{-2}$

Quotient: $a^{-5}b^{-5}+a^{-3}b^{-3}+a^{-1}b^{-1}$

$-a^{-12}b^{-11}+a^{-10}b^{-9}-a^{-8}b^{-7}$

$a^{-10}b^{-9}$

$-a^{-10}b^{-9}+a^{-8}b^{-7}-a^{-6}b^{-5}$

$a^{-8}b^{-7}-a^{-6}b^{-5}+a^{-4}b^{-3}$

$-a^{-8}b^{-7}+a^{-6}b^{-5}-a^{-4}b^{-3}$

9.

$$\begin{array}{l|l} m^{-4}n \quad +m^{-2}n^{-1} \quad +n^{-3} & m^{-4}-m^{-3}n^{-1}+m^{-2}n^{-2} \\ \hline & n+m+m^{2}n^{-1} \end{array}$$

$$\underline{-m^{-4}n+m^{-3}-m^{-2}n^{-1}}$$

$$m^{-3}$$

$$\underline{-m^{-3}+m^{-2}n^{-1}-m^{-1}n^{-2}}$$

$$m^{-2}n^{-1}-m^{-1}n^{-2}+n^{-3}$$

$$\underline{-m^{-2}n^{-1}+m^{-1}n^{-2}-n^{-3}}$$

10.

$$\begin{array}{l|l} 15a^{3}+a^{2}-19a+17-24a^{-1}+10a^{-2} & 3a+2-5a^{-1} \\ \hline & 5a^{2}-3a+4-2a^{-1} \end{array}$$

$$\underline{-15a^{3}-10a^{2}+25a}$$

$$-9a^{2}+6a+17$$

$$\underline{+9a^{2}+6a-15}$$

$$12a+2-24a^{-1}$$

$$\underline{-12a-8+20a^{-1}}$$

$$-6-4a^{-1}+10a^{-2}$$

$$\underline{+6+4a^{-1}-10a^{-2}}$$

11.

$$\begin{array}{l|l} a^{\frac{5}{4}}b^{-4}-a^{\frac{3}{4}}b^{-3} \quad +5a^{-\frac{1}{4}}b^{-1}-3a^{-\frac{3}{4}} & a^{\frac{1}{2}}b^{-1}-2+3a^{-\frac{1}{2}}b \\ \hline & a^{\frac{3}{4}}b^{-3}+a^{\frac{1}{4}}b^{-2}-a^{-\frac{1}{4}}b^{-1} \end{array}$$

$$\underline{-a^{\frac{5}{4}}b^{-4}+2a^{\frac{3}{4}}b^{-3}-3a^{\frac{1}{4}}b^{-2}}$$

$$a^{\frac{3}{4}}b^{-3}-3a^{\frac{1}{4}}b^{-2}+5a^{-\frac{1}{4}}b^{-1}$$

$$\underline{-a^{\frac{3}{4}}b^{-3}+2a^{\frac{1}{4}}b^{-2}-3a^{-\frac{1}{4}}b^{-1}}$$

$$-a^{\frac{1}{4}}b^{-2}+2a^{-\frac{1}{4}}b^{-1}-3a^{-\frac{3}{4}}$$

$$\underline{+a^{\frac{1}{4}}b^{-2}-2a^{-\frac{1}{4}}b^{-1}+3a^{-\frac{3}{4}}}$$

12.

$$\begin{array}{l|l} x^{-2}+x^{-\frac{3}{2}}y^{-\frac{1}{2}}+x^{-1}y^{-1} \quad +2y^{-2} & x^{-1}-x^{-\frac{1}{2}}y^{-\frac{1}{2}}+y^{-1} \\ \hline & x^{-1}+2x^{-\frac{1}{2}}y^{-\frac{1}{2}}+2y^{-1} \end{array}$$

$$\underline{-x^{-2}+x^{-\frac{3}{2}}y^{-\frac{1}{2}}-x^{-1}y^{-1}}$$

$$2x^{-\frac{3}{2}}y^{-\frac{1}{2}}$$

$$\underline{-2x^{-\frac{3}{2}}y^{-\frac{1}{2}}+2x^{-1}y^{-1}-2x^{-\frac{1}{2}}y^{-\frac{3}{2}}}$$

$$2x^{-1}y^{-1}-2x^{-\frac{1}{2}}y^{-\frac{3}{2}}+2y^{-2}$$

$$\underline{-2x^{-1}y^{-1}+2x^{-\frac{1}{2}}y^{-\frac{3}{2}}-2y^{-2}}$$

13.

$$\left(m - 6m^{\frac{1}{5}} + m^{-\frac{3}{5}}\right) : \left(m^{\frac{3}{5}} + 2m^{\frac{1}{5}} - m^{-\frac{1}{5}}\right) = m^{\frac{2}{5}} - 2 - m^{-\frac{2}{5}}$$

$$\begin{array}{l}
m - 6m^{\frac{1}{5}} + m^{-\frac{3}{5}} \\
\underline{-m - 2m^{\frac{3}{5}} + m^{\frac{1}{5}}} \\
-2m^{\frac{3}{5}} - 5m^{\frac{1}{5}} \\
\underline{2m^{\frac{3}{5}} + 4m^{\frac{1}{5}} - 2m^{-\frac{1}{5}}} \\
-m^{\frac{1}{5}} - 2m^{-\frac{1}{5}} + m^{-\frac{3}{5}} \\
\underline{+m^{\frac{1}{5}} + 2m^{-\frac{1}{5}} - m^{-\frac{3}{5}}}
\end{array}$$

14.

$$\left(2x + 4x^{\frac{2}{3}} + 2 + 4x^{-\frac{1}{3}}\right) : \left(x + 3x^{\frac{2}{3}} + 2x^{\frac{1}{3}}\right) = 2 - 2x^{-\frac{1}{3}} + 2x^{-\frac{2}{3}}$$

$$\begin{array}{l}
2x + 4x^{\frac{2}{3}} + 2 + 4x^{-\frac{1}{3}} \\
\underline{-2x - 6x^{\frac{2}{3}} - 4x^{\frac{1}{3}}} \\
-2x^{\frac{2}{3}} - 4x^{\frac{1}{3}} + 2 \\
\underline{+2x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 4} \\
2x^{\frac{1}{3}} + 6 + 4x^{-\frac{1}{3}} \\
\underline{-2x^{\frac{1}{3}} - 6 - 4x^{-\frac{1}{3}}}
\end{array}$$

15.

$$\left(4x^{\frac{5}{2}} + 3x^2y^{\frac{1}{2}} - x^{\frac{1}{2}}y^2\right) : \left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right) = 4x^2 - x^{\frac{3}{2}}y^{\frac{1}{2}} - xy - x^{\frac{1}{2}}y^{\frac{3}{2}}$$

$$\begin{array}{l}
4x^{\frac{5}{2}} + 3x^2y^{\frac{1}{2}} - x^{\frac{1}{2}}y^2 \\
\underline{-4x^{\frac{5}{2}} - 4x^2y^{\frac{1}{2}}} \\
-x^2y^{\frac{1}{2}} \\
\underline{+x^2y^{\frac{1}{2}} - x^{\frac{3}{2}}y} \\
-x^{\frac{3}{2}}y \\
\underline{+x^{\frac{3}{2}}y - xy^{\frac{3}{2}}} \\
-xy^{\frac{3}{2}} - x^{\frac{1}{2}}y^2 \\
\underline{+xy^{\frac{3}{2}} + x^{\frac{1}{2}}y^2}
\end{array}$$

16.

$$\left(x^{\frac{7}{3}} - 7ax^{\frac{4}{3}} - 3a^{\frac{4}{3}}x - 9a^{\frac{5}{3}}x^{\frac{2}{3}}\right) : \left(x^{\frac{4}{3}} + 2a^{\frac{1}{3}}x + 3a^{\frac{2}{3}}x^{\frac{2}{3}}\right) = x - 2a^{\frac{1}{3}}x^{\frac{2}{3}} + a^{\frac{2}{3}}x^{\frac{1}{3}} - 3a$$

$$\begin{array}{l}
x^{\frac{7}{3}} - 7ax^{\frac{4}{3}} - 3a^{\frac{4}{3}}x - 9a^{\frac{5}{3}}x^{\frac{2}{3}} \\
\underline{-x^{\frac{7}{3}} - 2a^{\frac{1}{3}}x^2 - 3a^{\frac{2}{3}}x^{\frac{5}{3}}} \\
-2a^{\frac{1}{3}}x^2 - 3a^{\frac{2}{3}}x^{\frac{5}{3}} - 7ax^{\frac{4}{3}} \\
\underline{+2a^{\frac{1}{3}}x^2 + 4a^{\frac{2}{3}}x^{\frac{5}{3}} + 6ax^{\frac{4}{3}}} \\
a^{\frac{2}{3}}x^{\frac{5}{3}} - ax^{\frac{4}{3}} - 3a^{\frac{4}{3}}x \\
\underline{-a^{\frac{2}{3}}x^{\frac{5}{3}} - 2ax^{\frac{4}{3}} - 3a^{\frac{4}{3}}x} \\
-3ax^{\frac{4}{3}} - 6a^{\frac{4}{3}}x - 9a^{\frac{5}{3}}x^{\frac{2}{3}} \\
\underline{+3ax^{\frac{4}{3}} + 6a^{\frac{4}{3}}x + 9a^{\frac{5}{3}}x^{\frac{2}{3}}}
\end{array}$$

17.

$$a^{\frac{3}{2}}+a^{\frac{1}{2}}b-b^{\frac{3}{2}}-a^{-1}b^{\frac{5}{2}} \;\Big|\; a^{\frac{1}{2}}+b^{\frac{1}{2}}+a^{-\frac{1}{2}}b$$

Quotient: $$a-a^{\frac{1}{2}}b^{\frac{1}{2}}+b-a^{-\frac{1}{2}}b^{\frac{3}{2}}$$

$$\underline{-a^{\frac{3}{2}}-ab^{\frac{1}{2}}-a^{\frac{1}{2}}b}$$

$$-ab^{\frac{1}{2}}-b^{\frac{3}{2}}$$

$$\underline{+ab^{\frac{1}{2}}+a^{\frac{1}{2}}b+b^{\frac{3}{2}}}$$

$$a^{\frac{1}{2}}b$$

$$\underline{-a^{\frac{1}{2}}b-b^{\frac{3}{2}}-a^{-\frac{1}{2}}b^{2}}$$

$$-b^{\frac{3}{2}}-a^{-\frac{1}{2}}b^{2}-a^{-1}b^{\frac{5}{2}}$$

$$\underline{+b^{\frac{3}{2}}+a^{-\frac{1}{2}}b^{2}+a^{-1}b^{\frac{5}{2}}}$$

18.

$$m^{-2}n^{2}-11m^{-1}n+1 \;\Big|\; m^{-\frac{3}{4}}n^{\frac{3}{2}}+3m^{-\frac{1}{4}}n-m^{\frac{1}{4}}n^{\frac{1}{2}}$$

Quotient: $$m^{-\frac{5}{4}}n^{\frac{1}{2}}-3m^{-\frac{3}{4}}-m^{-\frac{1}{4}}n^{-\frac{1}{2}}$$

$$\underline{-m^{-2}n^{2}-3m^{-\frac{3}{2}}n^{\frac{3}{2}}+m^{-1}n}$$

$$-3m^{-\frac{3}{2}}n^{\frac{3}{2}}-10m^{-1}n$$

$$\underline{+3m^{-\frac{3}{2}}n^{\frac{3}{2}}+9m^{-1}n-3m^{-\frac{1}{2}}n^{\frac{1}{2}}}$$

$$-m^{-1}n-3m^{-\frac{1}{2}}n^{\frac{1}{2}}+1$$

$$\underline{+m^{-1}n+3m^{-\frac{1}{2}}n^{\frac{1}{2}}-1}$$

19.

$$x^{-1}y^{2}+4+13x^{2}y^{-4}+6x^{3}y^{-6} \;\Big|\; x^{-3}y^{3}-x^{-2}y+3x^{-1}y^{-1}$$

Quotient: $$x^{2}y^{-1}+5x^{3}y^{-3}+2x^{4}y^{-5}$$

$$\underline{-x^{-1}y^{2}+1-3xy^{-2}}$$

$$5-3xy^{-2}+13x^{2}y^{-4}$$

$$\underline{-5+5xy^{-2}-15x^{2}y^{-4}}$$

$$2xy^{-2}-2x^{2}y^{-4}+6x^{3}y^{-6}$$

$$\underline{-2xy^{-2}+2x^{2}y^{-4}-6x^{3}y^{-6}}$$

20.

$$3+7a^{-\frac{2}{3}}b^{\frac{1}{2}}+a^{-2}b^{\frac{3}{2}}-a^{-\frac{8}{3}}b^{2} \;\Big|\; 3a^{\frac{2}{3}}b^{-\frac{1}{2}}+1+a^{-\frac{2}{3}}b^{\frac{1}{2}}$$

Quotient: $$a^{-\frac{2}{3}}b^{\frac{1}{2}}+2a^{-\frac{4}{3}}b-a^{-2}b^{\frac{3}{2}}$$

$$\underline{-3-a^{-\frac{2}{3}}b^{\frac{1}{2}}-a^{-\frac{4}{3}}b}$$

$$6a^{-\frac{2}{3}}b^{\frac{1}{2}}-a^{-\frac{4}{3}}b+a^{-2}b^{\frac{3}{2}}$$

$$\underline{-6a^{-\frac{2}{3}}b^{\frac{1}{2}}-2a^{-\frac{4}{3}}b-2a^{-2}b^{\frac{3}{2}}}$$

$$-3a^{-\frac{4}{3}}b-a^{-2}b^{\frac{3}{2}}-a^{-\frac{8}{3}}b^{2}$$

$$\underline{+3a^{-\frac{4}{3}}b+a^{-2}b^{\frac{3}{2}}+a^{-\frac{8}{3}}b^{2}}$$

EJERCICIO 227

1. $\left(a^{-1}\right)^2 = a^{-1\cdot 2} = a^{-2}$

2. $\left(a^{-2}b^{-1}\right)^3 = a^{-2\cdot 3}b^{-1\cdot 3} = a^{-6}b^{-3}$

3. $\left(a^{\frac{3}{2}}\right)^2 = a^{\frac{3}{2}\cdot 2} = a^{\frac{6}{2}} = a^3$

4. $\left(x^{\frac{3}{4}}\right)^3 = x^{\frac{3}{4}\cdot 3} = x^{\frac{9}{4}}$

5. $\left(m^{\frac{3}{4}}\right)^2 = m^{\frac{3}{4}\cdot 2} = m^{\frac{6}{4}} = m^{\frac{3}{2}}$

6. $\left(a^{-\frac{2}{3}}\right)^3 = a^{-\frac{2}{3}\cdot 3} = a^{-\frac{6}{3}} = a^{-2}$

7. $\left(x^{-4}y^{\frac{1}{4}}\right)^2 = x^{-4\cdot 2}y^{\frac{1}{4}\cdot 2} = x^{-8}y^{\frac{2}{4}} = x^{-8}y^{\frac{1}{2}}$

8. $\left(2a^{\frac{1}{2}}b^{\frac{1}{3}}\right)^2 = 2^2a^{\frac{1}{2}\cdot 2}b^{\frac{1}{3}\cdot 2} = 4a^{\frac{2}{2}}b^{\frac{2}{3}} = 4ab^{\frac{2}{3}}$

9. $\left(a^{-3}b^{-1}\right)^4 = a^{-3\cdot 4}b^{-1\cdot 4} = a^{-12}b^{-4}$

10. $\left(x^{\frac{2}{3}}y^{-\frac{1}{2}}\right)^6 = x^{\frac{2}{3}\cdot 6}y^{-\frac{1}{2}\cdot 6} = x^{\frac{12}{3}}y^{-\frac{6}{2}} = x^4y^{-3}$

11. $\left(3a^{\frac{2}{5}}b^{-3}\right)^5 = 3^5a^{\frac{2}{5}\cdot 5}b^{-3\cdot 5} = 243a^{\frac{10}{5}}b^{-15} = 243a^2b^{-15}$

12. $\left(2m^{-\frac{1}{2}}n^{-\frac{1}{3}}\right)^3 = 2^3m^{-\frac{1}{2}\cdot 3}n^{-\frac{1}{3}\cdot 3} = 8m^{-\frac{3}{2}}n^{-\frac{3}{3}} = 8m^{-\frac{3}{2}}n^{-1}$

EJERCICIO 228

1. $\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 + 2\left(a^{\frac{1}{2}}\right)\left(b^{\frac{1}{2}}\right) + \left(b^{\frac{1}{2}}\right)^2$

$= a + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b$

2. $\left(x^{\frac{3}{4}}-y^{\frac{1}{3}}\right)^2 = \left(x^{\frac{3}{4}}\right)^2 - 2\left(x^{\frac{3}{4}}\right)\left(y^{\frac{1}{3}}\right) + \left(y^{\frac{1}{3}}\right)^2$

$= x^{\frac{3}{2}} - 2x^{\frac{3}{4}}y^{\frac{1}{3}} + y^{\frac{2}{3}}$

3. $\left(m^{-\frac{1}{2}}+2m\right)^2 = \left(m^{-\frac{1}{2}}\right)^2 + 2(2m)\left(m^{-\frac{1}{2}}\right) + (2m)^2$

$= m^{-1} + 4m^{1-\frac{1}{2}} + 4m^2 = m^{-1} + 4m^{\frac{1}{2}} + 4m^2$

4. $\left(a^{-2}b^3 - a^3b^{-2}\right)^2$

$= \left(a^{-2}b^3\right)^2 - 2\left(a^{-2}b^3\right)\left(a^3b^{-2}\right) + \left(a^3b^{-2}\right)^2$

$= a^{-4}b^6 - 2a^{-2+3}b^{3-2} + a^6b^{-4}$

$= a^{-4}b^6 - 2ab + a^6b^{-4}$

5. $\left(a^{-1}-3b^{-\frac{3}{4}}\right)^2 = \left(a^{-1}\right)^2 - 2\left(a^{-1}\right)\left(3b^{-\frac{3}{4}}\right) + \left(3b^{-\frac{3}{4}}\right)^2$

$= a^{-2} - 6a^{-1}b^{-\frac{3}{4}} + 9b^{-\frac{3}{2}}$

6. $\left(a^{-2}+\sqrt{b}\right)^2 = \left(a^{-2}\right)^2 + 2\left(a^{-2}\right)\left(b^{\frac{1}{2}}\right) + \left(b^{\frac{1}{2}}\right)^2$

$= a^{-4} + 2a^{-2}b^{\frac{1}{2}} + b$

7. $\left(\sqrt[4]{x^3} - y^{-\frac{1}{2}}\right)^2 = \left(x^{\frac{3}{4}}\right)^2 - 2\left(x^{\frac{3}{4}}\right)\left(y^{-\frac{1}{2}}\right) + \left(y^{-\frac{1}{2}}\right)^2$

$= x^{\frac{3}{2}} - 2x^{\frac{3}{4}}y^{-\frac{1}{2}} + y^{-1}$

8. $\left(m^{-2}n^{\frac{1}{4}} - m^{\frac{1}{2}}n^{-1}\right)^2$

$= \left(m^{-2}n^{\frac{1}{4}}\right)^2 - 2\left(m^{-2}n^{\frac{1}{4}}\right)\left(m^{\frac{1}{2}}n^{-1}\right) + \left(m^{\frac{1}{2}}n^{-1}\right)^2$

$= m^{-4}n^{\frac{1}{2}} - 2m^{-2+\frac{1}{2}}n^{\frac{1}{4}-1} + mn^{-2}$

$= m^{-4}n^{\frac{1}{2}} - 2m^{-\frac{3}{2}}n^{-\frac{3}{4}} + mn^{-2}$

9. $\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)^3$

$= \left(a^{\frac{1}{3}}\right)^3 + 3\left(a^{\frac{1}{3}}\right)^2\left(b^{\frac{1}{3}}\right) + 3\left(a^{\frac{1}{3}}\right)\left(b^{\frac{1}{3}}\right)^2 + \left(b^{\frac{1}{3}}\right)^3$

$= a + 3a^{\frac{2}{3}}b^{\frac{1}{3}} + 3a^{\frac{1}{3}}b^{\frac{2}{3}} + b$

10. $\left(\sqrt[3]{x^2} - 3y^{-1}\right)^3$

$= \left(x^{\frac{2}{3}}\right)^3 - 3\left(x^{\frac{2}{3}}\right)^2\left(3y^{-1}\right) + 3\left(x^{\frac{2}{3}}\right)\left(3y^{-1}\right)^2 - \left(3y^{-1}\right)^3$

$= x^2 - 9x^{\frac{4}{3}}y^{-1} + 27x^{\frac{2}{3}}y^{-2} - 27y^{-3}$

11. $\left(m^{\frac{2}{3}}+4n^{-\frac{3}{2}}\right)^3=\left(m^{\frac{2}{3}}\right)^3+3\left(m^{\frac{2}{3}}\right)^2\left(4n^{-\frac{3}{2}}\right)+3\left(m^{\frac{2}{3}}\right)\left(4n^{-\frac{3}{2}}\right)^2+\left(4n^{-\frac{3}{2}}\right)^3=m^2+12m^{\frac{4}{3}}n^{-\frac{3}{2}}+48m^{\frac{2}{3}}n^{-3}+64n^{-\frac{9}{2}}$

12. $\left(2a^{-4}-3b^{-\frac{1}{2}}\right)^3=\left(2a^{-4}\right)^3-3\left(2a^{-4}\right)^2\left(3b^{-\frac{1}{2}}\right)+3\left(2a^{-4}\right)\left(3b^{-\frac{1}{2}}\right)^2-\left(3b^{-\frac{1}{2}}\right)^3=8a^{-12}-36a^{-8}b^{-\frac{1}{2}}+54a^{-4}b^{-1}-27b^{-\frac{3}{2}}$

13. $\left(\sqrt{x}-\sqrt[3]{y}\right)^3=\left(x^{\frac{1}{2}}\right)^3-3\left(x^{\frac{1}{2}}\right)^2\left(y^{\frac{1}{3}}\right)+3\left(x^{\frac{1}{2}}\right)\left(y^{\frac{1}{3}}\right)^2-\left(y^{\frac{1}{3}}\right)^3=x^{\frac{3}{2}}-3xy^{\frac{1}{3}}+3x^{\frac{1}{2}}y^{\frac{2}{3}}-y$

14. $\left(a^{\frac{1}{2}}+b^{\frac{2}{3}}\right)^4=\left(a^{\frac{1}{2}}\right)^4+4\left(a^{\frac{1}{2}}\right)^3\left(b^{\frac{2}{3}}\right)+6\left(a^{\frac{1}{2}}\right)^2\left(b^{\frac{2}{3}}\right)^2+4\left(a^{\frac{1}{2}}\right)\left(b^{\frac{2}{3}}\right)^3+\left(b^{\frac{2}{3}}\right)^4=a^2+4a^{\frac{3}{2}}b^{\frac{2}{3}}+6ab^{\frac{4}{3}}+4a^{\frac{1}{2}}b^2+b^{\frac{8}{3}}$

15. $\left(x^{-2}-y^{-\frac{1}{3}}\right)^4=\left(x^{-2}\right)^4-4\left(x^{-2}\right)^3\left(y^{-\frac{1}{3}}\right)+6\left(x^{-2}\right)^2\left(y^{-\frac{1}{3}}\right)^2-4\left(x^{-2}\right)\left(y^{-\frac{1}{3}}\right)^3+\left(y^{-\frac{1}{3}}\right)^4$

$=x^{-8}-4x^{-6}y^{-\frac{1}{3}}+6x^{-4}y^{-\frac{2}{3}}-4x^{-2}y^{-1}+y^{-\frac{4}{3}}$

16. $\left(x^{\frac{1}{3}}+y^{-\frac{3}{4}}\right)^5=\left(x^{\frac{1}{3}}\right)^5+5\left(x^{\frac{1}{3}}\right)^4\left(y^{-\frac{3}{4}}\right)+10\left(x^{\frac{1}{3}}\right)^3\left(y^{-\frac{3}{4}}\right)^2+10\left(x^{\frac{1}{3}}\right)^2\left(y^{-\frac{3}{4}}\right)^3+5\left(x^{\frac{1}{3}}\right)\left(y^{-\frac{3}{4}}\right)^4+\left(y^{-\frac{3}{4}}\right)^5$

$=x^{\frac{5}{3}}+5x^{\frac{4}{3}}y^{-\frac{3}{4}}+10xy^{-\frac{3}{2}}+10x^{\frac{2}{3}}y^{-\frac{9}{4}}+5x^{\frac{1}{3}}y^{-3}+y^{-\frac{15}{4}}$

17. $\left(\sqrt{m}-\sqrt[3]{n}\right)^5=\left(m^{\frac{1}{2}}\right)^5-5\left(m^{\frac{1}{2}}\right)^4\left(n^{\frac{1}{3}}\right)+10\left(m^{\frac{1}{2}}\right)^3\left(n^{\frac{1}{3}}\right)^2-10\left(m^{\frac{1}{2}}\right)^2\left(n^{\frac{1}{3}}\right)^3+5\left(m^{\frac{1}{2}}\right)\left(n^{\frac{1}{3}}\right)^4-\left(n^{\frac{1}{3}}\right)^5$

$=m^{\frac{5}{2}}-5m^2n^{\frac{1}{3}}+10m^{\frac{3}{2}}n^{\frac{2}{3}}-10mn+5m^{\frac{1}{2}}n^{\frac{4}{3}}-n^{\frac{5}{3}}$

18. $\left(a^2-2\sqrt{m}\right)^6$

$=\left(a^2\right)^6-6\left(a^2\right)^5\left(2m^{\frac{1}{2}}\right)+15\left(a^2\right)^4\left(2m^{\frac{1}{2}}\right)^2-20\left(a^2\right)^3\left(2m^{\frac{1}{2}}\right)^3+15\left(a^2\right)^2\left(2m^{\frac{1}{2}}\right)^4-6\left(a^2\right)\left(2m^{\frac{1}{2}}\right)^5+\left(2m^{\frac{1}{2}}\right)^6$

$=a^{12}-12a^{10}m^{\frac{1}{2}}+60a^8m-160a^6m^{\frac{3}{2}}+240a^4m^2-192a^2m^{\frac{5}{2}}+64m^3$

19. $\left(x^{-3}+\sqrt[4]{y}\right)^5=\left(x^{-3}\right)^5+5\left(x^{-3}\right)^4\left(y^{\frac{1}{4}}\right)+10\left(x^{-3}\right)^3\left(y^{\frac{1}{4}}\right)^2+10\left(x^{-3}\right)^2\left(y^{\frac{1}{4}}\right)^3+5\left(x^{-3}\right)\left(y^{\frac{1}{4}}\right)^4+\left(y^{\frac{1}{4}}\right)^5$

$=x^{-15}+5x^{-12}y^{\frac{1}{4}}+10x^{-9}y^{\frac{1}{2}}+10x^{-6}y^{\frac{3}{4}}+5x^{-3}y+y^{\frac{5}{4}}$

20. $\left(a^{-2}+3a^{-1}+2\right)^2=\left(a^{-2}\right)^2+\left(3a^{-1}\right)^2+(2)^2+2\left(a^{-2}\right)\left(3a^{-1}\right)+2\left(a^{-2}\right)(2)+2\left(3a^{-1}\right)(2)$

$=a^{-4}+9a^{-2}+4+6a^{-3}+4a^{-2}+12a^{-1}=a^{-4}+6a^{-3}+13a^{-2}+12a^{-1}+4$

21. $\left(x^{\frac{1}{2}}-x^{\frac{1}{4}}+2x^{-\frac{1}{4}}\right)^2=\left(x^{\frac{1}{2}}\right)^2+\left(-x^{\frac{1}{4}}\right)^2+\left(2x^{-\frac{1}{4}}\right)^2+2\left(x^{\frac{1}{2}}\right)\left(-x^{\frac{1}{4}}\right)+2\left(x^{\frac{1}{2}}\right)\left(2x^{-\frac{1}{4}}\right)+2\left(-x^{\frac{1}{4}}\right)\left(2x^{-\frac{1}{4}}\right)$

$=x+x^{\frac{1}{2}}+4x^{-\frac{1}{2}}-2x^{\frac{3}{4}}+4x^{\frac{1}{4}}-4=x-2x^{\frac{3}{4}}+x^{\frac{1}{2}}+4x^{\frac{1}{4}}-4+4x^{-\frac{1}{2}}$

22. $\left(a^{-\frac{1}{2}}+3+a^{\frac{1}{2}}\right)^2=\left(a^{-\frac{1}{2}}\right)^2+(3)^2+\left(a^{\frac{1}{2}}\right)^2+2\left(a^{-\frac{1}{2}}\right)(3)+2\left(a^{-\frac{1}{2}}\right)\left(a^{\frac{1}{2}}\right)+2(3)\left(a^{\frac{1}{2}}\right)$

$=a^{-1}+9+a+6a^{-\frac{1}{2}}+2+6a^{\frac{1}{2}}=a+6a^{\frac{1}{2}}+11+6a^{-\frac{1}{2}}+a^{-1}$

23. $\left(m+2m^{\frac{3}{4}}-3m^{\frac{1}{2}}\right)^2=(m)^2+\left(2m^{\frac{3}{4}}\right)^2+\left(-3m^{\frac{1}{2}}\right)^2+2(m)\left(2m^{\frac{3}{4}}\right)+2(m)\left(-3m^{\frac{1}{2}}\right)+2\left(2m^{\frac{3}{4}}\right)\left(-3m^{\frac{1}{2}}\right)$

$=m^2+4m^{\frac{3}{2}}+9m+4m^{\frac{7}{4}}-6m^{\frac{3}{2}}-12m^{\frac{5}{4}}=m^2+4m^{\frac{7}{4}}-2m^{\frac{3}{2}}-12m^{\frac{5}{4}}+9m$

24. $\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}-2+a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)^2=\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)^2+(-2)^2+\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)^2+2\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)(-2)+2\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)+2(-2)\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)$

$=ab^{-\frac{2}{3}}+4+a^{-1}b^{\frac{2}{3}}-4a^{\frac{1}{2}}b^{-\frac{1}{3}}+2-4a^{-\frac{1}{2}}b^{\frac{1}{3}}=ab^{-\frac{2}{3}}-4a^{\frac{1}{2}}b^{-\frac{1}{3}}+6-4a^{-\frac{1}{2}}b^{\frac{1}{3}}+a^{-1}b^{\frac{2}{3}}$

25. $\left(x^{\frac{1}{2}}+x^{\frac{1}{4}}-1\right)^3=\left(x^{\frac{1}{2}}\right)^3+\left(x^{\frac{1}{4}}\right)^3+(-1)^3+3\left(x^{\frac{1}{2}}\right)^2\left(x^{\frac{1}{4}}\right)+3\left(x^{\frac{1}{2}}\right)^2(-1)+3\left(x^{\frac{1}{4}}\right)^2\left(x^{\frac{1}{2}}\right)+3\left(x^{\frac{1}{4}}\right)^2(-1)$

$+3(-1)^2\left(x^{\frac{1}{2}}\right)+3(-1)^2\left(x^{\frac{1}{4}}\right)+6\left(x^{\frac{1}{2}}\right)\left(x^{\frac{1}{4}}\right)(-1)$

$=x^{\frac{3}{2}}+x^{\frac{3}{4}}-1+3x^{\frac{5}{4}}-3x+3x-3x^{\frac{1}{2}}+3x^{\frac{1}{2}}+3x^{\frac{1}{4}}-6x^{\frac{3}{4}}=x^{\frac{3}{2}}+3x^{\frac{5}{4}}-5x^{\frac{3}{4}}+3x^{\frac{1}{4}}-1$

26. $\left(a^{\frac{2}{3}}-2+a^{-\frac{2}{3}}\right)^3=\left(a^{\frac{2}{3}}\right)^3+(-2)^3+\left(a^{-\frac{2}{3}}\right)^3+3\left(a^{\frac{2}{3}}\right)^2(-2)+3\left(a^{\frac{2}{3}}\right)^2\left(a^{-\frac{2}{3}}\right)+3(-2)^2\left(a^{\frac{2}{3}}\right)+3(-2)^2\left(a^{-\frac{2}{3}}\right)$

$+3\left(a^{-\frac{2}{3}}\right)^2\left(a^{\frac{2}{3}}\right)+3\left(a^{-\frac{2}{3}}\right)^2(-2)+6\left(a^{\frac{2}{3}}\right)(-2)\left(a^{-\frac{2}{3}}\right)$

$=a^2-8+a^{-2}-6a^{\frac{4}{3}}+3a^{\frac{2}{3}}+12a^{\frac{2}{3}}+12a^{-\frac{2}{3}}+3a^{-\frac{2}{3}}-6a^{-\frac{4}{3}}-12$

$=a^2-6a^{\frac{4}{3}}+15a^{\frac{2}{3}}-20+15a^{-\frac{2}{3}}-6a^{-\frac{4}{3}}+a^{-2}$

27. $\left(m^{\frac{1}{6}}+2m^{\frac{1}{3}}+m^{\frac{1}{2}}\right)^3=\left(m^{\frac{1}{6}}\right)^3+\left(2m^{\frac{1}{3}}\right)^3+\left(m^{\frac{1}{2}}\right)^3+3\left(m^{\frac{1}{6}}\right)^2\left(2m^{\frac{1}{3}}\right)+3\left(m^{\frac{1}{6}}\right)^2\left(m^{\frac{1}{2}}\right)+3\left(2m^{\frac{1}{3}}\right)^2\left(m^{\frac{1}{6}}\right)$

$+3\left(2m^{\frac{1}{3}}\right)^2\left(m^{\frac{1}{2}}\right)+3\left(m^{\frac{1}{2}}\right)^2\left(m^{\frac{1}{6}}\right)+3\left(m^{\frac{1}{2}}\right)^2\left(2m^{\frac{1}{3}}\right)+6\left(m^{\frac{1}{6}}\right)\left(2m^{\frac{1}{3}}\right)\left(m^{\frac{1}{2}}\right)$

$=m^{\frac{1}{2}}+8m+m^{\frac{3}{2}}+6m^{\frac{2}{3}}+3m^{\frac{5}{6}}+12m^{\frac{5}{6}}+12m^{\frac{7}{6}}+3m^{\frac{7}{6}}+6m^{\frac{4}{3}}+12m$

$=m^{\frac{3}{2}}+6m^{\frac{4}{3}}+15m^{\frac{7}{6}}+20m+15m^{\frac{5}{6}}+6m^{\frac{2}{3}}+m^{\frac{1}{2}}$

EJERCICIO 229

1.

$$\begin{array}{r|l}
\sqrt{x^{-4}+6x^{-3}+13x^{-2}+12x^{-1}+4} & x^{-2}+3x^{-1}+2 \\ \hline
\underline{-x^{-4}\qquad\qquad\qquad} & \left(2x^{-2}+3x^{-1}\right)\left(3x^{-1}\right)=6x^{-3}+9x^{-2} \\
+6x^{-3}+13x^{-2}\qquad\qquad & \\
\underline{-6x^{-3}-\ 9x^{-2}\qquad\qquad} & \left(2x^{-2}+6x^{-1}+2\right)(2) \\
4x^{-2}+12x^{-1}+4 & \qquad =4x^{-2}+12x^{-1}+4 \\
\underline{-4x^{-2}-12x^{-1}-4} &
\end{array}$$

2.

$$
\begin{array}{l|l}
\sqrt{m+6m^{\frac{1}{2}}+11+6m^{-\frac{1}{2}}+m^{-1}} & m^{\frac{1}{2}}+3+m^{-\frac{1}{2}} \\ \hline
\underline{-m} & \left(2m^{\frac{1}{2}}+3\right)(3)=6m^{\frac{1}{2}}+9 \\
\quad 6m^{\frac{1}{2}}+11 & \\
\quad \underline{-6m^{\frac{1}{2}}-9} & \left(2m^{\frac{1}{2}}+6+m^{-\frac{1}{2}}\right)\left(m^{-\frac{1}{2}}\right) \\
\qquad 2+6m^{-\frac{1}{2}}+m^{-1} & \quad =2+6m^{-\frac{1}{2}}+m^{-1} \\
\qquad \underline{-2-6m^{-\frac{1}{2}}-m^{-1}} &
\end{array}
$$

3.

$$
\begin{array}{l|l}
\sqrt{9a^{\frac{4}{3}}-6a+25a^{\frac{2}{3}}-8a^{\frac{1}{3}}+16} & 3a^{\frac{2}{3}}-a^{\frac{1}{3}}+4 \\ \hline
\underline{-9a^{\frac{4}{3}}} & \left(6a^{\frac{2}{3}}-a^{\frac{1}{3}}\right)\left(-a^{\frac{1}{3}}\right) \\
\quad -6a+25a^{\frac{2}{3}} & \quad =-6a+a^{\frac{2}{3}} \\
\quad \underline{+6a-\ a^{\frac{2}{3}}} & \\
\qquad 24a^{\frac{2}{3}}-8a^{\frac{1}{3}}+16 & \left(6a^{\frac{2}{3}}-2a^{\frac{1}{3}}+4\right)(4) \\
\qquad \underline{-24a^{\frac{2}{3}}+8a^{\frac{1}{3}}-16} & \quad =24a^{\frac{2}{3}}-8a^{\frac{1}{3}}+16
\end{array}
$$

4.

$$
\begin{array}{l|l}
\sqrt{a^2+4a^{\frac{7}{4}}-2a^{\frac{3}{2}}-12a^{\frac{5}{4}}+9a} & a+2a^{\frac{3}{4}}-3a^{\frac{1}{2}} \\ \hline
\underline{-a^2} & \left(2a+2a^{\frac{3}{4}}\right)\left(2a^{\frac{3}{4}}\right) \\
\quad +4a^{\frac{7}{4}}-2a^{\frac{3}{2}} & \quad =4a^{\frac{7}{4}}+4a^{\frac{3}{2}} \\
\quad \underline{-4a^{\frac{7}{4}}-4a^{\frac{3}{2}}} & \\
\qquad -6a^{\frac{3}{2}}-12a^{\frac{5}{4}}+9a & \left(2a+4a^{\frac{3}{4}}-3a^{\frac{1}{2}}\right)\left(-3a^{\frac{1}{2}}\right) \\
\qquad \underline{+6a^{\frac{3}{2}}+12a^{\frac{5}{4}}-9a} & \quad =-6a^{\frac{3}{2}}-12a^{\frac{5}{4}}+9a
\end{array}
$$

5.

$\sqrt{mn^{-\frac{2}{3}} - 4m^{\frac{1}{2}}n^{-\frac{1}{3}} + 6 - 4m^{-\frac{1}{2}}n^{\frac{1}{3}} + m^{-1}n^{\frac{2}{3}}}$	$m^{\frac{1}{2}}n^{-\frac{1}{3}} - 2 + m^{-\frac{1}{2}}n^{\frac{1}{3}}$
$-mn^{-\frac{2}{3}}$	$\left(2m^{\frac{1}{2}}n^{-\frac{1}{3}} - 2\right)(-2)$
$-4m^{\frac{1}{2}}n^{-\frac{1}{3}} + 6$	$= -4m^{\frac{1}{2}}n^{-\frac{1}{3}} + 4$
$+4m^{\frac{1}{2}}n^{-\frac{1}{3}} - 4$	
$2 - 4m^{-\frac{1}{2}}n^{\frac{1}{3}} + m^{-1}n^{\frac{2}{3}}$	$\left(2m^{\frac{1}{2}}n^{-\frac{1}{3}} - 4 + m^{-\frac{1}{2}}n^{\frac{1}{3}}\right)\left(m^{-\frac{1}{2}}n^{\frac{1}{3}}\right)$
$-2 + 4m^{-\frac{1}{2}}n^{\frac{1}{3}} - m^{-1}n^{\frac{2}{3}}$	$= 2 - 4m^{-\frac{1}{2}}n^{\frac{1}{3}} + m^{-1}n^{\frac{2}{3}}$

6.

$\sqrt{a^{\frac{4}{5}} - 8a^{\frac{3}{5}} + 10a^{\frac{2}{5}} + 24a^{\frac{1}{5}} + 9}$	$a^{\frac{2}{5}} - 4a^{\frac{1}{5}} - 3$
$-a^{\frac{4}{5}}$	$\left(2a^{\frac{2}{5}} - 4a^{\frac{1}{5}}\right)\left(-4a^{\frac{1}{5}}\right)$
$-8a^{\frac{3}{5}} + 10a^{\frac{2}{5}}$	$= -8a^{\frac{3}{5}} + 16a^{\frac{2}{5}}$
$+8a^{\frac{3}{5}} - 16a^{\frac{2}{5}}$	
$-6a^{\frac{2}{5}} + 24a^{\frac{1}{5}} + 9$	$\left(2a^{\frac{2}{5}} - 8a^{\frac{1}{5}} - 3\right)(-3)$
$+6a^{\frac{2}{5}} - 24a^{\frac{1}{5}} - 9$	$= -6a^{\frac{2}{5}} + 24a^{\frac{1}{5}} + 9$

7.

$\sqrt[3]{a^{-3} - 6a^{-\frac{5}{2}} + 21a^{-2} - 44a^{-\frac{3}{2}} + 63a^{-1} - 54a^{-\frac{1}{2}} + 27}$	$a^{-1} - 2a^{-\frac{1}{2}} + 3$
$-a^{-3}$	$3\left(a^{-1}\right)^2 = 3a^{-2}$
$-6a^{-\frac{5}{2}} + 21a^{-2} - 44a^{-\frac{3}{2}}$	$3\left(a^{-1}\right)^2\left(-2a^{-\frac{1}{2}}\right) = -6a^{-\frac{5}{2}}$
$+6a^{-\frac{5}{2}} - 12a^{-2} + 8a^{-\frac{3}{2}}$	$3\left(a^{-1}\right)\left(-2a^{-\frac{1}{2}}\right)^2 = 12a^{-2}$
$9a^{-2} - 36a^{-\frac{3}{2}} + 63a^{-1} - 54a^{-\frac{1}{2}} + 27$	$\left(-2a^{-\frac{1}{2}}\right)^3 = -8a^{-\frac{3}{2}}$
$-9a^{-2} + 36a^{-\frac{3}{2}} - 63a^{-1} + 54a^{-\frac{1}{2}} - 27$	
	$3\left(a^{-1} - 2a^{-\frac{1}{2}}\right)^2 = 3a^{-2} - 12a^{-\frac{3}{2}} + 12a^{-1}$
	$3\left(a^{-1} - 2a^{-\frac{1}{2}}\right)^2(3) = 9a^{-2} - 36a^{-\frac{3}{2}} + 36a^{-1}$
	$3\left(a^{-1} - 2a^{-\frac{1}{2}}\right)(3)^2 = 27a^{-1} - 54a^{-\frac{1}{2}}$
	$(3)^3 = 27$

8. $$\sqrt[3]{x^2-6x^{\frac{4}{3}}+15x^{\frac{2}{3}}-20+15x^{-\frac{2}{3}}-6x^{-\frac{4}{3}}+x^{-2}} \quad \Big| \quad x^{\frac{2}{3}}-2+x^{-\frac{2}{3}}$$

$$-x^2$$

$$-6x^{\frac{4}{3}}+15x^{\frac{2}{3}}-20$$

$$+6x^{\frac{4}{3}}-12x^{\frac{2}{3}}+8$$

$$3x^{\frac{2}{3}}-12+15x^{-\frac{2}{3}}-6x^{-\frac{4}{3}}+x^{-2}$$

$$-3x^{\frac{2}{3}}+12-15x^{-\frac{2}{3}}+6x^{-\frac{4}{3}}-x^{-2}$$

$$3\left(x^{\frac{2}{3}}\right)^2=3x^{\frac{4}{3}}$$

$$3\left(x^{\frac{2}{3}}\right)^2(-2)=-6x^{\frac{4}{3}}$$

$$3\left(x^{\frac{2}{3}}\right)(-2)^2=12x^{\frac{2}{3}}$$

$$(-2)^3=-8$$

$$3\left(x^{\frac{2}{3}}-2\right)^2=3x^{\frac{4}{3}}-12x^{\frac{2}{3}}+12$$

$$3\left(x^{\frac{2}{3}}-2\right)^2\left(x^{-\frac{2}{3}}\right)=3x^{\frac{2}{3}}-12+12x^{-\frac{2}{3}}$$

$$3\left(x^{\frac{2}{3}}-2\right)\left(x^{-\frac{2}{3}}\right)^2=3x^{-\frac{2}{3}}-6x^{-\frac{4}{3}}$$

$$\left(x^{-\frac{2}{3}}\right)^3=x^{-2}$$

9. $$\sqrt[3]{a^{\frac{3}{2}}+3a^{\frac{5}{4}}\qquad-5a^{\frac{3}{4}}+3a^{\frac{1}{4}}-1} \quad \Big| \quad a^{\frac{1}{2}}+a^{\frac{1}{4}}-1$$

$$-a^{\frac{3}{2}}$$

$$+3a^{\frac{5}{4}}\qquad-5a^{\frac{3}{4}}+3a^{\frac{1}{4}}$$

$$-3a^{\frac{5}{4}}-3a-a^{\frac{3}{4}}$$

$$-3a-6a^{\frac{3}{4}}+3a^{\frac{1}{4}}-1$$

$$+3a+6a^{\frac{3}{4}}-3a^{\frac{1}{4}}+1$$

$$3\left(a^{\frac{1}{2}}\right)^2=3a$$

$$3\left(a^{\frac{1}{2}}\right)^2\left(a^{\frac{1}{4}}\right)=3a^{\frac{5}{4}}$$

$$3\left(a^{\frac{1}{2}}\right)\left(a^{\frac{1}{4}}\right)^2=3a$$

$$\left(a^{\frac{1}{4}}\right)^3=a^{\frac{3}{4}}$$

$$3\left(a^{\frac{1}{2}}+a^{\frac{1}{4}}\right)^2=3a+6a^{\frac{3}{4}}+3a^{\frac{1}{2}}$$

$$3\left(a^{\frac{1}{2}}+a^{\frac{1}{4}}\right)^2(-1)=-3a-6a^{\frac{3}{4}}-3a^{\frac{1}{2}}$$

$$3\left(a^{\frac{1}{2}}+a^{\frac{1}{4}}\right)(-1)^2=3a^{\frac{1}{2}}+3a^{\frac{1}{4}}$$

$$(-1)^3=-1$$

EJERCICIO 230

1. $\sqrt{a^2x^{-2}-\frac{2ax^{-1}}{3}+\frac{19}{9}-\frac{2xa^{-1}}{3}+x^2a^{-2}}$ | $ax^{-1}-\frac{1}{3}+a^{-1}x$

$$-a^2x^{-2}$$

$$-\frac{2ax^{-1}}{3}+\frac{19}{9}$$

$$+\frac{2ax^{-1}}{3}-\frac{1}{9}$$

$$2-\frac{2a^{-1}x}{3}+x^2a^{-2}$$

$$-2+\frac{2a^{-1}x}{3}-a^{-2}x^2$$

$$\left(2ax^{-1}-\frac{1}{3}\right)\left(-\frac{1}{3}\right)=-\frac{2ax^{-1}}{3}+\frac{1}{9}$$

$$\left(2ax^{-1}-\frac{2}{3}+a^{-1}x\right)\left(a^{-1}x\right)=2-\frac{2a^{-1}x}{3}+a^{-2}x^2$$

2. $\sqrt{x^2-4+2x^{-1}+4x^{-2}-4x^{-3}+x^{-4}}$ | $x-2x^{-1}+x^{-2}$

$$-x^2$$

$$-4+2x^{-1}+4x^{-2}$$

$$+4\qquad -4x^{-2}$$

$$2x^{-1}\qquad -4x^{-3}+x^{-4}$$

$$-2x^{-1}\qquad +4x^{-3}-x^{-4}$$

$$\left(2x-2x^{-1}\right)\left(-2x^{-1}\right)=-4+4x^{-2}$$

$$\left(2x-4x^{-1}+x^{-2}\right)\left(x^{-2}\right)=2x^{-1}-4x^{-3}+x^{-4}$$

3. $\sqrt{a^4-10a+4+25a^{-2}-20a^{-3}+4a^{-4}}$ | $a^2-5a^{-1}+2a^{-2}$

$$-a^4$$

$$-10a+4+25a^{-2}$$

$$+10a\qquad -25a^{-2}$$

$$+4\qquad -20a^{-3}+4a^{-4}$$

$$-4\qquad +20a^{-3}-4a^{-4}$$

$$\left(2a^2-5a^{-1}\right)\left(-5a^{-1}\right)=-10a+25a^{-2}$$

$$\left(2a^2-10a^{-1}+2a^{-2}\right)\left(2a^{-2}\right)=4-20a^{-3}+4a^{-4}$$

4. $\sqrt{\frac{m^4}{4}-5m^2+28-30m^{-2}+9m^{-4}}$ | $\frac{m^2}{2}-5+3m^{-2}$

$$-\frac{m^4}{4}$$

$$-5m^2+28$$

$$+5m^2-25$$

$$3-30m^{-2}+9m^{-4}$$

$$-3+30m^{-2}-9m^{-4}$$

$$\left(m^2-5\right)\left(-5\right)=-5m^2+25$$

$$\left(m^2-10+3m^{-2}\right)\left(3m^{-2}\right)=3-30m^{-2}+9m^{-4}$$

5. $$\sqrt{\frac{4x^2y^{-2}}{25}-\frac{2xy^{-1}}{5}+\frac{19}{12}-\frac{5x^{-1}y}{3}+\frac{25x^{-2}y^2}{9}}\quad\Big|\quad \frac{2xy^{-1}}{5}-\frac{1}{2}+\frac{5x^{-1}y}{3}$$

$$-\frac{4x^2y^{-2}}{25}$$

$$-\frac{2xy^{-1}}{5}+\frac{19}{12}$$

$$+\frac{2xy^{-1}}{5}-\frac{1}{4}$$

$$\frac{4}{3}-\frac{5x^{-1}y}{3}+\frac{25x^{-2}y^2}{9}$$

$$-\frac{4}{3}+\frac{5x^{-1}y}{3}-\frac{25x^{-2}y^2}{9}$$

$$\left(\frac{4xy^{-1}}{5}-\frac{1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{2xy^{-1}}{5}+\frac{1}{4}$$

$$\left(\frac{4xy^{-1}}{5}-1+\frac{5x^{-1}y}{3}\right)\left(\frac{5x^{-1}y}{3}\right)$$

$$=\frac{4}{3}-\frac{5x^{-1}y}{3}+\frac{25x^{-2}y^2}{9}$$

6. $$\sqrt{\frac{a^4}{9}+\frac{2a^3x^{-1}}{3}+a^2x^{-2}-\frac{2ax}{3}-2+a^{-2}x^2}\quad\Big|\quad \frac{a^2}{3}+ax^{-1}-a^{-1}x$$

$$-\frac{a^4}{9}$$

$$\frac{2a^3x^{-1}}{3}+a^2x^{-2}$$

$$-\frac{2a^3x^{-1}}{3}-a^2x^{-2}$$

$$-\frac{2ax}{3}-2+a^{-2}x^2$$

$$+\frac{2ax}{3}+2-a^{-2}x^2$$

$$\left(\frac{2a^2}{3}+ax^{-1}\right)\left(ax^{-1}\right)=\frac{2a^3x^{-1}}{3}+a^2x^{-2}$$

$$\left(\frac{2a^2}{3}+2ax^{-1}-a^{-1}x\right)\left(-a^{-1}x\right)$$

$$=-\frac{2ax}{3}-2+a^{-2}x^2$$

7. $$\sqrt{9m^4+30m^2+55+50m^{-2}+25m^{-4}}\quad\Big|\quad 3m^2+5+5m^{-2}$$

$$-9m^4$$

$$+30m^2+55$$

$$-30m^2-25$$

$$30+50m^{-2}+25m^{-4}$$

$$-30-50m^{-2}-25m^{-4}$$

$$\left(6m^2+5\right)(5)=30m^2+25$$

$$\left(6m^2+10+5m^{-2}\right)\left(5m^{-2}\right)$$

$$=30+50m^{-2}+25m^{-4}$$

8. $$\sqrt{\frac{4a^2b^2x^{-2}y^{-2}}{49}-\frac{2abx^{-1}y^{-1}}{7}+\frac{21}{20}-\frac{7xya^{-1}b^{-1}}{5}+\frac{49x^2y^2a^{-2}b^{-2}}{25}}\quad\Big|\quad \frac{2abx^{-1}y^{-1}}{7}-\frac{1}{2}+\frac{7a^{-1}b^{-1}xy}{5}$$

$$-\frac{4a^2b^2x^{-2}y^{-2}}{49}$$

$$-\frac{2abx^{-1}y^{-1}}{7}+\frac{21}{20}$$

$$\frac{2abx^{-1}y^{-1}}{7}-\frac{1}{4}$$

$$\frac{4}{5}-\frac{7xya^{-1}b^{-1}}{5}+\frac{49x^2y^2a^{-2}b^{-2}}{25}$$

$$-\frac{4}{5}+\frac{7xya^{-1}b^{-1}}{5}-\frac{49a^{-2}b^{-2}x^2y^2}{25}$$

$$\left(\frac{4abx^{-1}y^{-1}}{7}-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$=-\frac{2abx^{-1}y^{-1}}{7}+\frac{1}{4}$$

$$\left(\frac{4abx^{-1}y^{-1}}{7}-1+\frac{7a^{-1}b^{-1}xy}{5}\right)\left(\frac{7a^{-1}b^{-1}xy}{5}\right)$$

$$=\frac{4}{5}-\frac{7a^{-1}b^{-1}xy}{5}+\frac{49a^{-2}b^{-2}x^2y^2}{25}$$

9. $\sqrt{ab^{-\frac{2}{3}} - 4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 6 - 4b^{\frac{1}{3}}a^{-\frac{1}{2}} + a^{-1}b^{\frac{2}{3}}}$ $\quad\Big|\quad a^{\frac{1}{2}}b^{-\frac{1}{3}} - 2 + a^{-\frac{1}{2}}b^{\frac{1}{3}}$

$-ab^{-\frac{2}{3}}$

$-4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 6$

$+4a^{\frac{1}{2}}b^{-\frac{1}{3}} - 4$

$2 - 4a^{-\frac{1}{2}}b^{\frac{1}{3}} + a^{-1}b^{\frac{2}{3}}$

$-2 + 4a^{-\frac{1}{2}}b^{\frac{1}{3}} - a^{-1}b^{\frac{2}{3}}$

$\left(2a^{\frac{1}{2}}b^{-\frac{1}{3}} - 2\right)(-2) = -4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 4$

$\left(2a^{\frac{1}{2}}b^{-\frac{1}{3}} - 4 + a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)$

$= 2 - 4a^{-\frac{1}{2}}b^{\frac{1}{3}} + a^{-1}b^{\frac{2}{3}}$

10. $\sqrt{a^4b^4 + 6a^2b^2 + 7 - 6a^{-2}b^{-2} + a^{-4}b^{-4}}$ $\quad\Big|\quad a^2b^2 + 3 - a^{-2}b^{-2}$

$-a^4b^4$

$+6a^2b^2 + 7$

$-6a^2b^2 - 9$

$-2 - 6a^{-2}b^{-2} + a^{-4}b^{-4}$

$+2 + 6a^{-2}b^{-2} - a^{-4}b^{-4}$

$\left(2a^2b^2 + 3\right)(3) = 6a^2b^2 + 9$

$\left(2a^2b^2 + 6 - a^{-2}b^{-2}\right)\left(-a^{-2}b^{-2}\right)$

$= -2 - 6a^{-2}b^{-2} + a^{-4}b^{-4}$

11. $\sqrt{xy^{\frac{2}{3}} - 8x^{\frac{1}{2}}y^{\frac{1}{3}} + 18 - 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}}}$ $\quad\Big|\quad x^{\frac{1}{2}}y^{\frac{1}{3}} - 4 + x^{-\frac{1}{2}}y^{-\frac{1}{3}}$

$-xy^{\frac{2}{3}}$

$-8x^{\frac{1}{2}}y^{\frac{1}{3}} + 18$

$+8x^{\frac{1}{2}}y^{\frac{1}{3}} - 16$

$2 - 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}}$

$-2 + 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} - x^{-1}y^{-\frac{2}{3}}$

$\left(2x^{\frac{1}{2}}y^{\frac{1}{3}} - 4\right)(-4) = -8x^{\frac{1}{2}}y^{\frac{1}{3}} + 16$

$\left(2x^{\frac{1}{2}}y^{\frac{1}{3}} - 8 + x^{-\frac{1}{2}}y^{-\frac{1}{3}}\right)\left(x^{-\frac{1}{2}}y^{-\frac{1}{3}}\right)$

$= 2 - 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}}$

EJERCICIO 231

1. $\sqrt{18} = \sqrt{2\cdot 3^2} = 3\sqrt{2}$

2. $3\sqrt{48} = 3\sqrt{3\cdot 2^4} = 3\cdot 2^2\sqrt{3} = 12\sqrt{3}$

3. $\sqrt[3]{16} = \sqrt[3]{2\cdot 2^3} = 2\sqrt[3]{2}$

4. $\frac{1}{2}\sqrt[3]{128} = \frac{1}{2}\sqrt[3]{2^6\cdot 2} = \frac{1}{2}\cdot 2^2\sqrt[3]{2} = 2\sqrt[3]{2}$

5. $2\sqrt[4]{243} = 2\sqrt[4]{3^4\cdot 3} = 2\cdot 3\sqrt[4]{3} = 6\sqrt[4]{3}$

6. $\sqrt{50a^2b} = \sqrt{2\cdot 5^2a^2b} = 5a\sqrt{2b}$

7. $3\sqrt{81x^3y^4} = 3\sqrt{9^2\cdot x^2xy^4} = 3\cdot 9xy^2\sqrt{x} = 27xy^2\sqrt{x}$

8. $\frac{1}{2}\sqrt{108a^5b^7} = \frac{1}{2}\sqrt{2^2\cdot 3^2\cdot 3\cdot a^4ab^6b}$

$= \frac{2}{2}\cdot 3a^2b^3\sqrt{3ab} = 3a^2b^3\sqrt{3ab}$

9. $\frac{3}{5}\sqrt{125mn^6} = \frac{3}{5}\sqrt{5^2\cdot 5mn^6} = \frac{3\cdot 5}{5}n^3\sqrt{5m} = 3n^3\sqrt{5m}$

10. $2a\sqrt{44a^3b^7c^9} = 2a\sqrt{2^2\cdot 11a^2ab^6bc^8c}$

$= 2\cdot 2a\cdot ab^3c^4\sqrt{11abc} = 4a^2b^3c^4 = \sqrt{11abc}$

11. $2\sqrt[3]{16x^2y^7}$

$=2\sqrt[3]{2^3\cdot 2x^2y^6y}=2\cdot 2y^2\sqrt[3]{2x^2y}=4y^2\sqrt[3]{2x^2y}$

12. $\frac{2}{3}\sqrt[3]{27m^2n^8}$

$=\frac{2}{3}\sqrt[3]{3^3m^2n^6n^2}=\frac{2\cdot 3}{3}n^2\sqrt[3]{m^2n^2}=2n^2\sqrt[3]{m^2n^2}$

13. $5a\sqrt[3]{160x^7y^9z^{13}}$

$=5a\sqrt[3]{2^3\cdot 20x^6xy^9z^{12}z}$

$=5\cdot 2ax^2y^3z^4\sqrt[3]{20xz}=10ax^2y^3z^4\sqrt[3]{20xz}$

14. $\sqrt[4]{80a^4b^5c^{12}}=\sqrt[4]{2^4\cdot 5a^4b^4bc^{12}}=2abc^3\sqrt[4]{5b}$

15. $3\sqrt[4]{5x^8y^{14}z^{16}}=3\sqrt[4]{5x^8y^{12}y^2z^{16}}=3x^2y^3z^4\sqrt[4]{5y^2}$

16. $\frac{2}{5}\sqrt[5]{32x^2y^{11}}$

$=\frac{2}{5}\sqrt[5]{2^5x^2y^{10}y}=\frac{2\cdot 2}{5}y^2\sqrt[5]{x^2y}=\frac{4y^2}{5}\sqrt[5]{x^2y}$

17. $2xy\sqrt[3]{128x^2y^8}$

$=2xy\sqrt[3]{2^6\cdot 2x^2y^6y^2}$

$=2\cdot 2^2xyy^2\sqrt[3]{2x^2y^2}=8xy^3\sqrt[3]{2x^2y^2}$

18. $\frac{1}{3a}\sqrt{27a^3m^7}$

$=\frac{1}{3a}\sqrt{3^2\cdot 3a^2am^6m}=\frac{3am^3}{3a}\sqrt{3am}=m^3\sqrt{3am}$

19. $\frac{3}{5x}\sqrt[3]{375a^8b}$

$=\frac{3}{5x}\sqrt[3]{5^3\cdot 3a^6a^2b}=\frac{3\cdot 5a^2}{5x}\sqrt[3]{3a^2b}=\frac{3a^2}{x}\sqrt[3]{3a^2b}$

20. $\frac{1}{3}\sqrt[4]{81a^4b}=\frac{1}{3}\sqrt[4]{3^4a^4b}=\frac{3}{3}a\sqrt[4]{b}=a\sqrt[4]{b}$

21. $\sqrt{9a+18b}=\sqrt{9(a+2b)}=\sqrt{3^2(a+2b)}=3\sqrt{a+2b}$

22. $\sqrt{3a^3b^2-3a^2b^2}$

$=\sqrt{3a^2b^2(a-1)}=ab\sqrt{3(a-1)}=ab\sqrt{3a-3}$

23. $\sqrt{8x^2y^4+16xy^4}$

$=\sqrt{2^2\cdot 2xy^4(x+2)}=2y^2\sqrt{2x(x+2)}=2y^2\sqrt{2x^2+4x}$

24. $\sqrt{2x^2-4xy+2y^2}$

$=\sqrt{2(x^2-2xy+y^2)}=\sqrt{2(x-y)^2}=(x-y)\sqrt{2}$

25. $\sqrt{(a-b)(a^2-b^2)}$

$=\sqrt{(a-b)(a-b)(a+b)}$

$=\sqrt{(a-b)^2(a+b)}=(a-b)\sqrt{a+b}$

26. $\sqrt{2am^2+4amn+2an^2}$

$=\sqrt{2a(m^2+2mn+n^2)}=\sqrt{2a(m+n)^2}=(m+n)\sqrt{2a}$

27. $\sqrt{9a^3-36a^2+36a}$

$=\sqrt{3^2a(a^2-2a+4)}$

$=3\sqrt{a(a-2)^2}=3(a-2)\sqrt{a}=(3a-6)\sqrt{a}$

EJERCICIO 232

1. $\sqrt{\frac{1}{5}}=\sqrt{\frac{1\cdot 5}{5\cdot 5}}=\sqrt{\frac{5}{5^2}}=\frac{1}{5}\sqrt{5}$

2. $\sqrt{\frac{3}{8}}=\sqrt{\frac{3\cdot 2}{8\cdot 2}}=\sqrt{\frac{6}{2^4}}=\frac{1}{2^2}\sqrt{6}=\frac{1}{4}\sqrt{6}$

3. $2\sqrt{\frac{1}{2}}=2\sqrt{\frac{2\cdot 1}{2\cdot 2}}=2\sqrt{\frac{2}{2^2}}=\frac{2}{2}\sqrt{2}=\sqrt{2}$

4. $3\sqrt{\frac{1}{6}}=3\sqrt{\frac{1\cdot 6}{6\cdot 6}}=3\sqrt{\frac{6}{6^2}}=\frac{3}{6}\sqrt{6}=\frac{1}{2}\sqrt{6}$

5. $\frac{1}{2}\sqrt{\frac{2}{3}}=\frac{1}{2}\sqrt{\frac{2\cdot 3}{3\cdot 3}}=\frac{1}{2}\sqrt{\frac{6}{3^2}}=\frac{1}{2\cdot 3}\sqrt{6}=\frac{1}{6}\sqrt{6}$

6. $\sqrt{\frac{a^2}{8x}}=\sqrt{\frac{a^2x}{2^2\cdot 2x^2}}=\frac{a}{2x}\sqrt{\frac{x}{2}}=\frac{a}{2x}\sqrt{\frac{2x}{2^2}}=\frac{a}{4x}\sqrt{2x}$

7. $\frac{3}{2}\sqrt{\frac{4a^2}{27y^3}}=\frac{3}{2}\sqrt{\frac{3\cdot 2^2a^2y}{3^4y^4}}=\frac{3\cdot 2a}{2\cdot 3^2y^2}\sqrt{3y}=\frac{a}{3y^2}\sqrt{3y}$

8. $5\sqrt{\frac{9n}{5m^3}}=5\sqrt{\frac{3^2\cdot 5mn}{5^2m^4}}=\frac{5\cdot 3}{5m^2}\sqrt{5mn}=\frac{3}{m^2}\sqrt{5mn}$

9. $6\sqrt{\frac{5a^3}{24x^2}}=6\sqrt{\frac{5\cdot 6a^3}{2^4\cdot 3^2x^2}}=\frac{6}{2^2\cdot 3x}\sqrt{30a^2a}=\frac{a}{2x}\sqrt{30a}$

10. $\sqrt[3]{\frac{2}{3}}=\sqrt[3]{\frac{2\cdot 3^2}{3^3}}=\frac{1}{3}\sqrt[3]{18}$

11. $5\sqrt[3]{\frac{1}{5}}=5\sqrt[3]{\frac{5^2}{5^3}}=\frac{5}{5}\sqrt[3]{5^2}=\sqrt[3]{25}$

12. $\sqrt[3]{\frac{8}{9x^2}}=\sqrt[3]{\frac{2^3\cdot 3x}{3^3x^3}}=\frac{2}{3x}\sqrt[3]{3x}$

13. $2b^2\sqrt[3]{\frac{125}{4b^5}}=2b^2\sqrt[3]{\frac{5^3\cdot 2b}{2^3b^6}}=\frac{2\cdot 5b^2}{2b^2}\sqrt[3]{2b}=5\sqrt[3]{2b}$

14. $\frac{2}{3}\sqrt[3]{\frac{27x^2}{16a^2b^4}}=\frac{2}{3}\sqrt[3]{\frac{3^3\cdot 4ab^2x^2}{2^6a^3b^6}}=\frac{2\cdot 3}{3\cdot 2^2ab^2}\sqrt[3]{4ab^2x^2}=\frac{1}{2ab^2}\sqrt[3]{4ab^2x^2}$

15. $2xy\sqrt[4]{\frac{81a^2}{4x^3y}}=2xy\sqrt[4]{\frac{3^4\cdot 2^2a^2xy^3}{2^4x^4y^4}}=\frac{2\cdot 3xy}{2xy}\sqrt[4]{4a^2xy^3}=3\sqrt[4]{4a^2xy^3}$

EJERCICIO 233

1. $\sqrt[4]{9}=\sqrt[4]{3^2}=3^{\frac{2}{4}}=3^{\frac{1}{2}}=\sqrt{3}$

2. $\sqrt[6]{4}=\sqrt[6]{2^2}=2^{\frac{2}{6}}=2^{\frac{1}{3}}=\sqrt[3]{2}$

3. $\sqrt[9]{27}=\sqrt[9]{3^3}=3^{\frac{3}{9}}=3^{\frac{1}{3}}=\sqrt[3]{3}$

4. $\sqrt[8]{16}=\sqrt[8]{2^4}=2^{\frac{4}{8}}=2^{\frac{1}{2}}=\sqrt{2}$

5. $3\sqrt[12]{64}=3\sqrt[12]{2^6}=3\cdot 2^{\frac{6}{12}}=3\cdot 2^{\frac{1}{2}}=3\sqrt{2}$

6. $\sqrt[4]{25a^2b^2}$

$=\sqrt[4]{5^2a^2b^2}=5^{\frac{2}{4}}a^{\frac{2}{4}}b^{\frac{2}{4}}=5^{\frac{1}{2}}a^{\frac{1}{2}}b^{\frac{1}{2}}=\sqrt{5ab}$

7. $5\sqrt[6]{49a^2b^4}=5\sqrt[6]{7^2a^2b^4}=5\cdot 7^{\frac{1}{3}}a^{\frac{1}{3}}b^{\frac{2}{3}}=5\sqrt[3]{7ab^2}$

8. $\sqrt[8]{81x^4y^8}=\sqrt[8]{3^4x^4y^8}=3^{\frac{1}{2}}x^{\frac{1}{2}}y=y\sqrt{3x}$

9. $\sqrt[10]{32x^{10}y^{15}}$

$=\sqrt[10]{2^5x^{10}y^{15}}=2^{\frac{1}{2}}xy^{\frac{3}{2}}=x\sqrt{2y^3}=x\sqrt{2y^2y}=xy\sqrt{2y}$

10. $\sqrt[12]{64m^6n^{18}}$

$=\sqrt[12]{2^6m^6n^{18}}=2^{\frac{1}{2}}m^{\frac{1}{2}}n^{\frac{3}{2}}=\sqrt{2mn^3}=\sqrt{2mn^2n}=n\sqrt{2mn}$

11. $\sqrt[6]{343a^9x^{12}}$

$=\sqrt[6]{7^3a^9x^{12}}=7^{\frac{1}{2}}a^{\frac{3}{2}}x^2=x^2\sqrt{7a^3}=x^2\sqrt{7a^2a}=ax^2\sqrt{7a}$

12. $\sqrt[15]{m^{10}n^{15}x^{20}}$

$=m^{\frac{2}{3}}nx^{\frac{4}{3}}=n\sqrt[3]{m^2x^4}=n\sqrt[3]{m^2x^3x}=nx\sqrt[3]{m^2x}$

EJERCICIO 234

1. $2\sqrt{3}=\sqrt{2^2\cdot 3}=\sqrt{12}$

2. $3\sqrt{5}=\sqrt{3^2\cdot 5}=\sqrt{45}$

3. $5a\sqrt{b}=\sqrt{(5a)^2b}=\sqrt{25a^2b}$

4. $\frac{1}{2}\sqrt{2}=\sqrt{\left(\frac{1}{2}\right)^2\cdot 2}=\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{2}}$

5. $3a\sqrt{2a^2}=\sqrt{(3a)^2\cdot 2a^2}=\sqrt{9a^2\left(2a^2\right)}=\sqrt{18a^4}$

6. $5x^2y\sqrt{3}=\sqrt{\left(5x^2y\right)^2 3}=\sqrt{\left(25x^4y^2\right)3}=\sqrt{75x^4y^2}$

7. $ab^2\sqrt[3]{a^2b}$

$=\sqrt[3]{\left(ab^2\right)^3\left(a^2b\right)}=\sqrt[3]{\left(a^3b^6\right)\left(a^2b\right)}=\sqrt[3]{a^5b^7}$

8. $4m\sqrt[3]{2m^2}=\sqrt[3]{(4m)^3\left(2m^2\right)}=\sqrt[3]{\left(64m^3\right)\left(2m^2\right)}=\sqrt[3]{128m^5}$

9. $2a\sqrt[4]{8ab^3}=\sqrt[4]{(2a)^4\left(8ab^3\right)}=\sqrt[4]{\left(16a^4\right)\left(8ab^3\right)}=\sqrt[4]{128a^5b^3}$

10. $(a+b)\sqrt{\frac{a}{a+b}}=\sqrt{\frac{(a+b)^2(a)}{a+b}}=\sqrt{(a+b)(a)}=\sqrt{a^2+ab}$

11. $(x+1)\sqrt{\frac{2x}{x+1}}=\sqrt{\frac{(x+1)^2 2x}{x+1}}=\sqrt{(x+1)2x}=\sqrt{2x^2+2x}$

12. $(x-1)\sqrt{\frac{x-2}{x-1}}$

$=\sqrt{\frac{(x-1)^2(x-2)}{x-1}}=\sqrt{(x-1)(x-2)}=\sqrt{x^2-3x+2}$

EJERCICIO 235

1. $\sqrt{5}=\sqrt[6]{5^3}=\sqrt[6]{125}$

$\sqrt[3]{2}=\sqrt[6]{2^2}=\sqrt[6]{4}$

2. $\sqrt{2}=\sqrt[4]{2^2}=\sqrt[4]{4}$

$\sqrt[4]{3}=\sqrt[4]{3}$

3. $\sqrt{3}=\sqrt[12]{3^6}=\sqrt[12]{729}$

$\sqrt[3]{4}=\sqrt[12]{4^4}=\sqrt[12]{256}$

$\sqrt[4]{8}=\sqrt[12]{8^3}=\sqrt[12]{512}$

4. $\sqrt{2}=\sqrt[12]{2^6}=\sqrt[12]{64}$

$\sqrt[3]{3}=\sqrt[12]{3^4}=\sqrt[12]{81}$

$\sqrt[4]{5}=\sqrt[12]{5^3}=\sqrt[12]{125}$

$\sqrt[6]{7}=\sqrt[12]{7^2}=\sqrt[12]{49}$

5. $\sqrt{5x}=\sqrt[6]{(5x)^3}=\sqrt[6]{125x^3}$

$\sqrt[3]{4x^2y}=\sqrt[6]{\left(4x^2y\right)^2}=\sqrt[6]{16x^4y^2}$

$\sqrt[6]{7a^3b}=\sqrt[6]{7a^3b}$

6. $\sqrt[3]{2ab}=\sqrt[15]{(2ab)^5}=\sqrt[15]{32a^5b^5}$

$\sqrt[5]{3a^2x}=\sqrt[15]{\left(3a^2x\right)^3}=\sqrt[15]{27a^6x^3}$

$\sqrt[15]{5a^3x^2}=\sqrt[15]{5a^3x^2}$

7. $\sqrt[4]{8a^2x^3}=\sqrt[12]{\left(8a^2x^3\right)^3}=\sqrt[12]{512a^6x^9}$

$\sqrt[6]{3a^5m^4}=\sqrt[12]{\left(3a^5m^4\right)^2}=\sqrt[12]{9a^{10}m^8}$

8. $\sqrt[3]{x^2}=\sqrt[18]{\left(x^2\right)^6}=\sqrt[18]{x^{12}}$

$\sqrt[6]{2y^3}=\sqrt[18]{\left(2y^3\right)^3}=\sqrt[18]{8y^9}$

$\sqrt[9]{5m^7}=\sqrt[18]{\left(5m^7\right)^2}=\sqrt[18]{25m^{14}}$

9. $\sqrt[4]{3a}=\sqrt[20]{(3a)^5}=\sqrt[20]{243a^5}$

$\sqrt[5]{2b^2}=\sqrt[20]{\left(2b^2\right)^4}=\sqrt[20]{16b^8}$

$\sqrt[10]{7x^3}=\sqrt[20]{\left(7x^3\right)^2}=\sqrt[20]{49x^6}$

10. $2\sqrt[3]{a}=2\sqrt[12]{a^4}=2\sqrt[12]{a^4}$

$3\sqrt{2b}=3\sqrt[12]{(2b)^6}=3\sqrt[12]{64b^6}$

$4\sqrt[4]{5x^2}=4\sqrt[12]{\left(5x^2\right)^3}=4\sqrt[12]{125x^6}$

11. $3\sqrt[3]{a^2}=3\sqrt[18]{\left(a^2\right)^6}=3\sqrt[18]{a^{12}}$

$\frac{1}{2}\sqrt[6]{b^3}=\frac{1}{2}\sqrt[18]{\left(b^3\right)^3}=\frac{1}{2}\sqrt[18]{b^9}$

$4\sqrt[9]{x^5}=4\sqrt[18]{\left(x^5\right)^2}=4\sqrt[18]{x^{10}}$

12. $\sqrt{2m}=\sqrt[10]{(2m)^5}=\sqrt[10]{32m^5}$

$3\sqrt[5]{a^3x^4}=3\sqrt[10]{\left(a^3x^4\right)^2}=3\sqrt[10]{a^6x^8}$

$2\sqrt[10]{x^7y^2}=2\sqrt[10]{x^7y^2}$

EJERCICIO 236

1. $\sqrt{5}=\sqrt[6]{5^3}=\sqrt[6]{125}$

$\sqrt[3]{2}=\sqrt[6]{2^2}=\sqrt[6]{4}$

Luego el orden es: $\sqrt{5}$, $\sqrt[3]{2}$

2. $\sqrt[6]{15}=\sqrt[12]{(15)^2}=\sqrt[12]{225}$

$\sqrt[4]{7}=\sqrt[12]{7^3}=\sqrt[12]{343}$

Luego el orden es: $\sqrt[4]{7}$, $\sqrt[6]{15}$

3. $\sqrt{11}=\sqrt[6]{(11)^3}=\sqrt[6]{1.331}$

$\sqrt[3]{43}=\sqrt[6]{(43)^2}=\sqrt[6]{1.849}$

Luego el orden es: $\sqrt[3]{43}$, $\sqrt{11}$

4. $\sqrt{3}=\sqrt[6]{3^3}=\sqrt[6]{27}$

$\sqrt[3]{5}=\sqrt[6]{5^2}=\sqrt[6]{25}$

$\sqrt[6]{32}=\sqrt[6]{32}$

Luego el orden es: $\sqrt[6]{32}$, $\sqrt{3}$, $\sqrt[3]{5}$

5. $\sqrt[4]{3}=\sqrt[20]{3^5}=\sqrt[20]{243}$

$\sqrt[5]{4}=\sqrt[20]{4^4}=\sqrt[20]{256}$

$\sqrt[10]{15}=\sqrt[20]{15^2}=\sqrt[20]{225}$

Luego el orden es: $\sqrt[5]{4}$, $\sqrt[4]{3}$, $\sqrt[10]{15}$

6. $\sqrt[3]{2}=\sqrt[18]{2^6}=\sqrt[18]{64}$

$\sqrt[6]{3}=\sqrt[18]{3^3}=\sqrt[18]{27}$

$\sqrt[9]{9}=\sqrt[18]{9^2}=\sqrt[18]{81}$

Luego el orden es: $\sqrt[9]{9}$, $\sqrt[3]{2}$, $\sqrt[6]{3}$

EJERCICIO 237

1. $7\sqrt{2}-15\sqrt{2}=(7-15)\sqrt{2}=-8\sqrt{2}$

2. $4\sqrt{3}-20\sqrt{3}+19\sqrt{3}=(4-20+19)\sqrt{3}=3\sqrt{3}$

3. $\sqrt{5}-22\sqrt{5}-8\sqrt{5}=(1-22-8)\sqrt{5}=-29\sqrt{5}$

4. $\sqrt{2}-9\sqrt{2}+30\sqrt{2}-40\sqrt{2}=(1-9+30-40)\sqrt{2}=-18\sqrt{2}$

5. $\frac{3}{4}\sqrt{2}-\frac{1}{2}\sqrt{2}=\left(\frac{3}{4}-\frac{1}{2}\right)\sqrt{2}=\left(\frac{3-2}{4}\right)\sqrt{2}=\frac{1}{4}\sqrt{2}$

6. $\frac{3}{5}\sqrt{3}-\sqrt{3}=\left(\frac{3}{5}-1\right)\sqrt{3}=\left(\frac{3-5}{5}\right)\sqrt{3}=-\frac{2}{5}\sqrt{3}$

7. $2\sqrt{5}-\frac{1}{2}\sqrt{5}+\frac{3}{4}\sqrt{5}$

$=\left(2-\frac{1}{2}+\frac{3}{4}\right)\sqrt{5}=\left(\frac{8-2+3}{4}\right)\sqrt{5}=\frac{9}{4}\sqrt{5}$

8. $\frac{1}{4}\sqrt{3}+5\sqrt{3}-\frac{1}{8}\sqrt{3}$

$=\left(\frac{1}{4}+5-\frac{1}{8}\right)\sqrt{3}=\left(\frac{2+40-1}{8}\right)\sqrt{3}=\frac{41}{8}\sqrt{3}$

9. $a\sqrt{b}-3a\sqrt{b}+7a\sqrt{b}=(a-3a+7a)\sqrt{b}=5a\sqrt{b}$

10. $3x\sqrt{y}+(a-x)\sqrt{y}-2x\sqrt{y}$

$=(3x+a-x-2x)\sqrt{y}=a\sqrt{y}$

11. $(x-1)\sqrt{3}+(x-3)\sqrt{3}+4\sqrt{3}$

$=(x-1+x-3+4)\sqrt{3}=2x\sqrt{3}$

12. $\frac{1}{3}\sqrt[3]{2}-\frac{2}{3}\sqrt[3]{2}+2\sqrt[3]{2}$

$=\left(\frac{1}{3}-\frac{2}{3}+2\right)\sqrt[3]{2}=\left(\frac{1-2+6}{3}\right)\sqrt[3]{2}=\frac{5}{3}\sqrt[3]{2}$

13. $\frac{3}{5}\sqrt[3]{2}-\frac{1}{4}\sqrt[3]{2}+\frac{1}{6}\sqrt[3]{2}=\left(\frac{36-15+10}{60}\right)\sqrt[3]{2}=\frac{31}{60}\sqrt[3]{2}$

14. $x\sqrt[3]{a^2}-(a-2x)\sqrt[3]{a^2}+(2a-3x)\sqrt[3]{a^2}$

$=(x-a+2x+2a-3x)\sqrt[3]{a^2}=a\sqrt[3]{a^2}$

EJERCICIO 238

1. $\sqrt{45}=\sqrt{3^2\cdot 5}=3\sqrt{5}$

$-\sqrt{27}=-\sqrt{3^2\cdot 3}=-3\sqrt{3}$

$-\sqrt{20}=-\sqrt{2^2\cdot 5}=-2\sqrt{5}$

Entonces:

$=3\sqrt{5}-3\sqrt{3}-2\sqrt{5}$

$=(3-2)\sqrt{5}-3\sqrt{3}=\sqrt{5}-3\sqrt{3}$

2. $\sqrt{175}=\sqrt{5^2\cdot 7}=5\sqrt{7}$

$\sqrt{243}=\sqrt{3^4\cdot 3}=9\sqrt{3}$

$-\sqrt{63}=-\sqrt{3^2\cdot 7}=-3\sqrt{7}$

$-2\sqrt{75}=-2\sqrt{5^2\cdot 3}=-10\sqrt{3}$

Entonces:

$=5\sqrt{7}+9\sqrt{3}-3\sqrt{7}-10\sqrt{3}$

$=(5-3)\sqrt{7}+(9-10)\sqrt{3}=2\sqrt{7}-\sqrt{3}$

3. $\sqrt{80}=\sqrt{2^4\cdot 5}=2^2\sqrt{5}=4\sqrt{5}$

$-2\sqrt{252}=-2\sqrt{2^2\cdot 3^2\cdot 7}=-2\cdot 2\cdot 3\sqrt{7}=-12\sqrt{7}$

$3\sqrt{405}=3\sqrt{3^4\cdot 5}=3\cdot 3^2\sqrt{5}=27\sqrt{5}$

$-3\sqrt{500}=-3\sqrt{2^2\cdot 5^2\cdot 5}=-3\cdot 2\cdot 5\sqrt{5}=-30\sqrt{5}$

Entonces:

$=4\sqrt{5}+27\sqrt{5}-30\sqrt{5}-12\sqrt{7}$

$=(4+27-30)\sqrt{5}-12\sqrt{7}=\sqrt{5}-12\sqrt{7}$

4. $7\sqrt{450}=7\sqrt{3^2\cdot 5^2\cdot 2}=7\cdot 3\cdot 5\sqrt{2}=105\sqrt{2}$

$-4\sqrt{320}=-4\sqrt{2^6\cdot 5}=-4\cdot 2^3\sqrt{5}=-32\sqrt{5}$

$3\sqrt{80}=3\sqrt{2^4\cdot 5}=3\cdot 2^2\sqrt{5}=12\sqrt{5}$

$-5\sqrt{800}=-5\sqrt{2^4\cdot 2\cdot 5^2}=-5\cdot 2^2\cdot 5\sqrt{2}=-100\sqrt{2}$

Entonces:

$=105\sqrt{2}-100\sqrt{2}-32\sqrt{5}+12\sqrt{5}$

$=(105-100)\sqrt{2}+(-32+12)\sqrt{5}=5\sqrt{2}-20\sqrt{5}$

5. $\frac{1}{2}\sqrt{12}=\frac{1}{2}\sqrt{2^2\cdot 3}=\sqrt{3}$

$-\frac{1}{3}\sqrt{18}=-\frac{1}{3}\sqrt{3^2\cdot 2}=-\sqrt{2}$

$\frac{3}{4}\sqrt{48}=\frac{3}{4}\sqrt{2^4\cdot 3}=3\sqrt{3}$

$\frac{1}{6}\sqrt{72}=\frac{1}{6}\sqrt{2^2\cdot 2\cdot 3^2}=\sqrt{2}$

Entonces:

$=\sqrt{3}+3\sqrt{3}-\sqrt{2}+\sqrt{2}=(1+3)\sqrt{3}=4\sqrt{3}$

6. $\frac{3}{4}\sqrt{176}=\frac{3}{4}\sqrt{2^4\cdot 11}=\frac{2^2\cdot 3}{4}\sqrt{11}=3\sqrt{11}$

$-\frac{2}{3}\sqrt{45}=-\frac{2}{3}\sqrt{3^2\cdot 5}=-\frac{2\cdot 3}{3}\sqrt{5}=-2\sqrt{5}$

$\frac{1}{8}\sqrt{320}=\frac{1}{8}\sqrt{2^6\cdot 5}=\frac{2^3}{8}\sqrt{5}=\sqrt{5}$

$\frac{1}{5}\sqrt{275}=\frac{1}{5}\sqrt{5^2\cdot 11}=\frac{5}{5}\sqrt{11}=\sqrt{11}$

Entonces:

$=3\sqrt{11}+\sqrt{11}-2\sqrt{5}+\sqrt{5}$

$=(3+1)\sqrt{11}-(2+1)\sqrt{5}=4\sqrt{11}-\sqrt{5}$

7. $\frac{1}{7}\sqrt{147}= \qquad \frac{1}{7}\sqrt{3\cdot 7^2}=\sqrt{3}$

$-\frac{1}{5}\sqrt{700}=-\frac{1}{5}\sqrt{2^2\cdot 5^2\cdot 7}=-2\sqrt{7}$

$\frac{1}{10}\sqrt{28}= \qquad \frac{1}{10}\sqrt{2^2\cdot 7}=\frac{1}{5}\sqrt{7}$

$\frac{1}{3}\sqrt{2.187}= \qquad \frac{1}{3}\sqrt{3^6\cdot 3}=9\sqrt{3}$

Entonces:

$=\sqrt{3}+9\sqrt{3}-2\sqrt{7}+\frac{1}{5}\sqrt{7}$

$=(1+9)\sqrt{3}+\left(-2+\frac{1}{5}\right)\sqrt{7}$

$=10\sqrt{3}-\frac{9}{5}\sqrt{7}$

8. $\sqrt{\frac{1}{3}}=\sqrt{\frac{3}{3^2}}=\frac{1}{3}\sqrt{3}$

$-\sqrt{\frac{1}{2}}=\sqrt{\frac{2}{2^2}}=-\frac{1}{2}\sqrt{2}$

$\sqrt{\frac{3}{4}}=\sqrt{\frac{3}{2^2}=\frac{1}{2}}\sqrt{3}$

Entonces:

$=\frac{1}{3}\sqrt{3}+\frac{1}{2}\sqrt{3}-\frac{1}{2}\sqrt{2}$

$=\left(\frac{1}{3}+\frac{1}{2}\right)\sqrt{3}-\frac{1}{2}\sqrt{2}$

$=\frac{5}{6}\sqrt{3}-\frac{1}{2}\sqrt{2}$

9. $\sqrt{\frac{9}{5}}= \qquad \sqrt{\frac{3^2\cdot 5}{5^2}}=\frac{3}{5}\sqrt{5}$

$-\sqrt{\frac{1}{6}}= \qquad \sqrt{-\frac{6}{6^2}}=-\frac{1}{6}\sqrt{6}$

$-\sqrt{\frac{1}{20}}=-\sqrt{\frac{5}{2^2\cdot 5^2}}=-\frac{1}{10}\sqrt{5}$

$\sqrt{6}=\sqrt{6}$

Entonces:

$=\frac{3}{5}\sqrt{5}-\frac{1}{10}\sqrt{5}-\frac{1}{6}\sqrt{6}+\sqrt{6}$

$=\left(\frac{3}{5}-\frac{1}{10}\right)\sqrt{5}+\left(-\frac{1}{6}+1\right)\sqrt{6}$

$=\frac{1}{2}\sqrt{5}+\frac{5}{6}\sqrt{6}$

10. $\frac{5}{3}\sqrt{\frac{3}{5}}= \qquad \frac{5}{3}\sqrt{\frac{15}{5^2}}=\frac{1}{3}\sqrt{15}$

$-\frac{1}{2}\sqrt{\frac{3}{4}}= \qquad -\frac{1}{2}\sqrt{\frac{3}{2^2}}=-\frac{1}{4}\sqrt{3}$

$-5\sqrt{\frac{1}{15}}=-5\sqrt{\frac{15}{3^2\cdot 5^2}}=-\frac{1}{3}\sqrt{15}$

$3\sqrt{\frac{1}{12}}= \qquad 3\sqrt{\frac{3}{2^2\cdot 3^2}}=\frac{1}{2}\sqrt{3}$

Entonces:

$=\frac{1}{3}\sqrt{15}-\frac{1}{3}\sqrt{15}-\frac{1}{4}\sqrt{3}+\frac{1}{2}\sqrt{3}$

$=\left(-\frac{1}{4}+\frac{1}{2}\right)\sqrt{3}=\frac{1}{4}\sqrt{3}$

11. $5\sqrt{128}= \qquad 5\sqrt{2^6\cdot 2}=40\sqrt{2}$

$-\frac{1}{3}\sqrt{\frac{1}{3}}= \qquad -\frac{1}{3}\sqrt{\frac{3}{3^2}}=-\frac{1}{9}\sqrt{3}$

$-5\sqrt{98}=-5\sqrt{7^2\cdot 2}=-35\sqrt{2}$

$\sqrt{\frac{1}{27}}= \qquad \sqrt{\frac{3}{3^4}}=\frac{1}{9}\sqrt{3}$

Entonces:

$=40\sqrt{2}-35\sqrt{2}-\frac{1}{9}\sqrt{3}+\frac{1}{9}\sqrt{3}$

$=(40-35)\sqrt{2}=5\sqrt{2}$

12. $2\sqrt{700}= \qquad 2\sqrt{2^2\cdot 5^2\cdot 7}=20\sqrt{7}$

$-15\sqrt{\frac{1}{45}}=-15\sqrt{\frac{5}{3^2\cdot 5^2}}=-\sqrt{5}$

$4\sqrt{\frac{5}{16}}= \qquad 4\sqrt{\frac{5}{2^4}}=\sqrt{5}$

$-56\sqrt{\frac{1}{7}}= \qquad -56\sqrt{\frac{7}{7^2}}=-8\sqrt{7}$

Entonces:

$=20\sqrt{7}-8\sqrt{7}-\sqrt{5}+\sqrt{5}$

$=(20-8)\sqrt{7}=12\sqrt{7}$

13. $\sqrt{25ax^2}= \qquad \sqrt{5^2ax^2}=5x\sqrt{a}$

$\sqrt{49b}= \qquad \sqrt{7^2b}=7\sqrt{b}$

$-\sqrt{9ax^2}=-\sqrt{3^2ax^2}=-3x\sqrt{a}$

entonces:

$=5x\sqrt{a}-3x\sqrt{a}+7\sqrt{b}$

$=(5x-3x)\sqrt{a}+7\sqrt{b}$

$=2x\sqrt{a}+7\sqrt{b}$

14. $2\sqrt{m^2n} \qquad =2m\sqrt{n}$

$-\sqrt{9m^2n}=-\sqrt{3^2m^2n}=-3m\sqrt{n}$

$\sqrt{16mn^2}= \qquad \sqrt{2^4mn^2}=4n\sqrt{m}$

$-\sqrt{4mn^2}=-\sqrt{2^2mn^2}=-2n\sqrt{m}$

Entonces:

$=2m\sqrt{n}-3m\sqrt{n}+4n\sqrt{m}-2n\sqrt{m}$

$=(2m-3m)\sqrt{n}+(4n-2n)\sqrt{m}$

$=-m\sqrt{n}+2n\sqrt{m}$

15. $a\sqrt{320x}=a\sqrt{2^6\cdot 5x}=8a\sqrt{5x}$

$-7\sqrt{5a^2x} \qquad =-7a\sqrt{5x}$

$-(a-4b)\sqrt{5x}$

Entonces:

$=8a\sqrt{5x}-7a\sqrt{5x}-(a-4b)\sqrt{5x}$

$=(8a-7a-a+4b)\sqrt{5x}=4b\sqrt{5x}$

16. $\sqrt{9x-9}=\sqrt{3^2(x-1)}=3\sqrt{x-1}$

$\sqrt{4x-4}=\sqrt{2^2(x-1)}=2\sqrt{x-1}$

$-5\sqrt{x-1}$

Entonces:

$=3\sqrt{x-1}+2\sqrt{x-1}-5\sqrt{x-1}$

$=(3+2-5)\sqrt{x-1}=0$

17. $2\sqrt{a^4x+3a^4y}=\quad 2\sqrt{a^4\left(x+3y\right)}=2a^2\sqrt{x+3y}$

$-a^2\sqrt{9x+27y}=-a^2\sqrt{3^2\left(x+3y\right)}=-3a^2\sqrt{x+3y}$

$\sqrt{25a^4x+75a^4y}=\quad \sqrt{5^2a^4\left(x+3y\right)}=5a^2\sqrt{x+3y}$

Entonces:

$=2a^2\sqrt{x+3y}-3a^2\sqrt{x+3y}+5a^2\sqrt{x+3y}$

$=\left(2a^2-3a^2+5a^2\right)\sqrt{x+3y}=4a^2\sqrt{x+3y}$

18. $3a\sqrt{\frac{a+1}{a^2}}=\quad \frac{3a}{a}\sqrt{a+1}=3\sqrt{a+1}$

$-\sqrt{4a+4}=\quad -\sqrt{2^2\left(a+1\right)}=-2\sqrt{a+1}$

$\left(a+1\right)\sqrt{\frac{1}{a+1}}=\left(a+1\right)\sqrt{\frac{a+1}{\left(a+1\right)^2}}=\sqrt{a+1}$

Entonces:

$=3\sqrt{a+1}-2\sqrt{a+1}+\sqrt{a+1}=\left(3-2+1\right)\sqrt{a+1}=2\sqrt{a+1}$

19. $\left(a-b\right)\sqrt{\frac{a+b}{a-b}}=\quad \left(a-b\right)\sqrt{\frac{a^2-b^2}{\left(a-b\right)^2}}=\sqrt{a^2-b^2}$

$-\left(a+b\right)\sqrt{\frac{a-b}{a+b}}=-\left(a+b\right)\sqrt{\frac{a^2-b^2}{\left(a+b\right)^2}}=-\sqrt{a^2-b^2}$

$\left(2a-2b\right)\sqrt{\frac{1}{a-b}}=2\left(a-b\right)\sqrt{\frac{a-b}{\left(a-b\right)^2}}=2\sqrt{a-b}$

entonces:

$=\sqrt{a^2-b^2}-\sqrt{a^2-b^2}+2\sqrt{a-b}=2\sqrt{a-b}$

EJERCICIO 239

1. $\sqrt[3]{54}=\quad \sqrt[3]{3^3\cdot 2}=\sqrt[3]{2}$

$-\sqrt[3]{24}=-\sqrt[3]{2^3\cdot 3}=-2\sqrt[3]{3}$

$-\sqrt[3]{16}=-\sqrt[3]{2^3\cdot 2}=-2\sqrt[3]{2}$

Entonces:

$=3\sqrt[3]{2}-2\sqrt[3]{2}-2\sqrt[3]{3}=\sqrt[3]{2}-2\sqrt[3]{3}$

2. $\sqrt[3]{40}=\quad \sqrt[3]{2^3\cdot 5}=2\sqrt[3]{5}$

$\sqrt[3]{1.029}=\quad \sqrt[3]{7^3\cdot 3}=7\sqrt[3]{3}$

$-\sqrt[3]{625}=-\sqrt[3]{5^3\cdot 5}=-5\sqrt[3]{5}$

Entonces:

$=2\sqrt[3]{5}-5\sqrt[3]{5}+7\sqrt[3]{3}$

$=\left(2-5\right)\sqrt[3]{5}+7\sqrt[3]{3}=7\sqrt[3]{3}-3\sqrt[3]{5}$

3. $2\sqrt[3]{250}=\quad 2\sqrt[3]{5^3\cdot 2}=10\sqrt[3]{2}$

$-4\sqrt[3]{24}=-4\sqrt[3]{2^3\cdot 3}=-8\sqrt[3]{3}$

$-6\sqrt[3]{16}=-6\sqrt[3]{2^3\cdot 2}=-12\sqrt[3]{2}$

$\sqrt[3]{2.187}=\quad \sqrt[3]{3^6\cdot 3}=9\sqrt[3]{3}$

Entonces:

$=10\sqrt[3]{2}-12\sqrt[3]{2}+9\sqrt[3]{3}-8\sqrt[3]{3}$

$=\left(10-12\right)\sqrt[3]{2}+\left(9-8\right)\sqrt[3]{3}=\sqrt[3]{3}-2\sqrt[3]{2}$

4. $5\sqrt[3]{48}=\quad 5\sqrt[3]{2^3\cdot 6}=10\sqrt[3]{6}$

$-3\sqrt[3]{3.645}=-3\sqrt[3]{3^6\cdot 5}=-27\sqrt[3]{5}$

$-2\sqrt[3]{384}=-2\sqrt[3]{2^6\cdot 6}=-8\sqrt[3]{6}$

$4\sqrt[3]{1.715}=\quad 4\sqrt[3]{7^3\cdot 5}=28\sqrt[3]{5}$

Entonces:

$=10\sqrt[3]{6}-8\sqrt[3]{6}-27\sqrt[3]{5}+28\sqrt[3]{5}$

$=\left(10-8\right)\sqrt[3]{6}+\left(-27+28\right)\sqrt[3]{5}=2\sqrt[3]{6}+\sqrt[3]{5}$

5. $\sqrt[3]{81}=\quad \sqrt[3]{3^3\cdot 3}=3\sqrt[3]{3}$

$-3\sqrt[3]{375}=-3\sqrt[3]{5^3\cdot 3}=-15\sqrt[3]{3}$

$\sqrt[3]{686}=\quad \sqrt[3]{7^3\cdot 2}=7\sqrt[3]{2}$

$2\sqrt[3]{648}=2\sqrt[3]{2^3\cdot 3^3\cdot 3}=12\sqrt[3]{3}$

Entonces:

$=3\sqrt[3]{3}+12\sqrt[3]{3}-15\sqrt[3]{3}+7\sqrt[3]{2}=7\sqrt[3]{2}$

6. $\frac{1}{2}\sqrt[3]{24}=\quad \frac{1}{2}\sqrt[3]{2^3\cdot 3}=\sqrt[3]{3}$

$-\frac{2}{3}\sqrt[3]{54}=-\frac{2}{3}\sqrt[3]{3^3\cdot 2}=-2\sqrt[3]{2}$

$\frac{3}{5}\sqrt[3]{375}=\quad \frac{3}{5}\sqrt[3]{5^3\cdot 3}=3\sqrt[3]{3}$

$-\frac{1}{4}\sqrt[3]{128}=-\frac{1}{4}\sqrt[3]{2^6\cdot 2}=-\sqrt[3]{2}$

Entonces:

$=\sqrt[3]{3}+3\sqrt[3]{3}-2\sqrt[3]{2}-\sqrt[3]{2}$

$=\left(1+3\right)\sqrt[3]{3}+\left(-2-1\right)\sqrt[3]{2}=4\sqrt[3]{3}-3\sqrt[3]{2}$

7. $\frac{3}{5}\sqrt[3]{625}=\quad \frac{3}{5}\sqrt[3]{5^3\cdot 5}=3\sqrt[3]{5}$

$-\frac{3}{2}\sqrt[3]{192}=-\frac{3}{2}\sqrt[3]{2^6\cdot 3}=-6\sqrt[3]{3}$

$\frac{1}{7}\sqrt[3]{1.715}=\quad \frac{1}{7}\sqrt[3]{7^3\cdot 5}=\sqrt[3]{5}$

$-\frac{3}{8}\sqrt[3]{1.536}=-\frac{3}{8}\sqrt[3]{2^9\cdot 3}=-3\sqrt[3]{3}$

Entonces:

$=3\sqrt[3]{5}+\sqrt[3]{5}-6\sqrt[3]{3}-3\sqrt[3]{3}$

$=\left(3+1\right)\sqrt[3]{5}+\left(-6-3\right)\sqrt[3]{3}=4\sqrt[3]{5}-9\sqrt[3]{3}$

8. $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{\frac{2}{2^3}} = \frac{1}{2}\sqrt[3]{2}$

$\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{9}{3^3}} = \frac{1}{3}\sqrt[3]{9}$

$-\sqrt[3]{\frac{2}{27}} = -\sqrt[3]{\frac{2}{3^3}} = -\frac{1}{3}\sqrt[3]{2}$

Entonces:

$= \frac{1}{2}\sqrt[3]{2} - \frac{1}{3}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{9}$

$= \left(\frac{1}{2} - \frac{1}{3}\right)\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{9} = \frac{1}{6}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{9}$

9. $6\sqrt[3]{\frac{1}{24}} = 6\sqrt[3]{\frac{9}{2^3 \cdot 3^3}} = \sqrt[3]{9}$

$\sqrt[3]{\frac{1}{25}} = \sqrt[3]{\frac{5}{5^3}} = \frac{1}{5}\sqrt[3]{5}$

$-2\sqrt[3]{\frac{5}{64}} = -2\sqrt[3]{\frac{5}{2^3 \cdot 2^3}} = -\frac{1}{2}\sqrt[3]{5}$

Entonces:

$= \frac{1}{5}\sqrt[3]{2} - \frac{1}{2}\sqrt[3]{5} + \sqrt[3]{9}$

$= \left(\frac{1}{5} - \frac{1}{2}\right)\sqrt[3]{5} + \sqrt[3]{9} = -\frac{3}{10}\sqrt[3]{5} + \sqrt[3]{9}$

10. $7\sqrt[3]{\frac{1}{49}} = 7\sqrt[3]{\frac{7}{7^3}} = \sqrt[3]{7}$

$\sqrt[3]{\frac{1}{16}} = \sqrt[3]{\frac{4}{2^6}} = \frac{1}{4}\sqrt[3]{4}$

$\sqrt[3]{\frac{1}{2}} = \sqrt[3]{\frac{4}{2^3}} = \frac{1}{2}\sqrt[3]{4}$

$-2\sqrt[3]{\frac{7}{8}} = -2\sqrt[3]{\frac{7}{2^3}} = -\sqrt[3]{7}$

Entonces:

$= \frac{1}{4}\sqrt[3]{4} + \frac{1}{2}\sqrt[3]{4} + \sqrt[3]{7} - \sqrt[3]{7} = \left(\frac{1}{4} + \frac{1}{2}\right)\sqrt[3]{4} = \frac{3}{4}\sqrt[3]{4}$

11. $\frac{2}{3}\sqrt[3]{135} = \frac{2}{3}\sqrt[3]{3^3 \cdot 5} = 2\sqrt[3]{5}$

$\frac{1}{2}\sqrt[3]{\frac{1}{32}} = \frac{1}{2}\sqrt[3]{\frac{2}{2^6}} = \frac{1}{8}\sqrt[3]{2}$

$\frac{7}{4}\sqrt[3]{\frac{1}{4}} = \frac{7}{4}\sqrt[3]{\frac{2}{2^3}} = \frac{7}{8}\sqrt[3]{2}$

$-20\sqrt[3]{\frac{1}{200}} = -20\sqrt[3]{\frac{5}{2^3 \cdot 5^3}} = -2\sqrt[3]{5}$

Entonces:

$= 2\sqrt[3]{5} - 2\sqrt[3]{5} + \frac{1}{8}\sqrt[3]{2} + \frac{7}{8}\sqrt[3]{2} = \left(\frac{1}{8} + \frac{7}{8}\right)\sqrt[3]{2} = \sqrt[3]{2}$

12. $3\sqrt[3]{-24} = 3\sqrt[3]{(-2)^3 \cdot 3} = -6\sqrt[3]{3}$

$-4\sqrt[3]{-81} = -4\sqrt[3]{(-3)^3 \cdot 3} = 12\sqrt[3]{3}$

$-\sqrt[3]{-375} = -\sqrt[3]{(-5)^3 \cdot 3} = 5\sqrt[3]{3}$

Entonces:

$= -6\sqrt[3]{3} + 12\sqrt[3]{3} + 5\sqrt[3]{3}$

$= (-6 + 12 + 5)\sqrt[3]{3} = 11\sqrt[3]{3}$

13. $4\sqrt[3]{-320} = 4\sqrt[3]{(2)^6(-5)} = 16\sqrt[3]{-5}$

$-10\sqrt[3]{-40} = -10\sqrt[3]{(2)^3(-5)} = -20\sqrt[3]{-5}$

$-2\sqrt[3]{-54} = -2\sqrt[3]{(3)^3(-2)} = -6\sqrt[3]{-2}$

$3\sqrt[3]{-1.024} = 3\sqrt[3]{(2)^9(-2)} = 24\sqrt[3]{-2}$

Entonces:

$= 16\sqrt[3]{-5} - 20\sqrt[3]{-5} - 6\sqrt[3]{-2} + 24\sqrt[3]{-2}$

$= (16 - 20)\sqrt[3]{-5} + (-6 + 24)\sqrt[3]{-2}$

$= -4\sqrt[3]{-5} + 18\sqrt[3]{-2}$

14. $3\sqrt[3]{2a^3} = 3a\sqrt[3]{2}$

$-b\sqrt[3]{128} = -b\sqrt[3]{2^6 \cdot 2} = -4b\sqrt[3]{2}$

$(4b - 3a)\sqrt[3]{2}$

Entonces:

$= 3a\sqrt[3]{2} - 4b\sqrt[3]{2} + (4b - 3a)\sqrt[3]{2}$

$= (3a - 4b + 4b - 3a)\sqrt[3]{2} = 0$

15. $a\sqrt[3]{250b} = a\sqrt[3]{5^3\ 2b} = 5a\sqrt[3]{2b}$

$-\sqrt[3]{3ab^3} = -b\sqrt[3]{3a}$

$-5\sqrt[3]{2a^3b} = -5a\sqrt[3]{2b}$

$3b\sqrt[3]{3a}$

Entonces:

$5a\sqrt[3]{2b} - 5a\sqrt[3]{2b} - b\sqrt[3]{3a} + 3b\sqrt[3]{3a}$

$= (-b + 3b)\sqrt[3]{3a} = 2b\sqrt[3]{3a}$

EJERCICIO 240

1. $\sqrt{3}\cdot\sqrt{6}=\sqrt{18}=\sqrt{3^2\cdot 2}=3\sqrt{2}$

2. $5\sqrt{21}\cdot 2\sqrt{3}=10\sqrt{63}=10\sqrt{3^2\cdot 7}=30\sqrt{7}$

3. $\frac{1}{2}\sqrt{14}\cdot\frac{2}{7}\sqrt{21}=\frac{1}{7}\sqrt{294}=\frac{1}{7}\sqrt{7^2\cdot 6}=\sqrt{6}$

4. $\sqrt[3]{12}\cdot\sqrt[3]{9}=\sqrt[3]{108}=\sqrt[3]{3^3\cdot 4}=3\sqrt[3]{4}$

5. $\frac{5}{6}\sqrt[3]{15}\cdot 12\sqrt[3]{50}=10\sqrt[3]{750}=10\sqrt[3]{5^3\cdot 6}=50\sqrt[3]{6}$

6. $x\sqrt{2a}\cdot\frac{1}{a}\sqrt{5a}=\frac{x}{a}\sqrt{10a^2}=x\sqrt{10}$

7. $5\sqrt{12}\cdot 3\sqrt{75}=15\sqrt{900}=15\sqrt{2^2\cdot 3^2\cdot 5^2}=450$

8. $\frac{3}{4}\sqrt[3]{9a^2}\cdot 8\sqrt[3]{3ab}=6\sqrt[3]{27a^3b}=6a\sqrt[3]{3^3b}=18a\sqrt[3]{b}$

9. $3\sqrt{6}\cdot\sqrt{14}\cdot 2\sqrt{35}=6\sqrt{2.940}=6\sqrt{2^2\cdot 7^2\cdot 15}=84\sqrt{15}$

10. $\frac{1}{2}\sqrt{21}\cdot\frac{2}{3}\sqrt{42}\cdot\frac{3}{7}\sqrt{22}$

$=\frac{1}{7}\sqrt{19.404}=\frac{1}{7}\sqrt{2^2\cdot 7^2\cdot 3^2\cdot 11}=6\sqrt{11}$

11. $3\sqrt[3]{45}\cdot\frac{1}{6}\sqrt[3]{15}\cdot 4\sqrt[3]{20}=2\sqrt[3]{13.500}=2\sqrt[3]{5^3\cdot 3^3\cdot 4}=30\sqrt[3]{4}$

12. $\frac{5}{6}\sqrt{\frac{7}{8}}\cdot\frac{3}{5}\sqrt{\frac{4}{7}}=\frac{1}{2}\sqrt{\frac{1}{2}}=\frac{1}{2}\sqrt{\frac{2}{2^2}}=\frac{1}{4}\sqrt{2}$

13. $\frac{2}{x}\sqrt{a^2x}\cdot\frac{3}{2}\sqrt{\frac{1}{a^3}}=\frac{3}{x}\sqrt{\frac{x}{a}}=\frac{3}{x}\sqrt{\frac{ax}{a^2}}=\frac{3}{ax}\sqrt{ax}$

14. $\frac{1}{3}\sqrt{\frac{x}{y^2}}\cdot 6\sqrt{\frac{2}{y}}=2\sqrt{\frac{2x}{y^3}}=2\sqrt{\frac{2xy}{y^4}}=\frac{2}{y^2}\sqrt{2xy}$

EJERCICIO 241

1.
$$\begin{array}{l} \sqrt{2}-\sqrt{3} \\ \sqrt{2} \\ \hline =\sqrt{4}-\sqrt{6} \\ =\ 2-\sqrt{6} \end{array}$$

2.
$$\begin{array}{l} 7\sqrt{5}+5\sqrt{3} \\ 2\sqrt{3} \\ \hline =14\sqrt{15}+10\sqrt{9} \\ =14\sqrt{15}+30 \end{array}$$

3.
$$\begin{array}{l} 2\sqrt{3}+\sqrt{5}-5\sqrt{2} \\ 4\sqrt{15} \\ \hline =8\sqrt{45}+4\sqrt{75}-20\sqrt{30} \\ =4\left(2\sqrt{3^2\cdot 5}+\sqrt{5^2\cdot 3}-5\sqrt{30}\right) \\ =4\left(6\sqrt{5}+5\sqrt{3}-5\sqrt{30}\right) \end{array}$$

4.
$$\begin{array}{l} \sqrt{2}-\sqrt{3} \\ \sqrt{2}+2\sqrt{3} \\ \hline \sqrt{4}-\sqrt{6} \\ \qquad +2\sqrt{6}-2\sqrt{9} \\ \hline =\sqrt{4}+\sqrt{6}-2\sqrt{9} \\ =\ 2+\sqrt{6}-6 \\ =\ \sqrt{6}-4 \end{array}$$

5.
$$\begin{array}{l} \sqrt{5}+5\sqrt{3} \\ 2\sqrt{5}+3\sqrt{3} \\ \hline 2\sqrt{25}+10\sqrt{15} \\ \qquad +3\sqrt{15}+15\sqrt{9} \\ \hline =2\sqrt{25}+13\sqrt{15}+15\sqrt{9} \\ =\ 10+13\sqrt{15}+45 \\ =\ 55+13\sqrt{15} \end{array}$$

6.
$$\begin{array}{l} 3\sqrt{7}-2\sqrt{3} \\ 5\sqrt{3}+4\sqrt{7} \\ \hline 15\sqrt{21}-10\sqrt{9} \\ -8\sqrt{21}\qquad +12\sqrt{49} \\ \hline =7\sqrt{21}-10\sqrt{9}+12\sqrt{49} \\ =7\sqrt{21}-30+84 \\ =7\sqrt{21}+54 \end{array}$$

7.
$$\begin{array}{l} \sqrt{a}-2\sqrt{x} \\ 3\sqrt{a}+\sqrt{x} \\ \hline 3\sqrt{a^2}-6\sqrt{ax} \\ \qquad +\sqrt{ax}-2\sqrt{x^2} \\ \hline =3\sqrt{a^2}-5\sqrt{ax}-2\sqrt{x^2} \\ =3a-5\sqrt{ax}-2x \end{array}$$

8.
$$\begin{array}{l} 7\sqrt{5}-11\sqrt{7} \\ 5\sqrt{5}-8\sqrt{7} \\ \hline 35\sqrt{25}-55\sqrt{35} \\ \qquad -56\sqrt{35}+88\sqrt{49} \\ \hline =35\sqrt{25}-111\sqrt{35}+88\sqrt{49} \\ =175-111\sqrt{35}+616 \\ =791-111\sqrt{35} \end{array}$$

9.
$$\begin{array}{l} \sqrt{2}+\sqrt{3}+\sqrt{5} \\ \sqrt{2}-\sqrt{3} \\ \hline \sqrt{4}+\sqrt{6}+\sqrt{10} \\ \qquad -\sqrt{6}\qquad -\sqrt{9}-\sqrt{15} \\ \hline =\sqrt{4}\qquad +\sqrt{10}-\sqrt{9}-\sqrt{15} \\ =\ 2\qquad +\sqrt{10}-3-\sqrt{15} \\ =\sqrt{10}-\sqrt{15}-1 \end{array}$$

10.
$$\begin{array}{l} \sqrt{2}-3\sqrt{3}+\sqrt{5} \\ \sqrt{2}+2\sqrt{3}-\sqrt{5} \\ \hline \sqrt{4}-3\sqrt{6}+\sqrt{10} \\ \qquad +2\sqrt{6}\qquad -6\sqrt{9}+2\sqrt{15} \\ \qquad\qquad -\sqrt{10}\qquad +3\sqrt{15}-\sqrt{25} \\ \hline =\sqrt{4}-\sqrt{6}\qquad -6\sqrt{9}+5\sqrt{15}-\sqrt{25} \\ =\ 2-\sqrt{6}\qquad -18+5\sqrt{15}-5 \\ =5\sqrt{15}-\sqrt{6}-21 \end{array}$$

11.

$$\begin{array}{l} 2\sqrt{3}-\sqrt{6}+\sqrt{5} \\ \sqrt{3}+\sqrt{6}+3\sqrt{5} \\ \hline 2\sqrt{9}-\sqrt{18}+\sqrt{15} \\ \quad +2\sqrt{18}-\sqrt{36}+\sqrt{30} \\ \quad\quad +6\sqrt{15}-3\sqrt{30}+3\sqrt{25} \\ \hline =2\sqrt{9}+\sqrt{3^2\cdot 2}+7\sqrt{15}-\sqrt{36}-2\sqrt{30}+3\sqrt{25} \\ =6+3\sqrt{2}+7\sqrt{15}-6-2\sqrt{30}+15 \\ =7\sqrt{15}+3\sqrt{2}-2\sqrt{30}+15 \end{array}$$

12.

$$\begin{array}{l} \sqrt{a}+\sqrt{a+1} \\ \sqrt{a}+2\sqrt{a+1} \\ \hline \sqrt{a^2}+\sqrt{a^2+a} \\ \quad +2\sqrt{a^2+a}+2\sqrt{(a+1)^2} \\ \hline =\sqrt{a^2}+3\sqrt{a^2+a}+2\sqrt{(a+1)^2} \\ =a+3\sqrt{a^2+a}+2a+2 \\ =3a+3\sqrt{a^2+a}+2 \end{array}$$

13.

$$\begin{array}{l} 2\sqrt{a}-3\sqrt{a-b} \\ 3\sqrt{a}+\sqrt{a-b} \\ \hline 6\sqrt{a^2}-9\sqrt{a^2-ab} \\ \quad 2\sqrt{a^2-ab}-3\sqrt{(a-b)^2} \\ \hline =6\sqrt{a^2}-7\sqrt{a^2-ab}-3\sqrt{(a-b)^2} \\ =6a-7\sqrt{a^2-ab}-3a+3b \\ =3a+3b-7\sqrt{a^2-ab} \end{array}$$

14.

$$\begin{array}{l} \sqrt{1-x^2}+x \\ 2x+\sqrt{1-x^2} \\ \hline 2x\sqrt{1-x^2}+2x^2 \\ \quad x\sqrt{1-x^2}+\sqrt{(1-x^2)^2} \\ \hline =3x\sqrt{1-x^2}+2x^2+1-x^2 \\ =3x\sqrt{1-x^2}+x^2+1 \end{array}$$

15.

$$\begin{array}{l} \sqrt{a+1}+\sqrt{a-1} \\ \sqrt{a+1}+2\sqrt{a-1} \\ \hline \sqrt{(a+1)^2}+\sqrt{a^2-1} \\ \quad +2\sqrt{a^2-1}+2\sqrt{(a-1)^2} \\ \hline =a+1+3\sqrt{a^2-1}+2a-2 \\ =3a+3\sqrt{a^2-1}-1 \end{array}$$

16.

$$\begin{array}{l} 2\sqrt{x+2}-2 \\ \sqrt{x+2}-3 \\ \hline 2\sqrt{(x+2)^2}-2\sqrt{x+2} \\ \quad -6\sqrt{x+2}+6 \\ \hline =2\sqrt{(x+2)^2}-8\sqrt{x+2}+6 \\ =2x+4-8\sqrt{x+2}+6 \\ =2x+10-8\sqrt{x+2} \end{array}$$

17.

$$\begin{array}{l} 3\sqrt{a}-2\sqrt{a+x} \\ 2\sqrt{a}+3\sqrt{a+x} \\ \hline 6\sqrt{a^2}-4\sqrt{a^2+ax} \\ \quad +9\sqrt{a^2+ax}-6\sqrt{(a+x)^2} \\ \hline =6\sqrt{a^2}+5\sqrt{a^2+ax}-6\sqrt{(a+x)^2} \\ =6a+5\sqrt{a^2+ax}-6a-6x \\ =5\sqrt{a^2+ax}-6x \end{array}$$

18.

$$\begin{array}{l} \sqrt{a+x}-\sqrt{a-x} \\ \sqrt{a+x}-2\sqrt{a-x} \\ \hline \sqrt{(a+x)^2}-\sqrt{a^2-x^2} \\ \quad -2\sqrt{a^2-x^2}+2\sqrt{(a-x)^2} \\ \hline =\sqrt{(a+x)^2}-3\sqrt{a^2-x^2}+2\sqrt{(a-x)^2} \\ =a+x-3\sqrt{a^2-x^2}+2a-2x \\ =3a-x-3\sqrt{a^2-x^2} \end{array}$$

EJERCICIO 242

1. $\sqrt{x}=\sqrt[6]{x^3}$

$\sqrt[3]{2x^2}=\sqrt[6]{(2x^2)^2}=\sqrt[6]{4x^4}$

$\sqrt[6]{x^3}\cdot\sqrt[6]{4x^4}=\sqrt[6]{4x^7}$

$=\sqrt[6]{4x^6x}$

$=x\sqrt[6]{4x}$

2. $3\sqrt{2ab}=3\sqrt[4]{(2ab)^2}=3\sqrt[4]{4a^2b^2}$

$3\sqrt[4]{4a^2b^2}\cdot4\sqrt[4]{8a^3}=12\sqrt[4]{32a^5b^2}$

$=12\sqrt[4]{2^4\cdot2a^4ab^2}$

$=24a\sqrt[4]{2ab^2}$

3. $\sqrt[3]{9x^2y}=\sqrt[6]{(9x^2y)^2}$

$=\sqrt[6]{81x^4y^2}$

$\sqrt[6]{81x^4y^2}\cdot\sqrt[6]{81x^5}=\sqrt[6]{6.561x^9y^2}$

$=\sqrt[6]{3^6\cdot3^2x^6x^3y^2}$

$=3x\sqrt[6]{9x^3y^2}$

4. $\sqrt[3]{a^2b^2}=\sqrt[12]{(a^2b^2)^4}$

$=\sqrt[12]{a^8b^8}$

$2\sqrt[4]{3a^3b}=2\sqrt[12]{(3a^3b)^3}$

$=2\sqrt[12]{27a^9b^3}$

$\sqrt[12]{a^8b^8}\cdot2\sqrt[12]{27a^9b^3}=2\sqrt[12]{27a^{17}b^{11}}$

$=2\sqrt[12]{27a^{12}a^5b^{11}}$

$=2a\sqrt[12]{27a^5b^{11}}$

5. $\sqrt[4]{25x^2y^3}=\sqrt[12]{(25x^2y^3)^3}$

$=\sqrt[12]{15.625x^6y^9}$

$\sqrt[6]{125x^2}=\sqrt[12]{(125x^2)^2}$

$=\sqrt[12]{15.625x^4}$

$\sqrt[12]{15.625x^6y^9}\cdot\sqrt[12]{15.625x^4}$

$=\sqrt[12]{244.140.625x^{10}y^9}$

$=\sqrt[12]{5^{12}x^{10}y^9}=5\sqrt[12]{x^{10}y^9}$

6. $\frac{2}{3}\sqrt[3]{4m^2}=\frac{2}{3}\sqrt[15]{(4m^2)^5}$

$=\frac{2}{3}\sqrt[15]{4^5m^{10}}$

$\frac{3}{4}\sqrt[5]{16m^4n}=\frac{3}{4}\sqrt[15]{(4^2m^4n)^3}$

$=\frac{3}{4}\sqrt[15]{4^6m^{12}n^3}$

$\frac{2}{3}\sqrt[15]{4^5m^{10}}\cdot\frac{3}{4}\sqrt[15]{4^6m^{12}n^3}$

$=\frac{1}{2}\sqrt[15]{4^{11}m^{22}n^3}$

$=\frac{1}{2}\sqrt[15]{2^{22}m^{22}n^3}$

$=\frac{1}{2}\sqrt[15]{2^{15}\cdot2^7m^{15}m^7n^3}$

$=m\sqrt[15]{2^7m^7n^3}=m\sqrt[15]{128m^7n^3}$

7. $\sqrt{\frac{1}{2x}}=\sqrt[6]{\left(\frac{1}{2x}\right)^3}=\sqrt[6]{\frac{1}{8x^3}}$

$\sqrt[3]{2}=\sqrt[6]{(x^2)^2}=\sqrt[6]{x^4}$

$\sqrt[6]{\frac{x^{-3}}{8}}\cdot\sqrt[6]{x^4}=\sqrt[6]{\frac{x}{8}}=\sqrt[6]{\frac{8x}{2^6}}$

$=\frac{1}{2}\sqrt[6]{8x}$

8. $\sqrt{2x}=\sqrt[10]{(2x)^5}=\sqrt[10]{2^5x^5}$

$\sqrt[5]{4x}=\sqrt[10]{(2^2x)^2}=\sqrt[10]{2^4x^2}$

$\sqrt[10]{2^5x^5}\cdot\sqrt[10]{2^4x^2}\cdot\sqrt[10]{\frac{1}{16x^2}}$

$=\sqrt[10]{\frac{2^5\cdot2^4x^2x^5}{2^4x^2}}$

$=\sqrt[10]{2^5x^5}=(2x)^{\frac{5}{10}}=(2x)^{\frac{1}{2}}=\sqrt{2x}$

9. $\frac{2}{3}\sqrt{\frac{2b}{a}}=\frac{2}{3}\sqrt[6]{\left(\frac{2b}{a}\right)^3}=\frac{2}{3}\sqrt[6]{\frac{8b^3}{a^3}}$

$\frac{3}{8}\sqrt[3]{\frac{a^2}{4b^2}}=\frac{3}{8}\sqrt[6]{\left(\frac{a^2}{2^2b^2}\right)^2}=\frac{3}{8}\sqrt[6]{\frac{a^4}{2^4b^4}}$

$\frac{2}{3}\sqrt[6]{\frac{2^3b^3}{a^3}}\cdot\frac{3}{8}\sqrt[6]{\frac{a^4}{2^4b^4}}$

$=\frac{1}{4}\sqrt[6]{\frac{a}{2b}}=\frac{1}{4}\sqrt[6]{\frac{2^5ab^5}{2^6b^6}}=\frac{1}{8b}\sqrt{32ab^5}$

10. $\frac{1}{2}\sqrt{\frac{1}{3}}=\frac{1}{2}\sqrt[6]{\left(\frac{1}{3}\right)^3}=\frac{1}{2}\sqrt[6]{\frac{1}{3^3}}$

$\frac{3}{2}\sqrt[3]{\frac{1}{9}}=\frac{3}{2}\sqrt[6]{\left(\frac{1}{9}\right)^2}=\frac{3}{2}\sqrt[6]{\frac{1}{3^4}}$

$\frac{1}{2}\sqrt[6]{\frac{1}{3^3}}\cdot\frac{3}{2}\sqrt[6]{\frac{1}{3^4}}\cdot\sqrt[6]{243}$

$=\frac{3}{4}\sqrt[6]{\frac{3^5}{3^7}}$

$=\frac{3}{4}\sqrt[6]{\frac{1}{9}}$

$=\frac{3}{4}\sqrt[6]{\frac{3^4}{3^6}}$

$=\frac{1}{4}(3)^{\frac{4}{6}}=\frac{1}{4}(3)^{\frac{2}{3}}=\frac{1}{4}\sqrt[3]{3^2}=\frac{1}{4}\sqrt[3]{9}$

EJERCICIO 243

1. $\frac{4\sqrt{6}}{2\sqrt{3}}=2\sqrt{\frac{6}{3}}=2\sqrt{2}$

2. $\frac{2\sqrt{3a}}{10\sqrt{a}}=\frac{1}{5}\sqrt{\frac{3a}{a}}=\frac{1}{5}\sqrt{3}$

3. $\frac{\frac{1}{2}\sqrt{3xy}}{\frac{3}{4}\sqrt{x}}=\frac{4}{6}\sqrt{\frac{3xy}{x}}=\frac{2}{3}\sqrt{3y}$

4. $\frac{\sqrt{75x^2y^3}}{5\sqrt{3xy}}=\frac{1}{5}\sqrt{\frac{75x^2y^3}{3xy}}=\frac{1}{5}\sqrt{25xy^2}=\frac{1}{5}\sqrt{5^2xy^2}=y\sqrt{x}$

5. $\frac{3\sqrt[3]{16a^5}}{4\sqrt[3]{2a^2}}=\frac{3}{4}\sqrt[3]{\frac{16a^5}{2a^2}}=\frac{3}{4}\sqrt[3]{8a^3}=\frac{3}{4}\sqrt[3]{2^3a^3}=\frac{3}{2}a$

6. $\frac{\frac{5}{6}\sqrt{\frac{1}{2}}}{\frac{10}{3}\sqrt{\frac{2}{3}}}=\frac{15}{60}\sqrt{\frac{\frac{1}{2}}{\frac{2}{3}}}=\frac{1}{4}\sqrt{\frac{3}{4}}=\frac{1}{4}\sqrt{\frac{3}{2^2}}=\frac{1}{8}\sqrt{3}$

7. $\frac{4x\sqrt{a^3x^2}}{2\sqrt{a^2x^3}}=2x\sqrt{\frac{a^3x^2}{a^2x^3}}=2x\sqrt{\frac{a}{x}}=2x\sqrt{\frac{ax}{x^2}}=2\sqrt{ax}$

8. $$\frac{\frac{2a}{3}\sqrt[3]{x^2}}{\frac{a}{3x^2}\sqrt[3]{x^3}}=\frac{6ax^2}{3a}\sqrt[3]{\frac{x^2}{x^3}}=2x^2\sqrt[3]{\frac{1}{x}}=2x^2\sqrt[3]{\frac{x^2}{x^3}}=2x\sqrt[3]{x^2}$$

9. $$\frac{\frac{1}{3}\sqrt[3]{\frac{1}{2}}}{\frac{1}{6}\sqrt[3]{\frac{1}{3}}}=\frac{6}{3}\sqrt[3]{\frac{\frac{1}{2}}{\frac{1}{3}}}=2\sqrt[3]{\frac{3}{2}}=2\sqrt[3]{\frac{3\cdot 2^2}{2^3}}=\sqrt[3]{12}$$

EJERCICIO 244

1. $\sqrt[3]{2}=\sqrt[6]{2^2}$

$\sqrt{2}=\sqrt[6]{2^3}$

$$=\frac{\sqrt[6]{2^2}}{\sqrt[6]{2^3}}=\sqrt[6]{\frac{2^2}{2^3}}=\sqrt[6]{\frac{1}{2}}=\sqrt[6]{\frac{2^5}{2^6}}=\frac{1}{2}\sqrt[6]{32}$$

2. $\sqrt{9x}=\sqrt[6]{\left(3^2x\right)^3}=\sqrt[6]{3^6x^3}=3\sqrt[6]{x^3}$

$\sqrt[3]{3x^2}=\sqrt[6]{\left(3x^2\right)^2}=\sqrt[6]{9x^4}$

$$=\frac{3\sqrt[6]{x^3}}{\sqrt[6]{9x^4}}=3\sqrt[6]{\frac{x^3}{9x^4}}=3\sqrt[6]{\frac{1}{9x}}=3\sqrt[6]{\frac{3^4x^5}{3^6x^6}}=\frac{1}{x}\sqrt[6]{81x^5}$$

3. $\sqrt[3]{8a^3b}=\sqrt[12]{\left(2^3a^3b\right)^4}=\sqrt[12]{2^{12}a^{12}b^4}=2a\sqrt[12]{b^4}$

$\sqrt[4]{4a^2}=\sqrt[12]{\left(4a^2\right)^3}=\sqrt[12]{2^6a^6}$

$$=\frac{2a\sqrt[12]{b^4}}{\sqrt[12]{2^6a^6}}$$

$$=2a\sqrt[12]{\frac{2^6a^6b^4}{2^{12}a^{12}}}=\sqrt[12]{2^4\cdot 2^2a^4a^2b^4}$$

$$=(2ab)^{\frac{4}{12}}\cdot(2a)^{\frac{2}{12}}$$

$$=(2ab)^{\frac{1}{3}}\cdot(2a)^{\frac{1}{6}}=\sqrt[3]{2ab}\cdot\sqrt[6]{2a}$$

$$\Rightarrow \sqrt[3]{2ab}=\sqrt[6]{(2ab)^2}=\sqrt[6]{4a^2b^2}$$

$$=\sqrt[6]{4a^2b^2}\cdot\sqrt[6]{2a}=\sqrt[6]{8a^3b^2}$$

4. $\frac{1}{2}\sqrt{2x}=\frac{1}{2}\sqrt[6]{(2x)^3}=\frac{1}{2}\sqrt[6]{2^3x^3}$

$$\frac{\frac{1}{2}\sqrt[6]{2^3x^3}}{\frac{1}{4}\sqrt[6]{2^4x^4}}=2\sqrt[6]{\frac{1}{2x}}=2\sqrt[6]{\frac{2^5x^5}{2^6x^6}}=\frac{1}{x}\sqrt[6]{32x^5}$$

5. $\sqrt[3]{5m^2n}=\sqrt[15]{\left(5m^2n\right)^5}=\sqrt[15]{5^5m^{10}n^5}$

$\sqrt[5]{m^3n^2}=\sqrt[15]{\left(m^3n^2\right)^3}=\sqrt[15]{m^9n^6}$

$$=\frac{\sqrt[15]{5^5m^{10}n^5}}{\sqrt[15]{m^9n^6}}=\sqrt[15]{\frac{5^5m}{n}}=\sqrt[15]{\frac{5^5mn^{14}}{n^{15}}}=\frac{1}{n}\sqrt[15]{3.125mn^{14}}$$

6. $\sqrt[6]{18x^3y^4z^5}=\sqrt[12]{\left(3^2\cdot 2x^3y^4z^5\right)^2}=\sqrt[12]{3^4\cdot 2^2x^6y^8z^{10}}$

$\sqrt[4]{3x^2y^2z^3}=\sqrt[12]{\left(3x^2y^2z^3\right)^3}=\sqrt[12]{3^3x^6y^6z^9}$

$$=\frac{\sqrt[12]{3^4\cdot 2^2x^6y^8z^{10}}}{\sqrt[12]{3^3x^6y^6z^9}}=\sqrt[12]{12y^2z}$$

7. $\sqrt[3]{3m^4}=\sqrt[9]{\left(3m^4\right)^3}=\sqrt[9]{27m^{12}}=m\sqrt[9]{27m^3}$

$$=\frac{m\sqrt[9]{27m^3}}{\sqrt[9]{27m^2}}=m\sqrt[9]{m}$$

8. $\frac{4}{5}\sqrt[3]{4ab}=\frac{4}{5}\sqrt[6]{\left(2^2ab\right)^2}=\frac{4}{5}\sqrt[6]{2^4a^2b^2}$

$\frac{1}{10}\sqrt{2a^2}=\frac{1}{10}\sqrt[6]{\left(2a^2\right)^3}=\frac{1}{10}\sqrt[6]{2^3a^6}$

$$=\frac{\frac{4}{5}\sqrt[6]{2^4a^2b^2}}{\frac{1}{10}\sqrt[6]{2^3a^6}}=8\sqrt[6]{\frac{2b^2}{a^4}}=8\sqrt[6]{\frac{2a^2b^2}{a^6}}=\frac{8}{a}\sqrt[6]{2a^2b^2}$$

EJERCICIO 245

1. $\left(4\sqrt{2}\right)^2=4^2\sqrt{2^2}=16\cdot 2=32$

2. $\left(2\sqrt{3}\right)^2=2^2\sqrt{3^2}=4\cdot 3=12$

3. $\left(5\sqrt{7}\right)^2=5^2\sqrt{7^2}=25\cdot 7=175$

4. $\left(2\sqrt[3]{4}\right)^2=2^2\sqrt[3]{2^4}=4\sqrt[3]{2^3\cdot 2}=8\sqrt[3]{2}$

5. $\left(3\sqrt[3]{2a^2b}\right)^4=3^4\sqrt[3]{\left(2a^2b\right)^4}=81\sqrt[3]{2^3\cdot 2\cdot a^3a^3a^2b^3b}=162a^2b\sqrt{2a^2b}$

6. $\left(\sqrt[4]{8x^3}\right)^2=\sqrt[4]{\left(2^3x^3\right)^2}=\sqrt[4]{2^4\cdot 2^2x^4x^2}=2x\sqrt[4]{2^2x^2}=2x(2x)^{\frac{2}{4}}$

$=2x(2x)^{\frac{1}{2}}=2x\sqrt{2x}$

7. $\left(\sqrt[5]{81ab^3}\right)^3=\sqrt[5]{\left(3^4ab^3\right)^3}=\sqrt[5]{3^5\cdot 3^5\cdot 3^2a^3b^5b^4}=9b\sqrt[5]{9a^3b^4}$

8. $\left(\sqrt[6]{18}\right)^3=\sqrt[6]{18^3}=(18)^{\frac{3}{6}}=(18)^{\frac{1}{2}}=\sqrt{18}=\sqrt{3^2\cdot 2}=3\sqrt{2}$

9. $\left(4a\sqrt{2x}\right)^2=(4a)^2\sqrt{(2x)^2}=16a^2(2x)=32a^2x$

10. $\left(2\sqrt{x+1}\right)^2=2^2\sqrt{(x+1)^2}=4(x+1)=4x+4$

11. $\left(3\sqrt{x-a}\right)^2=3^2\sqrt{(x-a)^2}=9(x-a)=9x-9a$

12. $\left(4\sqrt[6]{9a^3b^4}\right)^3$

$=4^3\sqrt[6]{\left(3^2a^3b^4\right)^3}$

$=64\sqrt[6]{3^6a^6a^3b^6b^6}$

$=192ab^2\sqrt[6]{a^3}$

$=192ab^2(a)^{\frac{3}{6}}=192ab^2(a)^{\frac{1}{2}}=192ab^2\sqrt{a}$

13. $\left(\sqrt{2}-\sqrt{3}\right)^2=\sqrt{2^2}-2\sqrt{2}\sqrt{3}+\sqrt{3^2}$

$=2-2\sqrt{6}+3=5-2\sqrt{6}$

14. $\left(4\sqrt{2}+\sqrt{3}\right)^2=4^2\sqrt{2^2}+2\cdot 4\sqrt{2}\sqrt{3}+\sqrt{3^2}$

$=32+8\sqrt{6}+3=35+8\sqrt{6}$

15. $\left(\sqrt{5}-\sqrt{7}\right)^2=\sqrt{5^2}-2\sqrt{5}\sqrt{7}+\sqrt{7^2}$

$=5-2\sqrt{35}+7=12-2\sqrt{35}$

16. $\left(5\sqrt{7}-6\right)^2=5^2\sqrt{7^2}-2\cdot 5\cdot 6\sqrt{7}+6^2$

$=175-60\sqrt{7}+36=211-60\sqrt{7}$

17. $\left(\sqrt{x}+\sqrt{x-1}\right)^2=\sqrt{x^2}+2\sqrt{x}\sqrt{x-1}+\sqrt{(x-1)^2}$

$=x+2\sqrt{x^2-x}+x-1$

$=2x+2\sqrt{x^2-x}-1$

18. $\left(\sqrt{x+1}-4\sqrt{x}\right)^2$

$=\sqrt{(x+1)^2}-2\cdot 4\sqrt{x+1}\sqrt{x}+4^2\sqrt{x^2}$

$=x+1-8\sqrt{x^2+x}+16x=17x+1-8\sqrt{x^2+x}$

19. $\left(\sqrt{a+1}-\sqrt{a-1}\right)^2$

$=\sqrt{(a+1)^2}-2\sqrt{a+1}\sqrt{a-1}+\sqrt{(a-1)^2}$

$=a+1-2\sqrt{a^2-1}+a-1=2a-2\sqrt{a^2-1}$

20. $\left(2\sqrt{2x-1}+\sqrt{2x+1}\right)^2$

$=2^2\sqrt{(2x-1)^2}+2\cdot 2\sqrt{2x-1}\sqrt{2x+1}+\sqrt{(2x+1)^2}$

$=4(2x-1)+4\sqrt{4x^2-1}+2x+1$

$=8x-4+4\sqrt{4x^2-1}+2x+1$

$=10x-3+4\sqrt{4x^2-1}$

EJERCICIO 246

1. $\sqrt{\sqrt[3]{a^2}}=\sqrt[6]{a^2}=(a)^{\frac{2}{6}}=a^{\frac{1}{3}}=\sqrt[3]{a}$

2. $\sqrt[3]{\sqrt{8}}=\sqrt[6]{8}=(2)^{\frac{3}{6}}=(2)^{\frac{1}{2}}=\sqrt{2}$

3. $\sqrt[4]{\sqrt{81}}=\sqrt[8]{3^4}=(3)^{\frac{4}{8}}=(3)^{\frac{1}{2}}=\sqrt{3}$

4. $\sqrt{\sqrt{3a}}=\sqrt[4]{3a}$

5. $\sqrt{\sqrt[3]{4a^2}}=\sqrt[6]{2^2a^2}=(2a)^{\frac{2}{6}}=(2a)^{\frac{1}{3}}=\sqrt[3]{2a}$

6. $\sqrt[3]{2\sqrt{2}}=\sqrt[3]{\sqrt{2^2\cdot 2}}=\sqrt[6]{2^3}=(2)^{\frac{3}{6}}=(2)^{\frac{1}{2}}=\sqrt{2}$

7. $\sqrt{\sqrt[4]{25a^2}}=\sqrt[8]{5^2a^2}=(5a)^{\frac{2}{8}}=(5a)^{\frac{1}{4}}=\sqrt[4]{5a}$

8. $\sqrt[3]{\sqrt[4]{27a^3}}=\sqrt[12]{3^3a^3}=(3a)^{\frac{3}{12}}=(3a)^{\frac{1}{4}}=\sqrt[4]{3a}$

9. $\sqrt{3\sqrt[5]{3}}=\sqrt{\sqrt[5]{3^5\cdot 3}}=\sqrt[10]{3^6}=(3)^{\frac{6}{10}}=(3)^{\frac{3}{5}}=\sqrt[5]{3^3}=\sqrt[5]{27}$

10. $\sqrt[4]{\sqrt{a^4b^6}}=\sqrt[8]{a^4b^4b^2}=(ab)^{\frac{4}{8}}(b)^{\frac{2}{8}}=(ab)^{\frac{1}{2}}(b)^{\frac{1}{4}}$

$=\sqrt{ab}\cdot\sqrt[4]{b}$

$\Rightarrow\sqrt{ab}=\sqrt[4]{(ab)^2}=\sqrt[4]{a^2b^2}$

$=\sqrt[4]{a^2b^2}\ \sqrt[4]{b}=\sqrt[4]{a^2b^3}$

11. $\sqrt[5]{\sqrt[3]{x^{10}}}=\sqrt[15]{x^{10}}=x^{\frac{10}{15}}=x^{\frac{2}{3}}=\sqrt[3]{x^2}$

12. $\sqrt{\sqrt[3]{(a+b)^2}}=\sqrt[6]{(a+b)^2}=(a+b)^{\frac{2}{6}}=(a+b)^{\frac{1}{3}}=\sqrt[3]{a+b}$

EJERCICIO 247

1. $\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{3^2}}=\frac{\sqrt{3}}{3}$

2. $\frac{5}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{5\sqrt{2}}{\sqrt{2^2}}=\frac{5\sqrt{2}}{2}$

3. $\frac{3}{4\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{4\sqrt{5^2}}=\frac{3\sqrt{5}}{4\cdot 5}=\frac{3\sqrt{5}}{20}$

4. $\frac{2a}{\sqrt{2ax}}\cdot\frac{\sqrt{2ax}}{\sqrt{2ax}}=\frac{2a\sqrt{2ax}}{\sqrt{2^2a^2x^2}}=\frac{2a\sqrt{2ax}}{2ax}=\frac{\sqrt{2ax}}{x}$

5. $\frac{5}{\sqrt[3]{4a^2}}\cdot\frac{\sqrt[3]{2a}}{\sqrt[3]{2a}}=\frac{5\sqrt[3]{2a}}{\sqrt[3]{2^3a^3}}=\frac{5\sqrt[3]{2a}}{2a}$

6. $\frac{1}{\sqrt[3]{9x}}\cdot\frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}}=\frac{\sqrt[3]{3x^2}}{\sqrt[3]{3^3x^3}}=\frac{\sqrt[3]{3x^2}}{3x}$

7. $\frac{3}{\sqrt[4]{9a}}\cdot\frac{\sqrt[4]{3^2a^3}}{\sqrt[4]{3^2a^3}}=\frac{3\sqrt[4]{3^2a^3}}{\sqrt[4]{3^4a^4}}=\frac{3\sqrt[4]{9a^3}}{3a}=\frac{\sqrt[4]{9a^3}}{a}$

8. $\frac{6}{5\sqrt[3]{3x}}\cdot\frac{\sqrt[3]{3^2x^2}}{\sqrt[3]{3^2x^2}}=\frac{6\sqrt[3]{3^2x^2}}{5\sqrt[3]{3^3x^3}}=\frac{6\sqrt[3]{9x^2}}{15x}=\frac{2\sqrt[3]{9x^2}}{5x}$

9. $\frac{x}{\sqrt[4]{27x^2}}\cdot\frac{\sqrt[4]{3x^2}}{\sqrt[4]{3x^2}}=\frac{x\sqrt[4]{3x^2}}{\sqrt[4]{3^4x^4}}=\frac{x\sqrt[4]{3x^2}}{3x}=\frac{\sqrt[4]{3x^2}}{3}$

10. $\frac{1}{\sqrt[5]{8a^4}}\cdot\frac{\sqrt[5]{2^2a}}{\sqrt[5]{2^2a}}=\frac{\sqrt[5]{4a}}{\sqrt[5]{2^5a^5}}=\frac{\sqrt[5]{4a}}{2a}$

11. $\frac{5n^2}{3\sqrt{mn}}\cdot\frac{\sqrt{mn}}{\sqrt{mn}}=\frac{5n^2\sqrt{mn}}{3\sqrt{m^2n^2}}=\frac{5n^2\sqrt{mn}}{3mn}=\frac{5n\sqrt{mn}}{3m}$

12. $\frac{1}{5a\sqrt[4]{25x^3}}\cdot\frac{\sqrt[4]{5^2x}}{\sqrt[4]{5^2x}}=\frac{\sqrt[4]{25x}}{5a\sqrt[4]{5^4x^4}}=\frac{\sqrt[4]{25x}}{25ax}$

EJERCICIO 248

1. $\frac{3-\sqrt{2}}{1+\sqrt{2}}\cdot\frac{1-\sqrt{2}}{1-\sqrt{2}}=\frac{3-3\sqrt{2}-\sqrt{2}+\sqrt{2^2}}{1-\sqrt{2^2}}=\frac{3-4\sqrt{2}+2}{1-2}=\frac{5-4\sqrt{2}}{-1}=4\sqrt{2}-5$

2. $\frac{5+2\sqrt{3}}{4-\sqrt{3}}\cdot\frac{4+\sqrt{3}}{4+\sqrt{3}}=\frac{20+5\sqrt{3}+8\sqrt{3}+2\sqrt{3^2}}{4^2-\sqrt{3^2}}=\frac{20+13\sqrt{3}+6}{16-3}=\frac{26+13\sqrt{3}}{13}=2+\sqrt{3}$

3. $\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}}\cdot\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}=\frac{\sqrt{2^2}-2\sqrt{2}\sqrt{5}+\sqrt{5^2}}{\sqrt{2^2}-\sqrt{5^2}}=\frac{2-2\sqrt{10}+5}{2-5}=\frac{7-2\sqrt{10}}{-3}=\frac{2\sqrt{10}-7}{3}$

4. $\frac{\sqrt{7}+2\sqrt{5}}{\sqrt{7}-\sqrt{5}}\cdot\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{\sqrt{7^2}+\sqrt{35}+2\sqrt{35}+2\sqrt{5^2}}{\sqrt{7^2}-\sqrt{5^2}}=\frac{7+3\sqrt{35}+10}{7-5}=\frac{17+3\sqrt{35}}{2}$

5. $\frac{\sqrt{2}-3\sqrt{5}}{2\sqrt{2}+\sqrt{5}}\cdot\frac{2\sqrt{2}-\sqrt{5}}{2\sqrt{2}-\sqrt{5}}=\frac{2\sqrt{2^2}-\sqrt{10}-6\sqrt{10}+3\sqrt{5^2}}{2^2\sqrt{2^2}-\sqrt{5^2}}=\frac{4-7\sqrt{10}+15}{8-5}=\frac{19-7\sqrt{10}}{3}$

6. $\frac{19}{5\sqrt{2}-4\sqrt{3}}\cdot\frac{5\sqrt{2}+4\sqrt{3}}{5\sqrt{2}+4\sqrt{3}}=\frac{95\sqrt{2}+76\sqrt{3}}{5^2\sqrt{2^2}-4^2\sqrt{3^2}}=\frac{95\sqrt{2}+76\sqrt{3}}{50-48}=\frac{95\sqrt{2}+76\sqrt{3}}{2}$

7. $\frac{3\sqrt{2}}{7\sqrt{2}-6\sqrt{3}}\cdot\frac{7\sqrt{2}+6\sqrt{3}}{7\sqrt{2}+6\sqrt{3}}=\frac{21\sqrt{2^2}+18\sqrt{6}}{7^2\sqrt{2^2}-6^2\sqrt{3^2}}=\frac{42+18\sqrt{6}}{98-108}=\frac{42+18\sqrt{6}}{-10}=-\frac{21+9\sqrt{6}}{5}$

8. $\frac{4\sqrt{3}-3\sqrt{7}}{2\sqrt{3}+3\sqrt{7}}\cdot\frac{2\sqrt{3}-3\sqrt{7}}{2\sqrt{3}-3\sqrt{7}}=\frac{8\sqrt{3^2}-12\sqrt{21}-6\sqrt{21}+9\sqrt{7^2}}{2^2\sqrt{3^2}-3^2\sqrt{7^2}}=\frac{24-18\sqrt{21}+63}{12-63}=\frac{87-18\sqrt{21}}{-51}=\frac{6\sqrt{21}-29}{17}$

9. $\frac{5\sqrt{2}-6\sqrt{3}}{4\sqrt{2}-3\sqrt{3}}\cdot\frac{4\sqrt{2}+3\sqrt{3}}{4\sqrt{2}+3\sqrt{3}}=\frac{20\sqrt{2^2}+15\sqrt{6}-24\sqrt{6}-18\sqrt{3^2}}{4^2\sqrt{2^2}-3^2\sqrt{3^2}}=\frac{40-9\sqrt{6}-54}{32-27}=\frac{-14-9\sqrt{6}}{5}=-\frac{14+9\sqrt{6}}{5}$

10. $\frac{\sqrt{7}+3\sqrt{11}}{5\sqrt{7}+4\sqrt{11}}\cdot\frac{5\sqrt{7}-4\sqrt{11}}{5\sqrt{7}-4\sqrt{11}}=\frac{5\sqrt{7^2}-4\sqrt{77}+15\sqrt{77}-12\sqrt{11^2}}{5^2\sqrt{7^2}-4^2\sqrt{11^2}}=\frac{35+11\sqrt{77}-132}{175-176}=\frac{-97+11\sqrt{77}}{-1}=97-11\sqrt{77}$

11. $\frac{\sqrt{5}+\sqrt{2}}{7+2\sqrt{10}}\cdot\frac{7-2\sqrt{10}}{7-2\sqrt{10}}=\frac{7\sqrt{5}-2\sqrt{50}+7\sqrt{2}-2\sqrt{20}}{7^2-2^2\sqrt{10^2}}$

$=\frac{7\sqrt{5}-2\sqrt{5^2\cdot 2}+7\sqrt{2}-2\sqrt{2^2\cdot 5}}{49-40}=\frac{7\sqrt{5}-10\sqrt{2}+7\sqrt{2}-4\sqrt{5}}{9}=\frac{3\sqrt{5}-3\sqrt{2}}{9}=\frac{\sqrt{5}-\sqrt{2}}{3}$

12. $\frac{9\sqrt{3}-3\sqrt{2}}{6-\sqrt{6}}\cdot\frac{6+\sqrt{6}}{6+\sqrt{6}}=\frac{54\sqrt{3}+9\sqrt{18}-18\sqrt{2}-3\sqrt{12}}{6^2-\sqrt{6^2}}$

$=\frac{54\sqrt{3}+9\sqrt{3^2\cdot 2}-18\sqrt{2}-3\sqrt{2^2\cdot 3}}{36-6}=\frac{54\sqrt{3}+27\sqrt{2}-18\sqrt{2}-6\sqrt{3}}{30}=\frac{48\sqrt{3}+9\sqrt{2}}{30}=\frac{16\sqrt{3}+3\sqrt{2}}{10}$

13. $\dfrac{\sqrt{a}+\sqrt{x}}{2\sqrt{a}+\sqrt{x}}\cdot\dfrac{2\sqrt{a}-\sqrt{x}}{2\sqrt{a}-\sqrt{x}}$

$=\dfrac{2\sqrt{a^2}-\sqrt{ax}+2\sqrt{ax}-\sqrt{x^2}}{2^2\sqrt{a^2}-\sqrt{x^2}}=\dfrac{2a+\sqrt{ax}-x}{4a-x}$

14. $\dfrac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}\cdot\dfrac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}}$

$=\dfrac{\sqrt{x^2}-2\sqrt{x}\sqrt{x-1}+\sqrt{(x-1)^2}}{\sqrt{x^2}-\sqrt{(x-1)^2}}$

$=\dfrac{x-2\sqrt{x^2-x}+x-1}{x-(x-1)}=2x-1-2\sqrt{x^2-x}$

15. $\dfrac{\sqrt{a}-\sqrt{a+1}}{\sqrt{a}+\sqrt{a+1}}\cdot\dfrac{\sqrt{a}-\sqrt{a+1}}{\sqrt{a}-\sqrt{a+1}}$

$=\dfrac{\sqrt{a^2}-2\sqrt{a}\sqrt{a+1}+\sqrt{(a+1)^2}}{\sqrt{a^2}-\sqrt{(a+1)^2}}$

$=\dfrac{a-2\sqrt{a^2+a}+a+1}{a-(a+1)}$

$=\dfrac{2a+1-2\sqrt{a^2+a}}{-1}=2\sqrt{a^2+a}-2a-1$

16. $\dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}-\sqrt{2}}\cdot\dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$=\dfrac{\sqrt{(x+2)^2}+2\sqrt{x+2}\sqrt{2}+\sqrt{2^2}}{\sqrt{(x+2)^2}-\sqrt{2^2}}$

$=\dfrac{x+2+2\sqrt{2x+4}+2}{x+2-2}=\dfrac{x+4+2\sqrt{2x+4}}{x}$

17. $\dfrac{\sqrt{a+4}-\sqrt{a}}{\sqrt{a+4}+\sqrt{a}}\cdot\dfrac{\sqrt{a+4}-\sqrt{a}}{\sqrt{a+4}-\sqrt{a}}$

$=\dfrac{\sqrt{(a+4)^2}-2\sqrt{a+4}\sqrt{a}+\sqrt{a^2}}{\sqrt{(a+4)^2}-\sqrt{a^2}}$

$=\dfrac{a+4-2\sqrt{a^2+4a}+a}{a+4-a}=\dfrac{2a+4-2\sqrt{a^2+4a}}{4}$

$=\dfrac{a+2-\sqrt{a^2+4a}}{2}$

18. $\dfrac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}\cdot\dfrac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}-\sqrt{a-b}}$

$=\dfrac{\sqrt{(a+b)^2}-2\sqrt{a+b}\sqrt{a-b}+\sqrt{(a-b)^2}}{\sqrt{(a+b)^2}-\sqrt{(a-b)^2}}$

$=\dfrac{a+b-2\sqrt{a^2-b^2}+a-b}{a+b-a+b}=\dfrac{2a-2\sqrt{a^2-b^2}}{2b}=\dfrac{a-\sqrt{a^2-b^2}}{b}$

EJERCICIO 249

1. $\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}\cdot\dfrac{(\sqrt{2}+\sqrt{3})+\sqrt{5}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}$

$=\dfrac{\sqrt{6}+\sqrt{9}+\sqrt{15}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$=\dfrac{3+\sqrt{6}+\sqrt{15}}{2+2\sqrt{6}+3-5}=\dfrac{3+\sqrt{6}+\sqrt{15}}{2\sqrt{6}}\cdot\dfrac{2\sqrt{6}}{2\sqrt{6}}=\dfrac{6\sqrt{6}+2\sqrt{36}+2\sqrt{90}}{(2\sqrt{6})^2}=\dfrac{6\sqrt{6}+12+6\sqrt{10}}{24}=\dfrac{6(\sqrt{6}+2+\sqrt{10})}{24}=\dfrac{2+\sqrt{6}+\sqrt{10}}{4}$

2. $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{3}+\sqrt{6}}\cdot\dfrac{(\sqrt{2}+\sqrt{3})-\sqrt{6}}{(\sqrt{2}+\sqrt{3})-\sqrt{6}}$

$=\dfrac{2+\sqrt{6}-\sqrt{12}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{6})^2}$

$=\dfrac{2+\sqrt{6}-2\sqrt{3}}{2+2\sqrt{6}+3-6}=\dfrac{2+\sqrt{6}-2\sqrt{3}}{2\sqrt{6}-1}\cdot\dfrac{2\sqrt{6}+1}{2\sqrt{6}+1}=\dfrac{4\sqrt{6}+2+12+\sqrt{6}-4\sqrt{18}-2\sqrt{3}}{(2\sqrt{6})^2-1}=\dfrac{5\sqrt{6}+14-12\sqrt{2}-2\sqrt{3}}{23}$

3. $\frac{2-\sqrt{3}}{2+\sqrt{3}+\sqrt{5}}\cdot\frac{\left(2+\sqrt{3}\right)-\sqrt{5}}{\left(2+\sqrt{3}\right)-\sqrt{5}}$

$=\frac{4-3-2\sqrt{5}+\sqrt{15}}{\left(2+\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$

$=\frac{1-2\sqrt{5}+\sqrt{15}}{4+4\sqrt{3}+3-5}$

$=\frac{1-2\sqrt{5}+\sqrt{15}}{2+4\sqrt{3}}\cdot\frac{2-4\sqrt{3}}{2-4\sqrt{3}}$

$=\frac{2-4\sqrt{3}-4\sqrt{5}+8\sqrt{15}+2\sqrt{15}-4\sqrt{45}}{2^2-\left(4\sqrt{3}\right)^2}$

$=\frac{2-4\sqrt{3}-4\sqrt{5}+10\sqrt{15}-12\sqrt{5}}{-44}$

$=\frac{2\left(2\sqrt{3}+8\sqrt{5}-5\sqrt{15}-1\right)}{44}=\frac{2\sqrt{3}+8\sqrt{5}-5\sqrt{15}-1}{22}$

4. $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\cdot\frac{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}}{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}}$

$=\frac{\sqrt{6}+\sqrt{10}+3-5}{\left(\sqrt{2}+\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$

$=\frac{\sqrt{10}+\sqrt{6}-2}{2+2\sqrt{6}+3-5}$

$=\frac{\sqrt{10}+\sqrt{6}-2}{2\sqrt{6}}\cdot\frac{2\sqrt{6}}{2\sqrt{6}}$

$=\frac{2\sqrt{60}+2\sqrt{36}-4\sqrt{6}}{\left(2\sqrt{6}\right)^2}$

$=\frac{4\sqrt{15}+12-4\sqrt{6}}{24}$

$=\frac{4\left(\sqrt{15}+3-\sqrt{6}\right)}{24}=\frac{\sqrt{15}+3-\sqrt{6}}{6}$

5. $\frac{\sqrt{6}+\sqrt{3}+\sqrt{2}}{\sqrt{6}+\sqrt{3}-\sqrt{2}}\cdot\frac{\left(\sqrt{6}+\sqrt{3}\right)+\sqrt{2}}{\left(\sqrt{6}+\sqrt{3}\right)+\sqrt{2}}$

$=\frac{6+3+2+2\sqrt{18}+2\sqrt{12}+2\sqrt{6}}{\left(\sqrt{6}+\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$

$=\frac{11+6\sqrt{2}+4\sqrt{3}+2\sqrt{6}}{6+2\sqrt{18}+3-2}$

$=\frac{11+6\sqrt{2}+4\sqrt{3}+2\sqrt{6}}{2\sqrt{18}+7}\cdot\frac{2\sqrt{18}-7}{2\sqrt{18}-7}$

$=\frac{22\sqrt{18}-77+12\sqrt{36}-42\sqrt{2}+8\sqrt{54}-28\sqrt{3}+4\sqrt{108}-14\sqrt{6}}{\left(2\sqrt{18}\right)^2-49}$

$=\frac{66\sqrt{2}-77+72-42\sqrt{2}+24\sqrt{6}-28\sqrt{3}+24\sqrt{3}-14\sqrt{6}}{72-49}$

$=\frac{24\sqrt{2}-4\sqrt{3}+10\sqrt{6}-5}{23}$

6. $\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}-\sqrt{10}}\cdot\frac{\left(\sqrt{2}+\sqrt{5}\right)+\sqrt{10}}{\left(\sqrt{2}+\sqrt{5}\right)+\sqrt{10}}$

$=\frac{2-5+\sqrt{20}-\sqrt{50}}{2+2\sqrt{10}+5-10}$

$=\frac{2\sqrt{5}-5\sqrt{2}-3}{2\sqrt{10}-3}\cdot\frac{2\sqrt{10}+3}{2\sqrt{10}+3}$

$=\frac{4\sqrt{50}+6\sqrt{5}-10\sqrt{20}-15\sqrt{2}-6\sqrt{10-9}}{40-9}$

$=\frac{20\sqrt{2}+6\sqrt{5}-20\sqrt{5}-15\sqrt{2}-6\sqrt{10}-9}{31}$

$=\frac{5\sqrt{2}-14\sqrt{5}-6\sqrt{10}-9}{31}$

EJERCICIO 250

1. $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}\cdot\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}=\frac{2-\sqrt{6}}{2-3}=\frac{2-\sqrt{6}}{-1}=\sqrt{6}-2$

2. $\frac{\sqrt{3}}{\sqrt{3}-2\sqrt{5}}\cdot\frac{\sqrt{3}+2\sqrt{5}}{\sqrt{3}+2\sqrt{5}}$

$=\frac{3+2\sqrt{15}}{3-20}=\frac{3+2\sqrt{15}}{-17}=-\frac{3+2\sqrt{15}}{17}$

3. $\frac{2+\sqrt{5}}{1-\sqrt{5}}\cdot\frac{1+\sqrt{5}}{1+\sqrt{5}}$

$=\frac{2+2\sqrt{5}+\sqrt{5}+5}{1-5}=\frac{7+3\sqrt{5}}{-4}=-\frac{7+3\sqrt{5}}{4}$

4. $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}-\sqrt{5}}\cdot\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}$

$=\frac{2+2\sqrt{10}+5}{2-5}=\frac{7+2\sqrt{10}}{-3}=-\frac{7+2\sqrt{10}}{3}$

5. $\dfrac{2\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}\cdot\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{3}-\sqrt{7}}$

$=\dfrac{6-2\sqrt{21}-\sqrt{21}+7}{3-7}=\dfrac{13-3\sqrt{21}}{-4}=\dfrac{3\sqrt{21}-13}{4}$

6. $\dfrac{\sqrt{6}+2\sqrt{5}}{2\sqrt{6}-\sqrt{5}}\cdot\dfrac{2\sqrt{6}+\sqrt{5}}{2\sqrt{6}+\sqrt{5}}=\dfrac{12+5\sqrt{30}+10}{24-5}=\dfrac{22+5\sqrt{30}}{19}$

7. $\dfrac{5\sqrt{2}+3\sqrt{3}}{3\sqrt{2}-4\sqrt{3}}\cdot\dfrac{3\sqrt{2}+4\sqrt{3}}{3\sqrt{2}+4\sqrt{3}}$

$=\dfrac{30+29\sqrt{6}+36}{18-48}=\dfrac{66+29\sqrt{6}}{-30}=-\dfrac{66+29\sqrt{6}}{30}$

8. $\dfrac{\sqrt{7}-2\sqrt{11}}{2\sqrt{7}+\sqrt{11}}\cdot\dfrac{2\sqrt{7}-\sqrt{11}}{2\sqrt{7}-\sqrt{11}}=\dfrac{14-5\sqrt{77}+22}{28-11}=\dfrac{36-5\sqrt{77}}{17}$

EJERCICIO 251

1. $\sqrt{x-8}=2$

$\left(\sqrt{x-8}\right)^2=2^2$

$x-8=4$

$x=12$

2. $5-\sqrt{3x+1}=0$

$5=\sqrt{3x+1}$

$5^2=\left(3x+1\right)^2$

$25=3x+1$

$24=3x$

$8=x$

3. $7+\sqrt[3]{5x-2}=9$

$\sqrt[3]{5x-2}=2$

$\left(\sqrt[3]{5x-2}\right)^3=2^3$

$5x-2=8$

$5x=10$

$x=2$

4. $\sqrt{9x^2-5}-3x=-1$

$\sqrt{9x^2-5}=3x-1$

$\left(\sqrt{9x^2-5}\right)^2=\left(3x-1\right)^2$

$9x^2-5=9x^2-6x+1$

$-6=-6x$

$1=x$

5. $\sqrt{x^2-2x+1}=9-x$

$\sqrt{\left(x-1\right)^2}=9-x$

$x-1=9-x$

$2x=10$

$x=5$

6. $15-\sqrt[3]{7x-1}=12$

$3=\sqrt[3]{7x-1}$

$3^3=\left(\sqrt[3]{7x-1}\right)^3$

$27=7x-1$

$28=7x$

$4=x$

7. $\sqrt{x}+\sqrt{x+7}=7$

$\left(\sqrt{x+7}\right)^2=\left(7-\sqrt{x}\right)^2$

$x+7=49-14\sqrt{x}+x$

$7=49-14\sqrt{x}$

$1=7-2\sqrt{x}$

$-6=-2\sqrt{x}$

$3=\sqrt{x}$

$3^2=\left(\sqrt{x}\right)^2$

$9=x$

8. $\sqrt{3x-5}+\sqrt{3x-14}=9$

$\left(\sqrt{3x-5}\right)^2=\left(9-\sqrt{3x-14}\right)^2$

$3x-5=81-18\sqrt{3x-14}+3x-14$

$-72=-18\sqrt{3x-14}$

$4^2=\left(\sqrt{3x-14}\right)^2$

$16=3x-14$

$30=3x$

$10=x$

9. $\sqrt{x+10}-\sqrt{x+19}=-1$

$\left(\sqrt{x+10}\right)^2=\left(\sqrt{x+19}-1\right)^2$

$x+10=x+19-2\sqrt{x+19}+1$

$-10=-2\sqrt{x+19}$

$5=\sqrt{x+19}$

$5^2=\left(\sqrt{x+19}\right)^2$

$25=x+19$

$6=x$

10. $\sqrt{4x-11}=7\sqrt{2x-29}$

$\left(\sqrt{4x-11}\right)^2=\left(7\sqrt{2x-29}\right)^2$

$4x-11=49\left(2x-29\right)$

$4x-11=98x-1.421$

$1.410=94x$

$15=x$

11. $\sqrt{5x-19}-\sqrt{5x}=-1$

$\left(\sqrt{5x-19}\right)^2=\left(\sqrt{5x}-1\right)^2$

$5x-19=5x-2\sqrt{5x}+1$

$-20=-2\sqrt{5x}$

$10=\sqrt{5x}$

$100=5x$

$20=x$

12. $\sqrt{x-2}+5=\sqrt{x+53}$

$\left(\sqrt{x-2}+5\right)^2=\left(\sqrt{x+53}\right)^2$

$x-2+10\sqrt{x-2}+25=x+53$

$10\sqrt{x-2}=30$

$\sqrt{x-2}=3$

$x-2=9$

$x=11$

13. $\sqrt{9x-14}=3\sqrt{x+10}-4$

$$\left(\sqrt{9x-14}\right)^2=\left(3\sqrt{x+10}-4\right)^2$$
$$9x-14=9\left(x+10\right)-24\sqrt{x+10}+16$$
$$-14=90-24\sqrt{x+10}+16$$
$$-120=-24\sqrt{x+10}$$
$$5=\sqrt{x+10}$$
$$25=x+10$$
$$15=x$$

14. $\sqrt{x-16}-\sqrt{x+8}=-4$

$$\left(\sqrt{x-16}\right)^2=\left(\sqrt{x+8}-4\right)^2$$
$$x-16=x+8-8\sqrt{x+8}+16$$
$$-40=-8\sqrt{x+8}$$
$$5=\sqrt{x+8}$$
$$25=x+8$$
$$17=x$$

15. $\sqrt{5x-1}+3=\sqrt{5x+26}$

$$\left(\sqrt{5x-1}+3\right)^2=\left(\sqrt{5x+26}\right)^2$$
$$5x-1+6\sqrt{5x-1}+9=5x+26$$
$$6\sqrt{5x-1}=18$$
$$\sqrt{5x-1}=3$$
$$5x-1=9$$
$$5x=10$$
$$x=2$$

16. $13-\sqrt{13+4x}=2\sqrt{x}$

$$\left(13-2\sqrt{x}\right)^2=\left(\sqrt{13+4x}\right)^2$$
$$169-52\sqrt{x}+4x=13+4x$$
$$-52\sqrt{x}=-156$$
$$\sqrt{x}=3$$
$$x=9$$

17. $\sqrt{x-4}+\sqrt{x+4}=2\sqrt{x-1}$

$$\left(\sqrt{x-4}+\sqrt{x+4}\right)^2=\left(2\sqrt{x-1}\right)^2$$
$$x-4+2\sqrt{x^2-16}+x+4=4\left(x-1\right)$$
$$2x+2\sqrt{x^2-16}=4x-4$$
$$\left(2\sqrt{x^2-16}\right)^2=\left(2x-4\right)^2$$
$$4x^2-64=4x^2-16x+16$$
$$-80=-16x$$
$$5=x$$

18. $\sqrt{9x+7}-\sqrt{x}-\sqrt{16x-7}=0$

$$\left(\sqrt{9x+7}-\sqrt{x}\right)^2=\left(\sqrt{16x-7}\right)^2$$
$$9x+7-2\sqrt{9x^2+7x}+x=16x-7$$
$$-2\sqrt{9x^2+7x}=6x-14$$
$$\left(-2\sqrt{9x^2+7x}\right)^2=\left(6x-14\right)^2$$
$$4\left(9x^2+7x\right)=36x^2-168x+196$$
$$36x^2+28x=36x^2-168x+196$$
$$196x=196$$
$$x=1$$

19. $\sqrt{9x+10}-2\sqrt{x+3}=\sqrt{x-2}$

$$\left(\sqrt{9x+10}-2\sqrt{x+3}\right)^2=\left(\sqrt{x-2}\right)^2$$
$$9x+10-4\sqrt{9x^2+37x+30}+4\left(x+3\right)=x-2$$
$$13x+22-x+2=4\sqrt{9x^2+37x+30}$$
$$12x+24=4\sqrt{9x^2+37x+30}$$
$$3x+6=\sqrt{9x^2+37x+30}$$
$$\left(3x+6\right)^2=\left(\sqrt{9x^2+37x+30}\right)^2$$
$$9x^2+36x+36=9x^2+37x+30$$
$$36-30=37x-36x$$
$$6=x$$

20. $\sqrt{18x-8}-\sqrt{2x-4}-2\sqrt{2x+1}=0$

$$\left(\sqrt{18x-8}\right)^2=\left(\sqrt{2x-4}+2\sqrt{2x+1}\right)^2$$
$$18x-8=2x-4+4\sqrt{4x^2-6x-4}+4\left(2x+1\right)$$
$$18x-8=10x+4\sqrt{4x^2-6x-4}$$
$$8x-8=4\sqrt{4x^2-6x-4}$$
$$2x-2=\sqrt{4x^2-6x-4}$$
$$\left(2x-2\right)^2=\left(\sqrt{4x^2-6x-4}\right)^2$$
$$4x^2-8x+4=4x^2-6x-4$$
$$-2x=-8$$
$$x=4$$

21.

$$\sqrt{8x+9}-\sqrt{18x+34}+\sqrt{2x+7}=0$$

$$\left(\sqrt{8x+9}+\sqrt{2x+7}\right)^2=\left(\sqrt{18x+34}\right)^2$$

$$8x+9+2\sqrt{16x^2+74x+63}+2x+7=18x+34$$

$$10x+16+2\sqrt{16x^2+74x+63}=18x+34$$

$$2\sqrt{16x^2+74x+63}=8x+18$$

$$\sqrt{16x^2+74x+63}=4x+9$$

$$\left(\sqrt{16x^2+74x+63}\right)^2=\left(4x+9\right)^2$$

$$16x^2+74x+63=16x^2+72x+81$$

$$2x=18$$

$$x=9$$

22.

$$\sqrt{x-2}-\sqrt{x-5}=\sqrt{4x-23}$$

$$\left(\sqrt{x-2}-\sqrt{x-5}\right)^2=\left(\sqrt{4x-23}\right)^2$$

$$x-2-2\sqrt{x^2-7x+10}+x-5=4x-23$$

$$2x-7-4x+23=2\sqrt{x^2-7x+10}$$

$$-2x+16=2\sqrt{x^2-7x+10}$$

$$-x+8=\sqrt{x^2-7x+10}$$

$$\left(8-x\right)^2=\left(\sqrt{x^2-7x+10}\right)^2$$

$$64-16x+x^2=x^2-7x\ 10$$

$$54=9x$$

$$6=x$$

23.

$$\sqrt{x+6}-\sqrt{9x+70}=-2\sqrt{x+9}$$

$$\left(\sqrt{x+6}+2\sqrt{x+9}\right)^2=\left(\sqrt{9x+70}\right)^2$$

$$x+6+4\sqrt{x^2+15x+54}+4\left(x+9\right)=9x+70$$

$$5x+42+4\sqrt{x^2+15x+54}=9x+70$$

$$4\sqrt{x^2+15x+54}=4x+28$$

$$\sqrt{x^2+15x+54}=x+7$$

$$\left(\sqrt{x^2+15x+54}\right)^2=\left(x+7\right)^2$$

$$x^2+15x+54=x^2+14x+49$$

$$x=-5$$

24.

$$\sqrt{x-a}+\sqrt{x+a}=\sqrt{4x-2a}$$

$$\left(\sqrt{x-a}+\sqrt{x+a}\right)^2=\left(\sqrt{4x-2a}\right)^2$$

$$x-a+2\sqrt{x^2-a^2}+x+a=4x-2a$$

$$2x+2\sqrt{x^2-a^2}=4x-2a$$

$$2\sqrt{x^2-a^2}=2x-2a$$

$$\sqrt{x^2-a^2}=x-a$$

$$\left(\sqrt{x^2-a^2}\right)^2=\left(x-a\right)^2$$

$$x^2-a^2=x^2-2ax+a^2$$

$$2ax=2a^2$$

$$x=a$$

25.

$$\sqrt{x-4ab}=-2b+\sqrt{x}$$

$$\left(\sqrt{x-4ab}\right)^2=\left(\sqrt{x}+2b\right)^2$$

$$x-4ab=x-4b\sqrt{x}+4b^2$$

$$4b\sqrt{x}=4b^2+4ab$$

$$\sqrt{x}=b+a$$

$$\left(\sqrt{x}\right)^2=\left(a+b\right)^2$$

$$x=\left(a+b\right)^2$$

26.

$$\sqrt{x+4a}-\sqrt{x+2a-1}=1$$

$$\left(\sqrt{x+4a}-1\right)^2=\left(\sqrt{x+2a-1}\right)^2$$

$$x+4a-2\sqrt{x+4a}+1=x+2a-1$$

$$2a+2=2\sqrt{x+4a}$$

$$a+1=\sqrt{x+4a}$$

$$\left(a+1\right)^2=\left(\sqrt{x+4a}\right)^2$$

$$a^2+2a+1=x+4a$$

$$a^2-2a+1=x$$

$$\left(a-1\right)^2=x$$

EJERCICIO 252

1. $\sqrt{x}+\sqrt{x+5}=\frac{10}{\sqrt{x}}$

$\sqrt{x^2}+\sqrt{x^2+5x}=10$

$\left(\sqrt{x^2+5x}\right)^2=\left(10-\sqrt{x^2}\right)^2$

$x^2+5x=100-20x+x^2$

$25x=100$

$x=4$

2. $\sqrt{4x-11}+2\sqrt{x}=\frac{55}{\sqrt{4x-11}}$

$\sqrt{(4x-11)^2}+2\sqrt{x(4x-11)}=55$

$4x-11+2\sqrt{4x^2-11x}=55$

$2\sqrt{4x^2-11x}=66-4x$

$\sqrt{4x^2-11x}=33-2x$

$\left(\sqrt{4x^2-11x}\right)^2=(33-2x)^2$

$4x^2-11x=1.089-132x+4x^2$

$121x=1.089$

$x=9$

3. $\sqrt{x}-\sqrt{x-7}=\frac{4}{\sqrt{x}}$

$\sqrt{x^2}-\sqrt{x^2-7x}=4$

$(x-4)^2=\left(\sqrt{x^2-7x}\right)^2$

$x^2-8x+16=x^2-7x$

$16=x$

4. $\frac{\sqrt{x}-2}{\sqrt{x}+4}=\frac{\sqrt{x}+1}{\sqrt{x}+13}$

$\left(\sqrt{x}-2\right)\left(\sqrt{x}+13\right)=\left(\sqrt{x}+1\right)\left(\sqrt{x}+4\right)$

$\sqrt{x^2}+11\sqrt{x}-26=\sqrt{x^2}+5\sqrt{x}+4$

$6\sqrt{x}=30$

$\sqrt{x}=5$

$\left(\sqrt{x}\right)^2=5^2$

$x=25$

5. $\frac{6}{\sqrt{x+8}}=\sqrt{x+8}-\sqrt{x}$

$6=\sqrt{(x+8)^2}-\sqrt{x(x+8)}$

$6=x+8-\sqrt{x^2+8x}$

$\sqrt{x^2+8x}=x+2$

$\left(\sqrt{x^2+8x}\right)^2=(x+2)^2$

$x^2+8x=x^2+4x+4$

$4x=4$

$x=1$

6. $\sqrt{x-3}+\frac{8}{\sqrt{x+9}}=\sqrt{x+9}$

$\sqrt{(x-3)(x+9)}+8=\sqrt{(x+9)^2}$

$\sqrt{x^2+6x-27}+8=x+9$

$\left(\sqrt{x^2+6x-27}\right)^2=(x+1)^2$

$x^2+6x-27=x^2+2x+1$

$4x=28$

$x=7$

7. $\frac{\sqrt{x}+4}{\sqrt{x}-2}=\frac{\sqrt{x}+11}{\sqrt{x}-1}$

$\left(\sqrt{x}+4\right)\left(\sqrt{x}-1\right)=\left(\sqrt{x}+11\right)\left(\sqrt{x}-2\right)$

$\sqrt{x^2}+3\sqrt{x}-4=\sqrt{x^2}+9\sqrt{x}-22$

$18=6\sqrt{x}$

$3=\sqrt{x}$

$(3)^2=\left(\sqrt{x}\right)^2$

$9=x$

8. $2\sqrt{x+6}-\sqrt{4x-3}=\frac{9}{\sqrt{4x-3}}$

.

$2\sqrt{(x+6)(4x-3)}-\sqrt{(4x-3)^2}=9$

$2\sqrt{4x^2+21x-18}-(4x-3)=9$

$2\sqrt{4x^2+21x-18}-4x+3=9$

$\left(2\sqrt{4x^2+21x-18}\right)^2=(4x+6)^2$

$4\left(4x^2+21x-18\right)=16x^2+48x+36$

$16x^2+84x-72=16x^2+48x+36$

$36x=108$

$x=3$

9.

$$\frac{\sqrt{x}-2}{\sqrt{x}+2}=\frac{2\sqrt{x}-5}{2\sqrt{x}-1}$$

$$\left(\sqrt{x}-2\right)\left(2\sqrt{x}-1\right)=\left(2\sqrt{x}-5\right)\left(\sqrt{x}+2\right)$$

$$2\sqrt{x^2}-5\sqrt{x}+2=2\sqrt{x^2}-\sqrt{x}-10$$

$$12=4\sqrt{x}$$

$$3=\sqrt{x}$$

$$(3)^2=\left(\sqrt{x}\right)^2$$

$$9=x$$

10.

$$\sqrt{x+14}-\sqrt{x-7}=\frac{6}{\sqrt{x-7}}$$

$$\sqrt{(x+14)(x-7)}-\sqrt{(x-7)^2}=6$$

$$\sqrt{x^2+7x-98}-(x-7)=6$$

$$\sqrt{x^2+7x-98}-x+7=6$$

$$\left(\sqrt{x^2+7x-98}\right)^2=(x-1)^2$$

$$x^2+7x-98=x^2-2x+1$$

$$9x=99$$

$$x=11$$

EJERCICIO 253

1. $\sqrt{-a^2}=\sqrt{a^2(-1)}=a\sqrt{-1}=ai$

2. $\sqrt{-2}=\sqrt{2(-1)}=\sqrt{2}\sqrt{-1}=\sqrt{2}\,i$

3. $2\sqrt{-9}=2\sqrt{3^2(-1)}=6\sqrt{-1}=6i$

4. $\sqrt{-81}=\sqrt{9^2(-1)}=9\sqrt{-1}=9i$

5. $\sqrt{-6}=\sqrt{6(-1)}=\sqrt{6}\sqrt{-1}=\sqrt{6}\,i$

6. $3\sqrt{-b^4}=3\sqrt{b^2b^2(-1)}=3b^2\sqrt{(-1)}=3b^2\,i$

7. $\sqrt{-12}=\sqrt{2^2\cdot 3(-1)}=2\sqrt{3}\sqrt{-1}=2i\sqrt{3}$

8. $\sqrt{-7}=\sqrt{7(-1)}=\sqrt{7}\sqrt{(-1)}=\sqrt{7}\,i$

9. $\sqrt{-27}=\sqrt{3^2\cdot 3(-1)}=3\sqrt{3}\sqrt{-1}=3i\sqrt{3}$

10. $\sqrt{-4m^4}=\sqrt{2^2m^2m^2(-1)}=2m^2\sqrt{-1}=2im^2$

11. $\sqrt{-\frac{1}{16}}=\sqrt{\frac{1}{4^2}(-1)}=\frac{1}{4}\sqrt{-1}=\frac{1}{4}i$

12. $\sqrt{-a^2-b^2}=\sqrt{-1\left(a^2+b^2\right)}=\sqrt{-1}\sqrt{a^2+b^2}=i\sqrt{a^2+b^2}$

EJERCICIO 254

1. $\sqrt{-4}=\sqrt{2^2(-1)}=2i$

$\sqrt{-16}=\sqrt{4^2(-1)}=4i$

$=2i+4i=6i$

2. $\sqrt{-25}=\sqrt{5^2(-1)}=5i$

$\sqrt{-81}=\sqrt{9^2(-1)}=9i$

$-\sqrt{-49}=-\sqrt{7^2(-1)}=-7i$

$=5i+9i-7i=7i$

3. $2\sqrt{-9}=2\sqrt{3^2(-1)}=6i$

$3\sqrt{-100}=3\sqrt{10^2(-1)}=30i$

$=6i+30i=36i$

4. $3\sqrt{-64}=3\sqrt{8^2(-1)}=24i$

$-5\sqrt{-49}=-5\sqrt{7^2(-1)}=-35i$

$3\sqrt{-121}=3\sqrt{11^2(-1)}=33i$

$=24i-35i+33i=22i$

5. $2\sqrt{-a^2}=2\sqrt{a^2(-1)}=2ai$

$\sqrt{-a^4}=\sqrt{a^2a^2(-1)}=a^2\,i$

$\sqrt{-a^6}=\sqrt{a^4a^2(-1)}=a^3\,i$

$=2ai+a^2i+a^3i=i\left(2a+a^2+a^3\right)$

6. $\sqrt{-18}=\sqrt{3^2\cdot 2(-1)}=3\sqrt{2}\,i$

$\sqrt{-8}=\sqrt{2^2\cdot 2(-1)}=2\sqrt{2}\,i$

$2\sqrt{-50}=2\sqrt{5^2\cdot 2(-1)}=10\sqrt{2}\,i$

$=3\sqrt{2}\,i+2\sqrt{2}\,i+10\sqrt{2}\,i=15\sqrt{2}\,i$

7. $3\sqrt{-20}=3\sqrt{2^2\cdot 5(-1)}=6\sqrt{5}\,i$

$-2\sqrt{-45}=-2\sqrt{3^2\cdot 5(-1)}=-6\sqrt{5}\,i$

$3\sqrt{-125}=3\sqrt{5^2\cdot 5(-1)}=15\sqrt{5}\,i$

$=6\sqrt{5}\,i-6\sqrt{5}\,i+15\sqrt{5}\,i=15\sqrt{5}\,i$

8. $\sqrt{-a^4}=\sqrt{a^4(-1)}=a^2\,i$

$4\sqrt{-9a^4}=4\sqrt{3^2a^4(-1)}=12a^2\,i$

$-3\sqrt{-4a^4}=-3\sqrt{2^2a^4(-1)}=-6a^2\,i$

$=a^2\,i+12a^2\,i-6a^2\,i=7a^2\,i$

EJERCICIO 255

1. $\sqrt{-16}\cdot\sqrt{-25}$

$=4\sqrt{-1}\cdot 5\sqrt{-1}$

$=20\left(\sqrt{-1}\right)^2$

$=20(-1)$

$=-20$

2. $\sqrt{-81}\cdot\sqrt{-49}$

$=9\sqrt{-1}\cdot 7\sqrt{-1}$

$=63\left(\sqrt{-1}\right)^2$

$=63(-1)$

$=-63$

3. $5\sqrt{-36}\cdot 4\sqrt{-64}$

$=5\cdot 6\sqrt{-1}\cdot 4\cdot 8\sqrt{-1}$

$=30\sqrt{-1}\cdot 32\sqrt{-1}$

$=960\left(\sqrt{-1}\right)^2$

$=960(-1)$

$=-960$

4. $\sqrt{-3}\cdot\sqrt{-2}$

$=\sqrt{3}\sqrt{-1}\cdot\sqrt{2}\sqrt{-1}$

$=\sqrt{6}\left(\sqrt{-1}\right)^2$

$=\sqrt{6}(-1)$

$=-\sqrt{6}$

5. $2\sqrt{-5}\cdot 3\sqrt{-7}$

$=2\sqrt{5}\sqrt{-1}\cdot 3\sqrt{7}\sqrt{-1}$

$=6\sqrt{35}\left(\sqrt{-1}\right)^2$

$=6\sqrt{35}(-1)$

$=-6\sqrt{35}$

6. $\sqrt{-3}\cdot\sqrt{-75}$

$=\sqrt{3}\sqrt{-1}\cdot\sqrt{25}\sqrt{3}\sqrt{-1}$

$=5\sqrt{3^2}\left(\sqrt{-1}\right)^2$

$=15(-1)$

$=-15$

7. $2\sqrt{-7}\cdot 3\sqrt{-28}$

$=2\sqrt{7}\sqrt{-1}\cdot 3\sqrt{4}\sqrt{7}\sqrt{-1}$

$=6\sqrt{4}\sqrt{7^2}\left(\sqrt{-1}\right)^2$

$=12\cdot 7(-1)$

$=-84$

8. $\sqrt{-49}\cdot\sqrt{-4}\cdot\sqrt{-9}$

$=7\sqrt{-1}\cdot 2\sqrt{-1}\cdot 3\sqrt{-1}$

$=42\left(\sqrt{-1}\right)^3$

$=42\left(\sqrt{-1}\right)$

$=-42\sqrt{-1}$

$=-42i$

9. $\sqrt{-2}\cdot 3\sqrt{-5}\cdot\sqrt{-10}$

$=\sqrt{2}\sqrt{-1}\cdot 3\sqrt{5}\sqrt{-1}\cdot\sqrt{10}\sqrt{-1}$

$=3\sqrt{2\cdot 5\cdot 10}\left(\sqrt{-1}\right)^3$

$=3\sqrt{100}\left(-\sqrt{-1}\right)$

$=-30\sqrt{-1}$

$=-30i$

10. $\sqrt{-12}\cdot\sqrt{-27}\cdot\sqrt{-8}\cdot\sqrt{-50}$

$=2\sqrt{3}\sqrt{-1}\cdot 3\sqrt{3}\sqrt{-1}\cdot 2\sqrt{2}\sqrt{-1}\cdot 5\sqrt{2}\sqrt{-1}$

$=60\sqrt{9}\sqrt{4}\left(\sqrt{-1}\right)^4$

$=60\cdot 3\cdot 2(1)$

$=360$

11. $-5\sqrt{-x}\cdot 3\sqrt{-y}$

$=-5\sqrt{x}\sqrt{-1}\cdot 3\sqrt{y}\sqrt{-1}$

$=-15\sqrt{xy}\left(\sqrt{-1}\right)^2$

$=-15\sqrt{xy}(-1)$

$=15\sqrt{xy}$

12. $\sqrt{-4}+\sqrt{-9}$

$\sqrt{-25}-\sqrt{-16}$

$10\left(\sqrt{-1}\right)^2+15\left(\sqrt{-1}\right)^2$

$-\ 8\left(\sqrt{-1}\right)^2-12\left(\sqrt{-1}\right)^2$

$10(-1)\ +\ 7(-1)\ -\ 12(-1)$

$=\ -10-7+12\ =-5$

13. $\sqrt{-2}+3\sqrt{-5}$

$2\sqrt{-2}-6\sqrt{-5}$

$2\sqrt{4}\left(\sqrt{-1}\right)^2+6\sqrt{10}\left(\sqrt{-1}\right)^2$

$-6\sqrt{10}\left(\sqrt{-1}\right)^2-18\sqrt{25}\left(\sqrt{-1}\right)^2$

$4(-1)\ -\ 90(-1)$

$=\ -4+90$

$=\ 86$

14. $2\sqrt{-2}+5\sqrt{-3}$

$\sqrt{-2}-4\sqrt{-3}$

$2\sqrt{4}\left(\sqrt{-1}\right)^2+5\sqrt{6}\left(\sqrt{-1}\right)^2$

$-8\sqrt{6}\left(\sqrt{-1}\right)^2-20\sqrt{9}\left(\sqrt{-1}\right)^2$

$4(-1)\quad -3\sqrt{6}(-1)\quad -\ 60(-1)$

$=\ -4+3\sqrt{6}+60$

$=\ 56+3\sqrt{6}$

EJERCICIO 256

1. $\dfrac{\sqrt{-16}}{\sqrt{-4}}=\dfrac{4\sqrt{-1}}{2\sqrt{-1}}=2$ **2.** $\dfrac{\sqrt{-10}}{\sqrt{-2}}=\dfrac{\sqrt{10}\sqrt{-1}}{\sqrt{2}\sqrt{-1}}=\sqrt{\dfrac{10}{2}}=\sqrt{5}$ **3.** $\dfrac{\sqrt{-81}}{\sqrt{-3}}=\dfrac{9\sqrt{-1}}{\sqrt{3}\sqrt{-1}}=\dfrac{9}{\sqrt{3}}=\dfrac{9\sqrt{3}}{\sqrt{9}}=\dfrac{9\sqrt{3}}{3}=3\sqrt{3}$

4. $\frac{\sqrt{-90}}{\sqrt{-5}}=\frac{3\sqrt{10}\sqrt{-1}}{\sqrt{5}\sqrt{-1}}=3\sqrt{\frac{10}{5}}=3\sqrt{2}$

5. $\frac{\sqrt{-150}}{\sqrt{-3}}=\frac{5\sqrt{6}\sqrt{-1}}{\sqrt{3}\sqrt{-1}}=5\sqrt{\frac{6}{3}}=5\sqrt{2}$

6. $\frac{10\sqrt{-36}}{5\sqrt{-4}}=\frac{60\sqrt{-1}}{10\sqrt{-1}}=6$

7. $\frac{2\sqrt{-18}}{\sqrt{-6}}=\frac{6\sqrt{2}\sqrt{-1}}{\sqrt{6}\sqrt{-1}}=\frac{6\sqrt{2}}{\sqrt{6}}=\frac{6\sqrt{12}}{\sqrt{36}}=\frac{6\sqrt{2^2\cdot 3}}{6}=2\sqrt{3}$

8. $\frac{\sqrt{-315}}{\sqrt{-7}}=\frac{3\sqrt{35}\sqrt{-1}}{\sqrt{7}\sqrt{-1}}=3\sqrt{\frac{35}{7}}=3\sqrt{5}$

9. $\frac{\sqrt[4]{-27}}{\sqrt[4]{-3}}=\frac{\sqrt[4]{27}\sqrt{-1}}{\sqrt[4]{3}\sqrt{-1}}=\sqrt[4]{\frac{27}{3}}=\sqrt[4]{9}=\sqrt[4]{3^2}=3^{\frac{2}{4}}=3^{\frac{1}{2}}=\sqrt{3}$

10. $\frac{\sqrt[4]{-300}}{\sqrt[4]{-12}}=\frac{\sqrt[4]{300}\sqrt{-1}}{\sqrt[4]{12}\sqrt{-1}}=\sqrt[4]{\frac{300}{12}}=\sqrt[4]{25}=\sqrt[4]{5^2}=5^{\frac{2}{4}}=5^{\frac{1}{2}}=\sqrt{5}$

EJERCICIO 257

1.
$$\begin{array}{l} 2+3\sqrt{-1} \\ \underline{5-2\sqrt{-1}} \\ 7+\sqrt{-1} \\ 7+i \end{array}$$

2.
$$\begin{array}{l} -4-5\sqrt{-1} \\ \underline{-2+8\sqrt{-1}} \\ -6+3\sqrt{-1} \\ -6+3i \end{array}$$

3.
$$\begin{array}{l} 12-11\sqrt{-1} \\ \underline{8+7\sqrt{-1}} \\ 20-4\sqrt{-1} \\ 20-4i \end{array}$$

4.
$$\begin{array}{l} 5+\sqrt{-1} \\ 7+2\sqrt{-1} \\ \underline{9+7\sqrt{-1}} \\ 21+10\sqrt{-1} \\ 21+10i \end{array}$$

5.
$$\begin{array}{l} 3-2i \\ 5-8i \\ \underline{-10+13i} \\ -2+3i \end{array}$$

6.
$$\begin{array}{l} 1-i \\ 4+3i \\ \underline{\sqrt{2}+5i} \\ 5+\sqrt{2}+7i \end{array}$$

7.
$$\begin{array}{l} 2+\sqrt{-2} \\ \underline{4-\sqrt{-3}} \\ 6+\left(\sqrt{2}\sqrt{-1}-\sqrt{3}\sqrt{-1}\right) \\ 6+\left(\sqrt{2}i-\sqrt{3}i\right) \\ 6+\left(\sqrt{2}-\sqrt{3}\right)i \end{array}$$

8.
$$\begin{array}{l} 7+\sqrt{-5} \\ \sqrt{2}-\sqrt{-9} \\ \underline{-4+\sqrt{-16}} \\ \left(3+\sqrt{2}\right)+\left(\sqrt{5}\sqrt{-1}-\sqrt{9}\sqrt{-1}+\sqrt{16}\sqrt{-1}\right) \\ \left(3+\sqrt{2}\right)+\left(\sqrt{5}i-3i+4i\right) \\ \left(3+\sqrt{2}\right)+\left(\sqrt{5}i+i\right) \\ \left(3+\sqrt{2}\right)+\left(\sqrt{5}+1\right)i \end{array}$$

EJERCICIO 258

1. $\left(7-2\sqrt{-1}\right)+\left(7+2\sqrt{-1}\right)$
$=\left(7+7\right)+\left(-2+2\right)\sqrt{-1}$
$=7\cdot 2$
$=14$

2. $\left(-5-3\sqrt{-1}\right)+\left(-5+3\sqrt{-1}\right)$
$=\left(-5-5\right)+\left(-3+3\right)\sqrt{-1}$
$=-5\cdot 2$
$=-10$

3. $\left(9+i\sqrt{3}\right)+\left(9-i\sqrt{3}\right)$
$=\left(9+9\right)+\left(i-i\right)\sqrt{3}$
$=9\cdot 2$
$=18$

4. $\left(-7-5\sqrt{-1}\right)+\left(-7+5\sqrt{-1}\right)$
$=\left(-7-7\right)+\left(-5+5\right)\sqrt{-1}$
$=-7\cdot 2$
$=-14$

5. $\left(8-3\sqrt{-2}\right)+\left(8+3\sqrt{-2}\right)$
$=\left(8+8\right)+\left(-3+3\right)\sqrt{2}\sqrt{-1}$
$=8\cdot 2$
$=16$

6. $\left(\sqrt{2}+i\sqrt{3}\right)+\left(\sqrt{2}-i\sqrt{3}\right)$
$=\left(\sqrt{2}+\sqrt{2}\right)+\left(i-i\right)\sqrt{3}$
$=2\sqrt{2}$

EJERCICIO 259

1. $\left(3-2\sqrt{-1}\right)-\left(5+3\sqrt{-1}\right)$

$3-2\sqrt{-1}$

$\underline{-5-3\sqrt{-1}}$

$-2-5\sqrt{-1}$

$-2-5i$

2. $\left(8+4\sqrt{-1}\right)-\left(3-10\sqrt{-1}\right)$

$8+4\sqrt{-1}$

$\underline{-3+10\sqrt{-1}}$

$5+14\sqrt{-1}$

$5+14i$

3. $\left(-1-\sqrt{-1}\right)-\left(-7-8\sqrt{-1}\right)$

$-1-\sqrt{-1}$

$\underline{+7+8\sqrt{-1}}$

$6+7\sqrt{-1}$

$6+7i$

4. $\left(4-7\sqrt{-1}\right)-\left(5-3\sqrt{-1}\right)$

$4-7\sqrt{-1}$

$\underline{-5+3\sqrt{-1}}$

$-1-4\sqrt{-1}$

$-1-4i$

5. $\left(15-4\sqrt{-1}\right)-\left(8-7\sqrt{-1}\right)$

$15-4\sqrt{-1}$

$\underline{-8+7\sqrt{-1}}$

$7+3\sqrt{-1}$

$7+3i$

6. $\left(11+80\sqrt{-1}\right)-\left(3-50\sqrt{-1}\right)$

$11+80\sqrt{-1}$

$\underline{-3+50\sqrt{-1}}$

$8+130\sqrt{-1}$

$8+130i$

7. $\left(5-\sqrt{-25}\right)-\left(3+6i\right)$

$5-\sqrt{-25}$

$\underline{-3-6i}$

$2+\left(-\sqrt{25}\sqrt{-1}-6i\right)$

$2+\left(-5i-6i\right)$

$2+\left(-11i\right)$

$2-11i$

8. $\left(4+\sqrt{-5}\right)-\left(2+\sqrt{-3}\right)$

$4+\sqrt{-5}$

$\underline{-2-\sqrt{-3}}$

$2+\sqrt{5}\sqrt{-1}-\sqrt{3}\sqrt{-1}$

$2+\left(\sqrt{5}i-\sqrt{3}i\right)$

$2+\left(\sqrt{5}-\sqrt{3}\right)i$

9. $\left(\sqrt{2}-5\sqrt{-1}\right)-\left(\sqrt{3}+6\sqrt{-1}\right)$

$\sqrt{2}-5i$

$\underline{-\sqrt{3}-6i}$

$\left(\sqrt{2}-\sqrt{3}\right)+\left(-5i-6i\right)$

$\left(\sqrt{2}-\sqrt{3}\right)+\left(-11i\right)$

$\left(\sqrt{2}-\sqrt{3}\right)-11i$

10. $\left(8-\sqrt{-7}\right)-\left(-7+\sqrt{-3}\right)$

$8-\sqrt{-7}$

$\underline{7-\sqrt{-3}}$

$15+\left(-\sqrt{7}\sqrt{-1}-\sqrt{3}\sqrt{-1}\right)$

$15+\left(-\sqrt{7}i-\sqrt{3}i\right)$

$15-\left(\sqrt{7}+\sqrt{3}\right)i$

EJERCICIO 260

1. $\left(2-\sqrt{-1}\right)-\left(2+\sqrt{-1}\right)$

$2-\sqrt{-1}-2-\sqrt{-1}$

$=\left(2-2\right)+\left(-1-1\right)\sqrt{-1}$

$=-2\sqrt{-1}$

$=-2i$

2. $\left(7+3\sqrt{-1}\right)-\left(7-3\sqrt{-1}\right)$

$7+3\sqrt{-1}-7+3\sqrt{-1}$

$=\left(7-7\right)+\left(3+3\right)\sqrt{-1}$

$=6\sqrt{-1}$

$=6i$

3. $\left(-3-7\sqrt{-1}\right)-\left(-3+7\sqrt{-1}\right)$

$-3-7\sqrt{-1}+3-7\sqrt{-1}$

$=\left(-3+3\right)+\left(-7-7\right)\sqrt{-1}$

$=-7\cdot 2\sqrt{-1}$

$=-14i$

4. $\left(-5+\sqrt{-2}\right)-\left(-5-\sqrt{-2}\right)$

$-5+\sqrt{-2}+5+\sqrt{-2}$

$=\left(-5+5\right)+\left(1+1\right)\sqrt{2}\sqrt{-1}$

$=2\sqrt{2}\sqrt{-1}$

$=2\sqrt{2}i$

5. $\left(\sqrt{2}+\sqrt{-3}\right)-\left(\sqrt{2}-\sqrt{-3}\right)$

$\sqrt{2}+\sqrt{-3}-\sqrt{2}+\sqrt{-3}$

$=\left(\sqrt{2}-\sqrt{2}\right)+\left(1+1\right)\sqrt{3}\sqrt{-1}$

$=2\sqrt{3}\sqrt{-1}$

$=2\sqrt{3}i$

6. $\left(-\sqrt{5}-4\sqrt{-2}\right)-\left(-\sqrt{5}+4\sqrt{-2}\right)$

$-\sqrt{5}-4\sqrt{-2}+\sqrt{5}-4\sqrt{-2}$

$-\sqrt{5}-4\sqrt{-2}$

$\underline{+\sqrt{5}-4\sqrt{-2}}$

$-8\sqrt{-2}=-8\sqrt{2}\sqrt{-1}$

$=-8\sqrt{2}i$

EJERCICIO 261

1.
$$\begin{array}{l} 3-4\sqrt{-1} \\ \underline{5-3\sqrt{-1}} \\ 15-20\sqrt{-1} \\ \underline{\quad -\ 9\sqrt{-1}+12\left(\sqrt{-1}\right)^2} \\ 15-29\sqrt{-1}+12(-1) \\ 15-29i-12 \\ 3-29i \end{array}$$

2.
$$\begin{array}{l} 4+7\sqrt{-1} \\ \underline{-3-2\sqrt{-1}} \\ -12-21\sqrt{-1} \\ \underline{\quad -\ 8\sqrt{-1}-14\left(\sqrt{-1}\right)^2} \\ -12-29\sqrt{-1}-14(-1) \\ -12-29i+14 \\ 2-29i \end{array}$$

3.
$$\begin{array}{l} 7-\sqrt{-4} \\ \underline{5+\sqrt{-9}} \\ 35-5\sqrt{4}\sqrt{-1} \\ \underline{\quad +7\sqrt{9}\sqrt{-1}-\sqrt{4}\sqrt{9}\left(\sqrt{-1}\right)^2} \\ 35-10\sqrt{-1}+21\sqrt{-1}-\sqrt{36}(-1) \\ 35-10i+21i+6 \\ 41+11i \end{array}$$

4.
$$\begin{array}{l} 8-\sqrt{-9} \\ \underline{11+\sqrt{-25}} \\ 88-11\sqrt{9}\sqrt{-1} \\ \underline{\quad +8\sqrt{25}\sqrt{-1}-\sqrt{9}\sqrt{25}\left(\sqrt{-1}\right)^2} \\ 88-33i+40i-15(-1) \\ 88+7i+15 \\ 103+7i \end{array}$$

5.
$$\begin{array}{l} 3+\sqrt{-2} \\ \underline{5-\sqrt{-2}} \\ 15+5\sqrt{2}\sqrt{-1} \\ \underline{\quad -3\sqrt{2}\sqrt{-1}-\sqrt{2}\sqrt{2}\left(\sqrt{-1}\right)^2} \\ 15+2\sqrt{2}\sqrt{-1}-\sqrt{4}(-1) \\ 15+2\sqrt{2}\,i+2 \\ 17+2\sqrt{2}\,i \end{array}$$

6.
$$\begin{array}{l} 4+\sqrt{-3} \\ \underline{5-\sqrt{-2}} \\ 20+5\sqrt{3}\sqrt{-1} \\ \underline{\quad -4\sqrt{2}\sqrt{-1}-\sqrt{2}\sqrt{3}\left(\sqrt{-1}\right)^2} \\ 20+5\sqrt{3}\,i-4\sqrt{2}\,i-\sqrt{6}(-1) \\ 20+\left(5\sqrt{3}-4\sqrt{2}\right)i+\sqrt{6} \\ \left(20+\sqrt{6}\right)+\left(5\sqrt{3}-4\sqrt{2}\right)i \end{array}$$

7.
$$\begin{array}{l} \sqrt{2}+\sqrt{-5} \\ \underline{\sqrt{3}+\sqrt{-2}} \\ \sqrt{6}+\sqrt{3}\sqrt{5}\sqrt{-1} \\ \underline{\quad +\sqrt{2}\sqrt{2}\sqrt{-1}+\sqrt{5}\sqrt{2}\left(\sqrt{-1}\right)^2} \\ \sqrt{6}+\sqrt{15}\,i+\sqrt{4}\,i+\sqrt{10}(-1) \\ \sqrt{6}+\left(\sqrt{15}\,i+2i\right)-\sqrt{10} \\ \left(\sqrt{6}-\sqrt{10}\right)+\left(\sqrt{15}+2\right)i \end{array}$$

8.
$$\begin{array}{l} \sqrt{5}+\sqrt{-3} \\ \underline{\sqrt{5}+2\sqrt{-3}} \\ \sqrt{25}+\sqrt{5}\sqrt{3}\sqrt{-1} \\ \underline{\quad +2\sqrt{5}\sqrt{3}\sqrt{-1}+2\sqrt{9}\left(\sqrt{-1}\right)^2} \\ 5+\sqrt{15}\,i+2\sqrt{15}\,i+6(-1) \\ 5+3\sqrt{15}\,i-6 \\ 3\sqrt{15}\,i-1 \end{array}$$

EJERCICIO 262

1. $(1-i)(1+i)$

$=1-i^2$

$=1-\left(\sqrt{-1}\right)^2$

$=1-(-1)$

$=1+1=2$

2. $\left(3+2\sqrt{-1}\right)\left(3-2\sqrt{-1}\right)$

$=3^2-\left(2\sqrt{-1}\right)^2$

$=9-\left[4(-1)\right]$

$=9+4=13$

3. $\left(\sqrt{2}-5i\right)\left(\sqrt{2}+5i\right)$

$=\left(\sqrt{2}\right)^2-(5i)^2$

$=2-\left[25\left(\sqrt{-1}\right)^2\right]$

$=2-\left[25(-1)\right]$

$=2+25=27$

4. $\left(2\sqrt{3}+4i\right)\left(2\sqrt{3}-4i\right)$

$=\left(2\sqrt{3}\right)^2-(4i)^2$

$=4(3)-\left[16\left(\sqrt{-1}\right)^2\right]$

$=12-\left[16(-1)\right]$

$=12+16=28$

5. $\left(5-\sqrt{-2}\right)\left(5+\sqrt{-2}\right)$

$=25-\left(\sqrt{-2}\right)^2$

$=25-\left(\sqrt{2}\sqrt{-1}\right)^2$

$=25-\left[2\left(\sqrt{-1}\right)^2\right]$

$=25-\left[2(-1)\right]$

$=25+2=27$

6. $\left(-9-\sqrt{-5}\right)\left(-9+\sqrt{-5}\right)$

$=81-\left(\sqrt{-5}\right)^2$

$=81-\left(\sqrt{5}\sqrt{-1}\right)^2$

$=81-\left[5(-1)\right]$

$=81+5=86$

EJERCICIO 263

1. $\frac{1+\sqrt{-1}}{1-\sqrt{-1}}\cdot\frac{1+\sqrt{-1}}{1+\sqrt{-1}}$

$=\frac{1+2\sqrt{-1}+\left(\sqrt{-1}\right)^2}{1-\left(\sqrt{-1}\right)^2}=\frac{1+2i-1}{1+1}=\frac{2i}{2}=i$

2. $\frac{3+\sqrt{-1}}{3-\sqrt{-1}}\cdot\frac{3+\sqrt{-1}}{3+\sqrt{-1}}$

$=\frac{9+6\sqrt{-1}+(-1)}{9-\left(\sqrt{-1}\right)^2}=\frac{9+6i-1}{9+1}=\frac{8+6i}{10}=\frac{4+3i}{5}$

3. $\frac{5-3\sqrt{-1}}{3+4\sqrt{-1}}\cdot\frac{3-4\sqrt{-1}}{3-4\sqrt{-1}}$

$=\frac{15-29\sqrt{-1}+12(-1)}{9-\left(4\sqrt{-1}\right)^2}=\frac{15-29i-12}{9-\left[16(-1)\right]}=\frac{3-29i}{25}$

4. $\frac{8-5i}{7+6i}\cdot\frac{7-6i}{7-6i}=\frac{56-83i+30i^2}{49-36i^2}=\frac{56-83i-30}{49+36}=\frac{26-83i}{85}$

5. $\frac{4+\sqrt{-3}}{5-4\sqrt{-3}}\cdot\frac{5+4\sqrt{-3}}{5+4\sqrt{-3}}=\frac{20+21\sqrt{3}\sqrt{-1}+4\left(\sqrt{3}\right)^2(-1)}{25-\left(4\sqrt{3}\sqrt{-1}\right)^2}$

$=\frac{20+21\sqrt{3}i-12}{25-(-48)}=\frac{8+21\sqrt{3}i}{73}$

6. $\frac{\sqrt{2}+2\sqrt{-5}}{4\sqrt{2}-\sqrt{-5}}\cdot\frac{4\sqrt{2}+\sqrt{-5}}{4\sqrt{2}+\sqrt{-5}}$

$=\frac{4\left(\sqrt{2}\right)^2+9\sqrt{2}\sqrt{5}\sqrt{-1}+2\left(\sqrt{5}\right)^2(-1)}{16\left(\sqrt{2}\right)^2-\left(\sqrt{5}\sqrt{-1}\right)^2}$

$=\frac{8+9\sqrt{10}i-10}{32+5}=\frac{9\sqrt{10}i-2}{37}$

EJERCICIO 265

1. $3x^2-5x+2=0$

$x^2-\frac{5x}{3}+\frac{2}{3}=0$

$x^2-\frac{5x}{3}=-\frac{2}{3}$

$x^2-\frac{5x}{3}+\frac{25}{36}=\frac{25}{36}-\frac{2}{3}$

$\left(x-\frac{5}{6}\right)^2=\frac{25-24}{36}$

$x-\frac{5}{6}=\pm\sqrt{\frac{1}{6}}$

$x=\frac{5}{6}\pm\frac{1}{6}$

$x_1=\frac{5+1}{6}=1$

$x_2=\frac{5-1}{6}=\frac{4}{6}=\frac{2}{3}$

2. $4x^2+3x-22=0$

$x^2+\frac{3x}{4}=\frac{22}{4}$

$x^2+\frac{3x}{4}+\frac{9}{64}=\frac{11}{2}+\frac{9}{64}$

$\left(x+\frac{3}{8}\right)^2=\frac{352+9}{64}$

Continúa

2. Continuación

$x+\frac{3}{8}=\pm\sqrt{\frac{361}{64}}$

$x=-\frac{3}{8}\pm\frac{19}{8}$

$x_1=\frac{-3+19}{8}=\frac{16}{8}=2$

$x_2=\frac{-3-19}{8}=-\frac{22}{8}=-\frac{11}{4}$

3. $x^2+11x=-24$

$x^2+11x+\frac{121}{4}=\frac{121}{4}-24$

$\left(x+\frac{11}{2}\right)^2=\frac{121-96}{4}$

$x+\frac{11}{2}=\pm\sqrt{\frac{25}{4}}$

$x=-\frac{11}{2}\pm\frac{5}{2}$

$x_1=\frac{-11+5}{2}=\frac{-6}{2}=-3$

$x_2=\frac{-11-5}{2}=\frac{-16}{2}=-8$

4. $x^2=16x-63$

$x^2-16x=-63$

$x^2-16x+64=64-63$

$(x-8)^2=1$

$x-8=\pm1$

$x=8\pm1$

$x_1=8+1=9$

$x_2=8-1=7$

5. $12x-4-9x^2=0$

$-9x^2+12x=4$

$9x^2-12x=-4$

$x^2-\frac{12x}{9}=-\frac{4}{9}$

$x^2-\frac{4x}{3}+\frac{4}{9}=\frac{4}{9}-\frac{4}{9}$

$\left(x-\frac{2}{3}\right)^2=0$

$x-\frac{2}{3}=0$

$x_1=x_2=\frac{2}{3}$

6. $5x^2-7x-90=0$

$$x=\frac{7\pm\sqrt{7^2-4(5)(-90)}}{2(5)}$$

$$x=\frac{7\pm\sqrt{49+1.800}}{10}$$

$$x=\frac{7\pm\sqrt{1.849}}{10}=\frac{7\pm 43}{10}$$

$$x_1=\frac{7+43}{10}=\frac{50}{10}=5$$

$$x_2=\frac{7-43}{10}=-\frac{36}{10}=-3\tfrac{3}{5}$$

7. $6x^2=x+222$

$6x^2-x-222=0$

$$x=\frac{1\pm\sqrt{1^2+4(6)(222)}}{2(6)}$$

$$x=\frac{1\pm\sqrt{5.329}}{12}$$

$$x=\frac{1\pm 73}{12}$$

$$x_1=\frac{1+73}{12}=\frac{74}{12}=6\tfrac{1}{6}$$

$$x_2=\frac{1-73}{12}=\frac{-72}{12}=-6$$

8. $x+11=10x^2$

$10x^2-x-11=0$

$$x=\frac{1\pm\sqrt{1^2-4(10)(-11)}}{2(10)}$$

$$x=\frac{1\pm\sqrt{1+440}}{20}$$

$$x=\frac{1\pm 21}{20}$$

$$x_1=\frac{1+21}{20}=\frac{22}{20}=1\tfrac{1}{10}$$

$$x_2=\frac{1-21}{20}=\frac{-20}{20}=-1$$

9. $49x^2-70x+25=0$

$$x=\frac{70\pm\sqrt{(70)^2-4(49)(25)}}{2(49)}$$

$$x=\frac{70\pm\sqrt{4.900-4.900}}{98}$$

$$x_1=x_2=\frac{70}{98}=\frac{5}{7}$$

10. $12x-7x^2+64=0$

$-7x^2+12x+64=0$

$$x=\frac{-12\pm\sqrt{(12)^2-4(-7)(64)}}{2(-7)}$$

$$x=\frac{-12\pm\sqrt{144+1.792}}{-14}$$

$$x=\frac{-12\pm\sqrt{1.936}}{-14}$$

$$x=\frac{-12\pm 44}{-14}$$

$$x_1=\frac{-12+44}{-14}=-\frac{32}{14}=-2\tfrac{2}{7}$$

$$x_2=\frac{-12-44}{-14}=\frac{-56}{-14}=4$$

11. $x^2=15x-56$

$x^2+15x+56=0$

$$x=\frac{-15\pm\sqrt{(15)^2-4(1)(56)}}{2(1)}$$

$$x=\frac{-15\pm\sqrt{225-224}}{2}$$

$$x=\frac{-15\pm 1}{2}$$

$$x_1=\frac{-15+1}{2}=\frac{-14}{2}=-7$$

$$x_2=\frac{-15-1}{2}=\frac{-16}{2}=-8$$

12. $32x^2+18x-17=0$

$$x=\frac{-18\pm\sqrt{(18)^2-4(32)(-17)}}{2(32)}$$

$$x=\frac{-18\pm\sqrt{324+2.176}}{64}$$

$$x=\frac{-18\pm\sqrt{2.500}}{64}$$

$$x=\frac{-18\pm 50}{64}$$

$$x_1=\frac{-18+50}{64}=\frac{32}{64}=\frac{1}{2}$$

$$x_2=\frac{-18-50}{64}=\frac{-68}{64}=-1\tfrac{1}{16}$$

13. $176x=121+64x^2$

$64x^2-176x+121=0$

$$x=\frac{176\pm\sqrt{(176)^2-4(64)(121)}}{2(64)}$$

$$=\frac{176\pm\sqrt{30.976-30.976}}{128}$$

$$x_1=x_2=\frac{176}{128}=1\tfrac{3}{8}$$

14. $8x+5=36x^2$

$-36x^2+8x+5=0$

$$x=\frac{-8\pm\sqrt{(8)^2-4(-36)(5)}}{2(-36)}$$

$$x=\frac{-8\pm\sqrt{64+720}}{-72}$$

$$x=\frac{-8\pm\sqrt{784}}{-72}$$

$$x=\frac{-8\pm 28}{-72}$$

$$x_1=\frac{-8+28}{-72}=-\frac{20}{72}=-\frac{5}{18}$$

$$x_2=\frac{-8-28}{-72}=\frac{-36}{-72}=\frac{1}{2}$$

15. $27x^2+12x-7=0$

$$x=\frac{-12\pm\sqrt{(12)^2-4(27)(-7)}}{2(27)}$$

$$x=\frac{-12\pm\sqrt{144+756}}{54}$$

$$x=\frac{-12\pm\sqrt{900}}{54}$$

$$x_1=\frac{-12+30}{54}=\frac{18}{54}=\frac{1}{3}$$

$$x_2=\frac{-12-30}{54}=\frac{-42}{54}=-\frac{7}{9}$$

16. $15x=25x^2+2$

$25x^2-15x+2=0$

$x=\frac{15\pm\sqrt{(15)^2-4(25)(2)}}{2(25)}$

$x=\frac{15\pm\sqrt{225-200}}{50}$

$x=\frac{15\pm\sqrt{25}}{50}$

$x_1=\frac{15+5}{50}=\frac{20}{50}=\frac{2}{5}$

$x_2=\frac{15-5}{50}=\frac{10}{50}=\frac{1}{5}$

17. $8x^2-2x-3=0$

$x=\frac{2\pm\sqrt{(2)^2-4(8)(-3)}}{2(8)}$

$x=\frac{2\pm\sqrt{4+96}}{16}$

$x=\frac{2\pm\sqrt{100}}{16}$

$x=\frac{2\pm10}{16}$

$x_1=\frac{2+10}{16}=\frac{12}{16}=\frac{3}{4}$

$x_2=\frac{2-10}{16}=\frac{-8}{16}=-\frac{1}{2}$

18. $105=2x^2+x$

$2x^2+x-105=0$

$x=\frac{-1\pm\sqrt{(1)^2-4(2)(-105)}}{2(2)}$

$x=\frac{-1\pm\sqrt{1+840}}{4}$

$x=\frac{-1\pm\sqrt{841}}{4}$

$x=\frac{-1\pm29}{4}$

$x_1=\frac{-1+29}{4}=\frac{28}{4}=7$

$x_2=\frac{-1-29}{4}=\frac{-30}{4}=-7\tfrac{1}{2}$

EJERCICIO 266

1. $x(x+3)=5x+3$

$x^2+3x=5x+3$

$x^2+3x-5x-3=0$

$x^2-2x-3=0$

$x=\frac{2\pm\sqrt{(-2)^2-4(1)(-3)}}{2(1)}$

$x=\frac{2\pm\sqrt{4+12}}{2}$

$x=\frac{2\pm4}{2}$

$x_1=\frac{2+4}{2}=\frac{6}{2}=3$

$x_2=\frac{2-4}{2}=\frac{-2}{2}=-1$

2. $3(3x-2)=(x+4)(4-x)$

$9x-6=-x^2+16$

$x^2+9x-22=0$

$x=\frac{-9\pm\sqrt{(9)^2-4(1)(-22)}}{2(1)}$

$x=\frac{9\pm\sqrt{169}}{2}$

$x_1=\frac{-9+13}{2}=\frac{4}{2}=2$

$x_2=\frac{-9-13}{2}=\frac{-22}{2}=-11$

3. $9x+1=3(x^2-5)-(x-3)(x+2)$

$9x+1=3x^2-15-x^2+x+6$

$9x+1=2x^2+x-9$

$0=2x^2-8x-10$

$x=\frac{8\pm\sqrt{(-8)^2-4(2)(-10)}}{2(2)}$

$x=\frac{8\pm\sqrt{64+80}}{4}$

$x=\frac{8\pm12}{4}$

$x_1=\frac{8+12}{4}=\frac{20}{4}=5$

$x_2=\frac{8-12}{4}=\frac{-4}{4}=-1$

4. $(2x-3)^2-(x+5)^2=-23$

$4x^2-12x+9-x^2-10x-25+23=0$

$3x^2-22x+7=0$

$x=\frac{22\pm\sqrt{(22)^2-4(3)(7)}}{2(3)}$

$x=\frac{22\pm\sqrt{400}}{6}$

$x=\frac{22\pm20}{6}$

$x_1=\frac{22+20}{6}=\frac{42}{6}=7$

$x_2=\frac{22-20}{6}=\frac{2}{6}=\frac{1}{3}$

5. $25(x+2)^2=(x-7)^2-81$

$25x^2+100x+100=x^2-14x+49-81$

$24x^2+114x+132=0$

$4x^2+19x+22=0$

$x=\frac{-19\pm\sqrt{(19)^2-4(4)(22)}}{2(4)}$

$x=\frac{-19\pm\sqrt{361-352}}{8}$

$x=\frac{-19\pm3}{8}$

$x_1=\frac{-19+3}{8}=\frac{-16}{8}=-2$

$x_2=\frac{-19-3}{8}=\frac{-22}{8}=-2\tfrac{3}{4}$

6. $3x(x-2)-(x-6)=23(x-3)$

$3x^2-6x-x+6=23x-69$

$3x^2-30x+75=0$

$x^2-10x+25=0$

$x=\frac{10\pm\sqrt{(-10)^2-4(1)(25)}}{2(1)}$

$x=\frac{10\pm\sqrt{100-100}}{2}$

$x_1=x_2=\frac{10}{2}=5$

7. $7\left(x-3\right)-5\left(x^{2}-1\right)=x^{2}-5x\left(x+2\right)$

$$7x-21-5x^{2}+5=x^{2}-5x^{2}-10x$$

$$7x-16-5x^{2}=-4x^{2}-10x$$

$$-x^{2}+17x-16=0$$

$$x=\frac{-17\pm\sqrt{(17)^{2}-4\left(-1\right)\left(-16\right)}}{2\left(-1\right)}$$

$$x=\frac{-17\pm\sqrt{225}}{-2}$$

$$x=\frac{-17\pm 15}{-2}$$

$$x_{1}=\frac{-17+15}{-2}=\frac{-2}{-2}=1$$

$$x_{2}=\frac{-17-15}{-2}=\frac{-32}{-2}=16$$

8. $\left(x-5\right)^{2}-\left(x-6\right)^{2}=\left(2x-3\right)^{2}-118$

$$x^{2}-10x+25-x^{2}+12x-36=4x^{2}-12x+9-118$$

$$2x-11=4x^{2}-12x-109$$

$$4x^{2}-14x-98=0$$

$$2x^{2}-7x-49=0$$

$$x=\frac{7\pm\sqrt{\left(-7\right)^{2}-4(2)\left(-49\right)}}{2(2)}$$

$$x=\frac{7\pm\sqrt{441}}{4}$$

$$x_{1}=\frac{7+21}{4}=\frac{28}{4}=7$$

$$x_{2}=\frac{7-21}{4}=\frac{-14}{4}=-3\tfrac{1}{2}$$

9. $\left(5x-2\right)^{2}-\left(3x+1\right)^{2}-x^{2}-60=0$

$$25x^{2}-20x+4-9x^{2}-6x-1-x^{2}-60=0$$

$$15x^{2}-26x-57=0$$

$$x=\frac{26\pm\sqrt{\left(-26\right)^{2}-4(15)\left(-57\right)}}{2(15)}$$

$$x=\frac{26\pm\sqrt{676+3.420}}{30}$$

$$x=\frac{26\pm 64}{30}$$

$$x_{1}=\frac{26+64}{30}=\frac{90}{30}=3$$

$$x_{2}=\frac{26-64}{30}=\frac{-38}{30}=-1\tfrac{4}{15}$$

10. $\left(x+4\right)^{3}-\left(x-3\right)^{3}=343$

$$x^{3}+12x^{2}+48x+64-x^{3}+9x^{2}-27x+27-343=0$$

$$21x^{2}+21x-252=0$$

$$-x^{2}-x+12=0$$

$$x=\frac{1\pm\sqrt{\left(-1\right)^{2}-4\left(-1\right)(12)}}{2\left(-1\right)}$$

$$x=\frac{1\pm\sqrt{49}}{-2}$$

$$x=\frac{1\pm 7}{-2}$$

$$x_{1}=\frac{1+7}{-2}=\frac{8}{-2}=-4$$

$$x_{2}=\frac{1-7}{-2}=\frac{-6}{-2}=3$$

11. $\left(x+2\right)^{3}-\left(x-1\right)^{3}=x\left(3x+4\right)+8$

$$x^{3}+6x^{2}+12x+8-x^{3}+3x^{2}-3x+1=3x^{2}+4x+8$$

$$6x^{2}+5x+1=0$$

$$x=\frac{-5\pm\sqrt{(5)^{2}-4(6)(1)}}{2(6)}$$

$$x=\frac{-5\pm\sqrt{25-24}}{12}$$

$$x_{1}=\frac{-5+1}{12}=\frac{-4}{12}=-\frac{1}{3}$$

$$x_{2}=\frac{-5-1}{12}=\frac{-6}{12}=-\frac{1}{2}$$

12. $\left(5x-4\right)^{2}-\left(3x+5\right)\left(2x-1\right)=20x\left(x-2\right)+27$

$$25x^{2}-40x+16-6x^{2}-7x+5=20x^{2}-40x+27$$

$$19x^{2}-47x+21=20x^{2}-40x+27$$

$$-x^{2}-7x-6=0$$

$$x=\frac{7\pm\sqrt{\left(-7\right)^{2}-4\left(-1\right)\left(-6\right)}}{2\left(-1\right)}$$

$$x=\frac{7\pm\sqrt{49-24}}{-2}$$

$$x=\frac{7\pm\sqrt{25}}{-2}$$

$$x_{1}=\frac{7+5}{-2}=\frac{12}{-2}=-6$$

$$x_{2}=\frac{7-5}{-2}=\frac{2}{-2}=-1$$

EJERCICIO 267

1. $x^2-3x+2=0$

$x=-\frac{(-3)}{2}\pm\sqrt{\frac{(-3)^2}{4}-2}$

$x=\frac{3}{2}\pm\sqrt{\frac{9}{4}-2}$

$x=\frac{3}{2}\pm\sqrt{\frac{1}{4}}$

$x_1=\frac{3}{2}+\frac{1}{2}=\frac{4}{2}=2$

$x_2=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=1$

2. $x^2-2x-15=0$

$x=-\frac{(-2)}{2}\pm\sqrt{\frac{(-2)^2}{4}-(-15)}$

$x=1\pm\sqrt{1+15}$

$x=1\pm 4$

$x_1=1+4=5$

$x_2=1-4=-3$

3. $x^2=19x-88$

$x^2-19x+88=0$

$x=-\frac{(-19)}{2}\pm\sqrt{\frac{(-19)^2}{4}-88}$

$x=\frac{19}{2}\pm\sqrt{\frac{361}{4}-88}$

$x=\frac{19}{2}\pm\sqrt{\frac{9}{4}}$

$x_1=\frac{19}{2}+\frac{3}{2}=\frac{22}{2}=11$

$x_2=\frac{19}{2}-\frac{3}{2}=\frac{16}{2}=8$

4. $x^2+4x=285$

$x^2+4x-285=0$

$x=-\frac{4}{2}\pm\sqrt{\frac{(4)^2}{4}-(-285)}$

$x=-2\pm\sqrt{4+285}$

$x=-2\pm\sqrt{289}$

$x_1=-2+17=15$

$x_2=-2-17=-19$

5. $5x(x-1)-2(2x^2-7x)=-8$

$5x^2-5x-4x^2+14x+8=0$

$x^2+9x+8=0$

$x=-\frac{9}{2}\pm\sqrt{\frac{(9)^2}{4}-8}$

$x=-\frac{9}{2}\pm\sqrt{\frac{81}{4}-8}$

$x=-\frac{9}{2}\pm\sqrt{\frac{49}{4}}$

$x_1=-\frac{9}{2}+\frac{7}{2}=\frac{-2}{2}=-1$

$x_2=-\frac{9}{2}-\frac{7}{2}=\frac{-16}{2}=-8$

6. $x^2-(7x+6)=x+59$

$x^2-7x-6-x-59=0$

$x^2-8x-65=0$

$x=-\frac{(-8)}{2}\pm\sqrt{\frac{(-8)^2}{4}-(-65)}$

$x=4\pm\sqrt{16+65}$

$x=4\pm 9$

$x_1=4+9=13$

$x_2=4-9=-5$

7. $(x-1)^2+11x+199=3x^2-(x-2)^2$

$x^2-2x+1+11x+199=3x^2-x^2+4x-4$

$x^2+9x+200=2x^2+4x-4$

$x^2-5x-204=0$

$x=-\frac{(-5)}{2}\pm\sqrt{\frac{(5)^2}{4}-(-204)}$

$x=\frac{5}{2}\pm\sqrt{\frac{25}{4}+204}$

$x=\frac{5}{2}\pm\sqrt{\frac{841}{4}}$

$x_1=\frac{5}{2}+\frac{29}{2}=\frac{34}{2}=17$

$x_2=\frac{5}{2}-\frac{29}{2}=\frac{-24}{2}=-12$

8. $(x-2)(x+2)-7(x-1)=21$

$x^2-4-7x+7-21=0$

$x^2-7x-18=0$

$x=-\frac{(-7)}{2}\pm\sqrt{\frac{(-7)^2}{4}-(-18)}$

$x=\frac{7}{2}\pm\sqrt{\frac{49}{4}+18}$

$x=\frac{7}{2}\pm\sqrt{\frac{121}{4}}$

$x_1=\frac{7}{2}+\frac{11}{2}=\frac{18}{2}=9$

$x_2=\frac{7}{2}-\frac{11}{2}=\frac{-4}{2}=-2$

9. $2x^2-(x-2)(x+5)=7(x+3)$

$2x^2-x^2-3x+10=7x+21$

$x^2-10x-11=0$

$x=-\frac{(-10)}{2}\pm\sqrt{\frac{(-10)^2}{4}-(-11)}$

$x=5\pm\sqrt{25+11}$

$x=5\pm\sqrt{36}$

$x_1=5+6=11$

$x_2=5-6=-1$

10. $(x-1)(x+2)-(2x-3)(x+4)-x+14=0$

$x^2+x-2-2x^2-5x+12-x+14=0$

$-x^2-5x+24=0$

$x^2+5x-24=0$

$x=-\frac{5}{2}\pm\sqrt{\frac{(5)^2}{4}-(-24)}$

$x=-\frac{5}{2}\pm\sqrt{\frac{25}{4}+24}$

$x=-\frac{5}{2}\pm\sqrt{\frac{121}{4}}$

$x_1=-\frac{5}{2}+\frac{11}{2}=\frac{6}{2}=3$

$x_2=-\frac{5}{2}-\frac{11}{2}=\frac{-16}{2}=-8$

EJERCICIO 268

1. $\frac{x^2}{5}-\frac{x}{2}=\frac{3}{10}\quad mcm=10$

$2x^2-5x=3$

$2x^2-5x-3=0$

$x=\frac{5\pm\sqrt{(-5)^2-4(2)(-3)}}{2(2)}$

$x=\frac{5\pm\sqrt{25+24}}{4}$

$x=\frac{5\pm 7}{4}$

$x_1=\frac{5+7}{4}=\frac{12}{4}=3$

$x_2=\frac{5-7}{4}=\frac{-2}{4}=-\frac{1}{2}$

2. $4x-\frac{13}{x}=\frac{3}{2}\quad mcm=2x$

$8x^2-26=3x$

$8x^2-3x-26=0$

$x=\frac{3\pm\sqrt{(-3)^2-4(8)(-26)}}{2(8)}$

$x=\frac{3\pm\sqrt{9+832}}{16}$

$x=\frac{3\pm 29}{16}$

$x_1=\frac{3+29}{16}=\frac{32}{16}=2$

$x_2=\frac{3-29}{16}=\frac{-26}{16}=-1\frac{5}{8}$

3. $\frac{x^2}{6}-\frac{x}{2}=3(x-5)\quad mcm=6$

$x^2-3x=18(x-5)$

$x^2-3x=18x-90$

$x^2-21x+90=0$

$x=-\frac{(-21)}{2}\pm\sqrt{\frac{(-21)^2}{4}-90}$

$x=\frac{21}{2}\pm\sqrt{\frac{81}{4}}$

$x_1=\frac{21}{2}+\frac{9}{2}=\frac{30}{2}=15$

$x_2=\frac{21}{2}-\frac{9}{2}=\frac{12}{2}=6$

4. $\frac{1}{4}(x-4)+\frac{2}{5}(x-5)=\frac{1}{5}(x^2-53)$

$mcm=20$

$5(x-4)+8(x-5)=4(x^2-53)$

$5x-20+8x-40-4x^2+212=0$

$-4x^2+13x+152=0$

$x=\frac{-13\pm\sqrt{(13)^2-4(-4)(152)}}{2(-4)}$

$x=\frac{-13\pm\sqrt{169+2.432}}{-8}$

$x=\frac{-13\pm 51}{8}$

$x_1=\frac{-13+51}{-8}=\frac{38}{-8}=-4\frac{3}{4}$

$x_2=\frac{-13-51}{-8}=\frac{-64}{-8}=8$

5. $\frac{5}{x}-\frac{1}{x+2}=1\quad mcm=x+2$

$5\left(1+\frac{2}{x}\right)-1=x+2\quad mcm=x$

$5x+10-x=x^2+2x$

$x^2-2x-10=0$

$x=-\frac{(-2)}{2}\pm\sqrt{\frac{(-2)^2}{4}-(-10)}$

$x=1\pm\sqrt{1+10}$

$x=1\pm\sqrt{11}$

$x_1=1+\sqrt{11}$

$x_2=1-\sqrt{11}$

6. $\frac{15}{x}-\frac{11x+5}{x^2}=-1\quad mcm=x^2$

$15x-(11x+5)=-x^2$

$x^2+4x-5=0$

$x=-\frac{4}{2}\pm\sqrt{\frac{(4)^2}{4}-(-5)}$

$x=-2\pm\sqrt{4+5}$

$x=-2\pm 3$

$x_1=-2+3=1$

$x_2=-2-3=-5$

7. $\frac{8x}{3x+5}+\frac{5x-1}{x+1}=3\quad mcm=(3x+5)(x+1)$

$8x(x+1)+(5x-1)(3x+5)=3(3x+5)(x+1)$

$8x^2+8x+15x^2+22x-5=9x^2+24x+15$

$23x^2+30x-5=9x^2+24x+15$

$14x^2+6x-20=0$

$x=\frac{-6\pm\sqrt{(6)^2-4(14)(-20)}}{2(14)}$

$x=\frac{-6\pm\sqrt{1.156}}{28}=\frac{-6\pm 34}{28}$

$x_1=\frac{-6+34}{28}=\frac{28}{28}=1$

$x_2=\frac{-6-34}{28}=\frac{-40}{28}=-1\frac{3}{7}$

8. $\frac{1}{x-2}-\frac{1}{x-1}=\frac{1}{6}$

$mcm=6(x-2)(x-1)$

$6(x-1)-6(x-2)=(x-2)(x-1)$

$6x-6-6x+12=x^2-3x+2$

$6=x^2-3x+2$

$0=x^2-3x-4$

$x=-\frac{(-3)}{2}\pm\sqrt{\frac{(-3)^2}{4}-(-4)}$

$x=\frac{3}{2}\pm\sqrt{\frac{9}{4}+4}$

$x=\frac{3}{2}\pm\sqrt{\frac{25}{4}}=\frac{3}{2}\pm\frac{5}{2}$

$x_1=\frac{3}{2}+\frac{5}{2}=\frac{8}{2}=4$

$x_2=\frac{3}{2}-\frac{5}{2}=\frac{-2}{2}=-1$

9. $1-\frac{2x-3}{x+5}=\frac{x-2}{10}\quad mcm=10(x+5)$

$10(x+5)-10(2x-3)=(x+5)(x-2)$

$10x+50-20x+30=x^2+3x-10$

$-10x+80=x^2+3x-10$

$0=x^2+13x-90$

$x=-\frac{13}{2}\pm\sqrt{\frac{(13)^2}{4}-(-90)}$

Continúa

9. Continuación

$$x=-\frac{13}{2}\pm\sqrt{\frac{169}{4}+90}$$

$$x=-\frac{13}{2}\pm\sqrt{\frac{529}{4}}$$

$$x=\frac{-13}{2}\pm\frac{23}{2}$$

$$x_1=\frac{-13}{2}+\frac{23}{2}=\frac{10}{2}=5$$

$$x_2=\frac{-13}{2}-\frac{23}{2}=\frac{-36}{2}=-18$$

10. $\frac{x-13}{x}=5-\frac{10(5x+3)}{x^2}$ $\quad mcm=x^2$

$$x(x-13)=5x^2-10(5x+3)$$

$$x^2-13x=5x^2-50x-30$$

$$-4x^2+37x+30=0$$

$$x=\frac{-37\pm\sqrt{(37)^2-4\,(-4)(30)}}{2(-4)}$$

$$x=\frac{-37\pm\sqrt{1.369+480}}{-8}$$

$$x=\frac{37\pm\sqrt{1.849}}{-8}$$

$$x=\frac{-37\pm 43}{-8}$$

$$x_1=\frac{-37+43}{-8}=\frac{6}{-8}=-\frac{3}{4}$$

$$x_2=\frac{-37-43}{-8}=\frac{-80}{-8}=10$$

11. $\frac{x}{x-2}-\frac{x-2}{x}=\frac{5}{2}$ $\quad mcm=2x(x-2)$

$$2x(x)-2(x-2)(x-2)=5x(x-2)$$

$$2x^2-2(x^2-4x+4)=5x^2-10x$$

$$2x^2-2x^2+8x-8=5x^2-10x$$

$$0=5x^2-18x+8$$

$$x=\frac{-(-18)\pm\sqrt{(-18)^2-4(5)(8)}}{2(5)}$$

$$x=\frac{18\pm\sqrt{324-160}}{10}$$

$$x=\frac{18\pm\sqrt{164}}{10}$$

$$x=\frac{18\pm\sqrt{2^2\cdot 41}}{10}=\frac{18\pm 2\sqrt{41}}{10}=\frac{9\pm\sqrt{41}}{5}$$

$$x_1=\frac{9+\sqrt{41}}{5}\quad;\quad x_2=\frac{9-\sqrt{41}}{5}$$

12. $\frac{4x^2}{x-1}-\frac{1-3x}{4}=\frac{20x}{3}$ $\quad mcm=12(x-1)$

$$12(4x^2)-3(x-1)(1-3x)=4(x-1)(20x)$$

$$48x^2+9x^2-12x+3=80x^2-80x$$

$$-23x^2+68x+3=0$$

$$x=\frac{-68\pm\sqrt{(68)^2-4(-23)(3)}}{2(-23)}$$

$$x=\frac{-68\pm\sqrt{4.624+276}}{-46}=\frac{-68\pm\sqrt{4.900}}{-46}=\frac{-68\pm 70}{-46}$$

$$x_1=\frac{-68+70}{-46}=\frac{2}{-46}=-\frac{1}{23}\quad;\quad x_2=\frac{-68-70}{-46}=\frac{-138}{-46}=3$$

13. $\frac{3x-1}{x}-\frac{2x}{2x-1}-\frac{7}{6}=0$ $\quad mcm=6x(2x-1)$

$$6(2x-1)(3x-1)-6x(2x)-7x(2x-1)=0$$

$$6(6x^2-5x+1)-12x^2-14x^2+7x=0$$

$$36x^2-30x+6-12x^2-14x^2+7x=0$$

$$10x^2-23x+6=0$$

$$x=\frac{23\pm\sqrt{(23)^2-4(10)(6)}}{2(10)}$$

$$x=\frac{23\pm\sqrt{529-240}}{20}=\frac{23\pm\sqrt{289}}{20}=\frac{23\pm 17}{20}$$

$$x_1=\frac{23+17}{20}=\frac{40}{20}=2\quad;\quad x_2=\frac{23-17}{20}=\frac{6}{20}=\frac{3}{10}$$

14. $\frac{5x-8}{x-1}=\frac{7x-4}{x+2}$

$$(x+2)(5x-8)=(7x-4)(x-1)$$

$$5x^2+2x-16=7x^2-11x+4$$

$$0=2x^2-13x+20$$

$$x=\frac{13\pm\sqrt{(13)^2-4(2)(20)}}{2(2)}$$

$$x=\frac{13\pm\sqrt{169-160}}{4}=\frac{13\pm\sqrt{9}}{4}=\frac{13\pm 3}{4}$$

$$x_1=\frac{13+3}{4}=\frac{16}{4}=4\quad;\quad x_2=\frac{13-3}{4}=\frac{10}{4}=\frac{5}{2}=2\tfrac{1}{2}$$

15. $\dfrac{x+3}{2x-1}-\dfrac{5x-1}{4x+7}=0$

$mcm=(2x-1)(4x+7)$

$(x+3)(4x+7)-(5x-1)(2x-1)=0$

$4x^2+19x+21-10x^2+7x-1=0$

$-6x^2+26x+20=0$

$3x^2-13x-10=0$

$x=\dfrac{13\pm\sqrt{(-13)^2-4(3)(-10)}}{2(3)}$

$x=\dfrac{13\pm\sqrt{169+120}}{6}=\dfrac{13\pm\sqrt{289}}{6}=\dfrac{13\pm17}{6}$

$x_1=\dfrac{13+17}{6}=\dfrac{30}{6}=5$; $x_2=\dfrac{13-17}{6}=\dfrac{-4}{6}=-\dfrac{2}{3}$

18. $\dfrac{5}{x^2-1}-\dfrac{6}{x+1}=3\tfrac{5}{8}$ $\quad mcm=8(x^2-1)$

$40-48(x-1)=29(x^2-1)$

$40-48x+48=29x^2-29$

$-29x^2-48x+117=0$

$x=\dfrac{-(-48)\pm\sqrt{(48^2)-4(-29)(117)}}{2(-29)}$

$x=\dfrac{48\pm\sqrt{2.304+13.572}}{-58}=\dfrac{48\pm126}{-58}$

$x_1=\dfrac{48+126}{-58}=\dfrac{174}{-58}=-3$; $x_2=\dfrac{48-126}{-58}=\dfrac{-78}{-58}=1\tfrac{10}{29}$

16. $\dfrac{1}{4-x}-\dfrac{1}{6}=\dfrac{1}{x+1}$

$mcm=6(4-x)(x+1)$

$6(x+1)-(4-x)(x+1)=6(4-x)$

$6x+6+x^2-3x-4=24-6x$

$x^2+3x+2=24-6x$

$x^2+9x-22=0$

$x=-\dfrac{9}{2}\pm\sqrt{\dfrac{(9)^2}{4}-(-22)}$

$x=-\dfrac{9}{2}\pm\sqrt{\dfrac{81}{4}+22}=-\dfrac{9}{2}\pm\sqrt{\dfrac{169}{4}}=\dfrac{-9}{2}\pm\dfrac{13}{2}$

$x_1=\dfrac{-9}{2}+\dfrac{13}{2}=\dfrac{4}{2}=2$; $x_2=\dfrac{-9}{2}-\dfrac{13}{2}=\dfrac{-22}{2}=-11$

19. $\dfrac{x-1}{x+1}+\dfrac{x+1}{x-1}=\dfrac{2x+9}{x+3}$ $\quad mcm=(x+1)(x-1)(x+3)$

$(x-1)(x-1)(x+3)+(x+1)(x+1)(x+3)=(2x+9)(x+1)(x-1)$

$(x^2-2x+1)(x+3)+(x^2+2x+1)(x+3)=(2x+9)(x^2-1)$

$x^3+3x^2-2x^2-6x+x+3+x^3+3x^2+2x^2+6x+x+3=2x^3-2x+9x^2-9$

$2x^3+6x^2+2x+6=2x^3+9x^2-2x-9$

$0=3x^2-4x-15$

$x=\dfrac{4\pm\sqrt{(4)^2-4(3)(-15)}}{2(3)}$

$x=\dfrac{4\pm\sqrt{16+180}}{6}=\dfrac{4\pm\sqrt{196}}{6}=\dfrac{4\pm14}{6}$

$x_1=\dfrac{4+14}{6}=\dfrac{18}{6}=3$ $\quad x_2=\dfrac{4-14}{6}=\dfrac{-10}{6}=-1\tfrac{2}{3}$

17. $\dfrac{x+4}{x+5}-\dfrac{x+2}{x+3}=\dfrac{1}{24}$ $\quad mcm=24(x+5)(x+3)$

$24(x+3)(x+4)-24(x+2)(x+5)=(x+5)(x+3)$

$24x^2+168x+288-24x^2-168x-240=x^2+8x+15$

$48=x^2+8x+15$

$0=x^2+8x-33$

$x=-\dfrac{8}{2}\pm\sqrt{\dfrac{(8)^2}{4}-(-33)}$

$x=-4\pm\sqrt{16+33}=-4\pm\sqrt{49}=-4\pm7$

$x_1=-4+7=3$; $x_2=-4-7=-11$

20. $\dfrac{3}{x+2}-\dfrac{1}{x-2}=\dfrac{1}{x+1}$ $\quad mcm=(x+2)(x-2)(x+1)$

$3(x-2)(x+1)-(x+2)(x+1)=(x+2)(x-2)$

$3x^2-3x-6-x^2-3x-2=x^2-4$

$2x^2-6x-8=x^2-4$

$x^2-6x-4=0$

$x=-\dfrac{(-6)}{2}\pm\sqrt{\dfrac{(-6)^2}{4}-(-4)}=3\pm\sqrt{9+4}=3\pm\sqrt{13}$

$x_1=3+\sqrt{13}$; $x_2=3-\sqrt{13}$

EJERCICIO 269

1. $x^2-x-6=0$

$(x-3)(x+2)=0$

$x_1=3 \quad x_2=-2$

2. $x^2+7x=18$

$x^2+7x-18=0$

$(x+9)(x-2)=0$

$x_1=-9 \quad x_2=2$

3. $8x-65=-x^2$

$x^2+8x-65=0$

$(x+13)(x-5)=0$

$x_1=-13 \quad x_2=5$

4. $x^2=108-3x$

$x^2+3x-108=0$

108	2	
54	2	$2^2\cdot 3-3^2$
27	3	$12-9=3$
9	3	
3	3	
1		

$(x+12)(x-9)=0$

$x_1=-12 \quad x_2=9$

5. $2x^2+7x-4=0$

$(2x)^2+7(2x)-8=0$

$$\frac{(2x+8)(2x-1)}{2}=0$$

$(x+4)(2x\ 1)=0$

$x+4=0 \quad 2x-1=0$

$x_1=-4 \quad x_2=\frac{1}{2}$

6. $6x^2=10-11x$

$6x^2+11x-10=0$

$(6x)^2+11(6x)-60=0$

$$\frac{(6x+15)(6x-4)}{3\cdot 2}=0$$

$(2x+5)(3x-2)=0$

$2x+5=0 \qquad 3x-2=0$

$x_1=-\frac{5}{2}=-2\tfrac{1}{2} \quad x_2=\frac{2}{3}$

7. $20x^2-27x=14$

$20x^2-27x-14=0$

$(20x)^2-27(20x)-280=0$

$$\frac{(20x-35)(20x+8)}{5\cdot 4}=0$$

$(4x-7)(5x+2)=0$

$x_1=\frac{7}{4}=1\tfrac{3}{4} \qquad x_2=-\frac{2}{5}$

8. $7x=15-30x^2$

$30x^2+7x-15=0$

$(30x)^2+7(30x)-450=0$

$$\frac{(30x+25)(30x-18)}{5\cdot 6}=0$$

$(6x+5)(5x-3)=0$

$6x+5=0 \qquad 5x-3=0$

$6x=-5 \qquad 5x=3$

$x_1=\frac{-5}{6} \qquad x_2=\frac{3}{5}$

9. $60=8x^2+157x$

$8x^2+157x-60=0$

$(8x)^2+157(8x)-480=0$

$$\frac{(8x+160)(8x-3)}{8}=0$$

$(x+20)(8x-3)=0$

$x+20=0 \qquad 8x-3=0$

$x_1=-20 \qquad 8x=3$

$x_2=\frac{3}{8}$

10. $x(x-1)-5(x-2)=2$

$x^2-x-5x+10-2=0$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x-4=0 \qquad x-2=0$

$x_1=4 \qquad x_2=2$

11. $(x-2)^2-(2x+3)^2=-80$

$x^2-4x+4-4x^2-12x-9+80=0$

$-3x^2-16x+75=0$

$3x^2+16x-75=0$

$(3x)^2+16(3x)-225=0$

$$\frac{(3x+25)(3x-9)}{3}=0$$

Continúa

11. Continuación

$(3x+25)(x-3)=0$

$3x+25=0 \qquad x-3=0$

$3x=-25 \qquad x_2=3$

$x_1=-\frac{25}{3}=-8\tfrac{1}{3}$

12. $\frac{6}{x^2}-\frac{9}{x}=-\frac{4}{3} \qquad mcm=3x^2$

$18-27x=-4x^2$

$4x^2-27x+18=0$

$(4x)^2-27(4x)+72=0$

$$\frac{(4x-24)(4x-3)}{4}=0$$

$(x-6)(4x-3)=0$

$x-6=0 \qquad 4x-3=0$

$x_1=6 \qquad 4x=3$

$x_2=\frac{3}{4}$

13. $\frac{x+2}{x}+x=\frac{74}{x} \qquad mcm=x$

$x+2+x^2=74$

$x^2+x-72=0$

$(x+9)(x-8)=0$

$x+9=0 \quad x-8=0$

$x_1=-9 \qquad x_2=8$

14. $(x+2)^2-\frac{2x-5}{3}=3 \quad mcm=3$

$3(x+2)^2-(2x-5)=9$

$3x^2+12x+12-2x+5-9=0$

$3x^2+10x+8=0$

$(3x)^2+10(3x)+24=0$

$$\frac{(3x+6)(3x+4)}{3}=0$$

$(x+2)(3x+4)=0$

$x+2=0 \qquad 3x+4=0$

$x_1=-2 \qquad 3x=-4$

$x_2=\frac{-4}{3}=-1\tfrac{1}{3}$

15. $\frac{x}{x-2}+x=\frac{3x+15}{4} \quad mcm=4(x-2)$

$4x+4x(x-2)=(3x+15)(x-2)$

$4x+4x^2-8x=3x^2+9x-30$

$x^2-13x+30=0$

$(x-10)(x-3)=0$

$x-10=0 \qquad x-3=0$

$x_1=10 \qquad x_2=3$

16. $\frac{6}{x-4}-\frac{4}{x}=\frac{5}{12}\quad mcm=12x\left(x-4\right)$

$72x-48\left(x-4\right)=5x\left(x-4\right)$

$72x-48x+192=5x^2-20x$

$5x^2-44x-192=0$

$(5x)^2-44(5x)-960=0$

$\frac{\left(5x-60\right)\left(5x+16\right)}{5}=0$

$\left(x-12\right)\left(5x+16\right)=0$

$x-12=0\quad 5x+16=0$

$x_1=12\quad 5x=-16$

$x_2=\frac{-16}{5}=-3\tfrac{1}{5}$

17. $\left(x-2\right)^3-\left(x-3\right)^3=37$

$x^3-6x^2+12x-8-x^3+9x^2-27x+27-37=0$

$3x^2-15x-18=0$

$(3x)^2-15(3x)-54=0$

$\left(3x-18\right)\left(3x+3\right)=0$

$3x-18=0\quad 3x+3=0$

$3x=18\quad 3x=-3$

$x_1=6\quad x_2=-1$

18. $\frac{x-1}{x+1}-2=\frac{x+3}{3}\quad mcm=3\left(x+1\right)$

$3\left(x-1\right)-6\left(x+1\right)=\left(x+3\right)\left(x+1\right)$

$3x-3-6x-6=x^2+4x+3$

$x^2+7x+12=0$

$\left(x+4\right)\left(x+3\right)=0$

$x+4=0\quad x+3=0$

$x_1=-4\quad x_2=-3$

19. $\frac{4x-1}{2x+3}=\frac{2x+1}{6x+5}$

$\left(6x+5\right)\left(4x-1\right)=\left(2x+1\right)\left(2x+3\right)$

$24x^2+14x-5=4x^2+8x+3$

$20x^2+6x-8=0$

$10x^2+3x-4=0$

$(10x)^2+3(10x)-40=0$

$\frac{\left(10x+8\right)\left(10x-5\right)}{2\cdot 5}=0$

$\left(5x+4\right)\left(2x-1\right)=0$

$5x+4=0\quad 2x-1=0$

$5x=-4\quad 2x=1$

$x_1=\frac{-4}{5}\quad x_2=\frac{1}{2}$

20. $\frac{3x+2}{4}=5-\frac{9x+14}{12x}\quad mcm=12x$

$3x\left(3x+2\right)=60x-\left(9x+14\right)$

$9x^2+6x=60x-9x-14$

$9x^2-45x+14=0$

$(9x)^2-45(9x)+126=0$

$\frac{\left(9x-42\right)\left(9x-3\right)}{3\cdot 3}=0$

$\left(3x-14\right)\left(3x-1\right)=0$

$3x-14=0\quad 3x-1=0$

$3x=14\quad 3x=1$

$x_1=\frac{14}{3}\quad x_2=\frac{1}{3}$

$x_1=4\tfrac{2}{3}$

EJERCICIO 270

1. $x^2+2ax-35a^2=0$

$\left(x+7a\right)\left(x-5a\right)=0$

$x+7a=0\quad x-5a=0$

$x_1=-7a\quad x_2=5a$

2. $10x^2=36a^2-37ax$

$10x^2+37ax-36a^2=0$

$x=\frac{-37a\pm\sqrt{(37a)^2-4(10)\left(-36a^2\right)}}{2(10)}$

$x=\frac{-37a\pm\sqrt{2.809a^2}}{20}=\frac{-37a\pm 53a}{20}$

$x_1=\frac{-37a+53a}{20}=\frac{16a}{20}=\frac{4a}{5}$

$x_2=\frac{-37a-53a}{20}=\frac{-90a}{20}=\frac{-9a}{2}$

3. $a^2x^2+abx-2b^2=0$

$\left(a^2x\right)^2+ab\left(a^2x\right)-2a^2b^2=0$

$\left(a^2x+2ab\right)\left(a^2x-ab\right)=0$

$a^2x+2ab=0\quad a^2x-ab=0$

$a^2x=-2ab\quad a^2x=ab$

$x_1=\frac{-2ab}{a^2}\quad x_2=\frac{ab}{a^2}$

$x_1=-\frac{2b}{a}\quad x_2=\frac{b}{a}$

4. $$89bx = 42x^2 + 22b^2$$

$$42x^2 - 89bx + 22b^2 = 0$$

$$x = \frac{-(-89b) \pm \sqrt{(-89b)^2 - 4(42)(22b^2)}}{2(42)}$$

$$x = \frac{89 \pm \sqrt{4.225b^2}}{84} = \frac{89b \pm 65b}{84}$$

$$x_1 = \frac{89b - 65b}{84} = \frac{24b}{84} = \frac{2b}{7}$$

$$x_2 = \frac{89b + 65b}{84} = \frac{154b}{84} = \frac{11b}{6}$$

5. $$x^2 + ax = 20a^2$$

$$x^2 + ax - 20a^2 = 0$$

$$(x + 5a)(x - 4a) = 0$$

$$x + 5a = 0 \qquad x - 4a = 0$$

$$x_1 = -5a \qquad x_2 = 4a$$

6. $$2x^2 = abx + 3a^2b^2$$

$$2x^2 - abx - 3a^2b^2 = 0$$

$$(2x)^2 - ab(2x) - 6a^2b^2 = 0$$

$$\frac{(2x - 3ab)(2x + 2ab)}{2} = 0$$

$$(2x - 3ab)(x + ab) = 0$$

$$2x - 3ab = 0 \qquad x + ab = 0$$

$$2x = 3ab \qquad x = -ab$$

$$x_1 = \frac{3ab}{2} \qquad x_2 = -ab$$

7. $$b^2x^2 + 2abx = 3a^2$$

$$b^2x^2 + 2abx - 3a^2 = 0$$

$$(b^2x)^2 + 2ab(b^2x) - 3a^2b^2 = 0$$

$$\frac{(b^2x + 3ab)(b^2x - ab)}{b \cdot b} = 0$$

$$(bx + 3a)(bx - a) = 0$$

$$bx + 3a = 0 \qquad bx - a = 0$$

$$bx = -3a \qquad bx = a$$

$$x_1 = -\frac{3a}{b} \qquad x_2 = \frac{a}{b}$$

8. $$x^2 + ax - bx = ab$$

$$x^2 + ax - bx - ab = 0$$

$$x(x + a) - b(x + a) = 0$$

$$(x - b)(x + a) = 0$$

$$x - b = 0 \qquad x + a = 0$$

$$x_1 = b \qquad x_2 = -a$$

9. $$x^2 - 2ax = 6ab - 3bx$$

$$x^2 + 3bx - 2ax - 6ab = 0$$

$$x(x + 3b) - 2a(x + 3b) = 0$$

$$(x + 3b)(x - 2a) = 0$$

$$x + 3b = 0 \qquad x - 2a = 0$$

$$x_1 = -3b \qquad x_2 = 2a$$

10. $$3(2x^2 - mx) + 4nx - 2mn = 0$$

$$3x(2x - m) + 2n(2x - m) = 0$$

$$(2x - m)(3x + 2n) = 0$$

$$2x - m = 0 \qquad 3x + 2n = 0$$

$$2x = m \qquad 3x = -2n$$

$$x_1 = \frac{m}{2} \qquad x_2 = -\frac{2n}{3}$$

11. $$x^2 - a^2 - bx - ab = 0$$

$$(x^2 - a^2) - b(x + a) = 0$$

$$(x + a)(x - a) - b(x + a) = 0$$

$$(x - a - b)(x + a) = 0$$

$$x - a - b = 0 \qquad x + a = 0$$

$$x_1 = a + b \qquad x_2 = -a$$

12. $$abx^2 - x(b - 2a) = 2$$

$$abx^2 - bx + 2ax - 2 = 0$$

$$abx^2 + 2ax - bx - 2 = 0$$

$$ax(bx + 2) - (bx + 2) = 0$$

$$(bx + 2)(ax - 1) = 0$$

$$bx + 2 = 0 \qquad ax - 1 = 0$$

$$bx = -2 \qquad ax = 1$$

$$x_1 = \frac{-2}{b} \qquad x_2 = \frac{1}{a}$$

13. $$x^2 - 2ax + a^2 - b^2 = 0$$

$$x = \frac{-(-2a) \pm \sqrt{(2a)^2 - 4(a^2 - b^2)}}{2}$$

$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b^2}}{2} = \frac{2a \pm 2b}{2}$$

$$x_1 = \frac{2a + 2b}{2} = \frac{2(a + b)}{2} = a + b$$

$$x_2 = \frac{2a - 2b}{2} = \frac{2(a - b)}{2} = a - b$$

14. $4x(x-b)+b^2=4m^2$

$4x^2-4bx+b^2-4m^2=0$

$$x=\frac{-(-4b)\pm\sqrt{(-4b)^2-4(4)(b^2-4m^2)}}{2(4)}$$

$$x=\frac{4b\pm\sqrt{16b^2-16b^2+64m^2}}{8}$$

$$x=\frac{4b\pm\sqrt{64m^2}}{8}=\frac{4b\pm 8m}{8}$$

$$x_1=\frac{4b+8m}{8}=\frac{4(b+2m)}{8}=\frac{b+2m}{2}$$

$$x_2=\frac{4b-8m}{8}=\frac{4(b-2m)}{8}=\frac{b-2m}{2}$$

15. $x^2-b^2+4a^2-4ax=0$

$x^2-4ax-b^2+4a^2=0$

$$x=\frac{-(-4a)\pm\sqrt{(-4a)^2-4(-b^2+4a^2)}}{2}$$

$$x=\frac{4a\pm\sqrt{16a^2+4b^2-16a^2}}{2}=\frac{4a\pm 2b}{2}$$

$$x_1=\frac{4a+2b}{2}=\frac{2(2a+b)}{2}=2a+b$$

$$x_2=\frac{4a-2b}{2}=\frac{2(2a-b)}{2}=2a-b$$

16. $x^2-(a+2)x=-2a$

$x^2-ax-2x+2a=0$

$x(x-a)-2(x-a)=0$

$(x-a)(x-2)=0$

$x-a=0 \qquad x-2=0$

$x_1=a \qquad x_2=2$

17. $x^2+2x(4-3a)=48a$

$x^2+8x-6ax-48a=0$

$x(x+8)-6a(x+8)=0$

$(x+8)(x-6a)=0$

$x+8=0 \qquad x-6a=0$

$x_1=-8 \qquad x_2=6a$

18. $x^2-2x=m^2+2m$

$x^2-m^2-2x-2m=0$

$(x+m)(x-m)-2(x+m)=0$

$(x-m-2)(x+m)=0$

$x-m-2=0 \qquad x+m=0$

$x_1=m+2 \qquad x_2=-m$

19. $x^2+m^2x(m-2)=2m^5$

$x^2+xm^3-2xm^2-2m^5=0$

$x(x+m^3)-2m^2(x+m^3)=0$

$(x+m^3)(x-2m^2)=0$

$x+m^3=0 \qquad x-2m^2=0$

$x_1=-m^3 \qquad x_2=2m^2$

20. $6x^2-15ax=2bx-5ab$

$6x^2-2bx-15ax+5ab=0$

$2x(3x-b)-5a(3x-b)=0$

$(3x-b)(2x-5a)=0$

$3x-b=0 \qquad 2x-5a=0$

$3x=b \qquad 2x=5a$

$x_1=\frac{b}{3} \qquad x_2=\frac{5a}{2}$

21. $\frac{3x}{4}+\frac{a}{2}-\frac{x^2}{2a}=0 \qquad mcm=4a$

$3ax+2a^2-2x^2=0$

$2x^2-3ax-2a^2=0$

$(2x)^2-3a(2x)-4a^2=0$

$$\frac{(2x-4a)(2x+a)}{2}=0$$

$(x-2a)(2x+a)=0$

$x-2a=0 \qquad 2x+a=0$

$x_1=2a \qquad 2x=-a$

$x_2=-\frac{a}{2}$

22. $\frac{2x-b}{2}=\frac{2bx-b^2}{3x}$

$3x(2x-b)=2(2bx-b^2)$

$6x^2-3bx-4bx+2b^2=0$

$3x(2x-b)-2b(2x-b)=0$

$(2x-b)(3x-2b)=0$

$2x-b=0 \qquad 3x-2b=0$

$2x=b \qquad 3x=2b$

$x_1=\frac{b}{2} \qquad x_2=\frac{2b}{3}$

23. $\frac{a+x}{a-x}+\frac{a-2x}{a+x}=-4 \quad mcm=a^2-x^2$

$(a+x)(a+x)+(a-2x)(a-x)=-4(a^2-x^2)$

$a^2+2ax+x^2+a^2-3ax+2x^2=-4a^2+4x^2$

$6a^2-ax-x^2=0$

$x^2+ax-6a^2=0$

$(x+3a)(x-2a)=0$

$x+3a=0 \qquad x-2a=0$

$x_1=-3a \qquad x_2=2a$

24. $\frac{x^2}{x-1}=\frac{a^2}{2(a-2)}$

$2(a-2)x^2=a^2(x-1)$

$x^2(2a-4)=a^2x-a^2$

$x^2(2a-4)-a^2x+a^2=0$

$x=\frac{-(-a^2)\pm\sqrt{(-a^2)^2-4(2a-4)(a^2)}}{2(2a-4)}$

$x=\frac{a^2\pm\sqrt{a^4-8a^3+16a^2}}{2(2a-4)}$

$x=\frac{a^2\pm\sqrt{a^2(a^2-8a+16)}}{4a-8}$

$x=\frac{a^2\pm\sqrt{a^2(a-4)^2}}{4(a-2)}=\frac{a^2\pm a^2-4a}{4(a-2)}$

$x_1=\frac{a^2+a^2-4a}{4(a-2)}=\frac{2a^2-4a}{4(a-2)}=\frac{2a(a-2)}{4(a-2)}=\frac{a}{2}$

$x_2=\frac{a^2-(a^2-4a)}{4(a-2)}=\frac{4a}{4(a-2)}=\frac{a}{a-2}$

25. $x+\frac{2}{x}=\frac{1}{a}+2a \quad mcm=ax$

$ax^2+2a=x+2a^2x$

$ax^2-x+2a-2a^2x=0$

$x(ax-1)-2a(ax-1)=0$

$(ax-1)(x-2a)=0$

$ax-1=0 \qquad x-2a=0$

$ax=1 \qquad x=2a$

$x_1=\frac{1}{a} \qquad x_2=2a$

26. $\frac{2x-b}{b}-\frac{x}{x+b}=\frac{2x}{4b} \quad mcm=4b(x+b)$

$4(2x-b)(x+b)-4bx=2x(x+b)$

$8x^2+4bx-4b^2-4bx=2x^2+2bx$

$6x^2-2bx-4b^2=0$

$3x^2-bx-2b^2=0$

$(3x)^2-b(3x)-6b^2=0$

$\frac{(3x-3b)(3x+2b)}{3}=0$

$(x-b)(3x+2b)=0$

$x-b=0 \qquad 3x+2b=0$

$x_1=b \qquad 3x=-2b$

$x_2=-\frac{2b}{3}$

EJERCICIO 271

1. $3x^2=48$

$x^2=16$

$x=\sqrt{16}$

$x=\pm 4$

2. $5x^2-9=46$

$5x^2=55$

$x^2=11$

$x=\sqrt{11}$

$x=\pm\sqrt{11}$

3. $7x^2+14=0$

$x^2+2=0$

$x^2=-2$

$x=\sqrt{-2}$

$x=\sqrt{2}\sqrt{-1}$

$x=\pm\sqrt{2}\,i$

4. $9x^2-a^2=0$

$9x^2=a^2$

$x^2=\frac{a^2}{9}$

$x=\sqrt{\frac{a^2}{9}}$

$x=\pm\frac{a}{3}$

5. $(x+5)(x-5)=-7$

$x^2-25=-7$

$x^2=18$

$x=\sqrt{18}$

$x=\sqrt{3^2\cdot 2}$

$x=\pm 3\sqrt{2}$

6. $(2x-3)(2x+3)-135=0$

$$4x^2-9-135=0$$
$$4x^2=144$$
$$x^2=36$$
$$x=\sqrt{36}$$
$$x=\pm 6$$

7. $3(x+2)(x-2)=(x-4)^2+8x$

$$3x^2-12=x^2-8x+16+8x$$
$$2x^2-28=0$$
$$2x^2=28$$
$$x^2=14$$
$$x=\pm\sqrt{14}$$

8. $\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)=\frac{1}{3}$

$$x^2-\frac{1}{9}-\frac{1}{3}=0$$
$$x^2=\frac{4}{9}$$
$$x=\pm\sqrt{\frac{4}{9}}$$
$$x=\pm\frac{2}{3}$$

9. $(2x-1)(x+2)-(x+4)(x-1)+5=0$

$$2x^2+3x-2-x^2-3x+4+5=0$$
$$x^2+7=0$$
$$x^2=-7$$
$$x=\sqrt{7}\sqrt{-1}$$
$$x=\pm\sqrt{7}\,i$$

10. $\frac{5}{2x^2}-\frac{1}{6x^2}=\frac{7}{12}$ $\quad mcm=12x^2$

$$30-2=7x^2$$
$$28=7x^2$$
$$4=x^2$$
$$\sqrt{4}=x$$
$$\pm 2=x$$

11. $\frac{2x-3}{x-3}=\frac{x-2}{x-1}$ $\quad mcm=(x-3)(x-1)$

$$(2x-3)(x-1)=(x-2)(x-3)$$
$$2x^2-5x+3=x^2-5x+6$$
$$x^2-3=0$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

12. $\frac{x^2-5}{3}+\frac{4x^2-1}{5}-\frac{14x^2-1}{15}=0$

$$5x^2-25+12x^2-3-14x^2+1=0$$
$$3x^2-27=0$$
$$3x^2=27$$
$$x^2=9$$
$$x=\sqrt{9}$$
$$x=\pm 3$$

13. $2x-3-\frac{x^2+1}{x-2}=-7$ $\quad mcm=x-2$

$$2x(x-2)-3(x-2)-(x^2+1)=-7(x-2)$$
$$2x^2-4x-3x+6-x^2-1=-7x+14$$
$$x^2+5=14$$
$$x^2=9$$
$$x=\sqrt{9}$$
$$x=\pm 3$$

14. $3-\frac{3}{4x^2-1}=2$ $\quad mcm=4x^2-1$

$$3(4x^2-1)-3=2(4x^2-1)$$
$$12x^2-6=8x^2-2$$
$$4x^2=4$$
$$x^2=1$$
$$x=\sqrt{1}$$
$$x=\pm 1$$

EJERCICIO 272

1. $x^2=5x$

$$x^2-5x=0$$
$$x(x-5)=0$$
$$x_1=0 \quad x-5=0$$
$$x_2=5$$

2. $4x^2=-32x$

$$4x^2+32x=0$$
$$4x(x+8)=0$$
$$4x=0 \quad x+8=0$$
$$x_1=0 \quad x_2=-8$$

3. $x^2-3x=3x^2-4x$

$$0=2x^2-x$$
$$0=x(2x-1)$$
$$0=x_1 \quad 0=2x-1$$
$$1=2x\Rightarrow\frac{1}{2}=x_2$$

4. $5x^2+4=2(x+2)$

$5x^2+4=2x+4$

$5x^2-2x=0$

$x(5x-2)=0$

$x_1=0 \quad 5x-2=0$

$5x=2$

$x_2=\frac{2}{5}$

5. $(x-3)^2-(2x+5)^2=-16$

$x^2-6x+9-4x^2-20x-25+16=0$

$-3x^2-26x=0$

$x(3x+26)=0$

$x_1=0 \quad 3x+26=0$

$3x=-26$

$x_2=\frac{-26}{3}=-8\frac{2}{3}$

6. $\frac{x^2}{3}-\frac{x-9}{6}=\frac{3}{2} \quad mcm=12$

$4x^2-2x+18=18$

$2x(2x-1)=0$

$2x=0 \quad 2x-1=0$

$x_1=0 \quad 2x=1$

$x_2=\frac{1}{2}$

7. $(4x-1)(2x+3)=(x+3)(x-1)$

$8x^2+10x-3=x^2+2x-3$

$7x^2+8x=0$

$x(7x+8)=0$

$x_1=0 \quad 7x+8=0$

$7x=-8$

$x_2=-\frac{8}{7}$

$x_2=-1\frac{1}{7}$

8. $\frac{x+1}{x-1}-\frac{x+4}{x-2}=1 \quad mcm=(x-1)(x-2)$

$(x+1)(x-2)-(x+4)(x-1)=(x-1)(x-2)$

$x^2-x-2-x^2-3x+4=x^2-3x+2$

$0=x^2+x$

$0=x(x+1)$

$x_1=0 \quad x+1=0$

$x_2=-1$

EJERCICIO 273

1. $x+\sqrt{4x+1}=5$

$\sqrt{4x+1}=5-x$

$\left(\sqrt{4x+1}\right)^2=(5-x)^2$

$4x+1=25-10x+x^2$

$0=x^2-14x+24$

$0=(x-12)(x-2)$

$x-12=0$

$x_1=12 \rightarrow \text{Re}\textit{chazo sol extraña}$

$x-2=0$

$x_2=2 \rightarrow \textit{sol que satisface}$

2. $2x-\sqrt{x-1}=3x-7$

$7-x=\sqrt{x-1}$

$(7-x)^2=\sqrt{(x-1)^2}$

$49-14x+x^2=x-1$

$x^2-15x+50=0$

$(x-10)(x-5)=0$

$x-10=0$

$x_1=10 \rightarrow \text{Re}\textit{chazo sol extraña}$

$x-5=0$

$x_2=5 \rightarrow \textit{sol que satisface}$

3. $\sqrt{5x-1}+\sqrt{x+3}=4$

$\left(\sqrt{5x-1}+\sqrt{x+3}\right)^2=16$

$5x-1+2\sqrt{5x^2+14x-3}+x+3=16$

$2\sqrt{5x^2+14x-3}=14-6x$

$\left(\sqrt{5x^2+14x-3}\right)^2=(7-3x)^2$

$5x^2+14x-3=49-42x+9x^2$

$0=4x^2-56x+52$

$0=x^2-14x+13$

$0=(x-13)(x-1)$

$x-13=0$

$x_1=13 \rightarrow \text{Re}\textit{chazo por sol extraña}$

$x-1=0$

$x_2=1 \rightarrow \textit{sol que satisface}$

4. $2\sqrt{x}-\sqrt{x+5}=1$

$\left(2\sqrt{x}\right)^2=\left(1+\sqrt{x+5}\right)^2$

$4x=1+2\sqrt{x+5}+x+5$

$3x-6=2\sqrt{x+5}$

$\left(3x-6\right)^2=\left(2\sqrt{x+5}\right)^2$

$9x^2-36x+36=4x+20$

$9x^2-40x+16=0$

$(9x)^2-40(9x)+144=0$

$\dfrac{\left(9x-36\right)\left(9x-4\right)}{9}=0$

$\left(x-4\right)\left(9x-4\right)=0$

$x-4=0$

$x_1=4 \quad \rightarrow \textit{sol que satisface}$

$9x-4=0$

$9x=4$

$x_2=\dfrac{4}{9} \rightarrow \text{Re}\textit{chazo por sol extraña}$

5. $\sqrt{2x-1}+\sqrt{x+3}=3$

$\left(\sqrt{2x-1}\right)^2=\left(3-\sqrt{x+3}\right)^2$

$2x-1=9-6\sqrt{x+3}+x+3$

$x-13=-6\sqrt{x+3}$

$\left(6\sqrt{x+3}\right)^2=\left(13-x\right)^2$

$36\left(x+3\right)=169-26x+x^2$

$36x+108=169-26x+x^2$

$0=x^2-62x+61$

$0=\left(x-61\right)\left(x-1\right)$

$x-61=0$

$x_1=61 \quad \rightarrow \text{Re}\textit{chazo por sol extraña}$

$x-1=0$

$x_2=1 \rightarrow \textit{sol que satisface}$

6. $\sqrt{x-3}+\sqrt{2x+1}-2\sqrt{x}=0$

$\left(\sqrt{x-3}+\sqrt{2x+1}\right)^2=\left(2\sqrt{x}\right)^2$

$x-3+2\sqrt{2x^2-5x-3}+2x+1=4x$

$\left(2\sqrt{2x^2-5x-3}\right)^2=\left(x+2\right)^2$

$4\left(2x^2-5x-3\right)=x^2+4x+4$

$8x^2-20x-12=x^2+4x+4$

Continúa

6. Continuación

$7x^2-24x-16=0$

$(7x)^2-24(7x)-112=0$

$\dfrac{\left(7x-28\right)\left(7x+4\right)}{7}=0$

$\left(x-4\right)\left(7x+4\right)=0$

$x-4=0 \Rightarrow x_1=4 \quad \rightarrow \textit{sol factible}$

$7x+4=0$

$7x=-4 \Rightarrow x_2=-\dfrac{4}{7} \rightarrow \textit{sol inadmisible}$

7. $\sqrt{5x-1}-\sqrt{3-x}=\sqrt{2x}$

$\left(\sqrt{5x-1}-\sqrt{2x}\right)^2=\left(\sqrt{3-x}\right)^2$

$5x-1-2\sqrt{10x^2-2x}+2x=3-x$

$-2\sqrt{10x^2-2x}=-8x+4$

$8x-4=2\sqrt{10x^2-2x}$

$4\left(2x-1\right)=2\sqrt{10x^2-2x}$

$\left(4x-2\right)^2=\left(\sqrt{10x^2-2x}\right)^2$

$16x^2-16x+4=10x^2-2x$

$6x^2-14x+4=0$

$(6x)^2-14(6x)+24=0$

$\dfrac{\left(6x-12\right)\left(6x-2\right)}{6}=0$

$\left(x-2\right)\left(6x-2\right)=0$

$x-2=0$

$x_1=2 \quad \rightarrow \textit{sol factible}$

$6x-2=0$

$6x=2$

$x_2=\dfrac{1}{3} \quad \rightarrow \textit{sol inadmisible}$

8. $\sqrt{3x+1}+\sqrt{5x}=\sqrt{16x+1}$

$3x+1+2\sqrt{15x^2+5x}+5x=16x+1$

$2\sqrt{15x^2+5x}=8x$

$\left(\sqrt{15x^2+5x}\right)^2=(4x)^2$

$15x^2+5x=16x^2$

$0=x^2-5x$

$0=x\left(x-5\right)$

$x_1=0 \quad \rightarrow \textit{sol admisible}$

$x-5=0$

$x_2=5 \quad \rightarrow \textit{sol admisible}$

9. $\sqrt{2x+\sqrt{4x-3}}=3$

$2x+\sqrt{4x-3}=9$

$\left(\sqrt{4x-3}\right)^2=(9-2x)^2$

$4x-3=81-36x+4x^2$

$0=4x^2-40x+84$

$0=x^2-10x+21$

$0=(x-7)(x-3)$

$x-7=0$

$x_1=7 \rightarrow \text{Re}\textit{chazo por sol inadmisible}$

$x-3=0$

$x_2=3 \rightarrow \textit{sol aceptada}$

10. $\sqrt{x+3}+\dfrac{6}{\sqrt{x+3}}=5$

$\left(\sqrt{x+3}\right)^2+6=5\sqrt{x+3}$

$(x+9)^2=\left(5\sqrt{x+3}\right)^2$

$x^2-7x+6=0$

$(x-1)(x-6)=0$

$x-1=0$

$x_1=1 \rightarrow \textit{sol admisible}$

$x-6=0$

$x_2=6 \rightarrow \textit{sol admisible}$

11. $\sqrt{x}+\dfrac{4}{\sqrt{x}}=5$

$\left(\sqrt{x}\right)^2+4=5\sqrt{x}$

$(x+4)^2=\left(5\sqrt{x}\right)^2$

$x^2+8x+16=25x$

$x^2-17x+16=0$

$(x-16)(x-1)=0$

$x-16=0$

$x_1=16 \rightarrow \textit{sol admisible}$

$x-1=0$

$x_2=1 \rightarrow \textit{sol admisible}$

12. $2\sqrt{x}=\sqrt{x+7}+\dfrac{8}{\sqrt{x+7}}$

$2\sqrt{x^2+7x}=\sqrt{(x+7)^2}+8$

$\left(2\sqrt{x^2+7x}\right)^2=(x+15)^2$

$4x^2+28x=x^2+30x+225$

$3x^2-2x-225=0$

$(3x)^2-2(3x)-675=0$

$\dfrac{(3x-27)(3x+25)}{3}=0$

$(x-9)(3x+25)=0$

$x-9=0 \Rightarrow x_1=9 \rightarrow \textit{sol aceptada}$

$3x+25=0 \Rightarrow x_2=-\dfrac{25}{3} \rightarrow \textit{sol inadmisible}$

13. $\sqrt{x+\sqrt{x+8}}=2\sqrt{x}$

$x+\sqrt{x+8}=4x$

$\left(\sqrt{x+8}\right)^2=(3x)^2$

$x+8=9x^2$

$9x^2-x-8=0$

$(9x)^2-1(9x)-72=0$

$\dfrac{(9x-9)(9x+8)}{9}=0$

$x-1=0 \Rightarrow x_1=1 \rightarrow \textit{sol aceptada}$

$9x+8=0 \Rightarrow x_2=-\dfrac{8}{9} \rightarrow \textit{sol rechazada}$

14. $\sqrt{6-x}+\sqrt{x+7}-\sqrt{12x+1}=0$

$\left(\sqrt{6-x}+\sqrt{x+7}\right)^2=\left(\sqrt{12x+1}\right)^2$

$6-x+2\sqrt{42-x-x^2}+x+7=12x+1$

$2\sqrt{42-x-x^2}=12x-12$

$\left(\sqrt{42-x-x^2}\right)^2=(6x-6)^2$

$42-x-x^2=36x^2-72x+36$

$0=37x^2-71x-6$

$(37x)^2-71(37x)-222=0$

$\dfrac{(37x-74)(37x+3)}{37}=0$

$(x-2)(37x+3)=0$

$x-2=0 \Rightarrow x_1=2 \rightarrow \textit{sol aceptada}$

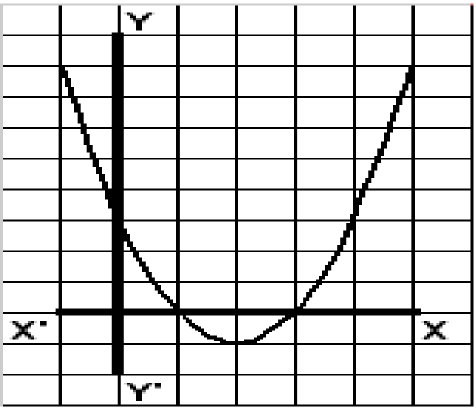
$37x+3=0$

$37x=-3 \Rightarrow x_2=-\dfrac{3}{37} \rightarrow \textit{sol inadmisible}$

EJERCICIO 274

11. $x^2-4x+3=0$

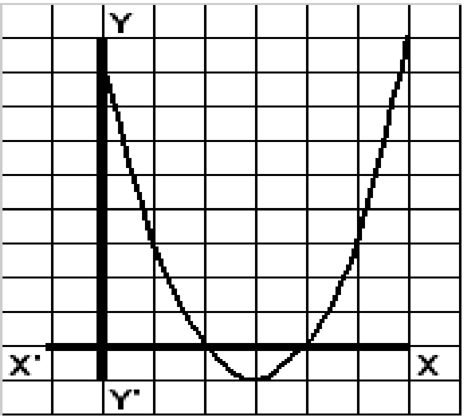
x	y
0	3
1	0
2	-1
3	0
-1	8



$sol: x_1=1 \quad x_2=3$

12. $x^2-6x+8=0$

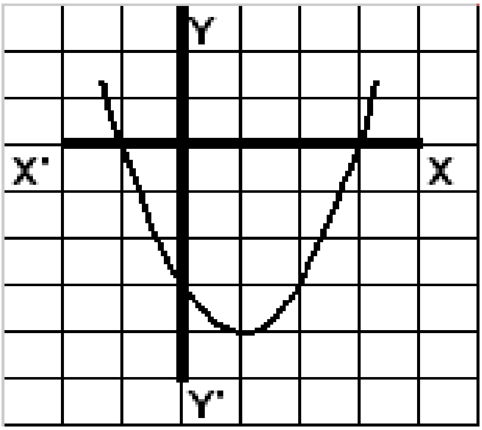
x	y
0	8
1	3
2	0
3	-1
4	0



$sol: x_1=2 \quad x_2=4$

13. $x^2-2x-3=0$

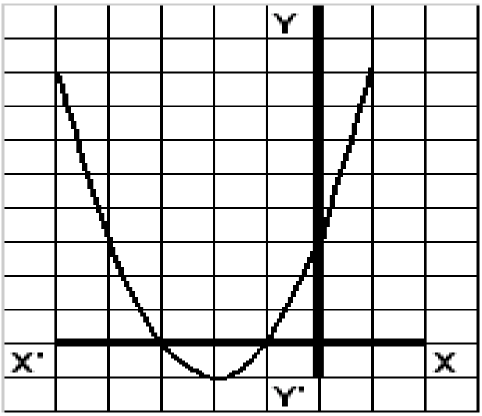
x	y
0	-3
1	-4
2	-3
3	0
-1	0



$sol: x_1=-1 \quad x_2=3$

14. $x^2+4x+3=0$

x	y
0	3
-1	0
-2	-1
-3	0
1	8

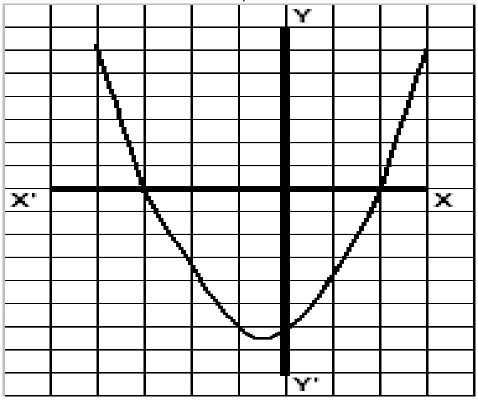


$sol: x_1=-3 \quad x_2=-1$

15. $x^2 = 6 - x$

$x^2 + x - 6 = 0$

x	y
0	-6
1	-4
2	0
3	6
-1	-6
-3	0

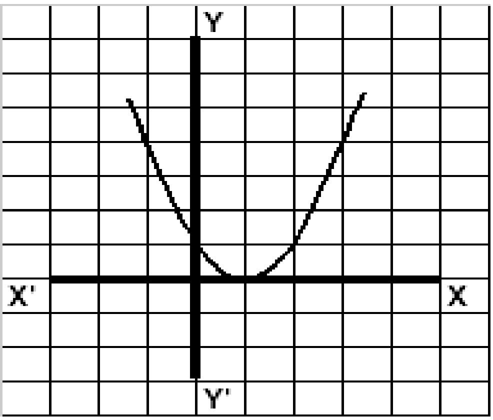


$sol: x_1 = -3 \quad x_2 = 2$

16. $x^2 = 2x - 1$

$x^2 - 2x + 1 = 0$

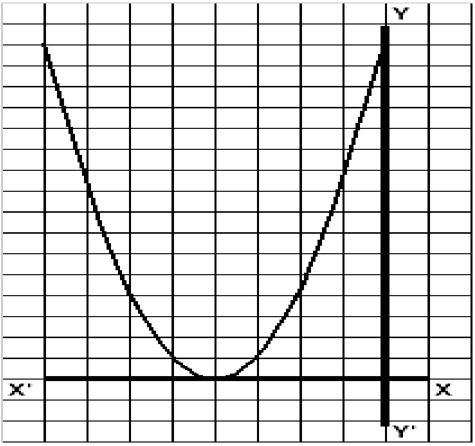
x	y
0	1
1	0
2	1
3	4
-1	4



$sol: x_1 = x_2 = 1$

17. $x^2 + 8x + 16 = 0$

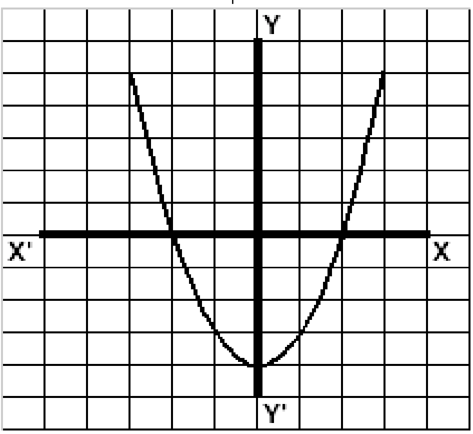
x	y
0	16
-2	4
-4	0
-6	4



$sol: x_1 = x_2 = -4$

18. $x^2 - 4 = 0$

x	y
1	-3
2	0
3	5
-1	-3
-2	0
-3	5
0	-4

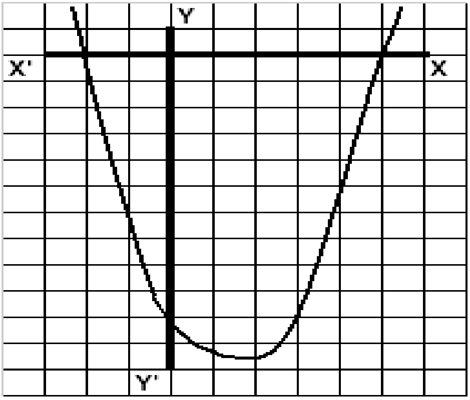


$sol: x_1 = -2 \quad x_2 = 2$

19. $x^2 = 3x + 10$

$x^2 - 3x - 10 = 0$

x	y
0	-10
3	-10
5	0
-1	-6
-2	0

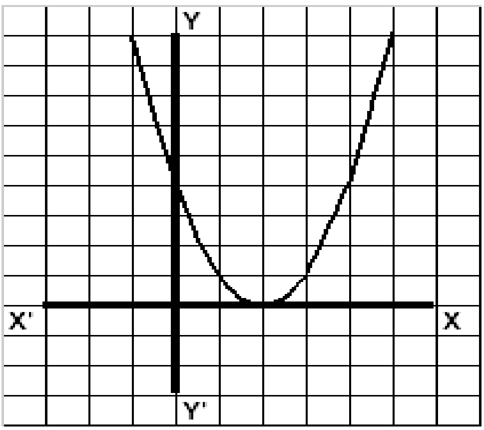


$sol: x_1 = -2 \quad x_2 = 5$

20. $x^2 - 4x = -4$

$x^2 - 4x + 4 = 0$

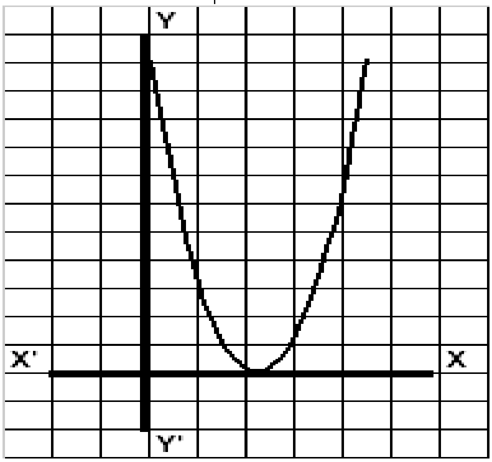
x	y
0	4
1	1
2	0
3	1
-1	9



$sol: x_1 = x_2 = 2$

21. $2x^2 - 9x + 10 = 0$

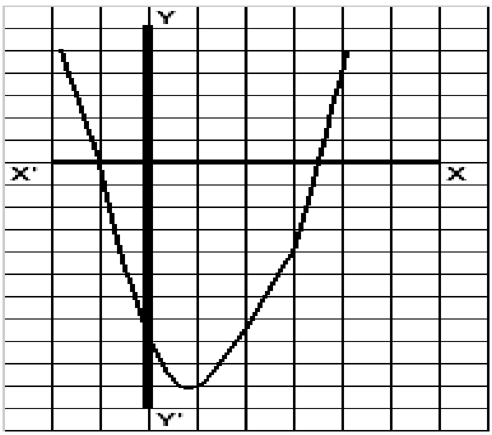
x	y
0	10
1	3
2	0
3	1
4	6



$sol: x_1 = 2 \quad x_2 = 2\frac{1}{2}$

22. $2x^2 - 5x - 7 = 0$

x	y
0	-7
1	-10
3	-4
4	5
-1	0



$sol: x_1 = -1 \quad x_2 = 3\frac{1}{2}$

EJERCICIO 275

1. $x \rightarrow N^{o}\ mayor$

$9-x \rightarrow N^{o}\ menor$

$x^2+(9-x)^2=53$

$x^2+81-18x+x^2=53$

$2x^2-18x+28=0$

$x^2-9x+14=0$

$(x-7)(x-2)=0$

$x_1=7 \quad x_2=2$

$7 \rightarrow N^{o}\ mayor$

$9-7=2 \rightarrow N^{o}\ menor$

2. $x \rightarrow N^{o}\ positivo$

$\frac{3x}{5} \rightarrow Otro\ Número$

$x\left(\frac{3x}{5}\right)=2.160$

$3x^2=10.800$

$x^2=3.600$

$x=\pm\sqrt{3.600}$

$x_1=60 \quad x_2=-60$

$60 \rightarrow N^{o}\ positivo$

$\frac{3(60)}{5}=36 \rightarrow Otro\ Número$

3. $x \rightarrow Edad\ de\ A$

$x-3 \rightarrow Edad\ de\ B$

$x^2+(x-3)^2=317$

$x^2+x^2-6x+9=317$

$2x^2-6x-308=0$

$x^2-3x-154=0$

$(x-14)(x+11)=0$

$x_1=14 \quad x_2=-11$

14 *años* $\rightarrow Edad\ de\ A$

$14-3=11$ *años* $\rightarrow Edad\ de\ B$

4. $x \rightarrow Un\ Número$

$3x \rightarrow El\ otro\ Número$

$(3x)^2-x^2=1.800$

$9x^2-x^2=1.800$

$8x^2=1.800$

$x^2=225$

$x=\pm\sqrt{225}$

$x_1=15 \quad x_2=-15$

$15 \rightarrow Un\ Número$

$3(15)=45 \rightarrow El\ otro\ Número$

5. $x \rightarrow N^{o}\ buscado$

$x^2-9=8(x-2)$

$x^2-9=8x-16$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x_1=7 \quad x_2=1$

$7 \rightarrow N^{o}\ buscado$

6. $x+1 \rightarrow N^{o}\ mayor$

$x \rightarrow N^{o}\ menor$

$(x+1)^2-57=3x$

$x^2+2x+1-57=3x$

$x^2-x-56=0$

$(x-8)(x+7)=0$

$x_1=8 \quad x_2=-7$

$8 \rightarrow N^{o}\ menor$

$8+1=9 \rightarrow N^{o}\ mayor$

7. $x+4 \rightarrow Largo$

$x \rightarrow Ancho$

$Area=x(x+4)$

$=x^2+4x$

$Doble\ del\ area=2x^2+8x$

$(x+8)(x+4)=2x^2+8x$

$x^2+12x+32=2x^2+8x$

$x^2-4x-32=0$

$(x-8)(x+4)=0$

$x_1=8 \quad x_2=-4$

$8m+4m=12m \rightarrow Largo$

$8m \rightarrow Ancho$

8. $x \rightarrow N^{o}\ sacos\ comp.$

$\frac{1.000}{x}bs. \rightarrow costo\ cada\ saco$

$\frac{1.000}{x+10}=\frac{1.000}{x}-5$

$1.000x=(x+10)1.000-5x(x+10)$

$10^3x=10^3x+10^4-5x^2-50x$

$5x^2+50x-10^4=0$

$x^2+10x-2.000=0$

$(x+50)(x-40)=0$

$x_1=-50 \quad x_2=40$

$40 \rightarrow N^{o}\ sacos\ comp.$

$\frac{1.000}{40}bs.=25bs. \rightarrow costo\ cada\ saco$

9. $4x \rightarrow Costo\ caballo$

$x \rightarrow Costo\ arreos$

$(4x)^2+x^2=860.625$

$16x^2+x^2=860.625$

$17x^2=860.625$

$x^2=50.625$

$x=\pm\sqrt{50.625}$

$x_1=225 \quad x_2=-225$

$4(225)=900$

900 *sucres* $\rightarrow Costo\ caballo$

225 *sucres* $\rightarrow Costo\ arreos$

10. $x-7 \rightarrow N^{o}\ menor$

$x \rightarrow N^{o}\ mayor$

$(x-7+x)(x-7)=184$

$(2x-7)(x-7)=184$

$2x^2-21x+49=184$

$2x^2-21x-135=0$

$(2x)^2-21(2x)-270=0$

$(2x-30)(2x+9)=0$

$(x-15)(2x+9)=0$

$x_1=15 \quad x_2=-\frac{9}{2}$

$15 \rightarrow N^{o}\ mayor$

$15-7=8 \rightarrow N^{o}\ menor$

11. $x \rightarrow Edad\ de\ A$

$23-x \rightarrow Edad\ de\ B$

$x(23-x)=102$

$23x-x^2=102$

$x^2-23x+102=0$

$(x-17)(x-6)=0$

$x_1=17 \quad x_2=6$

17 *años* $\rightarrow Edad\ de\ A$

6 *años* $\rightarrow Edad\ de\ B$

12. $x \rightarrow N^o$ *de libros*

$\frac{180}{x} \rightarrow$ *Costo c/u*

$$\frac{180}{x-6}=\frac{180}{x}+1$$

$$180x=180(x-6)+x(x-6)$$

$$180x=180x-1.080+x^2-6x$$

$$0=x^2-6x-1.080$$

$$0=(x-36)(x+30)$$

$x_1=36 \quad x_2=-30$

$36 \rightarrow$ *Libros*

$\frac{180}{36}=\$5 \rightarrow$ *Costo c/u*

13. $x \rightarrow N^o$ *de filas*

$\frac{180}{x} \rightarrow N^o$ *soldados cada fila*

$$\frac{180}{x}-8=x$$

$$180-8x=x^2$$

$$0=x^2+8x-180$$

$$0=(x+18)(x-10)$$

$x_1=-18 \quad x_2=10$

$10 \rightarrow N^o$ *de filas*

$\frac{180}{10}=18 \rightarrow N^o$ *soldados cada fila*

14. $x \rightarrow$ *Costo del reloj*

$x\%$ *de* $x=\frac{x^2}{100}$

$$\frac{x^2}{100}+x=75$$

$$x^2+100x=7.500$$

$$x^2+100x-7.500=0$$

$$(x+150)(x-50)=0$$

$x_1=-150 \quad x_2=50$

50 *soles* $\rightarrow$ *Costo del reloj*

15. $x \rightarrow$ *Pers. comp.*

$$\frac{1.200}{x}-194=x$$

$$1.200-194x=x^2$$

$$x^2+194x-1.200=0$$

$$(x+200)(x-6)=0$$

$x_1=-200 \quad x_2=6$

$6 \rightarrow$ *Pers. comp. el auto*

16. $x \rightarrow$ *Relojes comp.*

$\frac{192}{x} \rightarrow$ *Costo cada reloj*

$$\frac{192}{x}=\frac{3x}{4}$$

$$768=3x^2$$

$$256=x^2$$

$$\pm\sqrt{256}=x$$

$x_1=16 \quad x_2=-16$

$16 \rightarrow$ *Relojes comp.*

$\frac{192}{16}=\$12 \rightarrow$ *Costo cada reloj*

17. $x \rightarrow$ *Libros comp.*

$\frac{150}{x} \rightarrow$ *Costo cada lib.*

$$\frac{150}{x}+1=\frac{150}{x-5}$$

$$150(x-5)+x(x-5)=150x$$

$$150x-750+x^2-5x=150x$$

$$x^2-5x-750=0$$

$$(x-30)(x+25)=0$$

$x_1=30 \quad x_2=-25$

$30 \rightarrow$ *Libros comp.*

$\frac{150}{30}=\$5 \rightarrow$ *Costo cada lib.*

18. $x \rightarrow$ *Libros comp.*

$$\frac{200}{x}-10=x$$

$$200-10x=x^2$$

$$x^2+10x-200=0$$

$$(x+20)(x-10)=0$$

$x_1=-20 \quad x_2=10$

$10 \rightarrow$ *Libros comp.*

19. $x \rightarrow$ *Plumas comp.*

$\frac{24}{x} \rightarrow$ Pr*ecio c/u*

$$\frac{24}{x}-1=\frac{24}{x+4}$$

$$24(x+4)-x(x+4)=24x$$

$$24x+96-x^2-4x=24x$$

$$x^2+4x-96=0$$

$$(x+12)(x-8)=0$$

$x_1=-12 \quad x_2=8$

$8 \rightarrow$ *Plumas comp.*

$\frac{24}{8}=\$3 \rightarrow$ Pr*ecio c/u*

20. $x \rightarrow$ *Tiempo de recorrido*

$$\frac{240}{x}+20=\frac{240}{x-2}$$

$$240(x-2)+20x(x-2)=240x$$

$$240x-480+20x^2-40x=240x$$

$$20x^2-40x-480=0$$

$$x^2-2x-24=0$$

$$(x-6)(x+4)=0$$

$x_1=6 \quad x_2=-4$

6 *horas* $\rightarrow$ *Tiempo de recorrido*

21. $x \rightarrow$ *Caballos comp.*

$\frac{2.000}{x} \rightarrow$ Pr*ecio cada caballo*

$$\frac{2.000}{x}+60=\frac{2.000+80}{x-2}$$

$$2.000(x-2)+60x(x-2)=2.080x$$

$$2.000x-4.000+60x^2-120x=2.080x$$

$$60x^2-200x-4.000=0$$

$$3x^2-10x-200=0$$

$$(3x)^2-10(3x)-600=0$$

$$(3x-30)(3x+20)=0$$

$$(x-10)(3x+20)=0$$

$x_1=10 \quad x_2=-\frac{20}{3}$

$10 \rightarrow$ *Caballos comp.*

$\frac{2.000}{10}=\$200 \rightarrow$ Pr*ecio c/caballo*

22. $x \rightarrow N^o$ *menor*

$x+1 \rightarrow N^o$ int*ermedio*

$x+2 \rightarrow N^o$ *mayor*

$$\frac{x+2}{x+1}=\frac{3x}{10}$$

$$10(x+2)=3x(x+1)$$

$$10x+20=3x^2+3x$$

$$0=3x^2-7x-20$$

$$0=(3x)^2-7(3x)-60$$

$$0=(3x-12)(3x+5)$$

$$0=(x-4)(3x+5)$$

$x_1=4 \quad x_2=\frac{-5}{3}$

$4 \rightarrow N^o$ *menor*

$4+1=5 \rightarrow N^o$ int*ermedio*

$4+2=6 \rightarrow N^o$ *mayor*

23. $x \rightarrow$ *Un Nº.*

$\frac{180}{x} \rightarrow$ *El otro Nº.*

$\frac{\frac{180}{x}}{x} = 1\frac{1}{4}$

$\frac{180}{x^2} = \frac{5}{4}$

$720 = 5x^2$

$144 = x^2$

$\pm\sqrt{144} = x$

$x_1 = 12 \quad x_2 = -12$

$12 \rightarrow$ *Un Nº.*

$\frac{180}{12} = 15 \rightarrow$ *El otro Nº.*

24. $x \rightarrow$ *Nº naranjas comp.*

$\frac{150}{x} \rightarrow$ *Costo c / naranja*

$\left(\frac{150}{x} + 1\right)(x-5) = 150$

$(150 + x)(x-5) = 150x$

$150x - 750 + x^2 - 5x = 150x$

$x^2 - 5x - 750 = 0$

$(x-30)(x+25) = 0$

$x_1 = 30 \quad x_2 = -25$

$30 \rightarrow$ *Nº naranjas comp.*

$\frac{150}{30} = 5$ *ctvs.* $\rightarrow$ *Costo c / naranja*

25. $x \rightarrow$ *Costo del caballo*

$\frac{x^2}{100} + x = 171$

$x^2 + 100x = 17.100$

$x^2 + 100x - 17.100 = 0$

$(x+190)(x-90) = 0$

$x_1 = -190 \quad x_2 = 90$

90 *Q.* $\rightarrow$ *Costo del caballo*

26. $x \rightarrow$ *Nº mayor*

$\frac{352}{x} \rightarrow$ *Nº menor*

$\frac{\frac{x}{352}}{x} = 2 + \frac{\frac{10}{352}}{x}$

$\frac{x^2}{352} = 2 + \frac{10x}{352}$

$x^2 = 704 + 10x$

Continùa

26. Continuación

$x^2 - 10x - 704 = 0$

$(x-32)(x+22) = 0$

$x_1 = 32 \quad x_2 = -22$

$32 \rightarrow$ *Nº mayor*

$\frac{352}{32} = 11 \rightarrow$ *Nº menor*

27. $x \rightarrow$ *Long. pieza mayor*

$20 - x \rightarrow$ *Long. pieza menor*

$x \cdot x = 9(20-x)(20-x)$

$x^2 = 9(400 - 40x + x^2)$

$x^2 = 3.600 - 360x + 9x^2$

$0 = 8x^2 - 360x + 3.600$

$0 = x^2 - 45x + 450$

$0 = (x-30)(x-15)$

$x_1 = 30 \quad x_2 = 15$

$15m \rightarrow$ *Long. pieza mayor*

$20 - 15 = 5m \rightarrow$ *Long. pieza menor*

28. $x \rightarrow$ *Tiempo en horas*

$\frac{200}{x} \rightarrow$ *Veloc. del tren*

$\frac{200}{x} = \frac{200}{x-1} + 10$

$200(x-1) = 200x + 10x(x-1)$

$200x - 200 = 200x + 10x^2 - 10x$

$0 = 10x^2 - 10x + 200$

$0 = x^2 - x + 20$

$0 = (x-5)(x+4)$

$x_1 = 5 \quad x_2 = -4$

$\frac{200}{5} = 40 \, {Km}/{h} \rightarrow$ *Veloc. del tren*

29. $x \rightarrow$ *Dias trabj.*

$\frac{84}{x} \rightarrow$ *Valor del jornal*

$\frac{84}{x} - 1 = \frac{84}{x+2}$

$84(x+2) - x(x+2) = 84x$

$84x + 168 - x^2 - 2x = 84x$

$x^2 + 2x - 168 = 0$

$(x+14)(x-12) = 0$

$x_1 = -14 \quad x_2 = 12$

$12 \rightarrow$ *Dias trabj.*

$\frac{84}{12} = 7$ *col* $\rightarrow$ *Valor del jornal*

30. $x \rightarrow$ *Personas que van*

$\frac{90}{x} \rightarrow$ *Valor c / u*

$\frac{90}{x-3} - 1 = \frac{90}{x}$

$90x - x(x-3) = 90(x-3)$

$90x - x^2 + 3x = 90x - 270$

$x^2 - 3x - 270 = 0$

$(x-18)(x+15) = 0$

$x_1 = 18 \quad x_2 = -15$

$18 \rightarrow$ *Personas que van*

$\frac{90}{18} = \$5 \rightarrow$ *Valor c / u*

31. $x \rightarrow$ *Nº buscado*

$\frac{84}{x} - 5 = x$

$84 - 5x = x^2$

$x^2 + 5x - 84 = 0$

$(x+12)(x-7) = 0$

$x_1 = -12 \quad x_2 = 7$

$7 \rightarrow$ *Nº buscado*

32. $x \rightarrow$ *Edad actual A*

$x - 6 = \sqrt{x+6}$

$(x-6)^2 = \left(\sqrt{x+6}\right)^2$

$x^2 - 12x + 36 = x + 6$

$x^2 - 13x + 30 = 0$

$(x-10)(x-3) = 0$

$x_1 = 10 \quad x_2 = 3$

10 *años* $\rightarrow$ *Edad actual A*

33. $x \rightarrow$ *Libros comp.*

$\frac{40}{x} \rightarrow$ *Precio de c / libro*

$x - 2 \rightarrow$ *Nº de plumas*

$\frac{40}{x} = \frac{40}{x-2} - 1$

$40(x-2) = 40x - x(x-2)$

$40x - 80 = 40x - x^2 + 2x$

$x^2 - 2x - 80 = 0$

$(x-10)(x+8) = 0$

$x_1 = 10 \quad x_2 = -8$

$10 \rightarrow$ *Libros comp.*

$\frac{40}{10} = \$4 \rightarrow$ *Precio c / libro*

EJERCICIO 276

1. $3x^2+5x-2=0$

$b^2-4ac=5^2-4(3)(-2)=25+24=49$

Reales y desiguales, racionales

2. $2x^2-4x+1=0$

$b^2-4ac=(-4)^2-4(2)(1)=16-8=8$

Reales y desiguales, irracionales

3. $4x^2-4x+1=0$

$b^2-4ac=(-4)^2-4(4)(1)=16-16=0$

Reales e iguales

4. $3x^2-2x+5=0$

$b^2-4ac=(-2)^2-4(3)(5)=4-60=-56$

Imaginarias

5. $x^2-10x+25=0$

$b^2-4ac=(-10)^2-4(1)(25)=100-100=0$

Reales e iguales

6. $x^2-5x-5=0$

$b^2-4ac=(-5)^2-4(1)(-5)=25+20=45$

Reales y desiguales, irracionales

7. $2x^2-9x+7=0$

$b^2-4ac=(-9)^2-4(2)(7)=81-56=25$

Reales y desiguales, racionales

8. $36x^2+12x+1=0$

$b^2-4ac=(12)^2-4(36)(1)=144-144=0$

Reales e iguales

9. $4x^2-5x+3=0$

$b^2-4ac=(-5)^2-4(4)(3)=25-48=-23$

Imaginarias

10. $x^2+x-1=0$

$b^2-4ac=(1)^2-4(1)(-1)=1+4=5$

Reales y desiguales, irracionales

11. $5x^2-7x+8=0$

$b^2-4ac=(-7)^2-4(5)(8)=49-160=-111$

Imaginarias

12. $x^2-10x-11=0$

$b^2-4ac=(-10)^2-4(1)(-11)=100+44=144$

Reales y desiguales, racionales

EJERCICIO 277

1. $x^2+x-6=0$

Suma $2+(-3)=-1$

Producto $2(-3)=-6$

Si son raices

2. $x^2-4x-5=0$

Suma $1+5=6$

No son raices

3. $2x^2-x-1=0$

$x^2-\frac{x}{2}-\frac{1}{2}=0$

Suma $1-\frac{1}{2}=\frac{1}{2}$

Producto $1\left(-\frac{1}{2}\right)=-\frac{1}{2}$

Si son raices

4. $3x^2+8x-3=0$

$x^2+\frac{8x}{3}-1=0$

Suma $-3+\frac{1}{3}=-\frac{8}{3}$

Continúa

4. Continuación

Producto $-3\left(\frac{1}{3}\right)=-1$

Si son raices

5. $5x^2-11x+2=0$

$x^2-\frac{11x}{5}+\frac{2}{5}=0$

Suma $2-\frac{1}{5}=\frac{9}{5}$

No son raices

6. $4x^2+17x+4=0$

$x^2+\frac{17x}{4}+1=0$

Suma $-4-\frac{1}{4}=-\frac{17}{4}$

Producto $-4\left(-\frac{1}{4}\right)=1$

Si son raices

7. $5x^2+24x-5=0$

$x^2+\frac{24}{5}x-1=0$

Continúa

7. Continuación

Suma $-5-\frac{1}{5}=-\frac{26}{5}$

No son raices

8. $x^2+3x-28=0$

Suma $4-7=-3$

Producto $4(-7)=-28$

Si son raices

9. $6x^2+x-2=0$

$x^2+\frac{x}{6}-\frac{1}{3}=0$

Suma $\frac{1}{2}-\frac{2}{3}=-\frac{1}{6}$

Producto $\left(\frac{1}{2}\right)\left(-\frac{2}{3}\right)=-\frac{1}{3}$

Si son raices

10. $8x^2-2x-3=0$

$x^2-\frac{1}{4}x-\frac{3}{8}=0$

Suma $\frac{1}{2}-\frac{3}{4}=-\frac{1}{4}$

No son raices

EJERCICIO 278

1. Suma $3+4=7$
Producto $3(4)=12$
Luego $x^2-7x+12=0$

2. Suma $-1+3=2$
Producto $-1(3)=-3$
Luego $x^2-2x-3=0$

3. Suma $-5-7=-12$
Producto $-5(-7)=35$
Luego $x^2+12x+35=0$

4. Suma $-10+11=1$
Producto $-10(11)=-110$
Luego $x^2-x-110=0$

5. Suma $1+\frac{1}{2}=\frac{3}{2}$
Producto $1\left(\frac{1}{2}\right)=\frac{1}{2}$
Luego $x^2-\frac{3x}{2}+\frac{1}{2}=0$
$\Rightarrow 2x^2-3x+1=0$

6. Suma $-2-\frac{1}{5}=-\frac{11}{5}$
Producto $-2\left(-\frac{1}{5}\right)=\frac{2}{5}$
Luego $x^2+\frac{11x}{5}+\frac{2}{5}=0$
$\Rightarrow 5x^2+11x+2=0$

7. Suma $3-\frac{2}{3}=\frac{7}{3}$
Producto $3\left(-\frac{2}{3}\right)=-2$
Luego $x^2-\frac{7x}{3}-2=0$
$\Rightarrow 3x^2-7x-6=0$

8. Suma $-2-\frac{3}{2}=-\frac{7}{2}$
Producto $-2\left(-\frac{3}{2}\right)=3$
Luego $x^2+\frac{7x}{2}+3=0$
$\Rightarrow 2x^2+7x+6=0$

9. Suma $-\frac{1}{2}+\frac{3}{4}=\frac{1}{4}$
Producto $\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)=-\frac{3}{8}$
Luego $x^2-\frac{x}{4}-\frac{3}{8}=0$
$\Rightarrow 4x^2-x-\frac{3}{2}=0$
$\Rightarrow 8x^2-2x-3=0$

10. Suma $-5+\frac{2}{7}=-\frac{33}{7}$
Producto $-5\left(\frac{2}{7}\right)=-\frac{10}{7}$
Luego $x^2+\frac{33x}{7}-\frac{10}{7}=0$
$\Rightarrow 7x^2+33x-10=0$

11. Suma $6-\frac{5}{3}=\frac{13}{3}$
Producto $6\left(-\frac{5}{3}\right)=-10$
Luego $x^2-\frac{13}{3}x-10=0$
$\Rightarrow 3x^2-13x-30=0$

12. Suma $-2-\frac{1}{8}=-\frac{17}{8}$
Producto $-2\left(-\frac{1}{8}\right)=\frac{1}{4}$
Luego $x^2+\frac{17}{8}x+\frac{1}{4}=0$
$\Rightarrow 8x^2+17x+2=0$

13. Suma $18-52=-34$
Producto $18(-52)=-936$
Luego $x^2+34x-936=0$

14. Suma $-15-11=-26$
Producto $-15(-11)=165$
Luego $x^2+26x+165=0$

15. Suma $0+2=2$
Producto $0(2)=0$
Luego $x^2-2x=0$

16. Suma $0-\frac{1}{3}=-\frac{1}{3}$
Producto $0\left(-\frac{1}{3}\right)=0$
Luego $x^2+\frac{x}{3}=0$
$\Rightarrow 3x^2+x=0$

17. Suma $5-5=0$
Producto $5(-5)=-25$
Luego $x^2-25=0$

18. Suma $\frac{1}{2}-\frac{1}{2}=0$
Producto $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{1}{4}$
Luego $x^2-\frac{1}{4}=0$
$\Rightarrow 4x^2-1=0$

19. Suma $7+7=14$
Producto $7(7)=49$
Luego $x^2-14x+49=0$

20. Suma $8-\frac{11}{3}=\frac{13}{3}$
Producto $8\left(-\frac{11}{3}\right)=-\frac{88}{3}$
Luego $x^2-\frac{13}{3}x-\frac{88}{3}=0$
$\Rightarrow 3x^2-13x-88=0$

21. Suma $-\frac{5}{6}-\frac{9}{2}=-\frac{16}{3}$
Producto $\left(-\frac{5}{6}\right)\left(-\frac{9}{2}\right)=\frac{15}{4}$
Luego $x^2+\frac{16}{3}x+\frac{15}{4}=0$
$\Rightarrow 12x^2+64x+45=0$

22. Suma $-\frac{11}{2}+\frac{2}{7}=-\frac{73}{14}$
Producto $\left(-\frac{11}{2}\right)\left(\frac{2}{7}\right)=-\frac{11}{7}$
Luego $x^2+\frac{73}{14}x-\frac{11}{7}=0$
$\Rightarrow 14x^2+73x-22=0$

23. Suma $2a-a=a$
Producto $2a(-a)=-2a^2$
Luego $x^2-ax-2a^2=0$

24. $Suma \quad -\frac{2b}{3}+\frac{b}{4}=-\frac{5b}{12}$

$Producto \left(-\frac{2b}{3}\right)\left(\frac{b}{4}\right)=\frac{-b^2}{6}$

$Luego \quad x^2+\frac{5b}{12}x-\frac{b^2}{6}=0$

$\Rightarrow \quad 12x^2+5bx-2b^2=0$

25. $Suma \quad m-\frac{m}{2}=\frac{m}{2}$

$Producto \quad m\left(-\frac{m}{2}\right)=-\frac{m^2}{2}$

$Luego \quad x^2-\frac{m}{2}x-\frac{m^2}{2}=0$

$\Rightarrow \quad 2x^2-mx-m^2=0$

26. $Suma \quad b+a-b=a$

$Producto \; b(a-b)=ab-b^2$

$Luego \quad x^2-ax+ab-b^2=0$

27. $Suma \quad \frac{a}{2}-\frac{b}{3}=\frac{3a-2b}{6}$

$Producto \left(\frac{a}{2}\right)\left(-\frac{b}{3}\right)=\frac{-ab}{6}$

$Luego \quad x^2-\frac{3a-2b}{6}x-\frac{ab}{6}=0$

$\Rightarrow \quad 6x^2-(3a-2b)x-ab=0$

28. $Suma \quad 1+\sqrt{2}+1-\sqrt{2}=2$

$Producto \; (1+\sqrt{2})(1-\sqrt{2})=-1$

$Luego \quad x^2-2x-1=0$

29. $Suma \;\; 2+\sqrt{5}+2-\sqrt{5}=4$

$Producto \; (2+\sqrt{5})(2-\sqrt{5})=-1$

$Luego \quad x^2-4x-1=0$

30. $Suma \quad 3+\sqrt{-1}+3-\sqrt{-1}=6$

$Producto \; (3+\sqrt{-1})(3-\sqrt{-1})=10$

$Luego \quad x^2-6x+10=0$

EJERCICIO 279

1. $x^2-11x+30=0$

$(x-6)(x-5)=0$

$x_1=6 \qquad x_2=5$

2. $x^2+33x+260=0$

$x=-\frac{33}{2}\pm\sqrt{\frac{(33)^2}{4}-260}$

$x=\frac{-33}{2}\pm\sqrt{\frac{1.089-1.040}{4}}$

$x=\frac{-33}{2}\pm\sqrt{\frac{49}{4}}$

$x=\frac{-33}{2}\pm\frac{7}{2}$

$x_1=-\frac{33}{2}+\frac{7}{2}=\frac{-26}{2}=-13$

$x_2=\frac{-33}{2}-\frac{7}{2}=\frac{-40}{2}=-20$

3. $x^2+x-306=0$

$(x+18)(x-17)=0$

$x_1=-18 \qquad x_2=17$

4. $x^2+49x+294=0$

$(x+42)(x+7)=0$

$x_1=-42 \qquad x_2=-7$

5. $x^2-6x-247=0$

$(x-19)(x+13)=0$

$x_1=19 \qquad x_2=-13$

6. $x^2-\frac{3}{2}x-1=0$

$2x^2-3x-2=0$

$(2x)^2-3(2x)-4=0$

$(2x-4)(2x+1)=0$

$(x-2)(2x+1)=0$

$x_1=2 \qquad x_2=-\frac{1}{2}$

7. $x^2+\frac{22}{3}x+8=0$

$3x^2+22x+24=0$

$(3x)^2+22(3x)+72=0$

$(3x+18)(3x+4)=0$

$(x+6)(3x+4)=0$

$x_1=-6 \qquad x_2=-\frac{4}{3}$

8. $x^2-\frac{x}{4}-\frac{3}{8}=0$

$8x^2-2x-3=0$

$(8x)^2-2(8x)-24=0$

$(8x-6)(8x+4)=0$

$(4x-3)(2x+1)=0$

$x_1=\frac{3}{4} \qquad x_2=-\frac{1}{2}$

9. $x^2+\frac{95}{7}x-6=0$

$7x^2+95x-42=0$

$(7x)^2+95(7x)-294=0$

$(7x+98)(7x-3)=0$

$(x+14)(7x-3)=0$

$x_1=-14 \qquad x_2=\frac{3}{7}$

10. $x^2+\frac{10}{3}x+1=0$

$3x^2+10x+3=0$

$x=\frac{-10\pm\sqrt{(10)^2-4(3)(3)}}{2(3)}$

$x=\frac{-10\pm\sqrt{64}}{6}=\frac{-10\pm8}{6}$

$x_1=\frac{-10+8}{6}=\frac{-2}{6}=-\frac{1}{3}$

$x_2=\frac{-10-8}{6}=\frac{-18}{6}=-3$

11. $x^2-\frac{31}{40}x+\frac{3}{20}=0$

$40x^2-31x+6=0$

$(40x)^2-31(40x)+240=0$

$(40x-16)(40x-15)=0$

$(5x-2)(8x-3)=0$

$x_1=\frac{2}{5} \qquad x_2=\frac{3}{8}$

12. $x^2+\frac{x}{6}-\frac{5}{9}=0$

$18x^2+3x-10=0$

$(18x)^2+3(18x)-180=0$

$(18x+15)(18x-12)=0$

$(6x+5)(3x-2)=0$

$x_1=\frac{-5}{6}\quad x_2=\frac{2}{3}$

13. $x^2-\frac{7}{20}x-\frac{3}{10}=0$

$20x^2-7x-6=0$

$(20x)^2-7(20x)-120=0$

$(20x-15)(20x+8)=0$

$(4x-3)(5x+2)=0$

$x_1=\frac{3}{4}\quad x_2=\frac{-2}{5}$

14. $x^2-\frac{21x}{5}-4=0$

$5x^2-21x-20=0$

$(5x)^2-21(5x)-100=0$

$(5x-25)(5x+4)=0$

$(x-5)(5x+4)=0$

$x_1=5\quad x_2=-\frac{4}{5}$

15. $x^2-\frac{59}{72}x+\frac{1}{6}=0$

$72x^2-59x+12=0$

$(72x)^2-59(72x)+864=0$

$(72x-32)(72x-27)=0$

$(9x-4)(8x-3)=0$

$x_1=\frac{4}{9}\quad x_2=\frac{3}{8}$

16. $x^2-2x-4=0$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$

$x=\frac{2\pm\sqrt{20}}{2}=\frac{2\pm2\sqrt{5}}{2}$

$x_1=\frac{2+2\sqrt{5}}{2}=\frac{2(1+\sqrt{5})}{2}=1+\sqrt{5}$

$x_2=\frac{2-2\sqrt{5}}{2}=\frac{2(1-\sqrt{5})}{2}=1-\sqrt{5}$

17. $x^2-x-\frac{11}{4}=0$

$4x^2-4x-11=0$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(4)(-11)}}{2(4)}$

$x=\frac{4\pm\sqrt{64\cdot3}}{8}=\frac{4\pm8\sqrt{3}}{8}$

$x_1=\frac{4+8\sqrt{3}}{8}=\frac{4(1+2\sqrt{3})}{8}=\frac{1+2\sqrt{3}}{2}=\frac{1}{2}+\sqrt{3}$

$x_2=\frac{4-8\sqrt{3}}{8}=\frac{4(1-2\sqrt{3})}{8}=\frac{1-2\sqrt{3}}{2}=\frac{1}{2}-\sqrt{3}$

18. $x^2+\frac{4}{3}x-\frac{59}{9}=0$

$9x^2+12x-59=0$

$x=\frac{-12\pm\sqrt{(12)^2-4(9)(-59)}}{2(9)}$

$x=\frac{-12\pm\sqrt{2.268}}{18}=\frac{-12\pm\sqrt{2^2\cdot3^4\cdot7}}{18}=\frac{-12\pm18\sqrt{7}}{18}$

$x_1=\frac{-12+18\sqrt{7}}{18}=\frac{6(-2+3\sqrt{7})}{18}=\frac{-2+3\sqrt{7}}{3}=-\frac{2}{3}+\sqrt{7}$

$x_2=\frac{-12-18\sqrt{7}}{18}=\frac{-6(2+3\sqrt{7})}{18}=\frac{-2-3\sqrt{7}}{3}=-\frac{2}{3}-\sqrt{7}$

19. $x^2-ax-2a^2=0$

$(x-2a)(x+a)=0$

$x_1=2a\quad x_2=-a$

20. $x^2+7bx+10b^2=0$

$(x+5b)(x+2b)=0$

$x_1=-5b\quad x_2=-2b$

21. $x^2-\frac{mx}{2}-\frac{m^2}{9}=0$

$18x^2-9mx-2m^2=0$

$(18x)^2-9m(18x)-36m^2=0$

$(18x-12m)(18x+3m)=0$

$(6x-4m)(6x+m)=0$

$x_1=\frac{4m}{6}=\frac{2m}{3}\quad x_2=-\frac{m}{6}$

EJERCICIO 280

1. $x^2-16x+63=0$

$$x=\frac{-(-16)\pm\sqrt{(-16)^2-4(63)}}{2}$$
$$x=\frac{16\pm\sqrt{4}}{2}=\frac{16\pm 2}{2}$$
$$x_1=\frac{16+2}{2}=\frac{18}{2}=9$$
$$x_2=\frac{16-2}{2}=\frac{14}{2}=7$$
$$x^2-16x+63=(x-9)(x-7)$$

2. $x^2+24x+143=0$

$$x=\frac{-24\pm\sqrt{(24)^2-4(143)}}{2}$$
$$x=\frac{-24\pm\sqrt{4}}{2}=\frac{-24\pm 2}{2}$$
$$x_1=\frac{-24+2}{2}=\frac{-22}{2}=-11$$
$$x_2=\frac{-24-2}{2}=\frac{-26}{2}=-13$$
$$x^2+24x+143=[x-(-11)][x-(-13)]$$
$$=(x+11)(x+13)$$

3. $x^2-26x-155=0$

$$x=-\frac{(-26)}{2}\pm\sqrt{\frac{(26)^2}{4}-(-155)}$$
$$x=13\pm\sqrt{324}\ =\ 13\pm 18$$
$$x_1=13+18=31$$
$$x_2=13-18=-5$$
$$x^2-26x-155=(x-31)(x+5)$$

4. $2x^2+x-6=0$

$$x=\frac{-1\pm\sqrt{(1)^2-4(2)(-6)}}{2(2)}$$
$$x=\frac{-1\pm\sqrt{49}}{4}=\frac{-1\pm 7}{4}$$
$$x_1=\frac{-1+7}{4}=\frac{6}{4}=\frac{3}{2}$$
$$x_2=\frac{-1-7}{4}=-\frac{8}{4}=-2$$
$$2x^2+x-6=2\left(x-\frac{3}{2}\right)(x+2)$$
$$=2\left(\frac{2x-3}{2}\right)(x+2)$$
$$=(2x-3)(x+2)$$

5. $12x^2+5x-2=0$

$$x=\frac{-5\pm\sqrt{(5)^2-4(12)(-2)}}{2(12)}$$
$$x=\frac{-5\pm\sqrt{121}}{24}=\frac{-5\pm 11}{24}$$
$$x_1=\frac{-5+11}{24}=\frac{6}{24}=\frac{1}{4}$$
$$x_2=\frac{-5-11}{24}=\frac{-16}{24}=-\frac{2}{3}$$
$$12x^2+5x-2=12\left(x-\frac{1}{4}\right)\left(x+\frac{2}{3}\right)$$
$$=12\left(\frac{4x-1}{4}\right)\left(\frac{3x+2}{3}\right)$$
$$=\frac{12(4x-1)(3x+2)}{12}$$
$$=(4x-1)(3x+2)$$

6. $5x^2+41x+8$

$$x=\frac{-41\pm\sqrt{(41)^2-4(5)(8)}}{2(5)}$$
$$x=\frac{-41\pm\sqrt{1.521}}{10}=\frac{-41\pm 39}{10}$$
$$x_1=\frac{-41+39}{10}=-\frac{2}{10}=-\frac{1}{5}$$
$$x_2=\frac{-41-39}{10}=\frac{-80}{10}=-8$$
$$5x^2+41x+8=5\left(x+\frac{1}{5}\right)(x+8)$$
$$=(5x+1)(x+8)$$

7. $6x^2+7x-10$

$$x=\frac{-7\pm\sqrt{(7)^2-4(6)(-10)}}{2(6)}$$
$$x=\frac{-7\pm\sqrt{289}}{12}=\frac{-7\pm 17}{12}$$
$$x_1=\frac{-7+17}{12}=\frac{10}{12}=\frac{5}{6}$$
$$x_2=\frac{-7-17}{12}=\frac{-24}{12}=-2$$
$$6x^2+7x-10=6\left(x-\frac{5}{6}\right)(x+2)$$
$$=(6x-5)(x+2)$$

8. $12x^2-25x+12$

$$x=\frac{-(-25)\pm\sqrt{(-25)^2-4(12)(12)}}{2(12)}$$
$$x=\frac{25\pm\sqrt{49}}{24}=\frac{25\pm 7}{24}$$
$$x_1=\frac{25+7}{24}=\frac{32}{24}=\frac{4}{3}$$
$$x_2=\frac{25-7}{24}=\frac{18}{24}=\frac{3}{4}$$
$$12x^2-25x+12=12\left(x-\frac{4}{3}\right)\left(x-\frac{3}{4}\right)$$
$$=\frac{12(3x-4)(4x-3)}{12}$$
$$=(3x-4)(4x-3)$$

9. $8x^2+50x+63$

$$x=\frac{50\pm\sqrt{(50)^2-4(8)(63)}}{2(8)}$$
$$x=\frac{-50\pm\sqrt{484}}{16}=\frac{-50\pm 22}{16}$$
$$x_1=\frac{-50+22}{16}=\frac{-28}{16}=-\frac{7}{4}$$
$$x_2=\frac{-50-22}{16}=-\frac{72}{16}=-\frac{9}{2}$$
$$8x^2+50x+63=8\left(x+\frac{7}{4}\right)\left(x+\frac{9}{2}\right)$$
$$=(4x+7)(2x+9)$$

10. $27x^2+30x+7$

$$x=\frac{-30\pm\sqrt{(30)^2-4(27)(7)}}{2(27)}$$
$$x=\frac{-30\pm\sqrt{144}}{54}=\frac{-30\pm 12}{54}$$
$$x_1=\frac{-30+12}{54}=-\frac{18}{54}=-\frac{1}{3}$$
$$x_2=\frac{-30-12}{54}=-\frac{42}{54}=-\frac{7}{9}$$
$$27x^2+30x+7=27\left(x+\frac{1}{3}\right)\left(x+\frac{7}{9}\right)$$
$$=(3x+1)(9x+7)$$

11. $30x^2-61x+30$

$$x=\frac{-(-61)\pm\sqrt{(-61)^2-4(30)(30)}}{2(30)}$$

$$x=\frac{61\pm\sqrt{121}}{60}=\frac{61\pm 11}{60}$$

$$x_1=\frac{61+11}{60}=\frac{72}{60}=\frac{6}{5}$$

$$x_2=\frac{61-11}{60}=\frac{50}{60}=\frac{5}{6}$$

$$30x^2-61x+30=30\left(x-\frac{6}{5}\right)\left(x-\frac{5}{6}\right)$$

$$=(5x-6)(6x-5)$$

12. $11x^2-153x-180$

$$x=\frac{-(-153)\pm\sqrt{(-153)^2-4(11)(-180)}}{2(11)}$$

$$x=\frac{153\pm\sqrt{31.329}}{22}=\frac{153\pm 177}{22}$$

$$x_1=\frac{153+177}{22}=\frac{330}{22}=15$$

$$x_2=\frac{153-177}{22}=-\frac{24}{22}=-\frac{12}{11}$$

$$11x^2-153x-180=11(x-15)\left(x+\frac{12}{11}\right)$$

$$=(x-15)(11x+12)$$

13. $6-x-x^2$

x^2+x-6

$$x=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-(-6)}=-\frac{1}{2}\pm\sqrt{\frac{25}{4}}$$

$$x_1=-\frac{1}{2}+\frac{5}{2}=\frac{4}{2}=2$$

$$x_2=-\frac{1}{2}-\frac{5}{2}=-\frac{6}{2}=-3$$

$$x^2+x-6=(x-2)(x+3)$$

$$6-x-x^2=(2-x)(3+x)$$

14. $5-9x-2x^2$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(-2)(5)}}{2(-2)}$$

$$x=\frac{9\pm\sqrt{121}}{-4}=\frac{9\pm 11}{-4}$$

$$x_1=\frac{9+11}{-4}=\frac{20}{-4}=-5$$

$$x_2=\frac{9-11}{-4}=\frac{-2}{-4}=\frac{1}{2}$$

Continúa

14. Continuación

$$5-9x-2x^2=-2(x+5)\left(x-\frac{1}{2}\right)$$

$$=-(x+5)(2x-1)$$

$$=(x+5)(1-2x)$$

15. $15+4x-4x^2$

$$x=\frac{-4\pm\sqrt{(4)^2-4(15)(-4)}}{2(-4)}\quad x=\frac{-4\pm\sqrt{256}}{-8}$$

$$x_1=\frac{-4+16}{-8}=\frac{12}{-8}=-\frac{3}{2}$$

$$x_2=\frac{-4-16}{-8}=\frac{-20}{-8}=\frac{5}{2}$$

$$15+4x-4x^2=-4\left(x+\frac{3}{2}\right)\left(x-\frac{5}{2}\right)$$

$$=-(2x+3)(2x-5)$$

$$=(3+2x)(5-2x)$$

16. $4+13x-12x^2$

$12x^2-13x-4$

$$x=\frac{-(-13)\pm\sqrt{(-13)^2-4(12)(-4)}}{2(12)}\quad x=\frac{13\pm\sqrt{361}}{24}$$

$$x_1=\frac{13+19}{24}=\frac{32}{24}=\frac{4}{3}$$

$$x_2=\frac{13-19}{24}=\frac{-6}{24}=-\frac{1}{4}$$

$$12x^2-13x-4=12\left(x-\frac{4}{3}\right)\left(x+\frac{1}{4}\right)$$

$$=(3x-4)(1+4x)$$

$$4+13x-12x^2=(4-3x)(1+4x)$$

17. $72x^2-55x-7$

$$x=\frac{-(-55)\pm\sqrt{(-55)^2-4(72)(-7)}}{2(72)}\quad x=\frac{55\pm\sqrt{5.041}}{144}$$

$$x_1=\frac{55+71}{144}=\frac{126}{144}=\frac{7}{8}$$

$$x_2=\frac{55-71}{144}=\frac{-16}{144}=-\frac{1}{9}$$

$$72x^2-55x-7=72\left(x-\frac{7}{8}\right)\left(x+\frac{1}{9}\right)$$

$$=(8x-7)(9x+1)$$

18. $6+31x-30x^2$

$$x=\frac{-31\pm\sqrt{(31)^2-4(-30)(6)}}{2(-30)}$$

$$x=\frac{-31\pm\sqrt{1.681}}{-60}$$

$$x_1=\frac{-31+41}{-60}=\frac{10}{-60}=-\frac{1}{6}$$

$$x_2=\frac{-31-41}{-60}=\frac{-72}{-60}=\frac{6}{5}$$

$$6+31x-30x^2=-30\left(x+\frac{1}{6}\right)\left(x-\frac{6}{5}\right)$$

$$=-(6x+1)(5x-6)$$

$$=(1+6x)(6-5x)$$

19. $10x^2+207x-63$

$$x=\frac{-207\pm\sqrt{(207)^2-4(10)(-63)}}{2(10)}$$

$$x=\frac{-207\pm\sqrt{45.369}}{20}$$

$$x_1=\frac{-207+213}{20}=\frac{6}{20}=\frac{3}{10}$$

$$x_2=\frac{-207-213}{20}=\frac{-420}{20}=-21$$

$$10x^2+207x-63=10\left(x-\frac{3}{10}\right)(x+21)$$

$$=(10x-3)(x+21)$$

20. $100-15x-x^2$

$$x^2+15x-100$$

$$x=-\frac{15}{2}\pm\sqrt{\frac{(15)^2}{4}-(-100)}$$

$$x=\frac{-15}{2}\pm\sqrt{\frac{625}{4}}$$

$$x_1=\frac{-15}{2}+\frac{25}{2}=\frac{10}{2}=5$$

$$x_2=\frac{-15}{2}-\frac{25}{2}=\frac{-40}{2}=-20$$

$$x^2+15x-100=(x-5)(x+20)$$

$$-x^2-15x+100=-(x-5)(x+20)$$

$$=(5-x)(20+x)$$

21. $18x^2+31x-49$

$$x=\frac{-31\pm\sqrt{(31)^2-4(18)(-49)}}{2(18)}$$

$$x=\frac{-31\pm\sqrt{4.489}}{36}$$

$$x_1=\frac{-31+67}{36}=\frac{36}{36}=1$$

$$x_2=\frac{-31-67}{36}=\frac{-98}{36}=-\frac{49}{18}$$

$$18x^2+31x-49=18(x-1)\left(x+\frac{49}{18}\right)$$

$$=(x-1)(18x+49)$$

22. $6x^2-ax-2a^2$

$$x=\frac{-(-a)\pm\sqrt{(-a)^2-4(6)(-2a^2)}}{2(6)}=\frac{a\pm\sqrt{49a^2}}{12}$$

$$x_1=\frac{a+7a}{12}=\frac{8a}{12}=\frac{2a}{3}$$

$$x_2=\frac{a-7a}{12}=\frac{-6a}{12}=-\frac{1}{2}a$$

$$6x^2-ax-2a^2=6\left(x-\frac{2}{3}a\right)\left(x+\frac{a}{2}\right)$$

$$=(3x-2a)(2x+a)$$

23. $5x^2+22xy-15y^2$

$$x=\frac{-22y\pm\sqrt{(22y)^2-4(5)(-15y^2)}}{2(5)}$$

$$x=\frac{-22y\pm\sqrt{784y^2}}{10}=\frac{-22\pm28y}{10}$$

$$x_1=\frac{-22y+28y}{10}=\frac{6y}{10}=\frac{3}{5}y$$

$$x_2=\frac{-22y-28y}{10}=-\frac{50y}{10}=-5y$$

$$5x^2+22xy-15y^2=5\left(x-\frac{3y}{5}\right)(x+5y)$$

$$=(5x-3y)(x+5y)$$

24. $15x^2-32mx-7m^2$

$$x=\frac{-(-32m)\pm\sqrt{(-32m)^2-4(15)(-7m^2)}}{2(15)}$$

$$x=\frac{32m\pm\sqrt{1.444m^2}}{30}=\frac{32m\pm38m}{30}$$

$$x_1=\frac{32m+38m}{30}=\frac{70m}{30}=\frac{7m}{3}$$

$$x_2=\frac{32m-38m}{30}=\frac{-6m}{30}=-\frac{m}{5}$$

$$15x^2-32mx-7m^2=15\left(x+\frac{m}{5}\right)\left(x-\frac{7m}{3}\right)$$

$$=(5x+m)(3x-7m)$$

EJERCICIO 282

1. $x^4-1=0$

$(x^2+1)(x^2-1)=0$

$x^2+1=0 \qquad x^2-1=0$

$x^2=-1 \qquad x^2=1$

$x=\pm\sqrt{-1} \qquad x=\pm\sqrt{1}$

$x=\pm i \qquad x=\pm 1$

2. $x^3+1=0$

$(x+1)(x^2-x+1)=0$

$x+1=0$

$x_1=-1$

$x^2-x+1=0$

$$x=-\frac{(-1)}{2}\pm\sqrt{\frac{(-1)^2}{4}-1}$$

$$x=\frac{1}{2}\pm\sqrt{-\frac{3}{4}}=\frac{1}{2}\pm\frac{\sqrt{3}\sqrt{-1}}{2}$$

$$x_2=\frac{1}{2}+\frac{i\sqrt{3}}{2}=\frac{1+i\sqrt{3}}{2}$$

$$x_3=\frac{1}{2}-\frac{i\sqrt{3}}{2}=\frac{1-i\sqrt{3}}{2}$$

3. $x^4=81$

$x^4-81=0$

$(x^2-9)(x^2+9)=0$

$(x-3)(x+3)(x^2+9)=0$

$x-3=0 \qquad x+3=0$

$x_1=3 \qquad x_2=-3$

$x^2+9=0$

$x^2=-9$

$x=\pm\sqrt{9}\sqrt{-1}=\pm\sqrt{9}\,i$

$x_3=3i \qquad x_4=-3i$

4. $x^4-256=0$

$(x^2-16)(x^2+16)=0$

$(x+4)(x-4)(x^2+16)=0$

$x+4=0 \qquad x-4=0$

$x_1=-4 \qquad x_2=4$

$x^2+16=0$

$x^2=-16$

$x=\pm\sqrt{16}\sqrt{-1}=\pm 4i$

$x_3=4i \qquad x_4=-4i$

5. $(x^3+8)=0$

$(x+2)(x^2-2x+4)=0$

$x+2=0$

$x_1=-2$

$x^2-2x+4=0$

$$x=\frac{-(-2)}{2}\pm\sqrt{\frac{(-2)^2}{4}-4}$$

$$x=1\pm\sqrt{-\frac{12}{4}}=1\pm\frac{2\sqrt{3}\sqrt{-1}}{2}=1+i\sqrt{3}$$

$x_2=1+i\sqrt{3} \qquad x_3=1-i\sqrt{3}$

6. $x^4-625=0$

$(x^2-25)(x^2+25)=0$

$(x+5)(x-5)(x^2+25)=0$

$x+5=0 \qquad x-5=0$

$x_1=-5 \qquad x_2=5$

$x^2+25=0$

$x^2=-25$

$x=\pm\sqrt{25}\sqrt{-1}=\pm 5i$

$x_3=5i \qquad x_4=-5i$

7. $x^3+64=0$

$(x+4)(x^2-4x+16)=0$

$x+4=0$

$x_1=-4$

$x^2-4x+16=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(16)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-64}}{2}$$

$$x_2=\frac{4+\sqrt{-48}}{2}=\frac{4+4\sqrt{3}\sqrt{-1}}{2}=2+2\sqrt{3i}$$

$$x_3=\frac{4-\sqrt{-48}}{2}=\frac{4-4\sqrt{3}\sqrt{-1}}{2}=2-2\sqrt{3i}$$

8. $x^6-729=0$

$(x^3-27)(x^3+27)=0$

$(x-3)(x^2+3x+9)(x+3)(x^2-3x+9)=0$

$x-3=0 \qquad x+3=0$

$x_1=3 \qquad x_2=-3$

$x^2+3x+9=0$

Continúa

8. Continuación

$$x=\frac{-3}{2}\pm\sqrt{\frac{(3)^2}{4}-9}$$

$$x=\frac{-3}{2}\pm\frac{\sqrt{27}\sqrt{-1}}{\sqrt{4}}=\frac{-3}{2}\pm\frac{3\sqrt{3i}}{2}$$

$$x_3=\frac{-3-3\sqrt{3}\,i}{2} \qquad x_4=\frac{-3+3\sqrt{3i}}{2}$$

$x^2-3x+9=0$

$$x=\frac{-(-3)}{2}\pm\sqrt{\frac{(-3)^2}{4}-9}$$

$$x=\frac{3}{2}\pm\frac{\sqrt{27}\sqrt{-1}}{\sqrt{4}}=\frac{3}{2}\pm\frac{3\sqrt{3}i}{2}$$

$$x_5=\frac{3+3\sqrt{3}i}{2} \qquad x_6=\frac{3-3\sqrt{3}i}{2}$$

9. $x^3=8$

$x^3-8=0$

$(x-2)(x^2+2x+4)=0$

$x-2=0$

$x_1=2$

$x^2+2x+4=0$

$$x=\frac{-2}{2}\pm\sqrt{\frac{(2)^2}{4}-4}$$

$x=-1\pm\sqrt{3}\sqrt{-1}$

$x=-1\pm\sqrt{3}i$

$x_2=-1+\sqrt{3}i$

$x_3=-1-\sqrt{3}i$

10. $x^4=64$

$x^4-64=0$

$(x^2-8)(x^2+8)=0$

$x^2=8 \qquad x^2=-8$

$x=\sqrt{2^2\cdot 2} \qquad x=\sqrt{8}\sqrt{-1}$

$x=\pm 2\sqrt{2} \qquad x=\pm 2\sqrt{2}\,i$

$x_1=2\sqrt{2} \qquad x_3=2\sqrt{2}\,i$

$x_2=-2\sqrt{2} \qquad x_4=-2\sqrt{2}\,i$

EJERCICIO 283

1. $x^4-10x^2+9=0$

$(x^2-9)(x^2-1)=0$

$(x+3)(x-3)(x+1)(x-1)=0$

$x+3=0 \quad x-3=0$

$x_1=-3 \quad x_2=3$

$x+1=0 \quad x-1=0$

$x_3=-1 \quad x_4=1$

2. $x^4-13x^2+36=0$

$(x^2-4)(x^2-9)=0$

$x^2-4=0 \quad x^2-9=0$

$x^2=4 \quad x^2=9$

$x=\pm\sqrt{4} \quad x=\pm\sqrt{9}$

$x_1=2 \quad x_3=3$

$x_2=-2 \quad x_4=-3$

3. $x^4-29x^2+100=0$

$(x^2-25)(x^2-4)=0$

$(x+5)(x-5)(x+2)(x-2)=0$

$x+5=0 \quad x-5=0$

$x_1=-5 \quad x_2=5$

$x+2=0 \quad x-2=0$

$x_3=-2 \quad x_4=2$

4. $x^4-61x^2+900=0$

$(x^2-36)(x^2-25)=0$

$x^2-36=0 \quad x^2-25=0$

$x^2=36 \quad x^2=25$

$x=\pm\sqrt{36} \quad x=\pm\sqrt{25}$

$x_1=6 \quad x_3=5$

$x_2=-6 \quad x_4=-5$

5. $x^4+3x^2-4=0$

$(x^2+4)(x^2-1)=0$

$(x^2+4)(x+1)(x-1)=0$

$x+1=0 \quad x-1=0$

$x_1=-1 \quad x_2=1$

$x^2+4=0$

$x^2=-4$

$x=\sqrt{4}\sqrt{-1}=\pm 2i$

$x_3=2i \quad x_4=-2i$

6. $x^4+16x^2-225=0$

$(x^2+25)(x^2-9)=0$

$(x^2+25)(x+3)(x-3)=0$

$x+3=0 \quad x-3=0$

$x_1=-3 \quad x_2=3$

$x^2+25=0$

$x=\pm\sqrt{25}\sqrt{-1}$

$x_3=5i \quad x_4=-5i$

7. $x^4-45x^2-196=0$

$(x^2-49)(x^2+4)=0$

$(x+7)(x-7)(x^2+4)=0$

$x+7=0 \quad x-7=0$

$x_1=-7 \quad x_2=7$

$x^2+4=0$

$x^2=-4$

$x=\sqrt{4}\sqrt{-1}=\pm 2i$

$x_3=2i \quad x_4=-2i$

8. $x^4-6x^2+5=0$

$(x^2-5)(x^2-1)=0$

$(x^2-5)(x+1)(x-1)=0$

$x+1=0 \quad x-1=0$

$x_1=-1 \quad x_2=1$

$x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

$x_3=\sqrt{5} \quad x_4=-\sqrt{5}$

9. $4x^4-37x^2+9=0$

$(4x^2)^2-37(4x^2)+36=0$

$(4x^2-36)(4x^2-1)=0$

$(x^2-9)(4x^2-1)=0$

$x^2-9=0 \quad 4x^2-1=0$

$x^2=9 \quad x=\pm\sqrt{\frac{1}{4}}$

$x=\pm\sqrt{9}$

$x_1=3 \quad x_3=\frac{1}{2}$

$x_2=-3 \quad x_4=-\frac{1}{2}$

10. $9x^4-40x^2+16=0$

$(9x^2)^2-40(9x^2)+144=0$

$(9x^2-36)(9x^2-4)=0$

$(x^2-4)(9x^2-4)=0$

$(x+2)(x-2)(9x^2-4)=0$

$x+2=0 \quad x-2=0$

$x_1=-2 \quad x_2=2$

$9x^2-4=0$

$x=\pm\sqrt{\frac{4}{9}}$

$x_3=\frac{2}{3} \quad x_4=-\frac{2}{3}$

11. $25x^4+9x^2-16=0$

$25(x^2)^2+9x^2-16=0$

$x^2=\dfrac{-9\pm\sqrt{(9)^2-4(25)(-16)}}{2(25)}$

$x^2=\dfrac{-9\pm\sqrt{1.681}}{50}=\dfrac{-9\pm 41}{50}$

$x^2=\dfrac{-9+41}{50}=\dfrac{32}{50}=\dfrac{16}{25}$

$x=\pm\sqrt{\frac{16}{25}} \quad x_1=\frac{4}{5} \quad x_2=-\frac{4}{5}$

$x^2=\dfrac{-9-41}{50}=\dfrac{-50}{50}=-1$

$x=\pm\sqrt{-1} \quad x_3=i \quad x_4=-i$

12. $4x^4+11x^2-3=0$

$(4x^2)^2+11(4x^2)-12=0$

$(4x^2+12)(4x^2-1)=0$

$(x^2+3)(4x^2-1)=0$

$x^2+3=0$

$x^2=-3$

$x=\pm\sqrt{-3}=\pm\sqrt{3}\sqrt{-1}=\pm\sqrt{3}i$

$x_1=\sqrt{3}i \quad x_2=-\sqrt{3}i$

$4x^2-1=0$

$(2x+1)(2x-1)=0$

$2x+1=0 \quad 2x-1=0$

$x_3=-\frac{1}{2} \quad x_4=\frac{1}{2}$

13. $\left(2x^2+1\right)^2-\left(x^2-3\right)^2=80$

$4x^4+4x^2+1-x^4+6x^2-9=80$

$3x^4+10x^2-88=0$

$\left(3x^2\right)^2+10\left(3x^2\right)-264=0$

$\left(3x^2+22\right)\left(3x^2-12\right)=0$

$\left(3x^2+22\right)\left(x^2-4\right)=0$

$\left(3x^2+22\right)\left(x+2\right)\left(x-2\right)=0$

$x+2=0 \qquad x-2=0$

$x_1=-2 \qquad x_2=2$

$3x^2+22=0$

$x^2=\frac{-22}{3}$

$x=\pm\frac{\sqrt{22}\sqrt{-1}}{\sqrt{3}}$

$x_3=i\frac{\sqrt{22}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{i\sqrt{66}}{3}$

$x_4=-i\frac{\sqrt{22}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{-i\sqrt{66}}{3}$

14. $x^2\left(3x^2+2\right)=4\left(x^2-3\right)+13$

$3x^4+2x^2=4x^2-12+13$

$3x^4-2x^2-1=0$

$\left(3x^2\right)^2-2\left(3x^2\right)-3=0$

$\left(3x^2-3\right)\left(3x^2+1\right)=0$

$\left(x^2-1\right)\left(3x^2+1\right)=0$

$\left(x+1\right)\left(x-1\right)\left(3x^2+1\right)=0$

$x+1=0 \qquad x-1=0$

$x_1=-1 \qquad x_2=1$

$3x^2+1=0$

$x=\frac{\sqrt{-1}}{\sqrt{3}}=\pm\frac{i}{\sqrt{3}}$

$x_3=\frac{i}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{i\sqrt{3}}{3}$

$x_4=-\frac{i}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{i\sqrt{3}}{3}$

EJERCICIO 284

1. $x^6-7x^3-8=0$

$\left(x^3-8\right)\left(x^3+1\right)=0$

$x^3-8=0 \qquad x^3+1=0$

$x^3=8 \qquad x^3=-1$

$x=\sqrt[3]{8} \qquad x=\sqrt[3]{-1}$

$x_1=2 \qquad x_2=-1$

$\left(x^3-8\right)=0$

$\left(x-2\right)\left(x^2+2x+4\right)=0$

$x^2+2x+4=0$

$\left(ver\ ejerc.\ 282\ \ prob.9\right)$

$x_3=-1+\sqrt{3}i$

$x_4=-1-\sqrt{3}i$

$\left(x^3+1\right)=0$

$\left(x+1\right)\left(x^2-x+1\right)$

$\left(ver\ ejerc.\ 282\ \ prob.2\right)$

$x_5=\frac{1+\sqrt{3}i}{2} \qquad x_6=\frac{1-\sqrt{3}i}{2}$

2. $x^6+30x^3+81=0$

$\left(x^3+27\right)\left(x^3+3\right)=0$

$x^3+27=0 \qquad x^3+3=0$

$x^3=-27 \qquad x^3=-3$

$x=\sqrt[3]{-27} \qquad x=\sqrt[3]{-3}$

$x=-3 \qquad x=-\sqrt[3]{3}$

3. $8x^6+15x^3-2=0$

$\left(8x^3\right)^2+15\left(8x^3\right)-16=0$

$\left(8x^3+16\right)\left(8x^3-1\right)=0$

$\left(x^3+2\right)\left(8x^3-1\right)=0$

$x^3+2=0 \qquad 8x^3-1=0$

$x^3=-2 \qquad x=\sqrt{\frac{1}{8}}$

$x=\sqrt[3]{-2} \qquad x=\frac{1}{2}$

$x=-\sqrt[3]{2}$

4. $x^8-41x^4+400=0$

$\left(x^4-25\right)\left(x^4-16\right)=0$

$\left(x^2+5\right)\left(x^2-5\right)\left(x^2+4\right)\left(x^2-4\right)=0$

$x^2-5=0 \qquad x^2-4=0$

$x=\pm\sqrt{5} \qquad x=\pm\sqrt{4}$

$x=\pm 2$

5. $x^{10}-33x^5+32=0$

$\left(x^5-32\right)\left(x^5-1\right)=0$

$x^5-32=0 \qquad x^5-1=0$

$x=\sqrt[5]{32} \qquad x=\sqrt[5]{1}$

$x=2 \qquad x=1$

6. $x^{-4}-13x^{-2}+36=0$

$\left(x^{-2}-9\right)\left(x^{-2}-4\right)=0$

$x^{-2}-9=0 \qquad x^{-2}-4=0$

$x^{-2}=9 \qquad x^{-2}=4$

$\frac{1}{x^2}=9 \qquad \frac{1}{x^2}=4$

$\frac{1}{9}=x^2 \qquad \frac{1}{4}=x^2$

$\pm\sqrt{\frac{1}{9}}=x \qquad \pm\sqrt{\frac{1}{4}}=x$

$x_1=\frac{1}{3};\ x_2=-\frac{1}{3};\ x_3=\frac{1}{2};\ x_4=-\frac{1}{2}$

7. $x^{-6}+35x^{-3}=-216$

$x^{-6}+35x^{-3}+216=0$

$(x^{-3}+27)(x^{-3}+8)=0$

$x^{-3}+27=0 \qquad x^{-3}+8=0$

$x^{-3}=-27 \qquad x^{-3}=-8$

$\frac{1}{x^3}=-27 \qquad \frac{1}{x^3}=-8$

$\sqrt[3]{\frac{1}{-27}}=x \qquad \sqrt[3]{\frac{1}{-8}}=x$

$x=-\frac{1}{3} \qquad x=-\frac{1}{2}$

8. $x^{-10}=242x^{-5}+243$

$x^{-10}-242x^{-5}-243=0$

$(x^{-5}-243)(x^{-5}+1)=0$

$x^{-5}-243=0 \qquad x^{-5}+1=0$

$x^{-5}=243 \qquad x^{-5}=-1$

$\frac{1}{x^5}=243 \qquad \frac{1}{x^5}=-1$

$\sqrt[5]{\frac{1}{243}}=x \qquad \sqrt[5]{\frac{1}{-1}}=x$

$x=\frac{1}{3} \qquad x=-1$

9. $x^3-9x^{\frac{3}{2}}+8=0$

$\left(x^{\frac{3}{2}}-8\right)\left(x^{\frac{3}{2}}-1\right)=0$

$x^{\frac{3}{2}}-8=0 \qquad x^{\frac{3}{2}}-1=0$

$\sqrt{x^3}=8 \qquad \sqrt{x^3}=1$

$x^3=64 \qquad x^3=1$

$x=\sqrt[3]{64} \qquad x=1$

$x=4$

10. $x+x^{\frac{1}{2}}=6$

$x+x^{\frac{1}{2}}-6=0$

$\left(x^{\frac{1}{2}}+3\right)\left(x^{\frac{1}{2}}-2\right)=0$

$x^{\frac{1}{2}}+3=0 \qquad x^{\frac{1}{2}}-2=0$

$\sqrt{x}=-3 \qquad \sqrt{x}=2$

$x=(-3)^2 \qquad x=(2)^2$

$x=9 \qquad x=4$

11. $3x=16\sqrt{x}-5$

$3x-16x^{\frac{1}{2}}+5=0$

$9x-16\left(3x^{\frac{1}{2}}\right)+15=0$

$\left(3x^{\frac{1}{2}}-15\right)\left(3x^{\frac{1}{2}}-1\right)=0$

$3x^{\frac{1}{2}}-15=0 \qquad 3x^{\frac{1}{2}}-1=0$

$\sqrt{x}=\frac{15}{3} \qquad \sqrt{x}=\frac{1}{3}$

$x=(5)^2 \qquad x=\left(\frac{1}{3}\right)^2$

$x=25 \qquad x=\frac{1}{9}$

12. $2x^{\frac{1}{2}}-5x^{\frac{1}{4}}+2=0$

$4x^{\frac{1}{2}}-5\left(2x^{\frac{1}{4}}\right)+4=0$

$\left(2x^{\frac{1}{4}}-4\right)\left(2x^{\frac{1}{4}}-1\right)=0$

$\left(x^{\frac{1}{4}}-2\right)\left(2x^{\frac{1}{4}}-1\right)=0$

$x^{\frac{1}{4}}-2=0 \qquad 2x^{\frac{1}{4}}-1=0$

$x^{\frac{1}{4}}=2 \qquad 2\sqrt[4]{x}=1$

$\sqrt[4]{x}=2 \qquad x=\left(\frac{1}{2}\right)^4$

$x=(2)^4 \qquad x=\frac{1}{16}$

$x=16$

EJERCICIO 285

1. $\sqrt{5+\sqrt{24}}$

$m=\sqrt{(5)^2-24}$

$m=\sqrt{1}$

$m=1$

$=\sqrt{\frac{5+1}{2}}+\sqrt{\frac{5-1}{2}}$

$=\sqrt{3}+\sqrt{2}$

2. $\sqrt{8-\sqrt{60}}$

$m=\sqrt{(8)^2-60}$

$m=\sqrt{4}$

$m=2$

$=\sqrt{\frac{8+2}{2}}-\sqrt{\frac{8-2}{2}}$

$=\sqrt{5}-\sqrt{3}$

3. $\sqrt{8+\sqrt{28}}$

$m=\sqrt{(8)^2-28}$

$m=\sqrt{36}$

$m=6$

$=\sqrt{\frac{8+6}{2}}+\sqrt{\frac{8-6}{2}}$

$=\sqrt{7}+1$

4. $\sqrt{32-\sqrt{700}}$

$m=\sqrt{(32)^2-700}$

$m=\sqrt{324}=18$

$=\sqrt{\frac{32+18}{2}}-\sqrt{\frac{32-18}{2}}$

$=\sqrt{25}-\sqrt{7}$

$=5-\sqrt{7}$

5. $\sqrt{14+\sqrt{132}}$

$m=\sqrt{(14)^2-132}$

$m=\sqrt{64}=8$

$=\sqrt{\frac{14+8}{2}}+\sqrt{\frac{14-8}{2}}$

$=\sqrt{11}+\sqrt{3}$

6. $\sqrt{13+\sqrt{88}}$

$m=\sqrt{(13)^2-88}$

$m=\sqrt{81}=9$

$=\sqrt{\frac{13+9}{2}}+\sqrt{\frac{13-9}{2}}$

$=\sqrt{11}+\sqrt{2}$

7. $\sqrt{11+2\sqrt{30}}$

$\sqrt{11+\sqrt{120}}$

$m=\sqrt{(11)^2-120}=1$

$=\sqrt{\frac{11+1}{2}}+\sqrt{\frac{11-1}{2}}$

$=\sqrt{6}+\sqrt{5}$

8. $\sqrt{84-18\sqrt{3}}$

$\sqrt{84-\sqrt{972}}$

$m=\sqrt{(84)^2-972}$

$m=\sqrt{6.084}=78$

$=\sqrt{\frac{84+78}{2}}-\sqrt{\frac{84-78}{2}}$

$=\sqrt{81}-\sqrt{3}=9-\sqrt{3}$

9. $\sqrt{21+6\sqrt{10}}$

$\sqrt{21+\sqrt{360}}$

$m=\sqrt{(21)^2-360}$

$m=\sqrt{81}=9$

$=\sqrt{\frac{21+9}{2}}+\sqrt{\frac{21-9}{2}}=\sqrt{15}+\sqrt{6}$

10. $\sqrt{28+14\sqrt{3}}$

$\sqrt{28+\sqrt{588}}$

$m=\sqrt{(28)^2-588}$

$m=\sqrt{196}=14$

$=\sqrt{\frac{28+14}{2}}+\sqrt{\frac{28-14}{2}}$

$=\sqrt{21}+\sqrt{7}$

11. $\sqrt{14-4\sqrt{6}}$

$\sqrt{14-\sqrt{96}}$

$m=\sqrt{(14)^2-96}$

$m=\sqrt{100}=10$

$=\sqrt{\frac{14+10}{2}}-\sqrt{\frac{14-10}{2}}$

$=\sqrt{12}-\sqrt{2}$

$=\sqrt{2^2\cdot 3}-\sqrt{2}=2\sqrt{3}-\sqrt{2}$

12. $\sqrt{55+30\sqrt{2}}$

$\sqrt{55+\sqrt{1.800}}$

$m=\sqrt{(55)^2-1.800}$

$m=\sqrt{1.225}=35$

$=\sqrt{\frac{55+35}{2}}+\sqrt{\frac{55-35}{2}}$

$=\sqrt{45}+\sqrt{10}=3\sqrt{5}+\sqrt{10}$

13. $\sqrt{73-12\sqrt{35}}$

$\sqrt{73-\sqrt{5.040}}$

$m=\sqrt{(73)^2-5.040}$

$m=\sqrt{289}=17$

$=\sqrt{\frac{73+17}{2}}-\sqrt{\frac{73-17}{2}}$

$=\sqrt{45}-\sqrt{28}=3\sqrt{5}-2\sqrt{7}$

14. $\sqrt{253-60\sqrt{7}}$

$\sqrt{253-\sqrt{25.200}}$

$m=\sqrt{(253)^2-25.200}$

$m=\sqrt{38.809}=197$

$=\sqrt{\frac{253+197}{2}}-\sqrt{\frac{253-197}{2}}$

$=\sqrt{225}-\sqrt{28}=15-2\sqrt{7}$

15. $\sqrt{293-30\sqrt{22}}$

$\sqrt{293-\sqrt{19.800}}$

$m=\sqrt{(293)^2-19.800}$

$m=\sqrt{66.049}=257$

$=\sqrt{\frac{293+257}{2}}-\sqrt{\frac{293-257}{2}}$

$=\sqrt{275}-\sqrt{18}=5\sqrt{11}-3\sqrt{2}$

16. $\sqrt{\frac{5}{6}+\sqrt{\frac{2}{3}}}$

$m=\sqrt{\left(\frac{5}{6}\right)^2-\frac{2}{3}}$

$m=\sqrt{\frac{1}{36}}=\frac{1}{6}$

$=\sqrt{\frac{\frac{5}{6}+\frac{1}{6}}{2}}+\sqrt{\frac{\frac{5}{6}-\frac{1}{6}}{2}}$

$=\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{3}}=\frac{\sqrt{1}\sqrt{2}}{\sqrt{2}\sqrt{2}}+\frac{\sqrt{1}\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$=\frac{1}{2}\sqrt{2}+\frac{1}{3}\sqrt{3}$

17. $\sqrt{\frac{3}{4}-\sqrt{\frac{1}{2}}}$

$m=\sqrt{\left(\frac{3}{4}\right)^2-\frac{1}{2}}$

$m=\sqrt{\frac{1}{16}}=\frac{1}{4}$

$=\sqrt{\frac{\frac{3}{4}+\frac{1}{4}}{2}}-\sqrt{\frac{\frac{3}{4}-\frac{1}{4}}{2}}$

$=\sqrt{\frac{1}{2}}-\sqrt{\frac{1}{4}}=\frac{\sqrt{2}}{2}-\frac{1}{2}$

18. $\sqrt{\frac{9}{16}+\sqrt{\frac{1}{8}}}$

$m=\sqrt{\left(\frac{9}{16}\right)^2-\frac{1}{8}}$

$m=\sqrt{\frac{49}{256}}=\frac{7}{16}$

$=\sqrt{\frac{\frac{9}{16}+\frac{7}{16}}{2}}+\sqrt{\frac{\frac{9}{16}-\frac{7}{16}}{2}}$

$=\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{16}}=\frac{\sqrt{2}}{2}+\frac{1}{4}$

19. $\sqrt{6+4\sqrt{2}}$

$\sqrt{6+\sqrt{32}}$

$m=\sqrt{(6)^2-32}$

$m=\sqrt{4}=2$

$=\sqrt{\frac{6+2}{2}}+\sqrt{\frac{6-2}{2}}$

$=\sqrt{4}+\sqrt{2}=2+\sqrt{2}$

20. $\sqrt{7+4\sqrt{3}}$

$\sqrt{7+\sqrt{48}}$

$m=\sqrt{(7)^2-48}=1$

$=\sqrt{\frac{7+1}{2}}+\sqrt{\frac{7-1}{2}}$

$=\sqrt{4}+\sqrt{3}=2+\sqrt{3}$

21. $\sqrt{8+2\sqrt{7}}$

$\sqrt{8+\sqrt{28}}$

$m=\sqrt{(8)^2-28}$

$m=\sqrt{36}=6$

$=\sqrt{\frac{8+6}{2}}+\sqrt{\frac{8-6}{2}}$

$=\sqrt{7}+1$

22. $\sqrt{10+2\sqrt{21}}$

$\sqrt{10+\sqrt{84}}$

$m=\sqrt{(10)^2-84}$

$m=\sqrt{16}=4$

$=\sqrt{\dfrac{10+4}{2}}+\sqrt{\dfrac{10-4}{2}}$

$=\sqrt{7}+\sqrt{3}$

23. $\sqrt{18+6\sqrt{5}}$

$\sqrt{18+\sqrt{180}}$

$m=\sqrt{(18)^2-180}$

$m=\sqrt{144}=12$

$=\sqrt{\dfrac{18+12}{2}}+\sqrt{\dfrac{18-12}{2}}$

$=\sqrt{15}+\sqrt{3}$

24. $\sqrt{24-2\sqrt{143}}$

$\sqrt{24-\sqrt{572}}$

$m=\sqrt{(24)^2-572}$

$m=\sqrt{4}=2$

$=\sqrt{\dfrac{24+2}{2}}-\sqrt{\dfrac{24-2}{2}}$

$=\sqrt{13}-\sqrt{11}$

25. $\sqrt{30-20\sqrt{2}}$

$\sqrt{30-\sqrt{800}}$

$m=\sqrt{(30)^2-800}$

$m=\sqrt{100}=10$

$=\sqrt{\dfrac{30+10}{2}}-\sqrt{\dfrac{30-10}{2}}$

$=\sqrt{20}-\sqrt{10}=2\sqrt{5}-\sqrt{10}$

26. $\sqrt{9+6\sqrt{2}}$

$\sqrt{9+\sqrt{72}}$

$m=\sqrt{(9)^2-72}$

$m=\sqrt{9}=3$

$=\sqrt{\dfrac{9+3}{2}}+\sqrt{\dfrac{9-3}{2}}$

$=\sqrt{6}+\sqrt{3}$

27. $\sqrt{98-24\sqrt{5}}$

$\sqrt{98-\sqrt{2.880}}$

$m=\sqrt{(98)^2-2.880}$

$m=\sqrt{6.724}=82$

$=\sqrt{\dfrac{98+82}{2}}-\sqrt{\dfrac{98-82}{2}}$

$=\sqrt{90}-\sqrt{8}=3\sqrt{10}-2\sqrt{2}$

EJERCICIO 286

Los problemas de este ejercicio se resuelven aplicando la siguiente formula:
u = a + (n - 1) r donde:
u es el término enésimo ; a es el primer termino de la progresión ; n es la cantidad de términos ; r es la razón de cambio.

1. $a=7 \quad n=9 \quad r=10-7=3$

$u=7+8(3)=7+24=31$

2. $a=5 \quad n=12 \quad r=10-5=5$

$u=5+11(5)=5+55=60$

3. $a=9 \quad n=48 \quad r=15-12=3$

$u=9+47(3)=9+141=150$

4. $a=3 \quad n=63 \quad r=17-10=7$

$u=3+62(7)=3+434=437$

5. $a=11 \quad n=12 \quad r=6-11=-5$

$u=11+11(-5)=11-55=-44$

6. $a=19 \quad n=28 \quad r=5-12=-7$

$u=19+27(-7)=19-189=-170$

7. $a=3 \quad n=13 \quad r=-5-(-1)=-4$

$u=3+12(-4)=3-48=-45$

8. $a=8 \quad n=54 \quad r=0-8=-8$

$u=8+53(-8)=8-424=-416$

9. $a=-7 \quad n=31 \quad r=1-(-3)=4$

$u=-7+30(4)=-7+120=113$

10. $a=-8 \quad n=17 \quad r=12-2=10$

$u=-8+16(10)=-8+160=152$

11. $a=\dfrac{1}{2} \quad n=12 \quad r=\dfrac{3}{4}-\dfrac{1}{2}=\dfrac{1}{4}$

$u=\dfrac{1}{2}+11\left(\dfrac{1}{4}\right)=\dfrac{1}{2}+\dfrac{11}{4}=\dfrac{13}{4}=3\tfrac{1}{4}$

12. $a=\dfrac{2}{3} \quad n=17 \quad r=\dfrac{5}{6}-\dfrac{2}{3}=\dfrac{1}{6}$

$u=\dfrac{2}{3}+16\left(\dfrac{1}{6}\right)$

$u=\dfrac{2}{3}+\dfrac{8}{3}=\dfrac{10}{3}=3\tfrac{1}{3}$

13. $a=\dfrac{3}{8} \quad n=25 \quad r=\dfrac{11}{24}-\dfrac{3}{8}=\dfrac{1}{12}$

$u=\dfrac{3}{8}+24\left(\dfrac{1}{12}\right)$

$u=\dfrac{3}{8}+2=\dfrac{19}{8}=2\tfrac{3}{8}$

14. $a=\dfrac{1}{3} \quad n=19 \quad r=\dfrac{7}{8}-\dfrac{1}{3}=\dfrac{13}{24}$

$u=\dfrac{1}{3}+18\left(\dfrac{13}{24}\right)$

$u=\dfrac{1}{3}+\dfrac{39}{4}=\dfrac{121}{12}=10\tfrac{1}{12}$

15. $a=\dfrac{7}{2} \quad n=27 \quad r=\dfrac{21}{4}-\dfrac{7}{2}=\dfrac{7}{4}$

$u=\dfrac{7}{2}+26\left(\dfrac{7}{4}\right)=\dfrac{7}{2}+\dfrac{91}{2}=\dfrac{98}{2}=49$

16. $a=\dfrac{7}{9} \quad n=36 \quad r=\dfrac{1}{3}-\dfrac{7}{9}=-\dfrac{4}{9}$

$u=\dfrac{7}{9}+35\left(-\dfrac{4}{9}\right)$

$u=\dfrac{7}{9}-\dfrac{140}{9}=-\dfrac{133}{9}=-14\tfrac{7}{9}$

17. $a=\dfrac{2}{7} \quad n=15 \quad r=\dfrac{1}{8}-\dfrac{2}{7}=-\dfrac{9}{56}$

$u=\dfrac{2}{7}+14\left(-\dfrac{9}{56}\right)$

$u=\dfrac{2}{7}-\dfrac{126}{56}=-\dfrac{110}{56}=-1\tfrac{27}{28}$

18. $a=-\dfrac{3}{5} \quad n=21 \quad r=-\dfrac{14}{15}+\dfrac{3}{5}=-\dfrac{1}{3}$

$u=-\dfrac{3}{5}+20\left(-\dfrac{1}{3}\right)$

$u=-\dfrac{3}{5}-\dfrac{20}{3}=-\dfrac{109}{15}=-7\tfrac{4}{15}$

19. $a=-\frac{1}{4}\ \ n=13\ \ r=-\frac{9}{4}+\frac{1}{4}=-2$

$u=-\frac{1}{4}+12(-2)=-\frac{1}{4}-24=-\frac{97}{4}=-24\tfrac{1}{4}$

20. $a=-\frac{5}{6}\ \ n=19\ \ r=-\frac{1}{3}+\frac{5}{6}=\frac{1}{2}$

$u=-\frac{5}{6}+18\left(\frac{1}{2}\right)=-\frac{5}{6}+9=\frac{49}{6}=8\tfrac{1}{6}$

21. $a=\frac{11}{3}\ \ n=33\ \ r=\frac{35}{12}-\frac{11}{3}=-\frac{3}{4}$

$u=\frac{11}{3}+32\left(-\frac{3}{4}\right)=\frac{11}{3}-24=-\frac{61}{3}=-20\tfrac{1}{3}$

22. $a=\frac{14}{5}\ \ n=41\ \ r=\frac{27}{10}-\frac{14}{5}=-\frac{1}{10}$

$u=\frac{14}{5}+40\left(-\frac{1}{10}\right)$

$u=\frac{14}{5}-4=-\frac{6}{5}=-1\tfrac{1}{5}$

23. $a=-\frac{3}{5}\ \ n=26\ \ r=\frac{3}{10}+\frac{3}{5}=\frac{9}{10}$

$u=-\frac{3}{5}+25\left(\frac{9}{10}\right)$

$u=-\frac{3}{5}+\frac{45}{2}=\frac{219}{10}=21\tfrac{9}{10}$

24. $a=-4\ \ n=19$

$r=-\frac{2}{3}+4=\frac{10}{3}$

$u=-4+18\left(\frac{10}{3}\right)$

$u=-4+60=56$

25. $a=3\ \ n=39$

$r=-\frac{5}{4}-3=-\frac{17}{4}$

$u=3+38\left(-\frac{17}{4}\right)$

$u=3-\frac{323}{2}=-\frac{317}{2}=-158\tfrac{1}{2}$

EJERCICIO 287

Para resolver los problemas de este ejercicio se utilizan las siguientes formulas:

$a=u-(n-1)r \qquad r=\frac{u-a}{n-1} \qquad n=\frac{u-a+r}{r}$

Donde u es el término enésimo ; r es la razón de cambio ; n es la cantidad de términos ; a es el primer término de la progresión.

1. $u=20\ \ r=\frac{2}{7}\ \ n=15$

$a=20-(14)\left(\frac{2}{7}\right)$

$a=20-4=16$

2. $u=-18\ \ n=32\ \ r=3$

$a=-18-(31)(3)$

$a=-18-93=-111$

3. $u=1\ \ n=9\ \ r=1-\frac{3}{4}=\frac{1}{4}$

$a=1-(8)\left(\frac{1}{4}\right)=1-2=-1$

4. $u=\frac{25}{3}\ \ n=7$

Se plantea la ecuación para encontrar la razón

$\frac{25}{3}-2r=7$

$-2r=7-\frac{25}{3}$

$-2r=-\frac{4}{3}$

Continúa

4. Continuación

$r=\frac{4}{6}=\frac{2}{3}$

$a=\frac{25}{3}-(6)\left(\frac{2}{3}\right)$

$a=\frac{25}{3}-\frac{12}{3}=\frac{13}{3}=4\tfrac{1}{3}$

5. $a=3\ \ n=6\ \ u=8$

$r=\frac{8-3}{6-1}=\frac{5}{5}=1$

6. $a=-1\ \ n=10\ \ u=-4$

$r=\frac{-4-(-1)}{10-1}=-\frac{3}{9}=-\frac{1}{3}$

7. $a=\frac{1}{2}\ \ n=17\ \ u=-\frac{3}{8}$

$r=\frac{-\frac{3}{8}-\frac{1}{2}}{17-1}=\frac{-\frac{7}{8}}{16}=-\frac{7}{128}$

8. $a=5\ \ n=18\ \ u=-80$

$r=\frac{-80-5}{18-1}=\frac{-85}{17}=-5$

9. $a=-42\ \ n=92\ \ u=1.050$

$r=\frac{1.050-(-42)}{92-1}=\frac{1.092}{91}=12$

10. $a=4\ \ u=30$

$r=6-4=2$

$n=\frac{30-4+2}{2}=\frac{28}{2}=14$

11. $a=5\ \ u=18$

$r=\frac{16}{3}-5=\frac{1}{3}$

$n=\frac{18-5+\frac{1}{3}}{\frac{1}{3}}$

$n=\frac{13+\frac{1}{3}}{\frac{1}{3}}=\frac{\frac{40}{3}}{\frac{1}{3}}=40$

12. $a=\frac{26}{5}\ \ u=18$

$r=6-\frac{26}{5}=\frac{4}{5}$

$n=\frac{18-\frac{26}{5}+\frac{4}{5}}{\frac{4}{5}}$

$n=\frac{18-\frac{22}{5}}{\frac{4}{5}}=\frac{\frac{68}{5}}{\frac{4}{5}}=\frac{68}{4}=17$

EJERCICIO 288

1. $r=19-15=4$

$a=15 \quad n=8$

$u=15+(7)4$

$u=15+28=43$

$s=\dfrac{(15+43)8}{2}$

$s=(58)(4)=232$

2. $a=31 \quad n=19$

$r=7$

$u=31+(18)7$

$u=31+126=157$

$s=\dfrac{(31+157)(19)}{2}$

$s=\dfrac{3.572}{2}=1.786$

3. $a=42 \quad n=24 \quad r=-10$

$u=42+(23)(-10)$

$u=-188$

$s=\dfrac{(42-188)(24)}{2}$

$s=(-146)(12)=-1.752$

4. $a=-10 \quad n=80 \quad r=4$

$u=-10+(79)(4)$

$u=-10+316=306$

$s=\dfrac{(-10+306)(80)}{2}$

$s=(296)(40)=11.840$

5. $a=11 \quad n=60 \quad r=-10$

$u=11+(59)(-10)$

$u=11-590=-579$

$s=\dfrac{(11-579)(60)}{2}$

$s=(-568)(30)=-17.040$

6. $a=-5 \quad n=50 \quad r=-8$

$u=-5+(49)(-8)$

$u=-5-392=-397$

$s=\dfrac{(-5-397)(50)}{2}$

$s=(-402)(25)=-10.050$

7. $a=\dfrac{1}{2} \quad n=9 \quad r=\dfrac{1}{2}$

$u=\dfrac{1}{2}+(8)\left(\dfrac{1}{2}\right)$

$u=\dfrac{1}{2}+4=\dfrac{9}{2}$

$s=\dfrac{\left(\dfrac{1}{2}+\dfrac{9}{2}\right)(9)}{2}$

$s=\dfrac{45}{2}=22\tfrac{1}{2}$

8. $a=\dfrac{3}{10} \quad n=14 \quad r=\dfrac{1}{10}$

$u=\dfrac{3}{10}+(13)\left(\dfrac{1}{10}\right)$

$u=\dfrac{3}{10}+\dfrac{13}{10}=\dfrac{8}{5}$

$s=\dfrac{\left(\dfrac{3}{10}+\dfrac{8}{5}\right)(14)}{2}$

$s=\dfrac{133}{10}=13\tfrac{3}{10}$

9. $a=\dfrac{3}{4} \quad n=19 \quad r=\dfrac{3}{4}$

$u=\dfrac{3}{4}+18\left(\dfrac{3}{4}\right)$

$u=\dfrac{3}{4}+\dfrac{27}{2}=\dfrac{57}{4}$

$s=\dfrac{19\left(\dfrac{3}{4}+\dfrac{57}{4}\right)}{2}$

$s=\dfrac{19(15)}{2}=\dfrac{285}{2}=142\tfrac{1}{2}$

10. $a=\dfrac{2}{5} \quad n=34 \quad r=-\dfrac{3}{11}$

$u=\dfrac{2}{5}+33\left(-\dfrac{3}{11}\right)$

$u=\dfrac{2}{5}-9=-\dfrac{43}{5}$

$s=\dfrac{34\left(\dfrac{2}{5}-\dfrac{43}{5}\right)}{2}$

$s=-\dfrac{1.394}{10}=-139\tfrac{2}{5}$

11. $a=\dfrac{7}{3} \quad n=11 \quad r=\dfrac{4}{5}$

$r=\dfrac{47}{15}-\dfrac{7}{3}=\dfrac{47-35}{15}=\dfrac{4}{5}$

$u=\dfrac{7}{3}+10\left(\dfrac{4}{5}\right)$

$u=\dfrac{7}{3}+8=\dfrac{31}{3}$

$s=\dfrac{11\left(\dfrac{7}{3}+\dfrac{31}{3}\right)}{2}$

$s=\dfrac{418}{6}=69\tfrac{2}{3}$

12. $a=\dfrac{13}{4} \quad n=46$

$r=\dfrac{73}{20}-\dfrac{13}{4}=\dfrac{8}{20}=\dfrac{2}{5}$

$u=\dfrac{13}{4}+45\left(\dfrac{2}{5}\right)$

$u=\dfrac{13}{4}+18=\dfrac{85}{4}$

$s=\dfrac{46\left(\dfrac{13}{4}+\dfrac{85}{4}\right)}{2}$

$s=\dfrac{4.508}{8}=563\tfrac{1}{2}$

13. $a=-2 \quad n=17 \quad r=\dfrac{1}{4}+2=\dfrac{9}{4}$

$u=-2+16\left(\dfrac{9}{4}\right)=-2+36=34$

$s=\dfrac{17(-2+34)}{2}=17(16)=272$

14. $a=-5 \quad n=12 \quad r=-\dfrac{37}{8}+5=\dfrac{3}{8}$

$u=-5+11\left(\dfrac{3}{8}\right)=-\dfrac{7}{8}$

$s=\dfrac{12\left(-5-\dfrac{7}{8}\right)}{2}$

$s=6\left(-\dfrac{47}{8}\right)=-\dfrac{141}{4}=-35\tfrac{1}{4}$

EJERCICIO 289

1. $u=11 \quad a=3 \quad n=5$

$r=\frac{11-3}{5-1}=\frac{8}{4}=2$

$\Rightarrow 3+2=5$

$5+2=7$

$7+2=9$

Luego $\div 3.\ 5.\ 7.\ 9.\ 11.$

2. $u=-5 \quad a=19 \quad m=7$

$r=\frac{-5-19}{7+1}=\frac{-24}{8}=-3$

$\Rightarrow 19-3=16$

$16-3=13$

$13-3=10$

$10-3=7$

$7-3=4$

$4-3=1$

$1-3=-2$

Luego $\div 19.\ 16.\ 13.\ 10.\ 7.\ 4.\ 1.\ -2.\ -5.$

3. $u=-73 \quad a=-13 \quad n=7$

$r=\frac{-73+13}{7-1}=\frac{-60}{6}=-10$

$\Rightarrow -13-10=-23$

$-23-10=-33$

$-33-10=-43$

$-43-10=-53$

$-53-10=-63$

Luego $\div -13.\ -23.\ -33.\ -43.\ -53.\ -63.\ -73.$

4. $u=53 \quad a=-42 \quad m=4$

$r=\frac{53+42}{4+1}=\frac{95}{5}=19$

$\Rightarrow -42+19=-23$

$-23+19=-4$

$-4+19=15$

$15+19=34$

Luego $\div -42.\ -23.\ -4.\ 15.\ 34.\ 53.$

5. $u=-9 \quad a=-81 \quad n=7$

$r=\frac{-9+81}{7-1}=\frac{72}{6}=12$

$\Rightarrow -81+12=-69$

$-69+12=-57$

$-57+12=-45$

$-45+12=-33$

$-33+12=-21$

Luego $\div -81\ -69.\ -57.\ -45.\ -33.\ -21\ -9.$

6. $u=3 \quad a=1 \quad m=3$

$r=\frac{3-1}{3+1}=\frac{2}{4}=\frac{1}{2}$

$\Rightarrow 1+\frac{1}{2}=\frac{3}{2}$

$\frac{3}{2}+\frac{1}{2}=2$

$2+\frac{1}{2}=\frac{5}{2}$

Luego $\div 1\ \frac{3}{2}.\ 2.\ \frac{5}{2}.\ 3.$

$\div 1\ 1\frac{1}{2}.\ 2.\ 2\frac{1}{2}.\ 3.$

7. $u=12 \quad a=5 \quad n=6$

$r=\frac{12-5}{6-1}=\frac{7}{5}$

$\Rightarrow 5+\frac{7}{5}=\frac{32}{5}$

$\frac{32}{5}+\frac{7}{5}=\frac{39}{5}$

$\frac{39}{5}+\frac{7}{5}=\frac{46}{5}$

$\frac{46}{5}+\frac{7}{5}=\frac{53}{5}$

Luego $\div 5.\ \frac{32}{5}.\ \frac{39}{5}.\ \frac{46}{5}.\ \frac{53}{5}.\ 12.$

$\div 5.\ 6\frac{2}{5}.\ 7\frac{4}{5}.\ 9\frac{1}{5}.\ 10\frac{3}{5}.\ 12.$

8. $u=3 \quad a=-4 \quad m=5$

$r=\frac{3+4}{5+1}=\frac{7}{6}$

$\Rightarrow -4+\frac{7}{6}=-\frac{17}{6}$

$-\frac{17}{6}+\frac{7}{6}=-\frac{5}{3}$

$-\frac{5}{3}+\frac{7}{6}=-\frac{1}{2}$

$-\frac{1}{2}+\frac{7}{6}=\frac{2}{3}$

$\frac{2}{3}+\frac{7}{6}=\frac{11}{6}$

Luego $\div -4.\ -\frac{17}{6}.\ -\frac{5}{3}.\ -\frac{1}{2}.\ \frac{2}{3}.\ \frac{11}{6}.\ 3.$

$\div -4.\ -2\frac{5}{6}.\ -1\frac{2}{3}.\ -\frac{1}{2}.\ \frac{2}{3}.\ -1\frac{5}{6}.\ 3.$

9. $u=\frac{1}{8}\quad a=\frac{3}{4}\quad n=7$

$$r=\frac{\frac{1}{8}-\frac{3}{4}}{7-1}=\frac{-\frac{5}{8}}{6}=-\frac{5}{48}$$

$$\Rightarrow\frac{3}{4}-\frac{5}{48}=\frac{31}{48}$$

$$\frac{31}{48}-\frac{5}{48}=\frac{13}{24}$$

$$\frac{13}{24}-\frac{5}{48}=\frac{7}{16}$$

$$\frac{7}{16}-\frac{5}{48}=\frac{1}{3}$$

$$\frac{1}{3}-\frac{5}{48}=\frac{11}{48}$$

Luego $\div\frac{3}{4}.\frac{31}{48}.\frac{13}{24}.\frac{7}{16}.\frac{1}{3}.\frac{11}{48}.\frac{1}{8}.$

10. $u=3\quad a=-1\quad m=6$

$$r=\frac{3+1}{6+1}=\frac{4}{7}$$

$$\Rightarrow-1+\frac{4}{7}=-\frac{3}{7}$$

$$-\frac{3}{7}+\frac{4}{7}=\frac{1}{7}$$

$$\frac{1}{7}+\frac{4}{7}=\frac{5}{7}$$

$$\frac{5}{7}+\frac{4}{7}=\frac{9}{7}$$

$$\frac{9}{7}+\frac{4}{7}=\frac{13}{7}$$

$$\frac{13}{7}+\frac{4}{7}=\frac{17}{7}$$

Luego $\div-1\ -\frac{3}{7}.\frac{1}{7}.\frac{5}{7}.\frac{9}{7}.\frac{13}{7}.\frac{17}{7}.3.$

$\div-1\ -\frac{3}{7}.\frac{1}{7}.\frac{5}{7}.1\frac{2}{7}.1\frac{6}{7}.2\frac{3}{7}.3.$

11. $u=-\frac{1}{8}\quad a=\frac{2}{3}\quad n=7$

$$r=\frac{-\frac{1}{8}-\frac{2}{3}}{7-1}=\frac{-\frac{19}{24}}{6}=-\frac{19}{144}$$

$$\Rightarrow\frac{2}{3}-\frac{19}{144}=\frac{77}{144}$$

$$\frac{77}{144}-\frac{19}{144}=\frac{29}{72}$$

$$\frac{29}{72}-\frac{19}{144}=\frac{13}{48}$$

Continúa

11. Continuación

$$\frac{13}{48}-\frac{19}{144}=\frac{5}{36}$$

$$\frac{5}{36}-\frac{19}{144}=\frac{1}{144}$$

Luego $\div\frac{2}{3}.\frac{77}{144}.\frac{29}{72}.\frac{13}{48}.\frac{5}{36}.\frac{1}{144}.-\frac{1}{8}.$

12. $u=-5\quad a=-2\quad m=7$

$$r=\frac{-5+2}{7+1}=-\frac{3}{8}$$

$$\Rightarrow-2-\frac{3}{8}=-\frac{19}{8}$$

$$-\frac{19}{8}-\frac{3}{8}=-\frac{11}{4}$$

$$-\frac{11}{4}-\frac{3}{8}=-\frac{25}{8}$$

$$-\frac{25}{8}-\frac{3}{8}=-\frac{7}{2}$$

$$-\frac{7}{2}-\frac{3}{8}=-\frac{31}{8}$$

$$-\frac{31}{8}-\frac{3}{8}=-\frac{17}{4}$$

$$-\frac{17}{4}-\frac{3}{8}=-\frac{37}{8}$$

Luego $\div-2.-2\frac{3}{8}.-2\frac{3}{4}.-3\frac{1}{8}.-3\frac{1}{2}.-3\frac{7}{8}.-4\frac{1}{4}.-4\frac{5}{8}.-5.$

13. $u=-\frac{7}{10}\quad a=\frac{1}{2}\quad n=10$

$$r=\frac{-\frac{7}{10}-\frac{1}{2}}{10-1}=\frac{-\frac{6}{5}}{9}=-\frac{2}{15}$$

$$\Rightarrow\frac{1}{2}-\frac{2}{15}=\frac{11}{30}$$

$$\frac{11}{30}-\frac{2}{15}=\frac{7}{30}$$

$$\frac{7}{30}-\frac{2}{15}=\frac{1}{10}$$

$$\frac{1}{10}-\frac{2}{15}=-\frac{1}{30}$$

$$-\frac{1}{30}-\frac{2}{15}=-\frac{1}{6}$$

$$-\frac{1}{6}-\frac{2}{15}=-\frac{3}{10}$$

$$-\frac{3}{10}-\frac{2}{15}=-\frac{13}{30}$$

$$-\frac{13}{30}-\frac{2}{15}=-\frac{17}{30}$$

Luego

$\div\frac{1}{2}.\frac{11}{30}.\frac{7}{30}.\frac{1}{10}.-\frac{1}{30}.-\frac{1}{6}.-\frac{3}{10}.-\frac{13}{30}.-\frac{17}{30}.-\frac{7}{10}.$

EJERCICIO 290

1. $a=7 \quad n=20 \quad r=7$

$u=7+(19)(7)$

$u=7+133=140$

$s=\frac{(7+140)(20)}{2}$

$s=147\cdot 10=1.470$

2. $a=5 \quad n=80 \quad r=5$

$u=5+(79)(5)$

$u=5+395=400$

$s=\frac{(5+400)(80)}{2}$

$s=405\cdot 40=16.200$

3. $a=9 \quad n=43 \quad r=10$

$u=9+(42)(10)$

$u=9+420=429$

$s=\frac{(9+429)(43)}{2}$

$s=219\cdot 43=9.417$

4. $a=2 \quad n=100 \quad r=2$

$u=2+(99)(2)$

$u=2+198=200$

$s=\frac{(2+200)(100)}{2}$

$s=202\cdot 50=10.100$

5. $a=9 \quad n=100 \quad r=2$

$u=9+(99)(2)$

$u=9+198=207$

$s=\frac{(9+207)(100)}{2}$

$s=216\cdot 50=10.800$

6. $a=8 \quad n=50 \quad r=3$

$u=8+(49)(3)$

$u=8+147=155$

$s=\frac{(8+155)(50)}{2}$

$s=163\cdot 25$

$s=4.075\div 100$

$s=\$40,75$

7. $32 \rightarrow$ *Nº normal de dientes en la boca de un adulto*

$a=10 \quad n=32 \quad r=2$

$u=10+(31)(2)=72$

$s=\frac{(10+72)(32)}{2}$

$s=1.312\div 10$

$s=\$131,20$

8. $a=77 \quad n=72 \quad r=11$

$u=77+(71)(11)=858$

$s=\frac{(77+858)(72)}{2}$

$s=935\cdot 36=33.660$

9. $n=5\cdot 12=60$

$a=2 \quad r=3$

$u=2+(59)(3)$

$u=2+177=179$

$s=\frac{(2+179)(60)}{2}$

$s=181\cdot 30=5.430\ bs.$

10. $a=600 \quad r=25$

Si $n=8$

$u=600+(7)(25)$

$u=775\div 100$

Avanzó $7,75m$ *en el* 8º *segundo*

$s \rightarrow$ *Distancia recorrida en 8 seg.*

$s=\frac{(600+775)(8)}{2}$

$s=1.375\cdot 4$

$s=5.500\div 100=55m$

11. $s=2.400 \quad n=3$

$x \rightarrow$ *Ahorro el 2º año*

$\frac{x}{2} \rightarrow$ *Ahorro el 1º año*

$r=x-\frac{x}{2}=\frac{x}{2}$

Luego el 3º año Ahorro:

$x+\frac{x}{2}=\frac{3x}{2}$

Continúa

11. Continuación.

$\Rightarrow \frac{x}{2}+x+\frac{3x}{2}=2.400$

$x+2x+3x=4.800$

$6x=4.800$

$x=800$

El 1º año ahorro $\rightarrow \frac{x}{2}=\frac{800}{2}=400\,sucres$

Para el 2º año

$a=400 \quad n=2 \quad r=\frac{x}{2}=\frac{800}{2}=400$

$u=400+(1)(400)=800\ sucres$

Para el 3º año

$a=400 \quad n=3 \quad r=400$

$u=400+(2)(400)=1.200\ sucres$

12. 2º *y* 4º $=22$

3º *y* 7º $=34$

El término medio entre el 2º y el 4º es el 3º por tanto es una progresión impar. Luego el duplo del 3º término es 22

$x \rightarrow 3^{er}$ *término*

$2x=22$

$x=11$

Luego $11 \rightarrow 3^{er}$ *término*

Como $3º+7º=34$

$11+7º=34$

$7º=23$

$23 \rightarrow$ *es el 7º término*

Interpolación para conocer el 4º término

$\div 11...23$

Donde $n=5 \quad a=11 \quad u=23$

$r=\frac{23-11}{5-1}=\frac{12}{4}=3$

4º *término* $\rightarrow 3º+r=11+3=14$

2º *término* $\rightarrow 22-4º=22-14=8$

13. $n=32 \quad a=5 \quad r=3$

$u=5+(31)(3)$

$u=5+93=98$

$s=\frac{(5+98)32}{2}$

$s=103\cdot 16=\$1.648$

14. $a=50 \quad r=-5\tfrac{1}{2} \quad n=8$

$u=50+(7)\left(-\frac{11}{2}\right)$

$u=\frac{100-77}{2}=\frac{23}{2}=11\tfrac{1}{2}\,Km$

$s=\frac{\left(50+\frac{23}{2}\right)8}{2}=246\,Km$

15. 3° *y* 10° *términos son equidistantes a los extremos* 1° *y* 12°*, Luego la suma de estos extremos es igual a las suma de los términos equidistantes*

$3^{o}+10^{o}=1^{o}+12^{o}$

como $1^{o}+12^{o}=53\tfrac{1}{2}$

$\Rightarrow 3^{o}+10^{o}=53\tfrac{1}{2}$

16. $n=11 \quad a=-2 \quad u=-52$

$r=\frac{-52-(-2)}{11-1}=-\frac{50}{10}=-5$

2° *térm.* $\rightarrow -2-5=-7$

3° *térm.* $\rightarrow -7-5=-12$

4° *térm.* $\rightarrow -12-5=-17$

5° *térm.* $\rightarrow -17-5=-22$

6° *térm.* $\rightarrow -22-5=-27$

17. $a=500 \quad u=1.900 \quad r=200$

$n=\frac{1.900-500+200}{200}$

$n=8$ *años*

18. $n=11 \quad a=1.180 \quad u=6.180$

$r=\frac{6.180-1.180}{11-1}$

$r=\frac{5.000}{10}=\$500$

19. $n=5 \quad u=3.000 \quad r=-300$

$a=3.000-(4)(-300)$

$a=3.000+1.200=4.200$ *soles*

20. $a=161 \quad r=322 \quad n=5$

$u=161+(4)(322)=1.449$

$s=\frac{(1.449+161)(5)}{2}$

$s=\frac{(1.610)(5)}{2}$

$s=\frac{8.050}{2}=4.025$

$=4.025\div 10=402,5$ *pies*

21. $a=51 \quad r=2 \quad u=813$

$n=\frac{813-51+2}{2}=\frac{764}{2}=382$

$s=\frac{(813+51)(382)}{2}$

$s=(864)(191)=165.024$

22. $5^{o}\rightarrow 31 \quad 9^{o}\rightarrow 59$

interpolando

$n=5 \quad a=31 \quad u=59$

$r=\frac{59-31}{5-1}=\frac{28}{4}=7$

10° *térm.* $\rightarrow 59+7=66$

11° *térm.* $\rightarrow 66+7=73$

12° *térm.* $\rightarrow 73+7=80$

23. $n=3 \quad a=12.500 \quad u=20.500$

$a+u=12.500+20.500$

$a+u=33.000$

Luego $2\left(2^{o}\ term.\right)=33.000$

$2^{o}\ term.=16.500$ *colones*

EJERCICIO 291

1. $a=3 \quad n=7$

$r=6\div 3=2$

$u=3\left(2^{7-1}\right)$

$u=3\left(2^{6}\right)$

$u=3\cdot 64=192$

2. $a=\frac{1}{3} \quad n=8$

$r=3\div 1=3$

$u=\frac{1}{3}\left(3^{8-1}\right)$

$u=3^{6}=729$

3. $a=8 \quad n=9$

$r=4\div 8=\frac{1}{2}$

$u=8\left[\left(\frac{1}{2}\right)^{9-1}\right]$

$u=\frac{8}{2^{8}}=\frac{1}{2^{5}}=\frac{1}{32}$

4. $a=1 \quad n=6$

$r=\frac{4}{25}\cdot\frac{5}{2}=\frac{2}{5}$

$u=1\left[\left(\frac{2}{5}\right)^{6-1}\right]$

$u=\frac{2^{5}}{5^{5}}=\frac{32}{3.125}$

5. $a=3 \quad n=7$

$r=2\div 3=\frac{2}{3}$

$u=3\left[\left(\frac{2}{3}\right)^{7-1}\right]$

$u=\frac{2^{6}}{3^{5}}=\frac{64}{243}$

6. $a=\frac{1}{2} \quad n=6$

$r=\frac{1}{5}\cdot 2=\frac{2}{5}$

$u=\frac{1}{2}\left[\left(\frac{2}{5}\right)^{6-1}\right]$

$u=\frac{2^{4}}{5^{5}}=\frac{16}{3.125}$

7. $a=\frac{9}{4} \quad n=8$

$r=3\cdot\frac{4}{9}=\frac{4}{3}$

$u=\frac{9}{4}\left[\left(\frac{4}{3}\right)^{7}\right]$

$u=\frac{4^{6}}{3^{5}}=\frac{4.096}{243}=16\tfrac{208}{243}$

8. $a=-3 \quad n=6$

$r=6\div(-3)=-2$

$u=-3\left[(-2)^{5}\right]$

$u=-3(-32)=96$

9. $a=3 \quad n=9$

$r=-1\div 3=-\frac{1}{3}$

$u=3\left[\left(-\frac{1}{3}\right)^{8}\right]$

$u=\frac{1}{3^{7}}=\frac{1}{2.187}$

10. $a=\frac{5}{6} \quad n=5$

$r=\frac{1}{2}\cdot\frac{6}{5}=\frac{3}{5}$

$u=\frac{5}{6}\left[\left(\frac{3}{5}\right)^{4}\right]$

$u=\frac{3^{3}}{2\cdot 5^{3}}=\frac{27}{250}$

11. $a=16 \quad n=8$

$r=1\div(-4)=-\frac{1}{4}$

$u=16\left[\left(-\frac{1}{4}\right)^7\right]$

$u=-\frac{1}{4^5}=-\frac{1}{1.024}$

12. $a=\frac{3}{4} \quad n=8$

$r=\frac{1}{3}(-2)=-\frac{2}{3}$

$u=\frac{3}{4}\left[\left(-\frac{2}{3}\right)^7\right]$

$u=-\frac{2^5}{3^6}=-\frac{32}{729}$

13. $a=-\frac{3}{5} \quad n=5$

$r=\frac{3}{2}\left(-\frac{5}{3}\right)=-\frac{5}{2}$

$u=-\frac{3}{5}\left[\left(-\frac{5}{2}\right)^4\right]$

$u=-\frac{3\cdot 5^3}{2^4}=-\frac{375}{16}=-23\tfrac{7}{16}$

14. $a=-\frac{3}{4} \quad n=10$

$r=-\frac{1}{4}\left(-\frac{4}{3}\right)=\frac{1}{3}$

$u=-\frac{3}{4}\left[\left(\frac{1}{3}\right)^9\right]$

$u=-\frac{1}{4\cdot 3^8}=-\frac{1}{26.244}$

EJERCICIO 292

1. $r=\frac{1}{2} \quad u=\frac{1}{64} \quad n=7$

$a=\frac{\frac{1}{64}}{\left(\frac{1}{2}\right)^6}=\frac{\frac{1}{2^6}}{\frac{1}{2^6}}=\frac{2^6}{2^6}=1$

2. $r=\frac{2}{3} \quad u=\frac{64}{2.187} \quad n=9$

$a=\frac{\frac{64}{2.187}}{\left(\frac{2}{3}\right)^8} \quad a=\frac{\frac{2^6}{3^7}}{\frac{2^8}{3^8}}=\frac{2^6\cdot 3^8}{2^8\cdot 3^7}=\frac{3}{2^2}=\frac{3}{4}$

3. $r=\frac{32}{625}\cdot\frac{125}{16}=\frac{2}{5}$

$u=\frac{32}{625} \quad n=6$

$a=\frac{\frac{32}{625}}{\left(\frac{2}{5}\right)^5}=\frac{\frac{2^5}{5^4}}{\frac{2^5}{5^5}}=\frac{2^5\cdot 5^5}{2^5\cdot 5^4}=5$

4. $a=2 \quad u=64 \quad n=6$

$r=\sqrt[5]{\frac{64}{2}}=\sqrt[5]{\frac{2^6}{2}}=\sqrt[5]{2^5}=2$

5. $a=\frac{1}{3} \quad u=243 \quad n=7$

$r=\sqrt[6]{\frac{243}{\frac{1}{3}}}=\sqrt[6]{729}=\sqrt[6]{3^6}=\pm 3$

6. $a=-5 \quad u=640 \quad n=8$

$r=\sqrt[7]{\frac{640}{-5}}$

$r=\sqrt[7]{-128}=\sqrt[7]{-2^7}=-2$

7. $a=\frac{729}{2} \quad u=\frac{3}{2} \quad n=6$

$r=\sqrt[5]{\frac{\frac{3}{2}}{\frac{729}{2}}}=\sqrt[5]{\frac{1}{243}}=\frac{1}{3}$

8. $a=8 \quad u=\frac{1}{512} \quad n=7$

$r=\sqrt[6]{\frac{\frac{1}{512}}{8}}=\sqrt[6]{\frac{1}{4.096}}=\sqrt[6]{\frac{1}{4^6}}=\pm\frac{1}{4}$

9. $a=\frac{625}{16} \quad u=1 \quad n=5$

$r=\sqrt[4]{\frac{1}{\frac{625}{16}}}=\sqrt[4]{\frac{16}{625}}=\sqrt[4]{\frac{2^4}{5^4}}=\pm\frac{2}{5}$

10. $a=\frac{27}{64} \quad u=-\frac{2}{81} \quad n=8$

$r=\sqrt[7]{\frac{-\frac{2}{81}}{\frac{27}{64}}}$

$r=\sqrt[7]{-\frac{2\cdot 2^6}{3^4\cdot 3^3}}=\sqrt[7]{-\frac{2^7}{3^7}}=-\frac{2}{3}$

EJERCICIO 293

1. $a=6 \quad n=5$

$r=3\div 6=\frac{1}{2}$

$u=6\left[\left(\frac{1}{2}\right)^4\right]=6\left(\frac{1}{16}\right)=\frac{3}{8}$

$s=\frac{\left(\frac{3}{8}\right)\left(\frac{1}{2}\right)-6}{\frac{1}{2}-1}$

$s=-\frac{93}{16}(-2)=\frac{93}{8}=11\tfrac{5}{8}$

2. $a=4 \quad n=6$

$r=-8\div 4=-2$

$u=4\left[(-2)^5\right]$

$u=4(-32)=-128$

$s=\frac{-128(-2)-4}{-2-1}$

$s=\frac{252}{-3}=-84$

3. $a=12 \quad n=7$

$r=4\div 12=\frac{1}{3}$

$u=12\left[\left(\frac{1}{3}\right)^6\right]=12\left(\frac{1}{729}\right)=\frac{4}{243}$

$s=\frac{\left(\frac{4}{243}\right)\left(\frac{1}{3}\right)-12}{\frac{1}{3}-1}$

$s=-\frac{8.744}{729}\left(-\frac{3}{2}\right)$

$s=\frac{26.232}{1.458}=17\tfrac{1.446}{1.458}=17\tfrac{241}{243}$

4. $a=\frac{1}{4} \quad n=10$

$r=\frac{1}{2}\cdot 4=2$

$u=\frac{1}{4}\left[2^9\right]=128$

$s=\frac{128(2)-\frac{1}{4}}{2-1}$

$s=\frac{1.023}{4}=255\tfrac{3}{4}$

5. $a=\frac{9}{4} \quad n=8$

$r=\frac{3}{2}\cdot\frac{4}{9}=\frac{2}{3}$

$u=\frac{9}{4}\left[\left(\frac{2}{3}\right)^7\right]=\frac{2^5}{3^5}$

$s=\dfrac{\frac{2^5}{3^5}\left(\frac{2}{3}\right)-\frac{9}{4}}{\frac{2}{3}-1}$

$s=\frac{2^8-3^8}{3^6\cdot 2^2}(-3)$

$s=\frac{-6.305}{2.916}(-3)$

$s=\frac{18.915}{2.916}=6\tfrac{1.419}{2.916}=6\tfrac{473}{972}$

6. $a=-\frac{1}{10} \quad n=7$

$r=\frac{1}{5}(-10)=-2$

$u=-\frac{1}{10}\left[(-2)^6\right]=-\frac{64}{10}=-\frac{32}{5}$

$s=\dfrac{-\frac{32}{5}(-2)+\frac{1}{10}}{-2-1}$

$s=\dfrac{\frac{129}{10}}{-3}=-\frac{129}{30}=-4\tfrac{3}{10}$

7. $a=-6 \quad n=10$

$r=-3\div(-6)=\frac{1}{2}$

$u=-6\left[\left(\frac{1}{2}\right)^9\right]=-\frac{3}{2^8}$

$s=\dfrac{-\frac{3}{2^8}\left(\frac{1}{2}\right)+6}{\frac{1}{2}-1}$

$s=\dfrac{\frac{3.069}{512}}{-\frac{1}{2}}=\frac{6.138}{-512}=-11\tfrac{253}{256}$

8. $a=2 \quad n=8$

$r=-1\div 2=-\frac{1}{2}$

$u=2\left[\left(-\frac{1}{2}\right)^7\right]=-\frac{1}{2^6}=-\frac{1}{64}$

$s=\dfrac{-\frac{1}{64}\left(-\frac{1}{2}\right)-2}{-\frac{1}{2}-1}$

$s=\dfrac{\frac{255}{128}}{\frac{3}{2}}=\frac{510}{384}=1\tfrac{126}{384}=1\tfrac{21}{64}$

9. $a=\frac{3}{2} \quad n=6$

$r=1\div\frac{3}{2}=\frac{2}{3}$

$u=\frac{3}{2}\left(\frac{2}{3}\right)^5=\frac{2^4}{3^4}=\frac{16}{81}$

$s=\dfrac{\frac{16}{81}\left(\frac{2}{3}\right)-\frac{3}{2}}{\frac{2}{3}-1}$

$s=\dfrac{-1\frac{179}{486}}{-\frac{1}{3}}=4\tfrac{17}{162}$

10. $a=9 \quad n=6$

$r=1\div(-3)=-\frac{1}{3}$

$u=9\left(-\frac{1}{3}\right)^5=-\frac{1}{3^3}=-\frac{1}{27}$

$s=\dfrac{-\frac{1}{27}\left(-\frac{1}{3}\right)-9}{-\frac{1}{3}-1}$

$s=\dfrac{-\frac{728}{81}}{-\frac{4}{3}}$

$s=\frac{2.184}{324}=6\tfrac{240}{324}=6\tfrac{20}{27}$

EJERCICIO 294

1. $a=5 \quad u=3.125 \quad n=5$

$r=\sqrt[4]{\frac{3.125}{5}}=\sqrt{25}\pm 5$

$5(\pm 5)=\pm 25$

$\pm 25(\pm 5)=125$

$125(\pm 5)=\pm 625$

$\div\div 5{:}\pm 25{:}125{:}\pm 625{:}3.125$

2. $a=-7 \quad u=-224 \quad m=4$

$r=\sqrt[5]{\frac{-224}{-7}}=\sqrt[5]{32}=2$

$-7\cdot 2=-14$

$-14\cdot 2=-28$

$-28\cdot 2=-56$

$-56\cdot 2=-112$

$\div\div -7{:}-14{:}-28{:}-56{:}-112{:}-224$

3. $a=128 \quad u=2 \quad n=7$

$r=\sqrt[6]{\frac{2}{128}}=\sqrt[6]{\frac{1}{2^6}}=\pm\frac{1}{2}$

$128\left(\pm\frac{1}{2}\right)=\pm 64$

$\pm 64\left(\pm\frac{1}{2}\right)=32$

$32\left(\pm\frac{1}{2}\right)=\pm 16$

$\pm 16\left(\pm\frac{1}{2}\right)=8$

$8\left(\pm\frac{1}{2}\right)=\pm 4$

$\div\div 128{:}\pm 64{:}32{:}\pm 16{:}8{:}\pm 4{:}2$

4. $a=\frac{9}{2} \quad u=\frac{16}{27} \quad n=6$

$r=\sqrt[5]{\dfrac{\frac{16}{27}}{\frac{9}{2}}}=\sqrt[5]{\frac{2^5}{3^5}}=\frac{2}{3}$

$\frac{9}{2}\left(\frac{2}{3}\right)=3$

$3\left(\frac{2}{3}\right)=2$

$2\left(\frac{2}{3}\right)=\frac{4}{3}$

$\frac{4}{3}\left(\frac{2}{3}\right)=\frac{8}{9}$

$\div\div\frac{9}{2}{:}\ 3{:}\ 2{:}\ \frac{4}{3}{:}\ \frac{8}{9}{:}\ \frac{16}{27}$

$\div\div 4\tfrac{1}{2}{:}\ 3{:}\ 2{:}\ 1\tfrac{1}{3}{:}\ \frac{8}{9}{:}\ \frac{16}{27}$

5. $a=2 \quad u=\frac{2.187}{64} \quad m=6$

$$r=\sqrt[7]{\frac{\frac{2.187}{64}}{2}}=\sqrt[7]{\frac{3^7}{2^7}}=\frac{3}{2}$$

$$2\left(\frac{3}{2}\right)=3$$

$$3\left(\frac{3}{2}\right)=\frac{9}{2}$$

$$\frac{9}{2}\left(\frac{3}{2}\right)=\frac{27}{4}$$

$$\frac{27}{4}\left(\frac{3}{2}\right)=\frac{81}{8}$$

$$\frac{81}{8}\left(\frac{3}{2}\right)=\frac{243}{16}$$

$$\frac{243}{16}\left(\frac{3}{2}\right)=\frac{729}{32}$$

$$\div\div 2:3:4\tfrac{1}{2}:6\tfrac{3}{4}:10\tfrac{1}{8}:15\tfrac{3}{16}:22\tfrac{25}{32}:34\tfrac{11}{64}$$

6. $a=\frac{4}{9} \quad u=\frac{27}{256} \quad n=6$

$$r=\sqrt[5]{\frac{\frac{27}{256}}{\frac{4}{9}}}=\sqrt[5]{\frac{3^5}{4^5}}=\frac{3}{4}$$

$$\frac{4}{9}\left(\frac{3}{4}\right)=\frac{1}{3}$$

$$\frac{1}{3}\left(\frac{3}{4}\right)=\frac{1}{4}$$

$$\frac{1}{4}\left(\frac{3}{4}\right)=\frac{3}{16}$$

$$\frac{3}{16}\left(\frac{3}{4}\right)=\frac{9}{64}$$

$$\div\div\frac{4}{9}:\frac{1}{3}:\frac{1}{4}:\frac{3}{16}:\frac{9}{64}:\frac{27}{256}$$

7. $a=8 \quad u=\frac{1}{32} \quad m=7$

$$r=\sqrt[8]{\frac{\frac{1}{32}}{8}}=\sqrt[8]{\frac{1}{2^8}}=\pm\frac{1}{2}$$

$$8\left(\pm\frac{1}{2}\right)=\pm 4$$

$$\pm 4\left(\pm\frac{1}{2}\right)=2$$

$$2\left(\pm\frac{1}{2}\right)=\pm 1$$

$$\pm 1\left(\pm\frac{1}{2}\right)=\frac{1}{2}$$

$$\frac{1}{2}\left(\pm\frac{1}{2}\right)=\pm\frac{1}{4}$$

$$\pm\frac{1}{4}\left(\pm\frac{1}{2}\right)=\frac{1}{8}$$

$$\frac{1}{8}\left(\pm\frac{1}{2}\right)=\pm\frac{1}{16}$$

$$\div\div 8:\pm 4:2:\pm 1:\frac{1}{2}:\pm\frac{1}{4}:\frac{1}{8}:\pm\frac{1}{16}:\frac{1}{32}$$

EJERCICIO 295

1. $a=2 \quad r=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

$$s=\frac{2}{1-\frac{1}{4}}=\frac{2}{\frac{3}{4}}=\frac{8}{3}=2\tfrac{2}{3}$$

2. $a=\frac{1}{2} \quad r=\frac{1}{6}\cdot 2=\frac{1}{3}$

$$s=\frac{\frac{1}{2}}{1-\frac{1}{3}}=\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{3}{4}$$

3. $a=-5 \quad r=\frac{-2}{-5}=\frac{2}{5}$

$$s=\frac{-5}{1-\frac{2}{5}}=\frac{-5}{\frac{3}{5}}=-\frac{25}{3}$$

$$=-8\tfrac{1}{3}$$

4. $a=-4 \quad r=-\frac{8}{3}\left(-\frac{1}{4}\right)=\frac{2}{3}$

$$s=\frac{-4}{1-\frac{2}{3}}=\frac{-4}{\frac{1}{3}}=-12$$

5. $a=\frac{3}{4} \quad r=\frac{1}{4}\left(\frac{4}{3}\right)=\frac{1}{3}$

$$s=\frac{\frac{3}{4}}{1-\frac{1}{3}}=\frac{\frac{3}{4}}{\frac{2}{3}}=\frac{9}{8}=1\tfrac{1}{8}$$

6. $a=\frac{1}{6} \quad r=\frac{1}{7}\cdot 6=\frac{6}{7}$

$$s=\frac{\frac{1}{6}}{1-\frac{6}{7}}=\frac{\frac{1}{6}}{\frac{1}{7}}=\frac{7}{6}=1\tfrac{1}{6}$$

7. $a=2 \quad r=-\frac{2}{5}\cdot\frac{1}{2}=-\frac{1}{5}$

$$s=\frac{2}{1+\frac{1}{5}}=\frac{2}{\frac{6}{5}}=\frac{10}{6}=1\tfrac{2}{3}$$

8. $a=-14 \quad r=\frac{-6}{-14}=\frac{3}{7}$

$$s=\frac{-14}{1-\frac{3}{7}}=\frac{-14}{\frac{4}{7}}=-\frac{98}{4}$$

$$=-24\tfrac{1}{2}$$

EJERCICIO 296

1. $\frac{6}{10}+\frac{6}{100}+\frac{6}{1.000}\cdots$

$$a=\frac{6}{10} \quad r=\frac{6}{100}\left(\frac{10}{6}\right)=\frac{1}{10}$$

$$s=\frac{\frac{6}{10}}{1-\frac{1}{10}}=\frac{\frac{6}{10}}{\frac{9}{10}}=\frac{6}{9}=\frac{2}{3}$$

2. $\frac{12}{100}+\frac{12}{10.000}+\frac{12}{1.000.000}\cdots$

$$a=\frac{12}{100} \quad r=\frac{12}{10.000}\left(\frac{100}{12}\right)=\frac{1}{100}$$

$$s=\frac{\frac{12}{100}}{1-\frac{1}{100}}=\frac{\frac{12}{100}}{\frac{99}{100}}=\frac{12}{99}=\frac{4}{33}$$

3. $\frac{159}{1.000}+\frac{159}{1.000.000}\cdots$

$$a=\frac{159}{10^3} \quad r=\frac{159}{10^6}\left(\frac{10^3}{159}\right)=\frac{1}{10^3}$$

$$s=\frac{\frac{159}{10^3}}{1-\frac{1}{10^3}}=\frac{\frac{159}{10^3}}{\frac{999}{10^3}}=\frac{159}{999}=\frac{53}{333}$$

4. $\frac{32}{100}+\frac{32}{10.000}+\frac{32}{1.000.000}\ldots$

$a=\frac{32}{100}\quad r=\frac{32}{10.000}\left(\frac{100}{32}\right)=\frac{1}{100}$

$s=\frac{\frac{32}{100}}{1-\frac{1}{100}}=\frac{\frac{32}{100}}{\frac{99}{100}}=\frac{32}{99}$

5. $\frac{144}{10^3}+\frac{144}{10^6}+\frac{144}{10^9}\ldots$

$a=\frac{144}{10^3}\quad r=\frac{144}{10^6}\left(\frac{10^3}{144}\right)=\frac{1}{10^3}$

$s=\frac{\frac{144}{10^3}}{1-\frac{1}{10^3}}=\frac{\frac{144}{10^3}}{\frac{999}{10^3}}=\frac{144}{999}=\frac{16}{111}$

6. $\frac{3}{10}+\frac{5}{100}+\frac{5}{1.000}\ldots$

$a=\frac{5}{100}\quad r=\frac{5}{1.000}\left(\frac{100}{5}\right)=\frac{1}{10}$

$s=\frac{\frac{5}{100}}{1-\frac{1}{10}}=\frac{\frac{5}{100}}{\frac{9}{10}}=\frac{5}{90}=\frac{1}{18}$

$s=\frac{3}{10}+\frac{1}{18}=\frac{64}{180}=\frac{16}{45}$

7. $\frac{18}{100}+\frac{1}{10^3}+\frac{1}{10^4}\ldots$

$a=\frac{1}{10^3}\quad r=\frac{1}{10^4}\cdot 10^3=\frac{1}{10}$

$s=\frac{\frac{1}{10^3}}{1-\frac{1}{10}}=\frac{\frac{1}{10^3}}{\frac{9}{10}}=\frac{1}{900}$

$s=\frac{18}{100}+\frac{1}{900}=\frac{163}{900}$

8. $\frac{3}{10}+\frac{18}{10^3}+\frac{18}{10^5}\ldots$

$a=\frac{18}{10^3}\quad r=\frac{18}{10^5}\left(\frac{10^3}{18}\right)=\frac{1}{100}$

$s=\frac{\frac{18}{10^3}}{1-\frac{1}{100}}=\frac{\frac{18}{10^3}}{\frac{99}{100}}=\frac{1}{55}$

$s=\frac{3}{10}+\frac{1}{55}=\frac{175}{550}=\frac{7}{22}$

9. $2+\frac{18}{100}+\frac{18}{10^4}+\frac{18}{10^6}\ldots$

$a=\frac{18}{100}\quad r=\frac{18}{10^4}\left(\frac{100}{18}\right)=\frac{1}{100}$

$s=\frac{\frac{18}{100}}{1-\frac{1}{100}}=\frac{\frac{18}{100}}{\frac{99}{100}}=\frac{2}{11}$

$s=2+\frac{2}{11}=\frac{24}{11}$

EJERCICIO 297

1. $a=2\quad r=2$

Para saber cuanto gano el sábado $\rightarrow n=6$

$u=2\left(2^5\right)=64$ *Lempiras*

Para saber cuanto gané de lunes a sábado utilizo S para los 6 primeros términos de la progresión.

$\div\div 2:4:8\ldots$

Donde $a=2\quad r=2\quad u=64$

$s=\frac{64(2)-2}{2-1}=126$ *Lempiras*

2. $n=20\quad r=2\quad a=1$

$u=1\left(2^{19}\right)=524.288$

$s=\frac{524.288(2)-1}{2-1}$

$s=1.048.575\div 100$

$s=\$\ 10.485,75$

3. $n=8\quad r=\frac{1}{3}\quad u=1$

$a=\frac{1}{\left(\frac{1}{3}\right)^7}=\frac{1}{\frac{1}{2.187}}=2.187$ *balboas*

4. $n=9\ \rightarrow$ Im*par*

Como es una sucesión impar los términos equidistan, luego la multiplicación del primer y último término es igual al producto entre el 3º y el 7º término.

Entonces la respuesta es:

$=\frac{1}{216}$

5. $n=5\quad u=\frac{8}{81}$

3er *término* $=\sqrt{\frac{4}{81}}=\frac{2}{9}$

Se encuentra la razón para $n=3$, *entre el 3º y 5º término.*

$r=\sqrt{\frac{\frac{8}{81}}{\frac{2}{9}}}=\sqrt{\frac{36}{81}}=\frac{2}{3}$

$\frac{2}{9}\cdot\frac{3}{2}=\frac{1}{3}\ \rightarrow$ 2º *término*

$\frac{1}{3}\cdot\frac{3}{2}=\frac{1}{2}\ \rightarrow$ 1º *término*

6. $a=\frac{1}{4}\quad u=\frac{1}{32}\quad n=4$

$r=\sqrt[3]{\frac{\frac{1}{32}}{\frac{1}{4}}}=\sqrt[3]{\frac{1}{8}}=\frac{1}{2}$

$\frac{1}{32}\cdot 2=\frac{1}{16}\ \rightarrow$ 6º *término*

7. $r=\frac{2}{3}\quad u=160\quad n=5$

$a=\frac{160}{\left(\frac{2}{3}\right)^4}=\frac{160}{\frac{16}{81}}=810$

$s=\frac{160\left(\frac{2}{3}\right)-810}{\frac{2}{3}-1}$

$s=\frac{-\frac{2.110}{3}}{-\frac{1}{3}}=\$2.110$

8. $a=59.049\quad u=100.000\quad n=6$

$r=\sqrt[5]{\frac{100.000}{59.049}}=\sqrt[5]{\frac{10^5}{9^5}}=\frac{10}{9}$

9. $r=\frac{1}{3}\quad a=24.300\quad n=6$

$u=24.300\left(\frac{1}{3}\right)^5=\frac{24.300}{243}=100$

$s=\frac{100\left(\frac{1}{3}\right)-24.300}{\frac{1}{3}-1}=\frac{-\frac{72.800}{3}}{-\frac{2}{3}}=\frac{72.800}{2}=36.400\ bs.$

10. $n=15\quad a=1\quad r=3$

$u=(3)^{14}=4.782.969$

$s=\frac{4.782.969(3)-1}{3-1}=\frac{14.348.906}{2}=\$7.174.453$

EJERCICIO 298

1. $Log(532\cdot 0{,}184)$

$=Log\ 532+Log\ 0{,}184$

$=2{,}72591+\overline{1}{,}26482$

$=1{,}99073$

$\Rightarrow 10^{1{,}99073}=97{,}888$

2. $Log191{,}7+Log432$

$=2{,}28262+2{,}63548$

$=4{,}9181$

$\Rightarrow Anti\log 4{,}9181=82.814{,}4$

3. $Log0{,}7+Log0{,}013+Log0{,}9$

$=\overline{1}{,}84510+\overline{2}{,}11394+\overline{1}{,}95424$

$=-2{,}08672$

$\Rightarrow 10^{-2{,}08672}=0{,}00819$

4. $Log7{,}5+Log8{,}16+Log0{,}35+Log10.037$

$=0{,}875061+0{,}911690+\overline{1}{,}544068+4{,}001603$

$=4-1+2{,}332422$

$=5{,}332422$

$\Rightarrow 10^{5{,}332422}=214.991{,}85$

$\cong 214.992$

5. $Log3{,}2+Log4{,}3+Log7{,}8+Log103{,}4+Log0{,}019$

$=0{,}505149+0{,}633468+0{,}892094+2{,}014520+\overline{2}{,}278753$

$=2{,}323988\Rightarrow Anti\log 2{,}323988=210{,}857$

6. $Log95{,}18-Log7{,}23$

$=1{,}978317-0{,}859138$

$=1{,}119179\Rightarrow Anti\log 1{,}119179=13{,}15767031\cong 13{,}1577$

7. $Log8{,}125-Log0{,}9324$

$=Log8{,}125+Co\log 0{,}9324$

$=0{,}909823+0{,}030397$

$=0{,}940220\Rightarrow 10^{0{,}940220}=8{,}714063\cong 8{,}7141$

8. $Log7.653{,}95-Log12{,}354$

$=\log 7.653{,}95+Co\log 12{,}354$

$=3{,}883885+\overline{2}{,}908192$

$=2{,}792077\Rightarrow Anti\log 2{,}792077=619{,}55$

9. $Log0{,}72183-Log0{,}0095$

$=Log0{,}72183+Co\log 0{,}0095$

$=\overline{1}{,}858434+2{,}022276$

$=1{,}88071\Rightarrow 10^{1{,}88071}=75{,}981873\cong 75{,}982$

10. $Log9.114-Log0{,}02$

$=Log9.114+Co\log 0{,}02$

$=3{,}959709+1{,}698970$

$=5{,}658679\Rightarrow Anti\log 5{,}658679=455.699{,}97\cong 455.700$

11. $Log\,2^{10}=10(\log 2)$

$=10(0{,}301029)$

$=3{,}010299$

$Anti\log\ 3{,}010299=1.024$

12. $Log\,0{,}15^3=3(\log 0{,}15)$

$=3(\overline{1}{,}176091)$

$=3(-1)+3(0{,}176091)$

$=-2{,}471727$

$10^{-2{,}471727}=0{,}00337499\cong 0{,}003375$

13. $Log\,18{,}65^4=4(\log 18{,}65)$

$=4(1{,}270678)$

$=5{,}082715$

$Anti\log\ 5{,}082715=120.980{,}4916$

$\cong 120.980{,}5$

14. $Log(2\cdot 0{,}084)^2=\log(0{,}168)^2$

$=2(\log 0{,}168)$

$=2(\overline{1}{,}225309)$

$=2(-1)+2(0{,}225309)$

$=-1{,}549382=\overline{2}{,}450618$

$10^{-1{,}549382}=0{,}028223963\cong 0{,}028224$

$Anti\log\ \overline{2}{,}450618=0{,}028224$

15. $Log\,7{,}2^6=6(\log 7{,}2)$

$=6(0{,}857332)$

$=5{,}143994$

$Anti\log 5{,}143994=139.313{,}75$

16. $Log\sqrt{3}=\frac{\log 3}{2}=\frac{0,477121}{2}=0,238560$

$Anti\log 0,238560=1,7320483\cong 1,73205$

17. $Log\sqrt[3]{2}=\frac{\log 2}{3}=\frac{0,301029}{3}=0,100343$

$10^{0,100343}=1,259921$

18. $Log\sqrt[4]{5}=\frac{\log 5}{4}=\frac{0,698970}{4}=0,174742$

$Anti\log 0,174742=1,495348\cong 1,49535$

19. $Log\sqrt[5]{63}=\frac{\log 63}{5}=\frac{1,799340}{5}=0,359868$

$10^{0,359868}=2,290172$

20. $Log\sqrt[7]{815}=\frac{\log 815}{7}=\frac{2,911157}{7}=0,415879$

$Anti\log 0,415879=2,605431$

EJERCICIO 299

1. $Log 515+\log 78,19+co\log 6,13$

$=2,711807+1,893151+\overline{1},212539$

$=3,817497 \rightarrow Anti\log 3,817497=6.568,96\cong 6.569$

2. $Log 23,054+\log 934,5+co\log 8.164$

$=1,36275+2,97058+\overline{4},0881$

$=0,42143 \rightarrow Anti\log 0,42143=2,63894$

3. $Log 8,14+\log 9,73+co\log 0,6+co\log 7,8$

$=0,91062+0,98811+0,22185+\overline{1},10791$

$=1,22849 \rightarrow Anti\log 1,22849=16,923461\cong 16,9235$

4. $Log\ 513,4+\log 9,132+co\log 85,3+co\log 10,764$

$=2,71046+0,96057+\overline{2},06905+\overline{2},96802$

$=0,7081 \rightarrow Anti\log 0,7081=5,10622$

5. $Log 53,245+\log 4.325,6+co\log 32,815+co\log 91,79$

$=1,72628+3,63605+\overline{2},48392+\overline{2},03721$

$=1,88346 \rightarrow Anti\log 1,88346=76,4645$

6. $Log 32,6+\log 841,9+co\log 0,017+co\log 732,14$

$=1,51322+2,92526+1,76955+\overline{3},135406$

$=3,343436 \rightarrow Anti\log 3,343436=2.205,139151$

Como existe un signo negativo en la operación entonces este afecta el resultado total, luego la respuesta final es $\cong -2.205,14$

7. $Log 95,36+\log 0,14+co\log 83,7+co\log 2,936$

$=1,97937+\overline{1},14613+\overline{2},07727+\overline{1},53224$

$=-1,26499 \rightarrow Anti\log -1,26499=0,054326$

El signo negativo se anula puesto que se encuentra en el numerador y en el denominador luego la respuesta es positiva

8. $Log 7,2+\log 8,135+co\log 0,003+co\log 9.134,7$

$=0,85733+0,91036+2,52288+\overline{4},03930$

$=0,32987 \rightarrow Anti\log 0,32987=-2,137333\cong -2,13734$

9. $3^5\cdot 0,2^4=5\log 3+4\log 0,2$

$=5(0,47712)+4(\overline{1},30103)$

$=2,38561-2,79588$

$=-0,41027 \rightarrow 10^{-0,41027}=0,3888$

$*En\ la\ tabla\ se\ busca\ \overline{1},58973$

$\Rightarrow Anti\log 0,58973=0,3888$

10. $\frac{1}{2}\log 5+\frac{2}{3}\log 3$

$=\frac{1}{2}(0,69897)+\frac{2}{3}(0,47712)$

$=0,349485+0,31808$

$=0,66757$

$\rightarrow Anti\log 0,66757=4,651199\cong 4,6512$

11. $\frac{1}{5}\log 2+\frac{1}{2}\log 3+\frac{3}{4}\log 5$

$=\frac{1}{5}(0,30103)+\frac{1}{2}(0,47712)+\frac{3}{4}(0,69897)$

$=0,060206+0,23856+0,5242275$

$=0,82299$

$\rightarrow Anti\log 0,82299=6,65257\cong 6,6526$

12. $8\log 3+5co\log 5,6$

$=8(0,47712)+5(\overline{1},25181)$

$=3,81696-3,74095$

$=0,07601$

$\rightarrow Anti\log 0,07601=1,19127\cong 1,1913$

13. $7\log 0,53+3co\log 2,5$

$=7(\overline{1},72428)+3(\overline{1},60206)$

$=-1,93004-1,19382$

$=-3,12386 \rightarrow 10^{-3,12386}=0,000751865$

$*\ En\ la\ tabla\ se\ busca\ \overline{4},87614$

14. $\frac{2}{5}\log 3+\frac{5}{3}co\log 2$

$=\frac{2}{5}(0,47712)+\frac{5}{3}(\overline{1},69897)$

$=0,190848-0,501717$

$=-0,310869 \quad \rightarrow 10^{-0,310869}=0,488799\cong 0,4888$

$*$ *En la tabla se busca* $\overline{1},68914$

15. $\frac{Log 7,86+Log 8,14}{2}$

$=\frac{0,89542+0,91062}{2}$

$=\frac{1,80604}{2}$

$=0,90302 \quad \rightarrow Anti\log 0,90302=7,9987$

16. $\frac{Log 932,5+Log 813,6+Log 0,005}{2}$

$=\frac{2,96965+2,91041+\overline{3},69897}{2}$

$=\frac{3,57903}{2}$

$=1,78952 \quad Anti\log 1,78952=61,59138$

17. $\frac{Log 93,7+Log 104,2+co\log 8,35+co\log 7,3}{2}$

$=\frac{1,97174+2,01787+\overline{1},07831+\overline{1},13668}{2}$

$=\frac{2,2046}{2}=1,1023 \rightarrow Anti\log 1,1023=12,656174$

18. $\frac{Log 23,725+Log 9,182+Log 7,184}{3}$

$=\frac{1,37521+0,96294+0,85637}{3}$

$=\frac{3,19452}{3}=1,06484 \quad Anti\log 1,06484=-11,610207$

19. $\frac{Log 12.316+Log 0,25+co\log 931,8+co\log 0,07}{4}$

$=\frac{4,09047+\overline{1},39794+\overline{3},03068+1,15490}{4}$

$=\frac{1,67399}{4}=0,41849 \rightarrow Anti\log 0,41849=2,621125$

20. $\frac{Log 56.813+co\log 22.117}{5}$

$=\frac{4,75446+\overline{5},65527}{5}$

$=\frac{0,40973}{5}=0,081946 \rightarrow Anti\log 0,081946=1,20766$

21. $\frac{3}{2}\log 0,0316+\frac{3}{2}co\log 0,1615$

$=\frac{3}{2}(\overline{2},49969+0,79183)$

$=\frac{3}{2}(-0,70848)=-1,06272=\overline{2},93728$

$10^{-1,06272}=0,086552$

$Anti\log \overline{2},93728=0,08655625$

22. $\frac{3}{4}\log 3+\frac{2}{3}co\log 5$

$=\frac{3}{4}(0,47712)+\frac{2}{3}(\overline{1},30103)$

$=0,35784-0,46598$

$=-0,10814=\overline{1},89186$

$Anti\log \overline{1},89186=0,77958$

23. $\frac{Log 15+Co\log 4}{7}$

$=\frac{1,17609+\overline{1},39794}{7}$

$=\frac{0,57403}{7}=0,082$

$Anti\log 0,082=1,20782$

24. $\frac{Log 5+Co\log 3}{5}$

$=\frac{0,69897+\overline{1},52288}{5}=0,04437$

$Anti\log \quad 0,04437=-1,10756$

25. $\frac{6}{5}(Log 5+Co\log 8)$

$=\frac{6}{5}(0,69897+\overline{1},09691)$

$=\frac{6}{5}(-0,20412)=-0,24494=\overline{1},75506$

$Anti\log \overline{1},75506=0,568926\cong 0,56893$

26. $\frac{Log 3+Co\log 5}{2}+\frac{Log 5+Co\log 7}{3}$

$=\frac{3(0,47712+\overline{1},30103)+2(0,69897+\overline{1},15490)}{6}$

$=\frac{3(-0,22185)+2(-0,14613)}{6}$

$=\frac{-0,6655-0,29226}{6}$

$=\frac{-0,95781}{6}=-0,159635=\overline{1},840364$

$10^{-0,159635}=0,6924126$

$Anti\log \overline{1},840364=0,69241$

27. $\frac{Log\,2}{7}+\frac{Log\,3}{5}+\frac{Log\,0,2}{3}$

$=\frac{15(0,30103)+21(0,47712)+35(\overline{1},30103)}{105}$

$=\frac{4,51545+10,01952-24,46395}{105}$

$=\frac{-9,92898}{105}=-0,094562=\overline{1},905438$

$Anti\log\ \overline{1},905438=0,8043369\cong 0,80434$

28. $\frac{Log\,32,14}{2}+\frac{Log\,59,3}{3}+\frac{Co\log 317,6}{4}$

$=\frac{6(1,50705)+4(1,77305)+3(\overline{3},49812)}{12}$

$=\frac{9,0423+7,0922-7,50564}{12}$

$=\frac{8,62886}{12}=0,71907$

$Anti\log\ 0,71907=5,2368483\cong 5,23685$

29. $\frac{2\log 0,75+\log 39,162+co\log 0,07+co\log 3,89}{2}$

$=\frac{2(\overline{1},87506)+1,59286+1,15490+\overline{1},41005}{2}$

$=\frac{1,90793}{2}=0,953965=8,99425\cong 8,9943$

30. $\frac{3\log 0,2+2\log 0,3+4\,co\log 0,05+co\log 3,25}{2}$

$=\frac{3(\overline{1},30103)+2(\overline{1},47712)+4(1,30103)+\overline{1},48812}{2}$

$=\frac{-3+0,90309-2+0,95424+5,20412-1+0,48812}{2}$

$=\frac{1,54957}{2}=0,774785$

$Anti\log\ 0,774785=5,95366$

EJERCICIO 300

1. $36=2^2\cdot 3^2$

$Log\,36=2\log 2+2\log 3$

$=2(0,301030)+2(0,477121)$

$=0,60206+0,954242$

$Log\,36=1,556302$

2. $75=3\cdot 5^2$

$Log\,75=\log 3+2\log 5$

$=0,477121+2(0,698970)$

$=0,477121+1,39794$

$Log\,75=1,875061$

3. $30=2\cdot 3\cdot 5$

$Log\,30=\log 2+\log 3+\log 5$

$=0,301030+0,477121+0,698970$

$Log\,30=1,477121$

4. $48=2^4\cdot 3$

$Log\,48=4\log 2+\log 3$

$=4(0,301030)+0,477121$

$=1,20412+0,477121$

$Log\,48=1,681241$

5. $120=2^3\cdot 3\cdot 5$

$Log\,120=3\log 2+\log 3+\log 5$

$=3(0,30103)+0,477121+0,69897$

$=0,90309+1,176091$

$Log\,120=2,079181$

6. $98=7^2\cdot 2$

$Log\,98=2\log 7+\log 2$

$=2(0,845098)+0,30103$

$=1,690196+0,30103$

$Log\,98=1,991226$

7. $343=7^3$

$Log\,343=3\log 7$

$=3(0,845098)$

$Log\,343=2,535294$

$Log\,0,343=\overline{1},535294$

8. $225=5^2\cdot 3^2$

$Log\,225=2\log 5+2\log 3$

$=2(0,69897)+2(0,477121)$

$=1,39794+0,954242$

$=2,352182$

$Log\,22,5=1,352182$

9. $196=2^2\cdot 7^2$

$Log\,196=2(\log 2+\log 7)$

$=2(0,30103+0,845098)$

$=2(1,146128)$

$=2,292256$

$Log\,1,96=0,292256$

10. $875 = 5^3 \cdot 7$

$Log\,875 = 3\log 5 + \log 7$

$= 3(0,69897) + 0,845098$

$= 2,09691 + 0,845098$

$= 2,942008$

$Log\,0,875 = \overline{1},942008$

11. $2.025 = 3^4 \cdot 5^2$

$Log\,2.025 = 4\log 3 + 2\log 5$

$= 4(0,477121) + 2(0,69897)$

$= 1,908484 + 1,39794$

$= 3,306424$

$Log\,202,5 = 2,306424$

12. $448 = 2^6 \cdot 7$

$Log\,448 = 6\log 2 + \log 7$

$= 6(0,30103) + 0,845098$

$= 1,80618 + 0,845098$

$= 2,651278$

$Log\,44,8 = 1,651278$

13. $2\frac{1}{2} = \frac{5}{2} = 2,5$

$25 = 5^2$

$Log\,25 = 2(0,69897)$

$= 1,39794$

$Log\,2\frac{1}{2} = 0,397940$

14. $1\frac{1}{2} = \frac{3}{2} = 1,5$

$15 = 3 \cdot 5$

$Log 15 = \log 3 + \log 5$

$= 0,477121 + 0,698970$

$= 1,176091$

$Log 1\frac{1}{2} = 0,176091$

15. $1\frac{2}{5} = \frac{7}{5} = 1,4$

$14 = 7 \cdot 2$

$Log 14 = \log 7 + \log 2$

$= 0,845098 + 0,30103$

$= 1,146128$

$Log 1\frac{2}{5} = 0,146128$

16. $2\frac{1}{3} = \frac{7}{3}$

$Log\,2\frac{1}{3} = \log\frac{7}{3}$

$= \log 7 + co\log 3$

$= 0,845098 + \overline{1},522879$

$= 0,367977$

17. $13 = \frac{143}{11}$

$Log 13 = \log\frac{143}{11}$

$= \log 143 + co\log 11$

$= 2,155336 + \overline{2},958607$

$Log 13 = 1,113943$

18. $25 = \frac{225}{9}$

$Log\,25 = \log\frac{225}{9}$

$= \log 225 + co\log 9$

$= 2,352183 + \overline{1},045757$

$Log\,25 = 1,397940$

EJERCICIO 301

1. $5^x = 3$

$Log\,5^x = \log 3$

$x\log 5 = \log 3$

$x(0,69897) = 0,477121$

$x = \frac{0,477121}{0,69897} = 0,6826$

2. $7^x = 512$

$Log 7^x = \log 512$

$x\log 7 = \log 512$

$x = \frac{2,70927}{0,845098} = 3,205864$

$x \cong 3,2059$

3. $0,2^x = 0,0016$

$Log\,0,2^x = \log 0,0016$

$x\log\,0,2 = \log 0,0016$

$x(\overline{1},30103) = \overline{3},20412$

$x = \frac{\overline{3},20412}{\overline{1},30103}$

$x = \frac{-3}{-0,69897} + \frac{0,20412}{-0,69897}$

$x = 4,29203 - 0,29203$

$x = 4$

4. $9^x = 0,576$

$Log\,9^x = \log 0,576$

$x\log 9 = \overline{1},76042$

$x(0,95424) = \overline{1},76042$

$x = \frac{\overline{1},76042}{0,95424}$

$x = \frac{-0,23958}{0,95424}$

$x = -0,2510689$

$x \cong -0,25107$

5. $3^{x+1} = 729$

$Log\,3^{x+1} = 729$

$(x+1)\log 3 = \log 729$

$(x+1)(0,477121) = 2,86273$

$x + 1 = 6,00944$

$x = 6,00944 - 1$

$x = 5,00944 \Rightarrow x \cong 5$

6. $5^{x-2} = 625$

$Log\,5^{x-2} = \log 625$

$(x-2)\log 5 = 2,79588$

$x - 2 = \frac{2,79588}{0,69897}$

$x - 2 = 4 \Rightarrow x = 6$

7. $2^{3x+1} = 128$

$Log\,2^{3x+1} = \log 128$

$(3x+1)\log 2 = 2,10721$

$(3x+1)(0,30103) = 2,10721$

$3x + 1 = 7$

$3x = 6$

$x = 2$

8. $3^{2x-1} = 2.187$

$Log\,3^{2x-1} = \log 2.187$

$(2x-1)\log 3 = 3,33985$

$(2x-1)(0,477121) = 3,33985$

$2x - 1 = 7$

$2x = 8 \Rightarrow x = 4$

9. $11^{2x} = 915$

$Log 11^{2x} = \log 915$

$2x\log 11 = 2,96142$

$2x = \frac{2,96142}{1,04139}$

$2x = 2,84372$

$x = 1,42186$

EJERCICIO 302

1. $a=3 \quad u=48 \quad r=6\div 3=2$

$n=\frac{Log48+Colog3}{Log2}+1$

$n=\frac{1,68124+\overline{1},522879}{0,30103}+1$

$n=\frac{1,204119}{0,30103}+1=4+1=5$

2. $a=2 \quad u=\frac{243}{16} \quad r=\frac{3}{2}$

$n=\frac{Log\frac{243}{16}+Colog2}{Log\frac{3}{2}}+1$

$n=\frac{Log243+colog16+colog2}{Log1,5}+1$

$n=\frac{2,38461+\overline{2},79588+\overline{1},69897}{0,176091}+1$

$n=\frac{0,87946}{0,176091}+1=4,994+1 \quad n=5,994 \quad n\cong 6$

3. $a=4 \quad u=512 \quad r=8\div 4=2$

$n=\frac{Log512+Colog4}{Log2}+1$

$n=\frac{2,70927+\overline{1},39794}{0,30103}+1$

$n=\frac{2,10721}{0,30103}+1=7+1=8$

4. $a=6 \quad u=\frac{2.048}{81} \quad r=\frac{8}{6}=\frac{4}{3}$

$n=\frac{Log\frac{2.048}{81}+Colog6}{Log\frac{4}{3}}$

$n=\frac{Log2.048+Colog81+Colog6}{Log4+Colog3}$

$n=\frac{3,31133+\overline{2},09151+\overline{1},22184}{0,60206+\overline{1},522879}+1$

$n=\frac{0,62468}{0,124939}+1$

$n=4,999879+1$

$n=5,999879$

$n\cong 6$

5. $a=2 \quad u=\frac{625}{8} \quad r=\frac{5}{2}=2,5$

$n=\frac{Log\frac{625}{8}+Colog2}{Log2,5}+1$

$n=\frac{Log625+Colog8+Colog2}{Log2,5}+1$

$n=\frac{2,79588+\overline{1},09691+\overline{1},69897}{0,39794}+1$

$n=\frac{1,59176}{0,39794}+1=4+1=5$

EJERCICIO 303

1. $c=500 \quad r=0,06 \quad t=3$

$C=500(1+0,06)^3$

$C=500(1,06)^3$

$Log\,C=Log\,500(1,06)^3$

$Log\,C=Log\,500+3\log 1,06$

$Log\,C=2,69897+3(0,02531)$

$Log\,C=2,7749$

$Anti\log 2,7749=\$595,51$

2. $c=3.500 \quad r=0,07 \quad t=5$

$C=3.500(1+0,07)^5$

$Log\ C=Log\,3.500(1,07)^5$

$Log\,C=Log\,3.500+5\,Log\,1,07$

$=3,54407+5(0,02938)$

$=3,54407+0,1469$

$Log\,C=3,69097$

$Anti\log 3,69097=4.908,94$ *soles*

3. $c=8.132 \quad r=0,09 \quad t=10$

$C=8.132(1+0,09)^{10}$

$Log\,C=Log\,8.132(1,09)^{10}$

$=Log\,8.132+10\,Log\,1,09$

$=3,91020+10(0,03743)$

$=3,91020+0,3743$

$Log\,C=4,2845$

$Anti\log\ 4,2845=19.253,070$ *bs.*

4. $c = 930 \quad r = 0,035 \quad t = 7$

$C = 930\,(1 + 0,035)^7$

$Log\,C = Log\,930(1,035)^7$

$= Log\,930 + 7\,Log\,1,035$

$= 2,9684829 + 7(0,0149403)$

$= 2,9684829 + 0,01045821$

$Log\,C = 3,073065 \quad Anti\log 3,073065 = \$1.183,2186$

5. $C = 12.318 \quad t = 6$

$r = 4\tfrac{1}{4} = \dfrac{17}{4} = 4,25 \Rightarrow \dfrac{4,25}{100} = 0,0425$

$C = 12.318(1 + 0,0425)^6$

$Log\,C = Log\,12.318(1,0425)^6$

$= Log\,12.318 + 6\,Log\,1,0425$

$= 4,0905402 + 6(0,018076)$

$= 4,0905402 + 0,108456$

$Log\,C = 4,1989962$

$Anti\log 4,1989962 = \$15.812,34204$

6. $C = 24.186 \quad t = 7$

$r = 5\tfrac{1}{2} = \dfrac{11}{2} = 5,5 \Rightarrow \dfrac{5,5}{100} = 0,055$

$C = 24.186(1 + 0,055)^7$

$Log\,C = Log\,24.186(1,055)^7$

$= Log\,24.186 + 7\log 1,055$

$= 4,383564 + 7(0,0232524)$

$= 4,383564 + 0,1627668$

$Log\,C = 4,5463308$

$Anti\log\ 4,5463308 = 35.182,83244$ *sucres*

7. $C = 54.293 \quad t = 5$

$r = 3\tfrac{3}{4} = \dfrac{15}{4} = 3,75 \Rightarrow \dfrac{3,75}{100} = 0,0375$

$Log\,C = Log\,54.293(1,0375)^5$

$= Log\,54.293 + 5\log 1,0375$

$= 4,734743 + 5(0,015988)$

$= 4,734743 + 0,07994$

$= 4,814683$

$Anti\log\ 4,814683 = \$65.265,399$

8. $c = 800 \quad t = 4 \quad r = \dfrac{0,03}{2} = 0,015$

$C = 800(1 + 0,015)^4$

$Log\,C = Log\,800(1 + 0,015)^4$

$= Log\,800 + 4\log 1,015$

$= 2,90309 + 4(0,00647)$

$= 2,90309 + 0,02588$

$Log\,C = 2,92897 \quad Anti\log C = \$849,09$

9. $c = 900 \quad t = \dfrac{12}{3} = 4 \quad r = \dfrac{0,04}{4} = 0,01$

$C = 900(1 + 0,01)^4$

$Log\,C = Log\,900 + 4\log 1,01$

$= 2,954242 + 4(0,004321)$

$= 2,954242 + 0,017284$

$= 2,971526$

$Anti\log 2,971526 = \$936,54$

10. $r = 0,05 \quad C = 972,60 \quad t = 4$

$c = \dfrac{972,60}{(1 + 0,05)^4}$

$Log\,c = \log 972,6 + 4\,co\log 1,05$

$= 2,987934 + 4(\bar{1},978811)$

$= 2,987934 - 0,084756$

$Log\,c = 2,903178$

$Anti\log 2,903178 = \$800,16$

11. $C = 1.893,5 \quad t = 6$

$r = 4\tfrac{1}{2} = \dfrac{9}{2} = 4,5 \Rightarrow \dfrac{4,5}{100} = 0,045$

$c = \dfrac{1.893,5}{(1 + 0,045)^6}$

$Log\,c = \log 1.893,5 + 6\,co\log 1,045$

$= 3,277265 + 6(\bar{1},980883)$

$= 3,277265 - 0,114702$

$= 3,162563$

$Anti\log 3,162563 = \$1.454,02$

12. $C = 54.198,16 \quad r = 0,08 \quad t = 7$

$c = \dfrac{54.198,16}{(1 + 0,08)^7}$

$Log\,c = \log 54.198,16 + 7\,co\log 1,08$

$= 4,733984 + 7(\bar{1},966576)$

$= 4,733984 - 0,233968$

$Log\,c = 4,500016$

$Anti\log 4,500016 = 31.623,9 \cong 31.624\,Q.$

13. $c=600 \quad r=0,03 \quad C=695,56$

$$t=\frac{Log695,56-Log600}{Log1,03}$$

$$t=\frac{2,842334-2,778151}{0,012837}$$

$$t=\frac{0,064183}{0,012837}=4,9998\cong 5\ \textit{años}$$

14. $c=1.215 \quad C=1.709,61 \quad r=0,05$

$$t=\frac{Log1.709,61-\log 1.215}{Log\left(1+0,05\right)}$$

$$t=\frac{3,232897-3,084576}{0,021189}$$

$$t=\frac{0,148321}{0,021189}=7\ \textit{años}$$

15. $c=800 \quad t=4 \quad C=1.048,63$

$$Log\left(1+r\right)=\frac{\log 1.048,63-\log 800}{4}$$

$$Log\left(1+r\right)=\frac{3,020622-2,903089}{4}$$

$$Log\left(1+r\right)=\frac{0,117533}{4}=0,029384$$

$$Anti\log\ 0,029384=1,07$$

$$1+r=1,07$$

$$r=0,07 \ \Rightarrow 7\%$$

16. $c=6.354 \quad t=4 \quad C=7.151,46$

$$Log\left(1+r\right)=\frac{Log\,7.151,46-Log\,6.354}{4}$$

$$=\frac{3,854394-3,803047}{4}$$

$$Log\left(1+r\right)=0,012836$$

$$Anti\log 0,012836=1,03$$

$$1+r=1,03$$

$$r=0,03\Rightarrow 3\%$$

17. $c=900 \quad r=0,05 \quad t=2+\frac{1}{3}=2,33333$

$$C=900\left(1+0,05\right)^{2,33333}$$

$$Log\,C=Log\,900+2,33333\log 1,05$$

$$=2,954242+2,33333\left(0,0211892\right)$$

$$=2,954242+0,049441$$

$$=3,003683$$

$$Anti\log 3,003683=1.008,52$$

Los intereses producidos representan la diferencia entre la suma inicial y lo que se ha convertido

$\Rightarrow$ *Intereses producidos* $=1.008,52-900$

$= \$108,52$

EJERCICIO 304

1. $c=40.000 \quad r=0,05 \quad t=10$

$$a=\frac{40.000\left(0,05\right)\left(1,05\right)^{10}}{\left(1,05\right)^{10}-1}$$

$$Log\left(1,05\right)^{10}=10\,Log\,1,05$$

$$=10\left(0,0211892\right)$$

$$=0,211892$$

$$Anti\log 0,211892=1,62889$$

$$Log\,a=\log 40.000+\log 0,05+\log 1,62889+co\log 0,62889$$

$$=4,602059+\overline{2},69897+0,211892+0,2014253$$

$$=3,7143463$$

$$Anti\log\ 3,7143463=\$5.180,21$$

2. $c=85.000 \quad r=0,03 \quad t=12$

$$a=\frac{85.000(0,03)(1+0,03)^{12}}{(1+0,03)^{12}-1}$$

$$Log(1,03)^{12}=12\log 1,03$$
$$=12(0,012837)$$
$$=0,154044$$

$Anti\log\ 0,154044=1,425761$

$$Log\ a=\log 85.000+\log 0,03+\log 1,425761+co\log 0,425761$$
$$=4,929418+\bar{2},477121+0,154044+0,370834$$
$$=3,931417$$

$Anti\log\ 3,931417=8.539,2\quad soles$

3. $c=600.000 \quad r=0,05 \quad t=20$

$$a=\frac{600.000(0,05)(1+0,05)^{20}}{(1+0,05)^{20}-1}$$

$$Log(1,05)^{20}=20\log 1,05$$
$$=20(0,0211892)$$
$$=0,423784$$

$Anti\log 0,423784=2,65328$

$$Log\ a=\log 600.000+\log 0,05+\log 2,65328+co\log 1,65328$$
$$=5,778151+\bar{2},69897+0,423784+\bar{1},781653$$
$$=4,682558$$

$Anti\log\ 4,682558=\$48.145,75$

4. $c=5'000.000 \quad r=0,06 \quad t=30$

$$a=\frac{5'000.000(0,06)(1+0,06)^{30}}{(1+0,06)^{30}-1}$$

$$Log(1,06)^{30}=30\log 1,06$$
$$=30(0,025306)$$
$$=0,75918$$

$Anti\log 0,75918=5,7435$

$$\log\ a=\log 5'000.000+\log 0,06+\log 5,7435+co\log 4,7435$$
$$=6,69897+\bar{2},778151+0,75918+\bar{1},323901$$
$$=5,5602$$

$Anti\log\ 5,5602=363.245,2\quad bs.$

5. $c=3.000 \quad r=0,06 \quad t=5$

$a=3.000\cdot 0,237396$

$a=712,188\cong 712,19\ bs.$

$$a=\frac{3.000(0,06)(1,06)^{5}}{(1,06)^{5}-1}$$

$$a=\frac{180(1,338226)}{0,338226}$$

$$a=\frac{240,88068}{0,338226}$$

$a=712,1885367\cong 712,19\ bs.$

NOTA:

0,237396 → *Vr. que aparece en la tabla de interés compuesto decreciente para el* 6% *en* 5 *años.*

1,338226 → *Vr. que aparece en la tabla de interés compuesto para el* 6% *a* 5 *años.*

6. $c=12.000 \quad r=0,07 \quad t=12$

$a=12.000\cdot 0,125902$

$a=1.510,824\ bs.$

7. $c=350.000 \quad r=4\tfrac{1}{2}\% \quad t=3$

$a=350.000\cdot 0,363773$

$a=127.320,55$ *sucres*

8. $c=425.000 \quad r=6\% \quad t=10$

$a=425.000\cdot 0,135868$

$a=57.743,9$ *soles*

9. $c=90.750 \quad r=5\% \quad t=25$

$a=90.750\cdot 0,070952$

$a=6.438,894\ bs.$

10. $c=73.550 \quad r=5\tfrac{1}{2}\% \quad t=30$

$a=73.550\cdot 0,068805$

$a=5.060,60775\cong 5.060,61\ bs.$

11. $c=473.000 \quad r=3\tfrac{1}{2}\% \quad t=9$

$a=473.000\cdot 0,131446$

$a=62.173,958\cong 62.173,96$ *sucres*

12. $c = 45.800 \quad r = 4\% \quad t = 30$

$a = 45.800 \cdot 0,057830$

$a = 2.648,614$ *soles*

$$a = \frac{45.800(0,04)(1,04)^{30}}{(1,04)^{30} - 1}$$

$Log(1,04)^{30} = 30 \log 1,04$

$= 30(0,017033)$

$= 0,51099$

$Anti \log\ 0,51099 = 3,243321$

$Log\, a = Log\, 45.800 + \log 0,04 + \log 3,243321 + co \log 2,243321$

$= 4,660865 + \overline{2},60206 + 0,510989 + \overline{1},649108$

$= 3,423022$

$Anti \log 3,423022 = 2.648,61$ *soles*

EJERCICIO 305

1. $c = 30.000 \quad r = 0,06 \quad t = 9$

$$i = \frac{30.000(0,06)}{(1+0,06)^{10} - (1+0,06)}$$

$Log(1,06)^{10} = 10 \log 1,06$

$= 10(0,02531)$

$= 0,2531$

$Anti \log\ 0,2531 = 1,791018$

$Log\ i = Log\, 30.000 + \log 0,06 + co \log 0,731018$

$= 4,477121 + \overline{2},778151 + 0,136072$

$= 3,391344$

$Anti \log\ 3,391344 = \$2.462,38$

2. $c = 90.000 \quad r = 0,04 \quad t = 20$

$$i = \frac{90.000(0,04)}{(1,04)^{21} - 1,04}$$

En la tabla de interés

compuesto $(1,04)^{21} = 2,278768$

$Log\ i = Log\, 90.000 + \log 0,04 + co \log 1,238768$

$= 4,95424 + \overline{2},60206 + \overline{1},90701$

$= 3,4633$

$Anti \log 3,4633 = 2.906,03$ *sucres*

3. $c = 200.000 \quad r = 0,05 \quad t = 40$

$$i = \frac{200.000(0,05)}{(1,05)^{41} - 1,05}$$

$Log(1,05)^{41} = 41 \log 1,05$

$= 41(0,0211892)$

$= 0,868757$

$Anti \log 0,868757 = 7,391915$

$Log\ i = Log\, 200.000 + \log 0,05 + co \log 6,341915$

$= 5,30103 + \overline{2},69897 + \overline{1},19777$

$= 3,19777$

$Anti \log 3,19777 = \$1.576,79$

4. $c = 40.000 \quad r = 0,06 \quad t = 25$

$$i = \frac{40.000(0,06)}{(1,06)^{26} - 1,06}$$

$(1,06)^{26} \rightarrow 4,549383$

$Log\ i = \log 40.000 + \log 0,06 + co \log 3,489383$

$= 4,60206 + \overline{2},778151 + \overline{1},457251$

$= 2,837462$

$Anti \log\ 2,337462 = \$687,79$